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Higher Curvature Gravity and its implication

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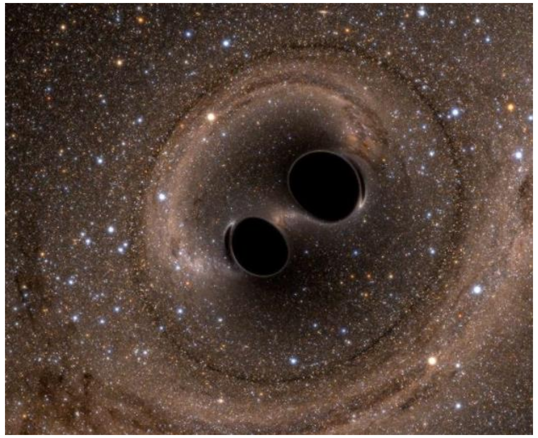
1. Motivation :



Q :Why Higher Curvatures - Gauss-Bonnet Term?

1) Low energy effective theory from string theory
→ Einstein Gravity + higher curvature terms
Gauss-Bonnet term is the simplest leading term.

2) Recent observations of gravitational wave from the mergers of compact binary sources have opened the new horizons in astrophysics as well as cosmology.

These may lead to the tests and constraints to theories beyond Einstein gravity.

Q : What is the physical effects of the Higher Curvature terms?

1) Effects to the Black Holes.

In BHs in the dilaton-Einstein-Gauss-Bonnnet theory, there exists

hairs :
minimum mass,
instability, etc.

2) Effects in the Early Universe.
during the inflation, etc.

3) Holography

(asymptotic) AdS geometry ↔ Quantum System
in d+1 dim. in d dim.

Black Holes ↔ Quantum System with finite T

Ex) (global) 5dim AdS geometry ↔ N=4 SUSY Yang-Mills in 4-dim.
instability of AdS BH ↔ instability/phase transitions in Quantum System

Review : AdS BH geometry

is described by the following action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{g}\,(-\mathcal{R}+2\Lambda)$$

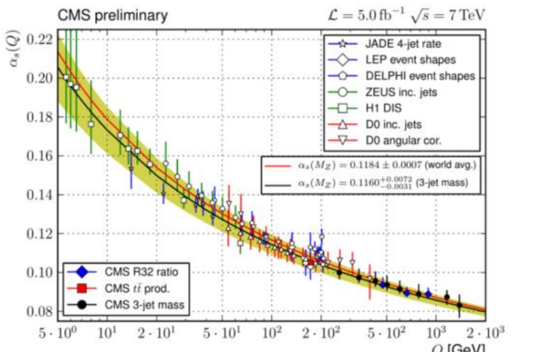
$\Lambda = -\frac{6}{R^2}$: cosmological constant

$$ds^2 = -N^2(r)\,dt^2 + f^{-2}(r)\,dr^2 + r^2 d\Omega^2$$

$$N^2(r) = f^2(r) = (1 - \frac{2M}{r} + \frac{r^2}{l^2})$$

No lower bound on the black hole mass

$$\beta(g^2) = \frac{dg^2(\mu)}{d\ln\mu}$$



Holographic QCD

(asymptotic AdS Space) ⟷ QCD

$$g_s = e^{\phi(r)} = g^2_{YM}(\mu)$$

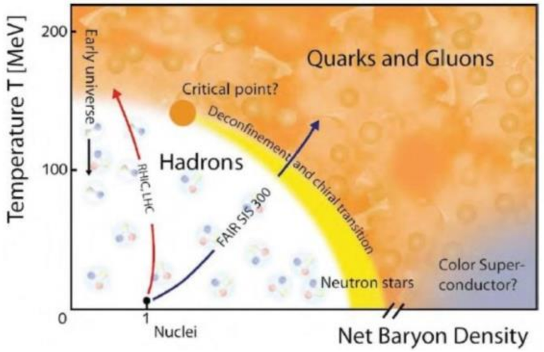
Holography :

QCD Phase transition

	Confinement	Deconfinement
QCD (4Dim)	Hadron	Quark-Gluon
Gravity (5Dim)	Thermal AdS	AdS Black Hole

Hawking-Page phase transition

=Transition of bulk geometry



Metric of Thermal AdS)

= **Slice of AdS metric**

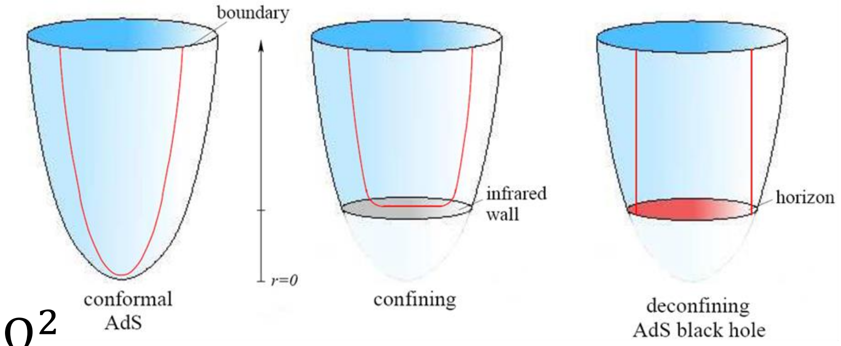
$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu), \qquad 0 < z \leq z_m$$

AdS BH geometry

$$ds^2 = -N^2(r)\, dt^2 + f^{-2}(r)\, dr^2 + r^2 d\Omega^2$$

$$N^2(r) = f^2(r) = (1 - \frac{2M}{r} + \frac{r^2}{l^2})$$

Ex) Confinement –Deconfinement



Horizon $r_H = 2M$

Erlich,Katz,Son,Stephanov PRL(2005),

Da Rold, Pomarol NPB(2005)

[Herzog , Phys.Rev.Lett.98:091601,2007]

Figure from Erdmenger et.al, EPJA (2008)

The geometry with smaller action is the stable one for given T.

$$\Delta S = \lim_{\epsilon \to 0}(S_{AdSBH} - S_{tAdS}) = \frac{\pi z_h R^3}{\kappa^2}\left(\frac{1}{z_{IR}^4} - \frac{1}{2z_h^4}\right)$$

