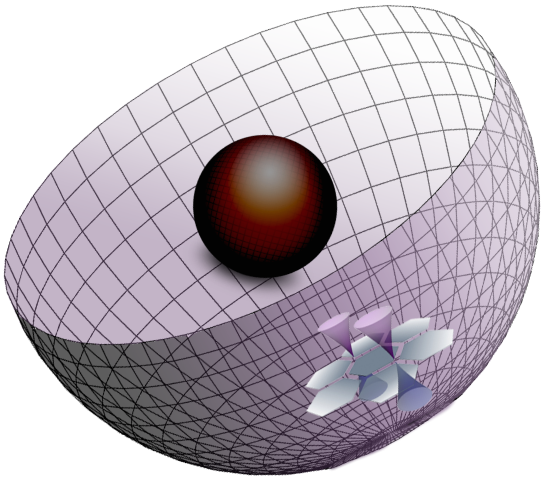
Why light quark mass is so small?



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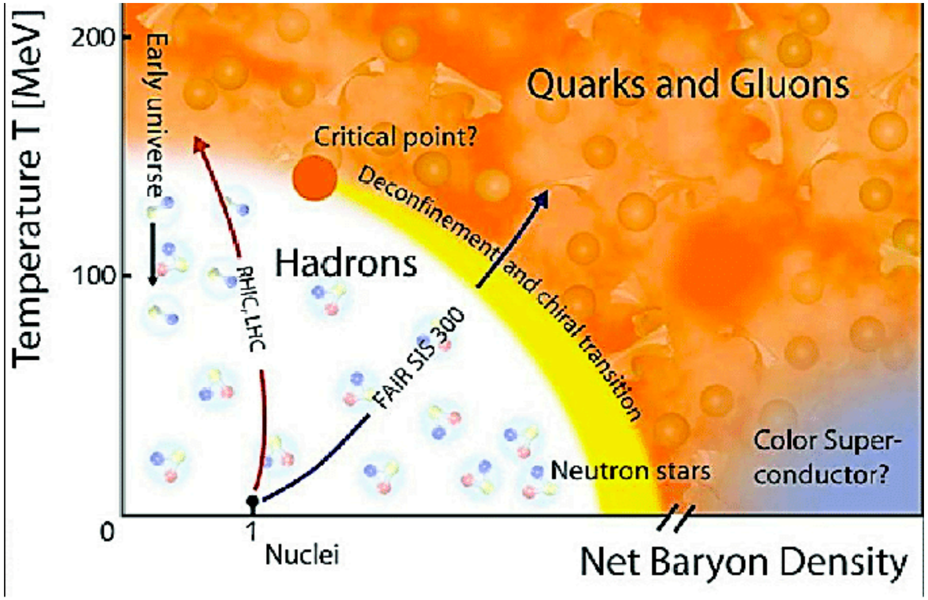
2019.12@Ibs+Pusan

Motivation

- Proton mass = 947MeV vs.
  Bare quark (u,d) =2-4 MeV → Quark mass is less than 1%
- Confinement and deconfiment phase transition almost overlapping. Why two completely different concepts gives related phase diagram?
- 

Chiral symmetry and the confinement

- Regge trajectory : ←Dynamics of QCD
- Chiral symmetry need m_q=0: Initial condition

However, chiral transition

~ confinement/dc transition

- Is there any relation?

Story line

1. Take holographic QCD in 2+1.
2. Show that if $m_q = 0$, we have a Regge traj.
3. Show that if $m_q \neq 0$, we have non-linear traj.
4. Since low E QCD dynamics has Regge traj, having non-zero quark mass is not consistent with low E QCD. That is, QCD vac forbid the light quark mass.

