Lattice NRQCD study of heavy quark and anti-quark annihilations in QGP and heavy dark matter annihilations in early universe

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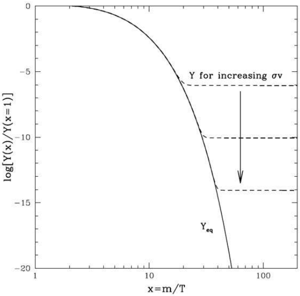
in collaboration with M. Laine and S. Biondini
JCAP1910(2019) 078, and
with M. Laine, PLB 795(2019) 469, JCAP1701(2017) 013,
JHEP1607(2016) 143

Outline

Heavy quark in QGP and Heavy DM in early universe

- heavy quarks in QGP
- heavy DM (stronlgy interacting via Nonabelian GT) in early universe
- “chemical equilibration in thermal environment”



- freeze-out

Heavy quark in QGP and Heavy DM in early universe

• importance of $\frac{M}{k_B T}$ and $\alpha(M)$

• and rough estimate of decoupling temperature is for dark matter is, Hubble rate $\sim$ annihilation rate

$$H \sim n < \sigma v > \quad \rightarrow \quad \frac{T^2}{M_{\rm pl}} \sim (\frac{MT}{2\pi})^{\frac{3}{2}} e^{-\frac{M}{T}} \frac{\alpha^2}{M^2}$$

for $\alpha \sim 0.01,\ T \sim \frac{M}{25}$

• bottom quark ($\sim$ 4.5 GeV) and the temperature reachable in heavy ion collisions ($\sim$ 600 MeV) and $\alpha_s(M) \sim v \sim 0.3$

• heavy particle in this situation is “non-relativistic

Heavy quark in QGP and Heavy DM in early universe

• "Sommerfeld effect" enhances annihilation (heavy quark annihilation in QGP and dark matter (WIMP/SIMP) annihilation in cosmology) (e.g, Hisano et al, hep-ph/0612049)

• thermal effect (producing mass shift, thermal width, mixing angle modification) can be $O(1)$ effect

• bound states can be disturbed by this $O(1)$ effect

• such effects can be studied through the change of spectral function/or thermal correlator

• these effects can be "strong"

Heavy quark in QGP and Heavy DM in early universe

• for example, modification of heavy quark potential in thermal environment (cf. M. Laine et al, hep-ph/0611300).

$$V(r) = -\alpha_s \left[m_D + \frac{\exp(-m_D r)}{r} \right]$$

and

$$\Gamma(r) = 2\alpha_s T \int_0^\infty dx \frac{x}{(1+x^2)^2} \left[1 - \frac{\sin(x m_D r)}{x m_D r} \right]$$

Number density evolution

• B.W. Lee and S. Weinberg, PRL39 (1977) 165 (Lee-Weinberg equation)

$$\frac{\partial n}{\partial t} + 3Hn = - < \sigma v > (n^2 - n_{eq}^2)$$

• SK and M. Laine, JHEP1607 (2016) 143

$$\frac{\partial n}{\partial t} + 3Hn = -\Gamma_{\rm chem}(n - n_{eq}) + O(n - n_{eq})^2$$

with $\Gamma_{\rm chem} = 2 < \sigma v > n_{eq}$

cf. linear response theory

Number density evolution

- T. Binder, L. Covi, and K. Mukaida, PRD98 (2018) 115023

$$\frac{\partial n}{\partial t} + 3Hn = - < \sigma v > (e^{\frac{2\mu(n)}{T}} - 1) n_{eq}^2$$

cf. Saha equation

Number density evolution

- for QCD, non-perturbative definition for the chemical/kinetic equilibration rate is necessary

- equilibriation rate is a real-time quantity

- lattice gauge theory is a method which can calculate non-perturbative quantities using first principles of quantum field theory

- lattice gauge theory is defined on a Euclidean space and has difficulty in calculating real-time quantity

Number density evolution

• lattice study for heavy quark/anti-quark pair annihilation in QGP $\rightarrow$ NRQCD (E.Braaten et al, hep-ph/9407339) in non-zero temperature