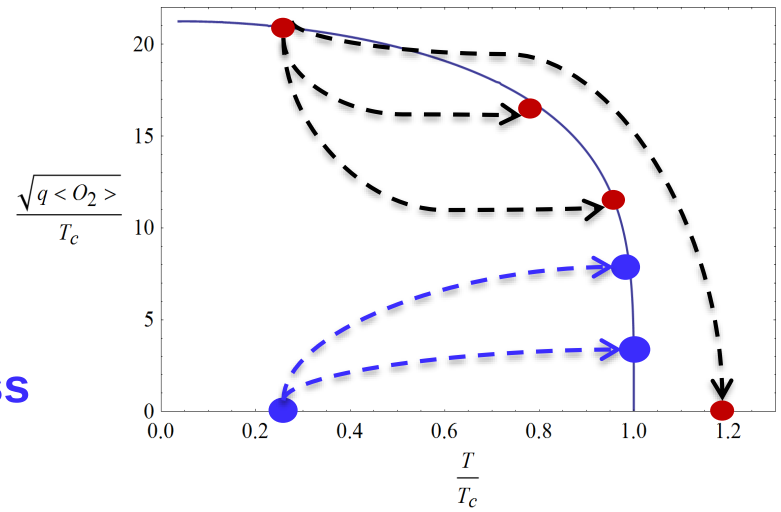
> 0 for T < Tc

< 0 for T>Tc

Q: Can we find more natural BH geometry (asymptotic AdS) towards realistic holography, with natural realization of Tc, etc?

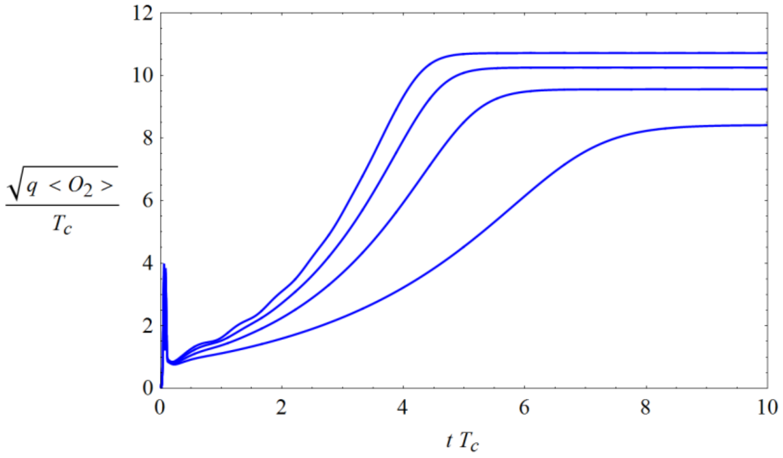
Holographic Approach to the nonequilibrium physics

Two scenarios of Far-from-equilibrium dynamics



Blue routes : Condensation Process

· Non-equilibrium evolution of an unstable configuration

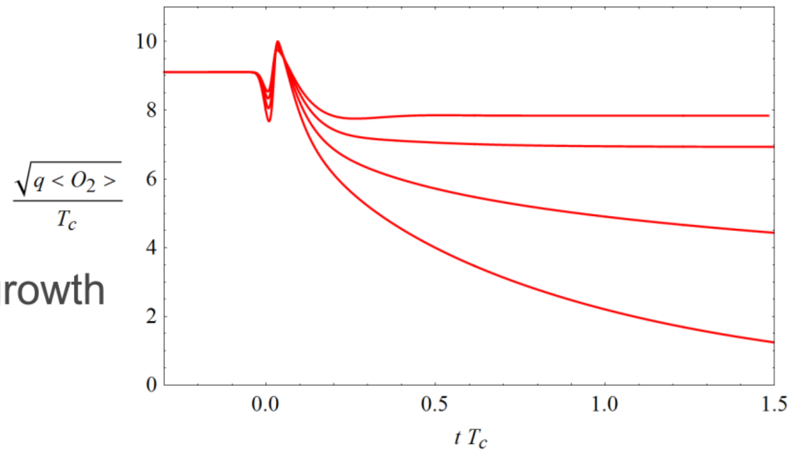


· Condensate undergoes an exponential growth

- Phase transition in "real" time !

Red routes: Quantum Quenching

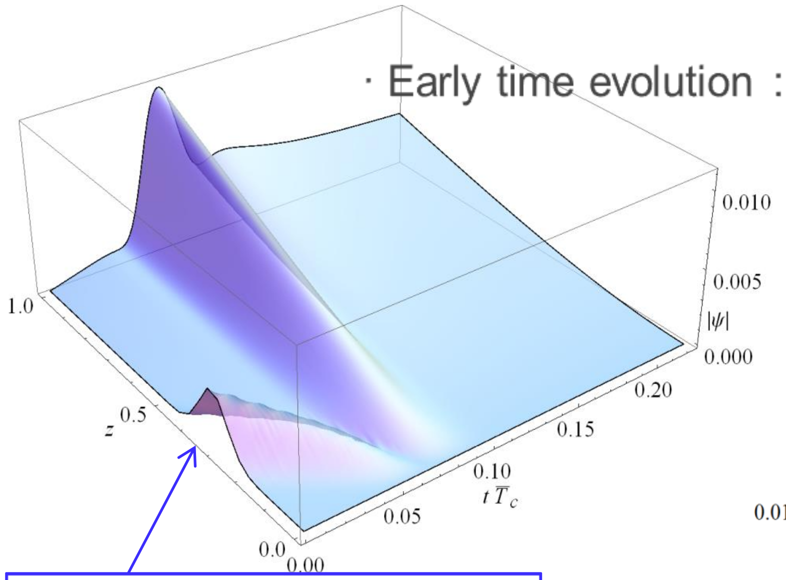
· Dynamical response to a sudden injection of energy