Idea: consider the constituent quark mass

$$S_D = i \int d^d x \sqrt{-g}\bar{\psi}(\Gamma^M \mathcal{D}_M - m - \Phi)\psi + S_{bd},$$

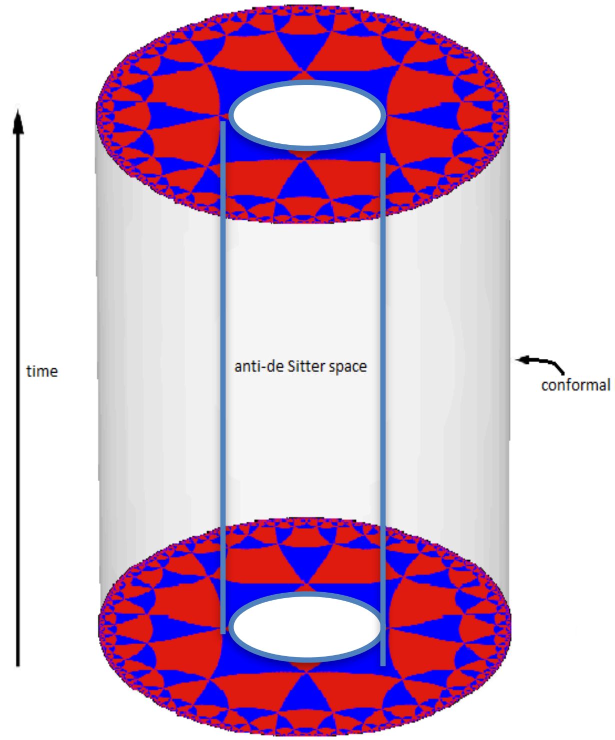
We could take a hard wall condition

Then hard wall =dual to a Bag !

—> BUT, will get $m_n^2 = n^2\tau$.

So, we consider soft wall instead, using a scalar Φ:

If Φ cost a lot in the central region, it works!



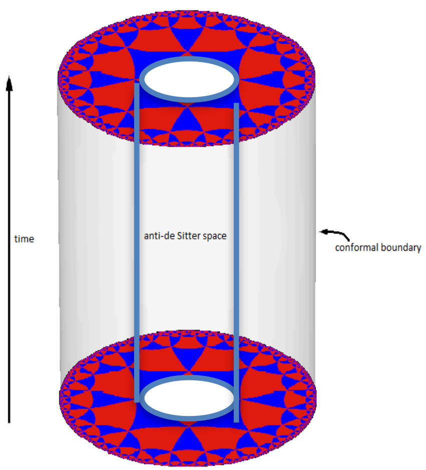
The scalar in AdS4

For any conserved U(1), if there is charged field

$$S = \int d^{d+1}x\sqrt{-g}\Big(\qquad - |D_\mu \Phi|^2 - m_\Phi^2 |\Phi|^2 \Big)$$

$$ds^2 = (dz^2 + \eta^{\mu\nu} dx^u dx^\nu)/z^2, \text{ with } \eta^{00} = -1.$$

$$m_\Phi^2 = \Delta(\Delta - d) \qquad d = 2+1 \qquad \Delta = 2$$



$$\Phi = M_0 z + M z^2, \text{ in AdS}_4,$$

we set the source $M_0 = 0$ so that $\Phi = M z^2$.

$z \to \infty$ at the core!

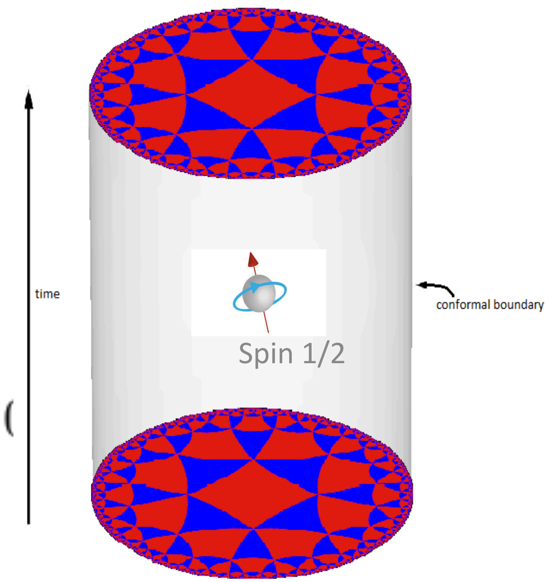
Idea: consider constituent quark mass

$$\psi_\pm = (-gg^{rr})^{-\frac{1}{4}}\phi_\pm, \quad \phi_\pm = \begin{pmatrix} y_\pm \\ z_\pm \end{pmatrix}$$