• applying QCD result to heavy dark matter annihilation in early universe

• strongly interacting system

• caveat: continuum extrapolation not performed

Non-linear susceptibility

- T. Binder, L. Covi, and K. Mukaida, PRD98 (2018) 115023

$$\frac{\partial n}{\partial t} + 3Hn = - < \sigma v > (e^{\frac{2\mu(n)}{T}} - 1) n_{eq}^2$$

Non-linear susceptibility

$$n(\mu) = \frac{\partial p}{\partial \mu}, \quad \chi = T\frac{\partial n}{\partial \mu}$$

since $p = p_0 + p_1 e^{\beta\mu} + p_2 e^{2\beta\mu} + \cdots$,

$$2p_2 e^{2\beta\mu} \simeq T(\chi - n).$$

with $p_2 = \hat{p}_2 n_{eq}^2 T$,

$$\hat{p}_2 = \frac{\int_{\vec{x}} \left\{ \left\langle \mathrm{Re\ Tr} G_{\vec{x}} \mathrm{Re\ Tr} G_{\vec{0}} \right\rangle - \left\langle \mathrm{Re\ Tr} G_{\vec{0}} \right\rangle^2 \right\}}{2\left\langle \mathrm{Re\ Tr} G_{\vec{0}} \right\rangle^2}$$

since $n_{eq} = 2\langle \mathrm{Re\ Tr} G_{\vec{0}} \rangle$

Method

- use lattice NRQCD for heavy quark in thermal equilibrium (light quarks and gluons are in thermal equilibrium)

- calculate "quarkonium" correlators

- obtain "matrix elements"

Method

• anisotropic Euclidean lattices (i.e., the time direction lattice spacing is different from the space direction lattice spacing, $a_s/a_t = 3.5$), $a_s = 0.1227(8)$ fm

• $N_f = 2 + 1$ light quark flavors ($M_\pi \simeq 400$ MeV, $M_K \simeq 500$ MeV)

• $24^3 \times N_t$ lattices

• T_c = 185 MeV, $a_s M = 2.92$

• lattices used for bottomonium at $T \neq 0$ study (G. Aarts et al, JHEP07 (2014) 097) and electric conductivity of QGP (G. Aarts et al, JHEP-2 (2015) 186)

Method

$$G^{\theta}(0,\mathbf{x};\cdot) = \frac{\delta_{\mathbf{x},\vec{0}}}{a_s^3}\,,$$

$$G^{\theta}(a_t,\mathbf{x};\cdot) = \left(1-\frac{a_t\mathcal{H}_0}{2n}\right)^n U_0^{\dagger}(0,\mathbf{x})\left(1-\frac{a_t\mathcal{H}_0}{2n}\right)^n G^{\theta}(0,\mathbf{x};\cdot)\,,$$

$$G^{\theta}(\tau+a_t,\mathbf{x};\cdot) = \left(1-\frac{a_t\mathcal{H}_0}{2n}\right)^n U_0^{\dagger}(\tau,\mathbf{x})\left(1-\frac{a_t\mathcal{H}_0}{2n}\right)^n \left(1-a_t\,\delta\mathcal{H}\right) G^{\theta}(\tau,$$

where U_0 is a time-direction gauge link. The lowest-order Hamiltonian reads

$$\mathcal{H}_0 = -\frac{\Delta^{(2)}}{2M}\,,$$

where $\Delta^{(2)}$ is a discretized gauge Laplacian.

Method

The higher order correction is

$$\delta\mathcal{H} = -\frac{(\Delta^{(2)})^2}{8M^3} + \frac{ig_0\,(\nabla\cdot\mathbf{E}-\mathbf{E}\cdot\nabla)}{8M^2} - \frac{g_0\,\sigma\cdot(\nabla\times\mathbf{E}-\mathbf{E}\times\nabla)}{8M^2} - \frac{g_0\,\sigma\cdot\mathbf{B}}{2M} + \frac{a_s^2\Delta^{(4)}}{24M} - \frac{a_t(\Delta^{(2)})^2}{16nM^2}\,,$$