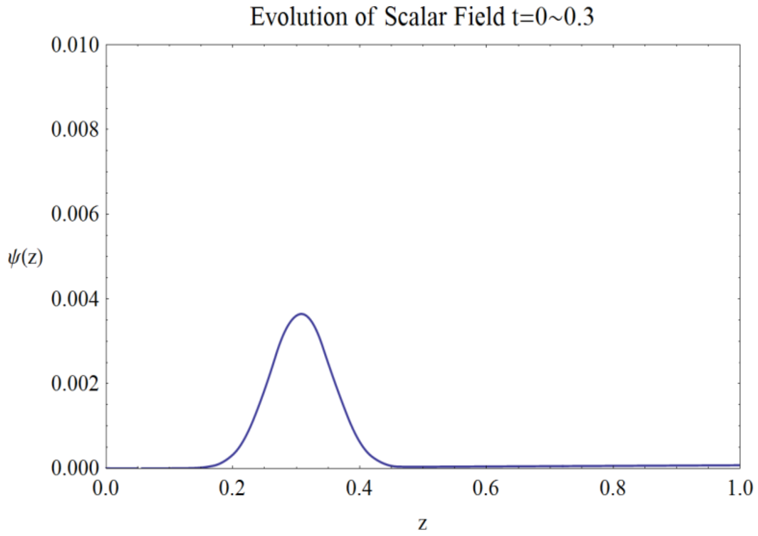
Condensation Process on Anisotropic Background - evolution of scalar field

X. Bai, B-HL, M.Park, and K. Sunly JHEP09(2014)054

- Early time evolution : $t = 0 \sim 0.2$

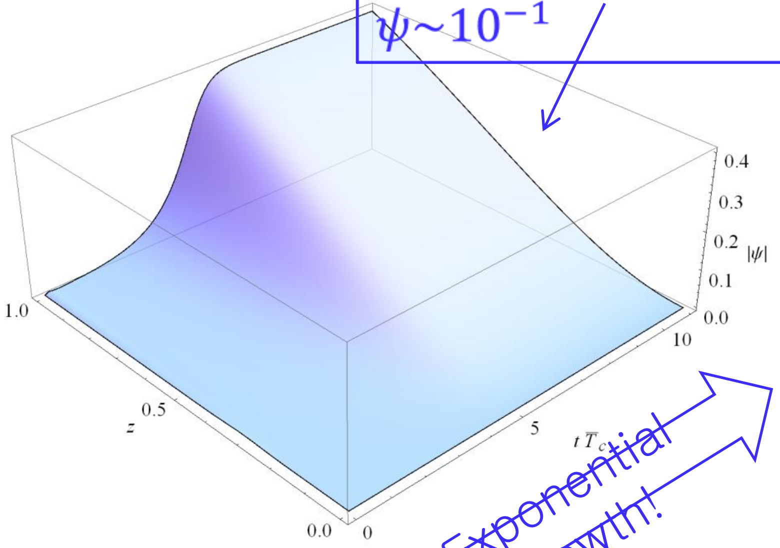


initial perturbation
- scalar field $\psi \sim 10^{-3}$



- Late time evolution : $t = 0.2 \sim 10$

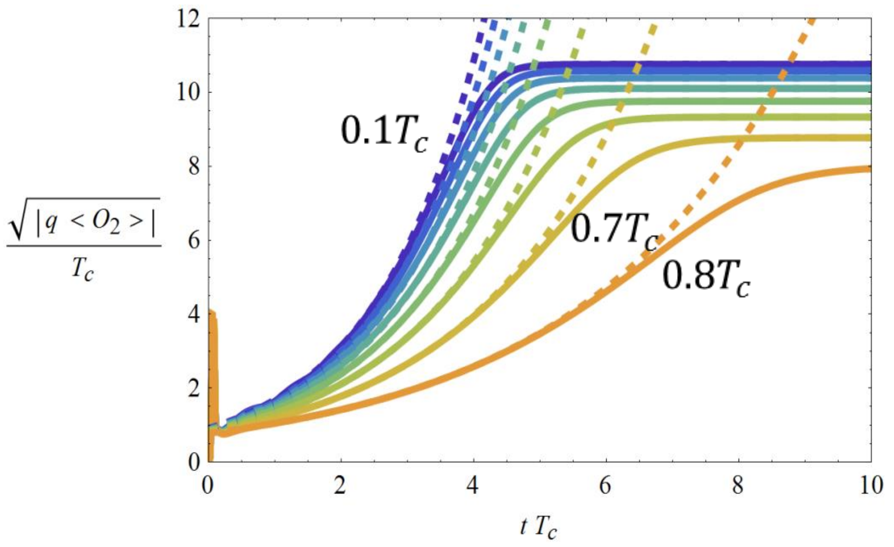
final configuration
- scalar field $\psi \sim 10^{-1}$



Exponential growth!

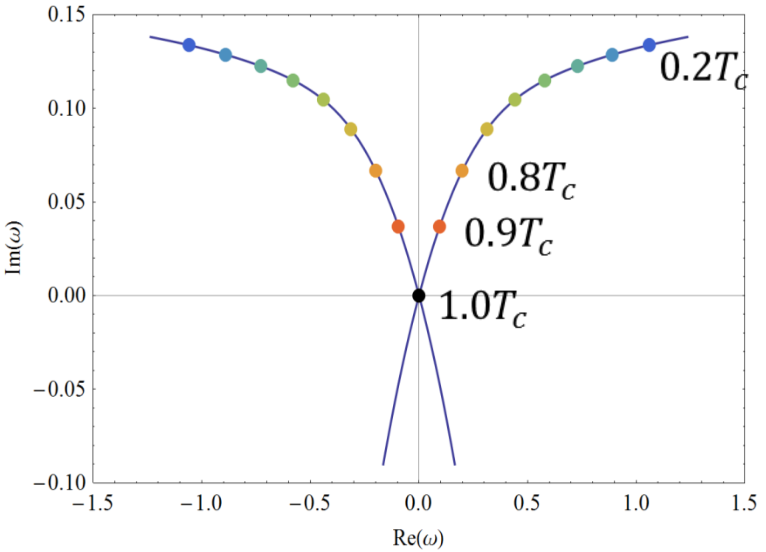
Quasinormal modes (QNMs) - linearized fluctuation

· Matching between QNMs (dashed) and non-equilibrium evolution (solid)



· Initial temperature : $0.1T_c \sim 0.8T_c$

· Leading three orders of QNMs $T = 0.1T_c \sim 1.2T_c$



· QNMs move upward as T is lowered.
· Points indicate critical temperature T_c.

Q: How to describe the nonequilibrium physics?
May the Instability of BH play the important role?