$$\xi_+ = i\frac{y_-}{z_+}, \text{ and } \quad \xi_- = -i\frac{z_-}{y_+}.$$

$$r^2\partial_r\xi_- + (2mr + 2\frac{M}{r})\xi_- + (k_1 - w)\xi_-^2 - k_1 - w =$$

$$G_R = \lim_{r\to\infty} r^{2m}\text{diag}(\xi_+, \xi_-),$$



Linear spectrum

$$G_{22} = \frac{M^{-\frac{1}{2}+m}(k_1+w)\Gamma(\frac{1}{2}-m)\Gamma(m+\frac{1}{2}+\frac{k_1^2-w^2}{4M})}{2\Gamma(\frac{1}{2}+m)\Gamma(1+\frac{k_1^2-w^2}{4M})},$$

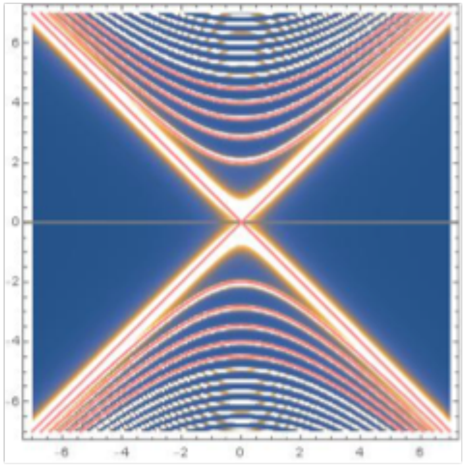
$$\frac{k^2-\omega^2}{4M}+m+\frac{1}{2}=-n, \quad \text{for} \quad n\in\mathbf{Z},$$

$$\alpha' m_n^2 = (n+m+\frac{1}{2}),$$

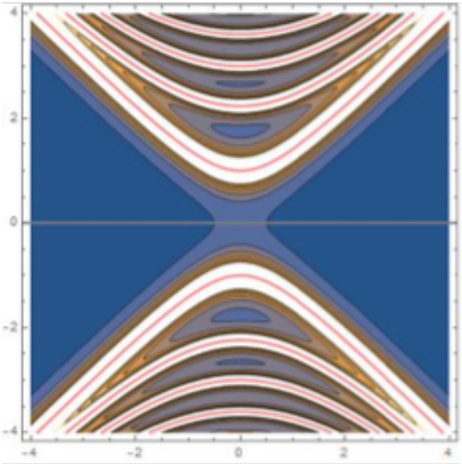
→ Open string with tension

$T=1/(2\pi\alpha')$ with $\alpha'=\frac{1}{4M}$, if $M\neq 0$.

→ Zero tension in zero condensation $M\to 0$



(a) gapless $m=-\frac{1}{2}, M=1$



(b) gaped $m=-\frac{1}{4}, M=1$

: Fermion on top of the Scalar condensation

2.2. Fermion with scalar interaction in holography

For the baryon spectrum, we consider following fermion
tion in AdS space.

$$S_\psi = \int d^{d+1}x\sqrt{-g}\,i\bar{\psi}(\Gamma^\mu \mathcal{D}_\mu - m - \Phi)\psi, \qquad ($$

where $\mathcal{D}_\mu = \partial_\mu + \frac{1}{4}\omega_{ab\mu}\Gamma^{ab}$. In this paper, we consider (
$d = 3$. The equation of motion of (10) is given by

$$(\Gamma^\mu \mathcal{D}_\mu - m - \Phi)\psi = 0, \quad \Rightarrow \quad -\Psi_n'' + V\Psi_n = E_n\Psi_n,$$

which can be written as a Schrödinger form Eq.(7) with

$$V(z) = \frac{m(m-1) + \Phi^2}{z^2} = \frac{m(m-1)}{z^2} + (Mz + M_0)^2. \qquad (\qquad \psi = e^{ik.x}, E_n = k^2$$