Method

$$
\begin{aligned}
P_1 &\equiv \frac{1}{2N_c}\mathrm{re},\left\langle G^{\theta}_{\alpha\alpha;ii}(\beta,\vec{0};0,\vec{0})\right\rangle ,\\
P_2 &\equiv \frac{1}{2N_c}\left\langle G^{\theta}_{\alpha\gamma;ij}(\beta,\vec{0};0,\vec{0})G^{\theta\dagger}_{\gamma\alpha;ji}(\beta,\vec{0};0,\vec{0})\right\rangle ,\\
P_3 &\equiv \frac{1}{2N_c^2}\left\langle G^{\theta}_{\alpha\alpha;ij}(\beta,\vec{0};0,\vec{0})G^{\theta\dagger}_{\gamma\gamma;ji}(\beta,\vec{0};0,\vec{0})\right\rangle .
\end{aligned}
$$

- singlet Sommerefeld factor

$$
\bar{S}_1 = \frac{P_2}{P_1^2} .
$$

- octet channel Sommerefeld factor

$$
\bar{S}_8 = \frac{N_c^2 P_3 - P_2}{(N_c^2-1)P_1^2} .
$$

Method

- P-wave Sommerfeld factor

$$\bar{S}_p = \frac{P_p}{M^2 P_1^2}$$

with

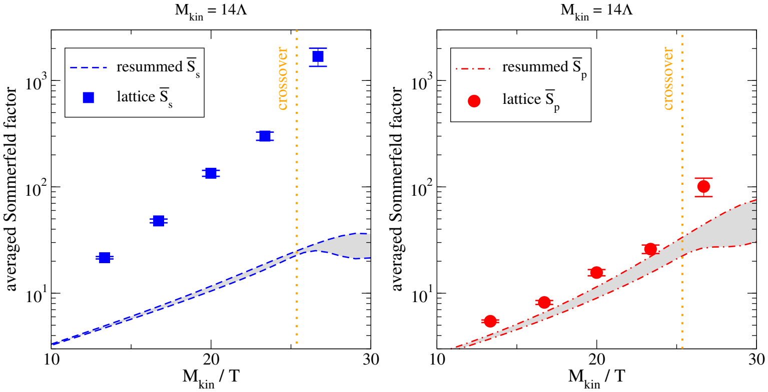
$$p_p = \mathrm{Tr}\langle \Delta_i G_V(\beta,\vec{0};0,\vec{0};i) G^\dagger(\beta,\vec{0};0,\vec{0})\rangle - \mathrm{Tr}\langle G_V(\beta,\vec{0};0,\vec{0};i) \Delta_i G^\dagger(\beta,\vec{0};0,\vec{0}))$$

- non-linear susceptibility

$$\hat{p}_2 = \frac{\int_{\vec{x}} \left\{ \left\langle \mathrm{Re}\,\mathrm{Tr} G_{\vec{x}} \mathrm{Re}\,\mathrm{Tr} G_{\vec{0}} \right\rangle - \left\langle \mathrm{Re}\,\mathrm{Tr} G_{\vec{0}} \right\rangle^2 \right\}}{2 \left\langle \mathrm{Re}\,\mathrm{Tr} G_{\vec{0}} \right\rangle^2}$$

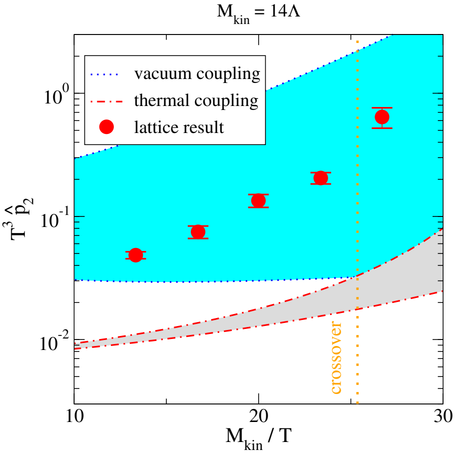
Thermally averaged Sommerfeld factor

- "chemical equilibration rate" as a transport peak
- Sommerfeld factor at $T \neq 0$
- SK, M. Laine (JHEP1607 (2016) 143, PLB795 (2019) 469)



NR particle number susceptibility

- Non-relativistic particle number susceptibility
- S.Biondini, SK, M.Laine, arXiv:1908.07541 (to be published in JCAP)



Discussion

• lattice NRQCD study of heavy quark chemical equilibration rate in QGP is possible

• subtle interplay of thermal effect in strongly interacting system with bound states in $T \neq 0$

• applicable to heavy dark matter annihilation in early universe $\rightarrow$ insight to SIMP?

• continuum extrapolation is needed due to the limitation of the current lattice setup