Einstein-Gauss-Bonnet Gravity

Simplest Higher Curvature Extension of the Einstein Gravity

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2} R + \alpha R_{GB}^2 - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

$$R_{GB}^2 = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$$

Gauss-Bonnet term, topological in 4-dim.

Lovelock theory

- 2^{nd} order equation of motion, no ghosts

Lovelock term of order m = Euler characteristic of dimension 2m

$$L_m = \delta^{b_1 b_2 b_3 b_4 \cdots}_{a_1 a_2 a_3 a_4 \cdots} R_{a_1 a_2}{}^{b_1 b_2} R_{a_3 a_4}{}^{b_3 b_4} \cdots$$

Ex) Order 1

$$L_1 = R$$

Einstein-Hilbert term, topological in 2-dim.

Order 2

$$L_2 = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd} = R_{GB}^2$$

Gauss-Bonnet term, topological in 4-dim.

cf). Horndeski theory

We will work on the Dilaton-Einstein-Gauss-Bonnet Gravity

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2} R - \frac{1}{2}\xi(\phi) R_{GB}^2 + \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

Contents

V 1. Motivation

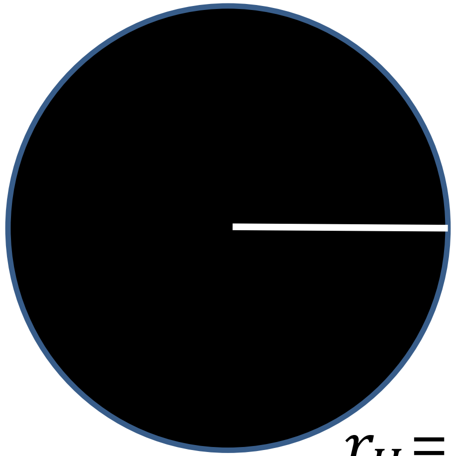
2. Black Holes in the Dilaton Einstein Gauss-Bonnet(DEGB) theory
   - Black Hole solutions (asymptotic flat & asymptotic AdS)
   - Stability of the DEGB Black holes under fragmentation

3. Cosmological implication of the Gauss-Bonnet term
   - Effects to Inflation
   - Reconstruction of the Scalar Field Potential

4. Summary

2. Black Holes in the Dilaton Einstein Gauss-Bonnet (DEGB) theory

Review : Einsten theory – Schwarzschild Black Hole



Action

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

where $g = \det g_{\mu\nu}$ and $\kappa \equiv 8\pi G$

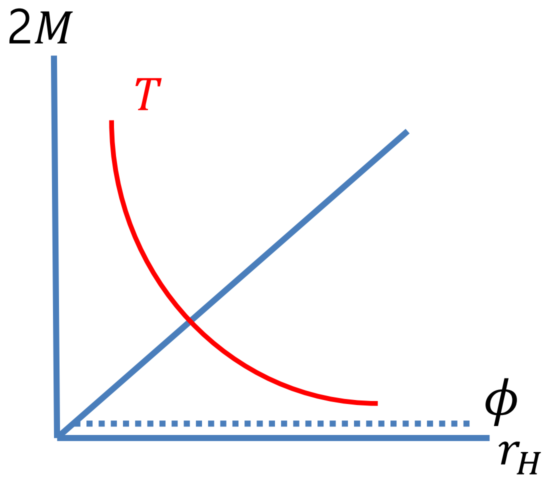
Black Hole solution

$$ds^2 = -\left(1-\frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1-\frac{2M}{r}\right)} + r^2 d\Omega^2$$

Horizon

$r_H = 2M$

$\phi = 0$ No hair

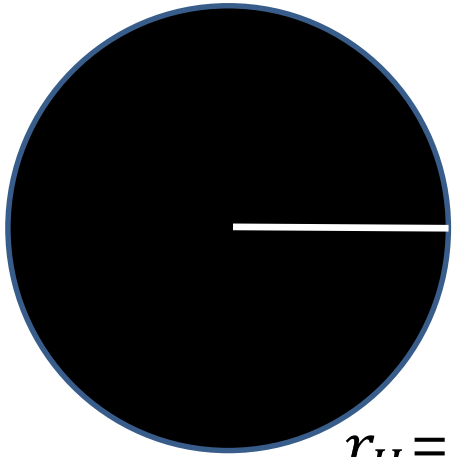


Note :

1. No minimum mass of BH :
   there is no lower bound on $r_H = 2M$
2. No (scalar) Hair
   $\phi = 0$ [Chase, CMR 19, 276 (1970)]

Review : AdS Black Holes

Brown, Creighton & Mann (1994)



$r_H = 2M$

Action

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2} R + \Lambda - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

where $g = \det g_{\mu\nu}$ and $\kappa \equiv 8\pi G$

Black Hole solution

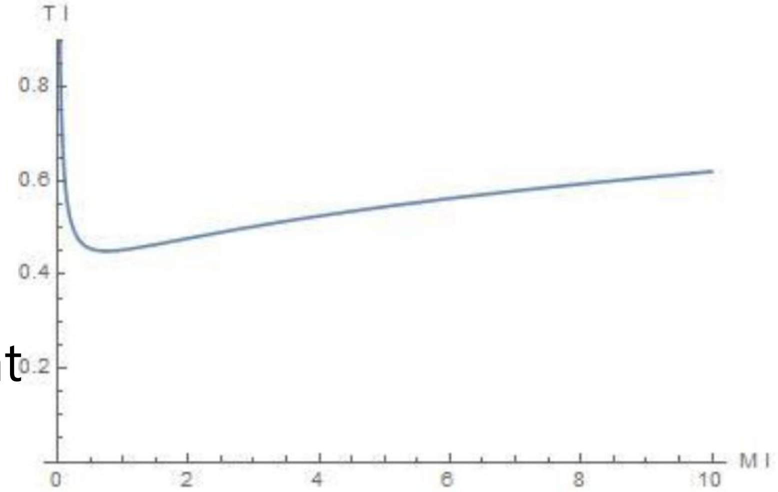
$$ds^2 = -N^2(r)\, dt^2 + f^{-2}(r)\, dr^2 + r^2 d\Omega^2$$

$$N^2(r) = f^2(r) = (1 - \frac{2M}{r} + \frac{r^2}{l^2})$$

$\phi = 0$ No hair

$$T = \frac{r_+}{\pi\ell^2} + \frac{1}{2\pi r_+}$$

$$r_+^2 = -\frac{\ell^2}{2} + \ell\sqrt{\frac{\ell^2}{4} + \omega M} \qquad \omega \equiv \frac{16\pi G_N}{3\mathrm{vol}(S^3)}$$



- Small BH : Similar to BH's in flat spacetime, Negative specific heat
- Large BH : Stable, important in AdS thermodynamics

Note :

For $\Lambda = 0$, the theory becomes the Einstein gravity.