If $M_0 = 0,$ $\quad m_n^2 = 4M^2(n + m + 1/2)$

The Question

$$What\ if\ M_0 \neq 0?$$

$$-\Psi_n'' + V\Psi_n = E_n\Psi_n,$$

$$V(z) = \frac{m(m-1)+\Phi^2}{z^2} = \frac{m(m-1)}{z^2} + (Mz + M_0)^2.$$

⇨ Some 3d QM problem

$$-u'' + Vu = \frac{E^2}{4}u,$$

$$V = \left(m + \frac{1}{2}br\right)^2 + \frac{L(L+1)}{r^2}.$$

$$m \leftrightarrow M_0, \quad b/2 \leftrightarrow M, \quad \text{and } E^2/4 \leftrightarrow E_n.$$

Heun’s equation

$$u = r\Psi$$

$$\Psi(r) = \exp\left(-\frac{b}{4}\left(r + \frac{2m}{b}\right)^2\right) r^L y(r)$$

$$r\frac{\partial^2 y}{\partial r^2} + \left(-br^2 - 2mr + 2(l+1)\right)\frac{\partial y}{\partial r} + \left(\left(\frac{E^2}{4} - b\left(L + \frac{3}{2}\right)\right) r - 2m(L+1)\right) y = 0$$

$$\rho\frac{d^2 y}{d\rho^2} + \left(\mu\rho^2 + \varepsilon\rho + \nu\right)\frac{dy}{d\rho} + (\Omega\rho + \varepsilon\omega)\, y = 0$$

Heun’s equation

$$y(\rho) = \sum_{n=0}^{\infty} d_n \rho^n$$

$$d_{n+1} = A_n\, d_n + B_n\, d_{n-1} \qquad \text{for } n \geq 1,$$

$$A_n = -\frac{\varepsilon(n+\omega)}{(n+1)(n+\nu)}, \qquad B_n = -\frac{\Omega + \mu(n-1)}{(n+1)(n+\nu)},$$

Two vs Three term recurrence rel.

after factoring asymptotic behavior

Hypergeometric → Two term recurrence $d_{n+2} = B_{n+1} d_n$

Heun → Three term recurrence $d_{n+2} = A_{n+1} d_{n+1} + B_{n+1} d_n$

⇨ Request

$B_{N+1} = d_{N+1} = 0$ for some $N \in \mathbb{N}_0$

Then $d_{N+2} = d_{N+3} = d_{N+4} = \cdots = 0$

Need expression of d_{N+1} = polynomial $\mathscr{P}_{N+1}$

$$
\begin{aligned}
\mathcal{P}_1(a_0,b_0) &= b_0(L+1)-a_0,\\
\mathcal{P}_2(a_0,b_0) &= (b_0(L+1)-a_0)(b_0(L+2)-a_0)-4(L+1),\\
\mathcal{P}_3(a_0,b_0) &= (L+1)(L+2)(L+3)b_0^3-(3L(L+4)+11)a_0b_0^2\\
&\quad+\Big(3(L+2)a_0^2-4(L+1)(4L+9)\Big)b_0-a_0^3+4(4L+5)a_0,\\
\mathcal{P}_4(a_0,b_0) &= (L+1)(L+2)(L+3)(L+4)b_0^4-2(2L+5)(L(L+5)+5)a_0b_0^3\\
&\quad+\Big((6L(L+5)+35)a_0^2-(L+1)(5L(2L+11))+72\Big)b_0^2\\
&\quad-\Big(2(2L+5)a_0^3+4(20L(L+4)+69)a_0\Big)b_0-20(2L+3)a_0^2\\
&\quad+144(L+1)(L+2),\\
\mathcal{P}_5(a_0,b_0) &= (L+1)(L+2)(L+3)(L+4)(L+5)b_0^5\\
&\quad-\Big(5L(L+6)(L(L+6)+15)+274\Big)a_0b_0^4\\
&\quad+\Big(5(L+3)(2L(L+6)+15)a_0^2\\
&\qquad-4(L+1)(L(5L(4L+39)+607)+600)\Big)b_0^3\\
&\quad-\Big(5(2L(L+6)+17)a_0^3-4(L(15L(4L+31)+1096)+763)a_0\Big)b_0^2\\
&\quad+\Big(5(L+3)a_0^4-12(5L(4L+19)+98)a_0^2\\
&\qquad+32(L+1)(16L(2L+11)+225)\Big)b_0\\
&\quad+20(4L+7)a_3-32(16L(2L+7)+89)a_0-a_0^5.
\end{aligned}
\qquad (25)
$$