2-2) Dilaton-Einstein-Gauss-Bonnet (DEGB) theory : Hairy black holes

Action

$$I = \int_{\mathcal{M}} \sqrt{-g}\, d^4x \left[\frac{R}{2\kappa} - \frac{1}{2}\nabla_\alpha \Phi \nabla^\alpha \Phi + \alpha e^{-\gamma\Phi} R^2_{\rm GB} \right] + \oint_{\partial\mathcal{M}} \sqrt{-h}\, d^3x \frac{K - K_o}{\kappa}, \quad (1)$$

where $g = \det g_{\mu\nu}$ and $\kappa \equiv 8\pi G$

Guo,N.Ohta & T.Torii, Prog.Theor.Phys. 120,581(2008);121 ,253 (2009);
N.Ohta &Torii,Prog.Theor.Phys.121,959; 122,1477(2009);124,207 (2010);
K.i.Maeda,N.Ohta Y.Sasagawa, PRD80, 104032(2009); 83,044051 (2011)
N. Ohta and T. Torii, Phys.Rev. D 88 ,064002 (2013).

The GB term :

$$R^2_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

Note (for α and γ)

1) For $\gamma = 0$, DEGB theory becomes the Einstein-Gauss-Bonnet (EGB) theory, with the GB term becoming the boundary term

2) The symmetry under $\gamma \to -\gamma, \Phi \to -\Phi$. allows choosing γ positive without loss of generality.

3) α scaling :The coupling α dependency could be absorbed by the r → r/$\sqrt{\alpha}$ transformation. However, the behaviors for the α = 0 case cannot be generated in this way. Hence, we keep the parameter α, to show a continuous change to α = 0.

Q : signature of α ?

2-1) Einsten Gauss-Bonnet (EGB) theory

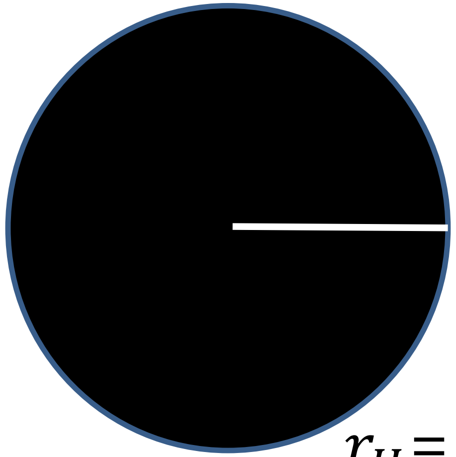
W.Ahn, B. Gwak, BHL, W.Lee, Eur.Phys.J.C (2015)

Action

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2} R + \alpha R^2_{GB} - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

where $g = \det g_{\mu\nu}$ and $\kappa \equiv 8\pi G$ $R^2_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

The Gauss-Bonnet term



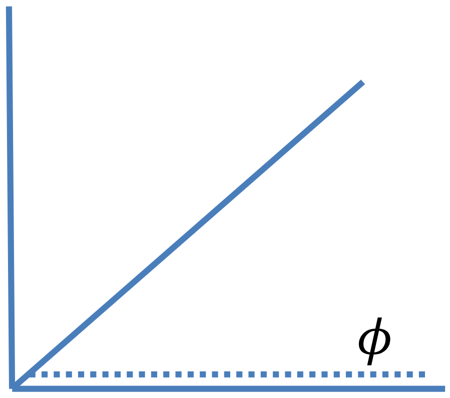
Black Hole solution

$$ds^2 = -\left(1-\frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1-\frac{2M}{r}\right)} + r^2 d\Omega^2$$

$\phi = 0$ No hair

Horizon

$r_H = 2M$



Note :

1) For the coupling $\alpha = 0$, the theory becomes the Einstein gravity.

2) GB term is a surface term, not affecting the e.o.m. Hence,
The black hole solution is the same as that of the Schwarzschild one.

3) However, the GB term contributes to the black hole entropy and influence stability.

The Einstein equations and the scalar field equation are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa \left(\partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} \partial_\rho \Phi \partial^\rho \Phi + T^{GB}_{\mu\nu} \right) , \quad (2)$$

$$\frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi] - \alpha \gamma e^{-\gamma \Phi} R^2_{GB} = 0 , \quad (3)$$

Note :

1) All the black holes in the DEGB theory with given non-zero couplings α and γ have hairs.
I.e., there does not exist black hole solutions without a hair in DEGB theory.
(If we have Φ = 0, dilaton e.o.m. reduces to R^2_{GB} = 0. so it cannot satisfy the dilaton e.o.m..)

2) Hair Charge Q is not zero, and is not independent charge either : secondary hair.

Question) How can it be consistant with the no hair theorem?

Note :