$d_{N+1}(a_0, b_0) = 0$

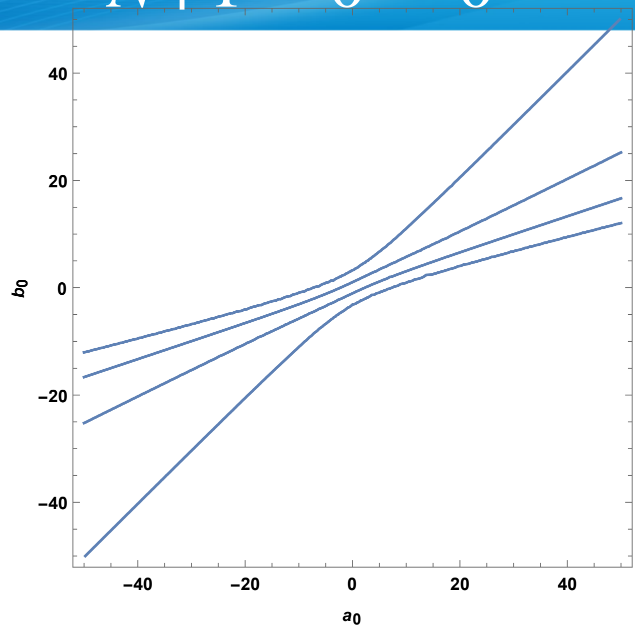


Figure 3: Contours of $\mathcal{P}_3(a_0, b_0) = 0$.

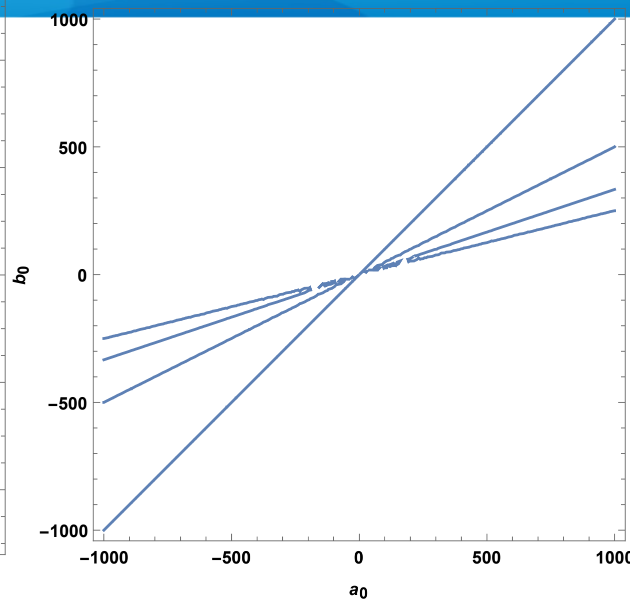
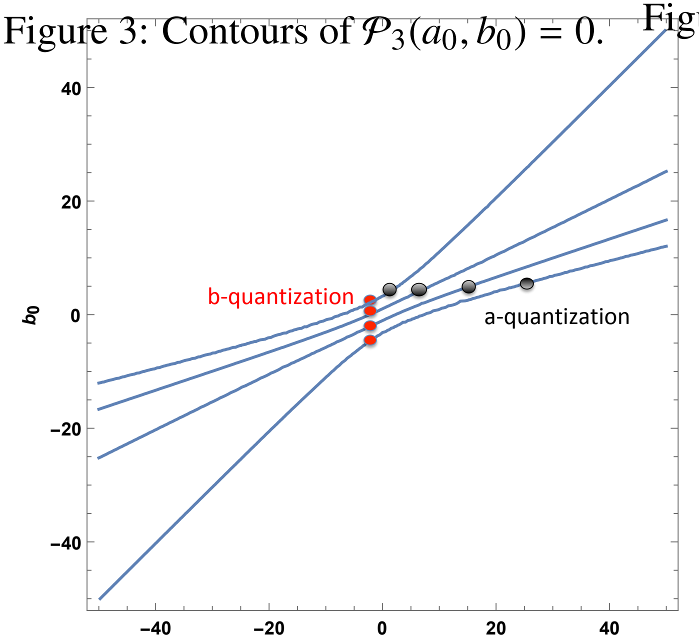