1. Primary hair : mass, angular momentum, charge

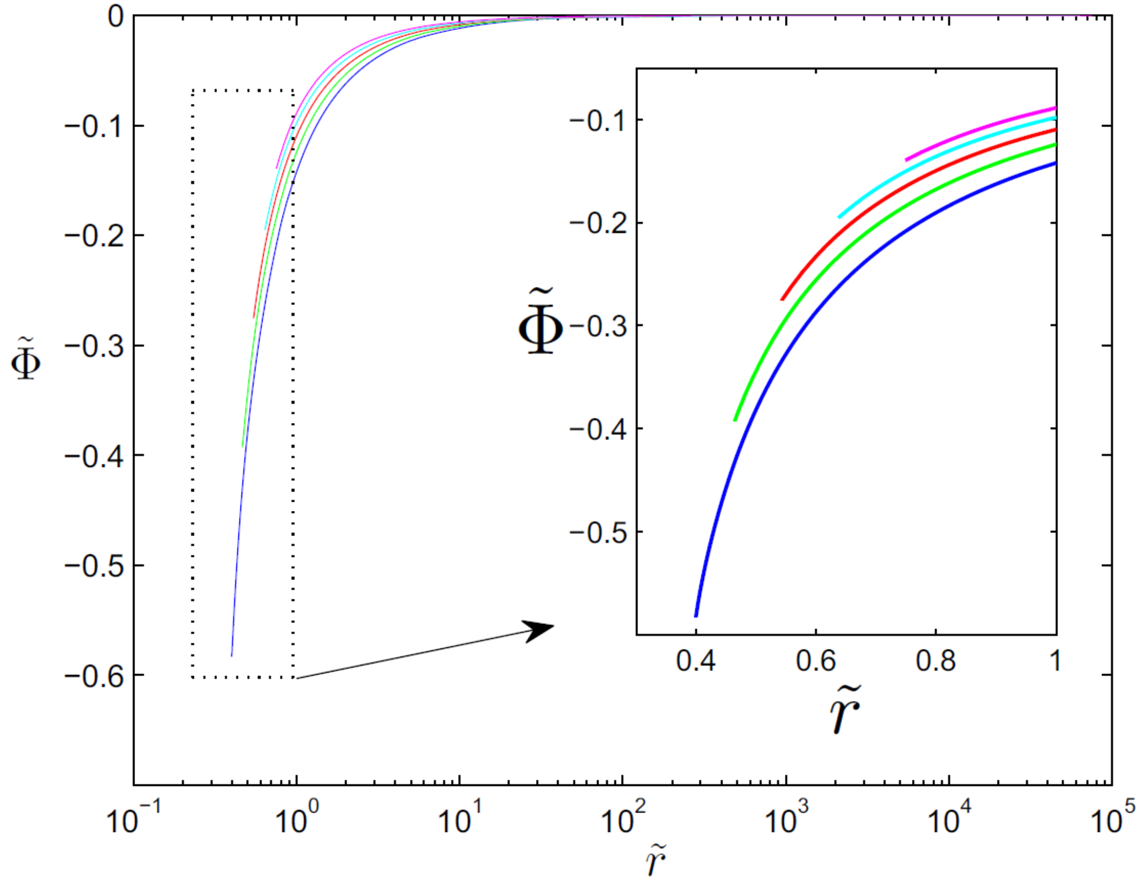
Global charges are given by a surface integral of flux density over a sphere at infinity. In other words, global charges are quantities which can be measured at infinity.

2. Secondary hair : scalar(dilaton) hair, … : is determined by the primary hair.

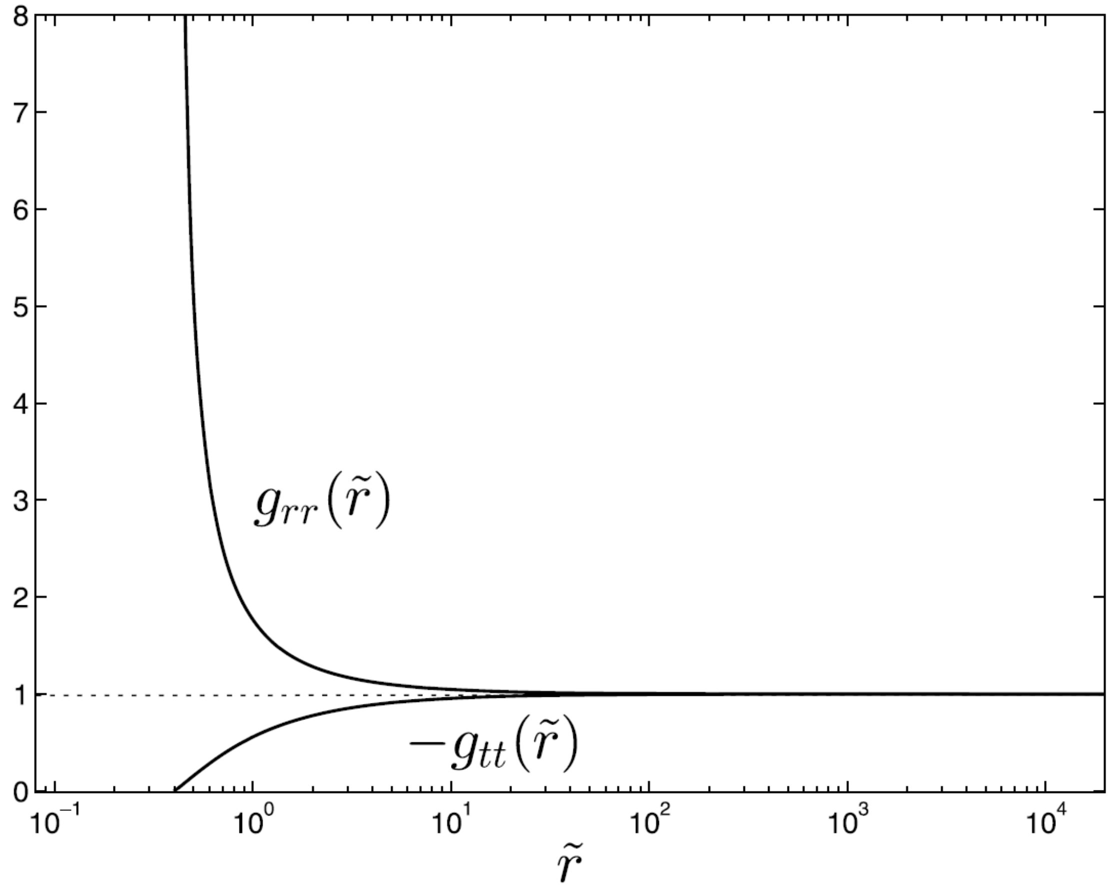
DEGB black hole solutions (α > 0)