Figure 4: Asymptotic view of Fig. 3



Parameters of V

$$d_{N+1}(a_0, b_0) = 0$$

A degree N+1 polynomial eq.

Consequence of three term recurrence rel.

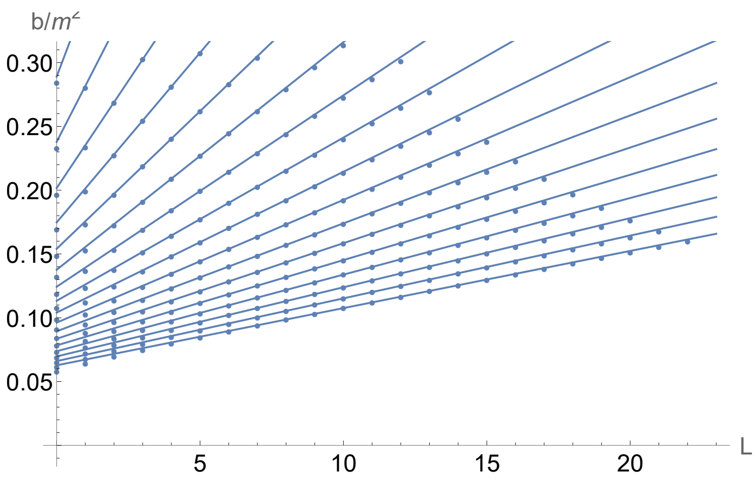
$$B_{N+1} = d_{N+1} = 0 \qquad \text{for some } \; N \in \mathbb{N}_0$$

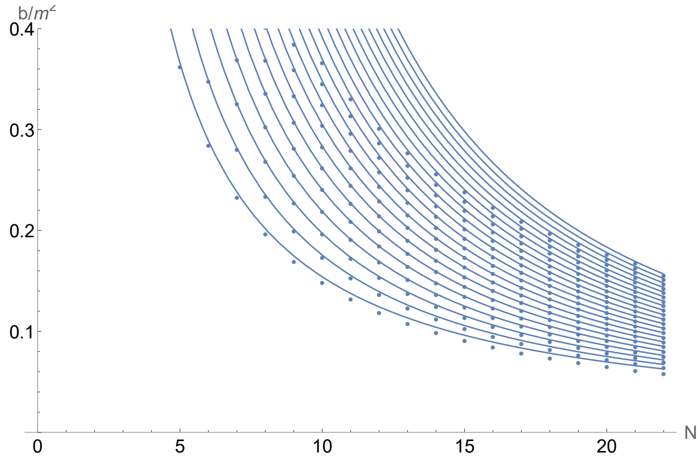
$$\frac{b}{m^2} = \frac{2.18(\frac{4}{7}N + L + \frac{10}{7})}{N^2 + \frac{1}{9}N - \frac{1}{40}}$$

Extra quantization

$$E^2 = 4b\left(N + L + \frac{3}{2}\right)$$

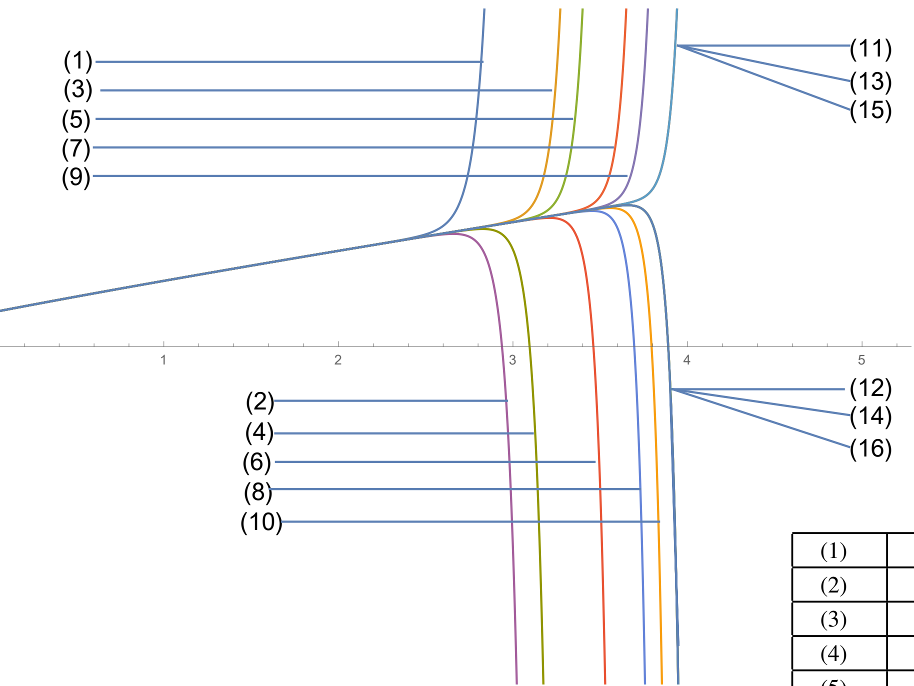
Becomes Non-linear

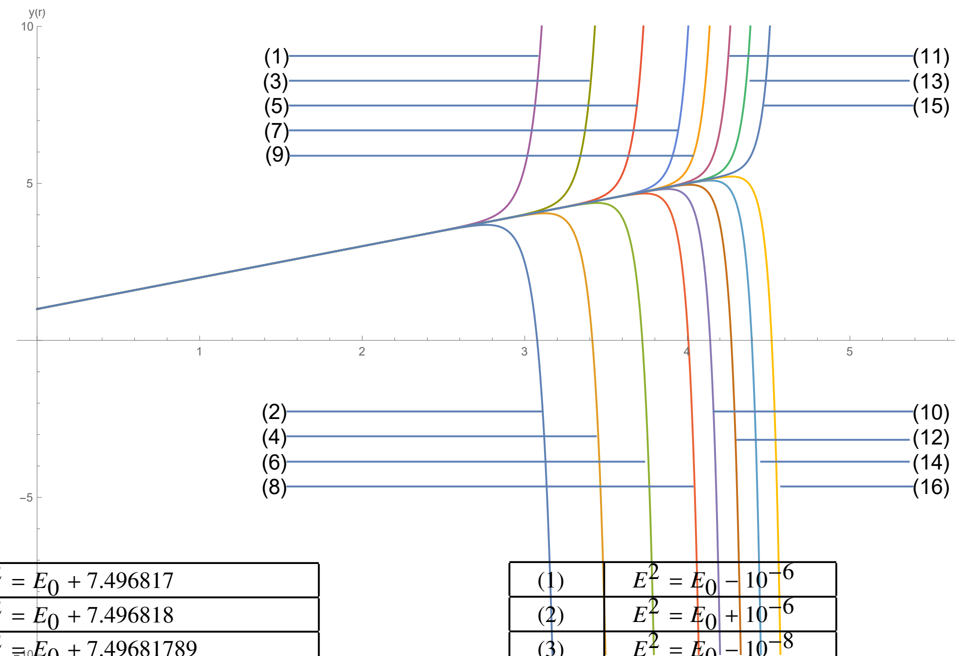




Is extra quantization Really Necessary?