γ = 1/6, and α = 1/16

Scalar field profiles



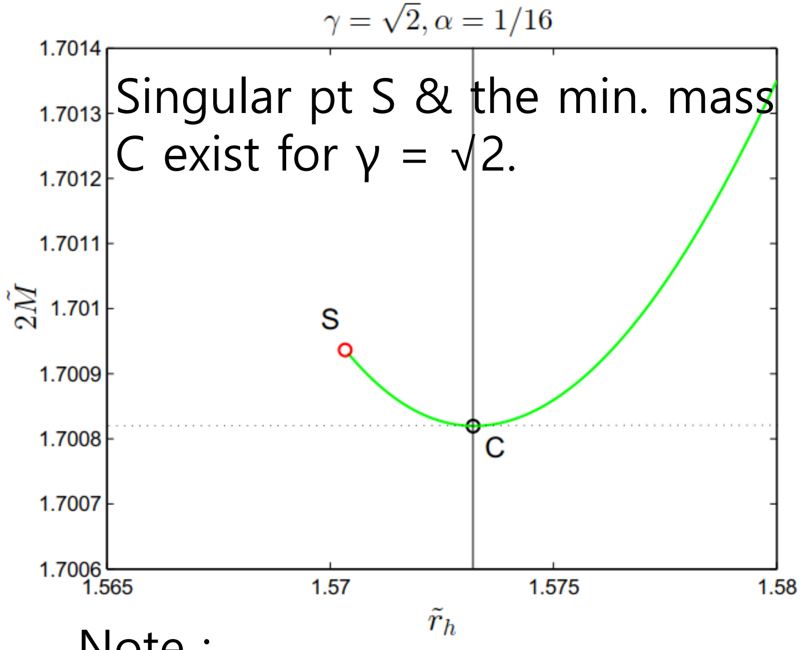
the metric components for r_h = 1

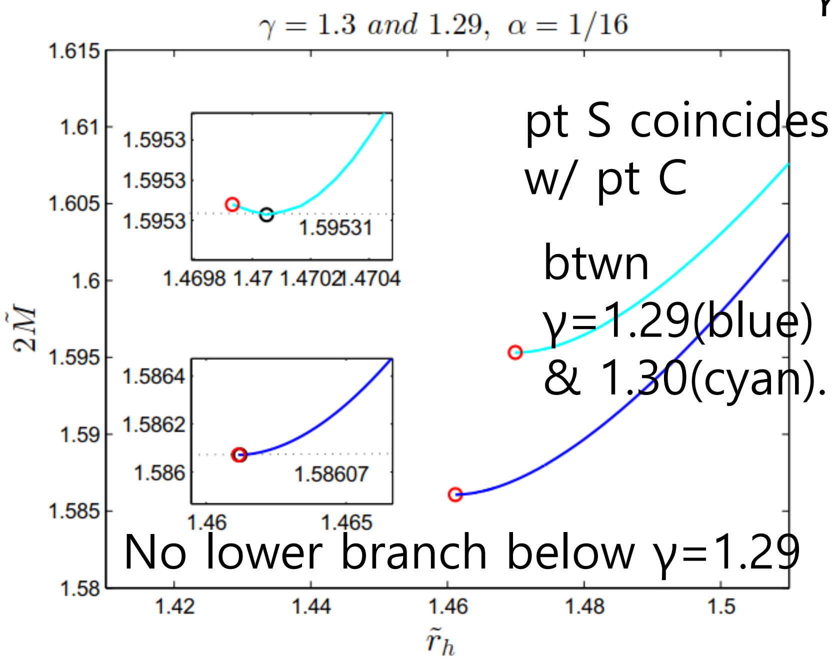


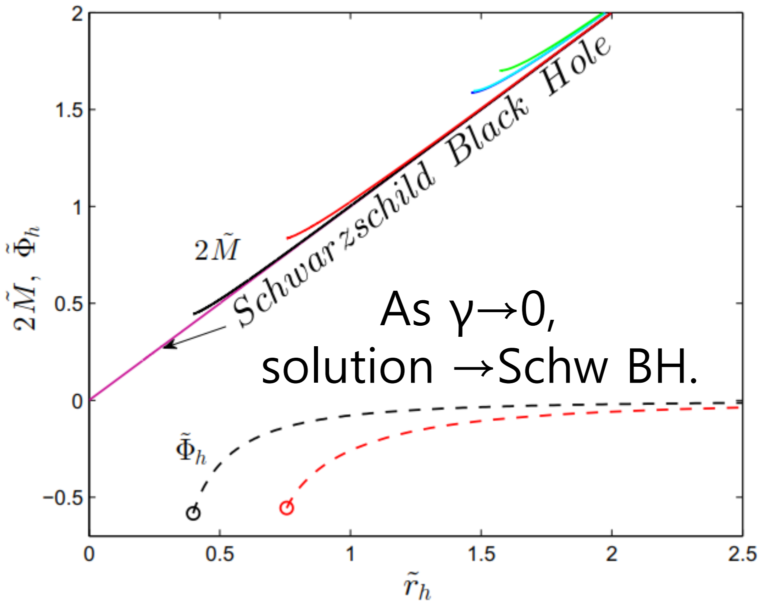
The five solid lines correspond to different DEGB black hole solutions.

Coupling γ dependency of the minimum mass for fixed α = 1/16 >0.

γ=√2(green),γ=1.3(cyan),γ=1.29(blue)
γ=1/2(red), γ=1/6(black), γ=0(purple)







Note :