True solution : y=1+r





(1)	$E^2 = E_0 + 7.496817$
(2)	$E^2 = E_0 + 7.496818$
(3)	$E^2 = E_0 + 7.49681789$
(4)	$E^2 = E_0 + 7.49681790$
(5)	$E^2 = E_0 + 7.4968178907$
(6)	$E^2 = E_0 + 7.4968178908$
(7)	$E^2 = E_0 + 7.496817890781$
(8)	$E^2 = E_0 + 7.496817890782$
(9)	$E^2 = E_0 + 7.4968178907817$
(10)	$E^2 = E_0 + 7.4968178907818$
(11)	$E^2 = E_0 + 7.49681789078176$
(12)	$E^2 = E_0 + 7.49681789078177$
(13)	$E^2 = E_0 + 7.496817890781766$
(14)	$E^2 = E_0 + 7.496817890781767$
(15)	$E^2 = E_0 + 7.4968178907817661$
(16)	$E^2 = E_0 + 7.4968178907817662$

(1)	$E^2 = E_0 - 10^{-6}$
(2)	$E^2 = E_0 + 10^{-6}$
(3)	$E^2 = E_0 - 10^{-8}$
(4)	$E^2 = E_0 + 10^{-8}$
(5)	$E^2 = E_0 - 10^{-10}$
(6)	$E^2 = E_0 + 10^{-10}$
(7)	$E^2 = E_0 - 10^{-12}$
(8)	$E^2 = E_0 + 10^{-12}$
(9)	$E^2 = E_0 - 10^{-13}$
(10)	$E^2 = E_0 + 10^{-13}$
(11)	$E^2 = E_0 - 10^{-14}$
(12)	$E^2 = E_0 + 10^{-14}$
(13)	$E^2 = E_0 - 10^{-15}$
(14)	$E^2 = E_0 + 10^{-15}$
(15)	$E^2 = E_0 - 10^{-16}$
(16)	$E^2 = E_0 + 10^{-16}$

- Where the equation

$$-u'' + Vu = \frac{E^2}{4}u,$$

$$V = \left(m + \frac{1}{2}br\right)^2 + \frac{L(L+1)}{r^2}.$$

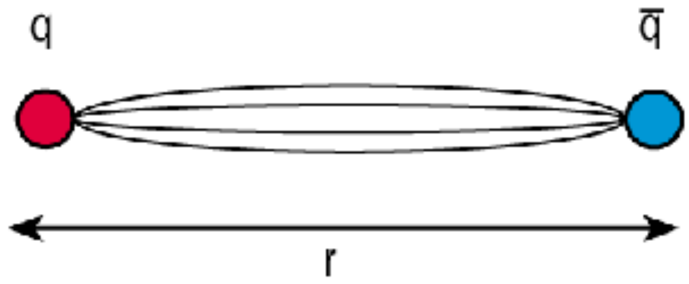
came from?

Quark-antiquark system with scalar interaction

- Lichtenberg spinless Bag Model (1982)

$$H = \sqrt{\vec{\mathbf{p}}^2 + m_1^2} + \sqrt{\vec{\mathbf{p}}^2 + m_2^2}$$

$$m_i \to m_i + \frac{1}{2}S,$$



$$H^2 = 4\left[(m + \frac{1}{2}br)^2 + P_r^2 + \frac{L(L+1)}{r^2}\right]$$

$$H^2\Psi = E^2\Psi$$

- Notice that this is 3d eq.

For $m = 0$, $E^2 = 4b\,(L + N_r + 3/2)$ Gursey : 1992

Conclusion

- if the bare quark mass is not zero, the string tension is also quantized (apart from the energy), which ruin the Regge trajectory, which is the requirement of confinement.

- So confinement request bare quark mass =0

- Then, chiral symmetry is consequence of confinement.

• 감사합니다.

- Thank you
- 谢谢
- ございます