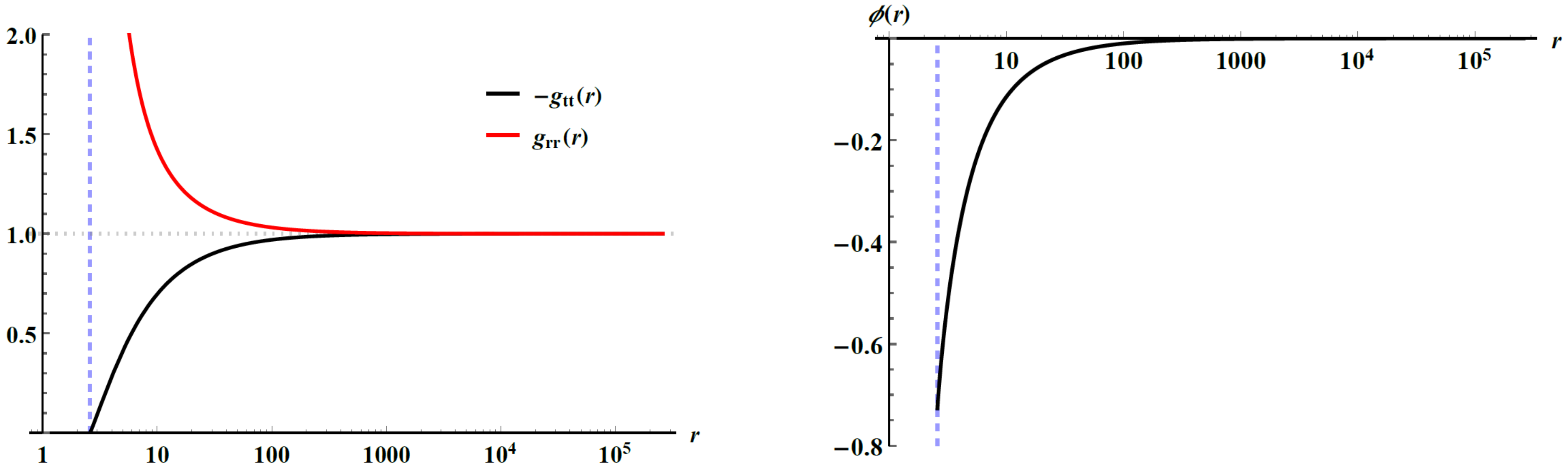
1. For large γ, sing. pt S & extremal pt C (with minimum mass $\tilde{M}$) exist.
2. The solutions between point S and C are unstable for perturbations and end at the singular point S , I.e., there are two black holes for a given mass in which the smaller one is unstable under perturbations.
3. As γ smaller, the singular point S gets closer to the minimum mass point C.
4. Below γ=1.29, the solutions are perturbatively stable and approach the Schwarzschild black hole in the limit of γ going to zero. These solutions depend on the coupling γ.
5. If DEGB BH horizon becomes larger, the scalar field goes to 0, and the BH becomes a Schwarzschild BH.

GB term → makes gravity "less attractive" (for α >0) !!!

Q: How about the properties, such as Stability & implication to the cosmology, etc ?

DEGB Black Hole solutions (α < 0)

BHL, W. Lee, D. Rho, PRD (2019)



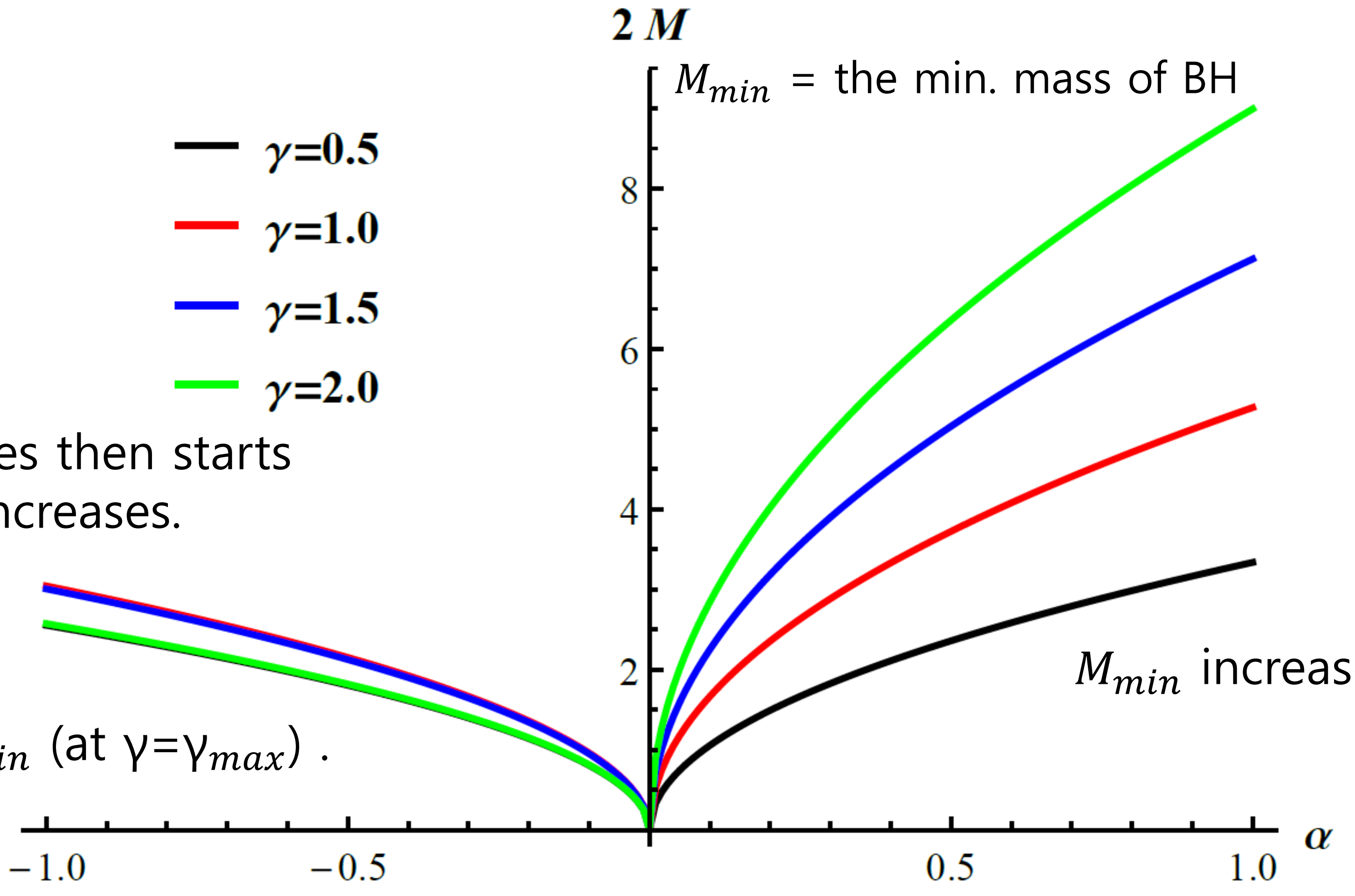
(a) $-g_{tt}(r)$ and $g_{rr}(r)$ vs. r

(b) $\phi(r)$ vs. r

The metric components and the profile of the dilaton field for a black hole solution.

The role of the signature α : the mass of BH

BHL, W. Lee, D. Rho, PRD (2019)



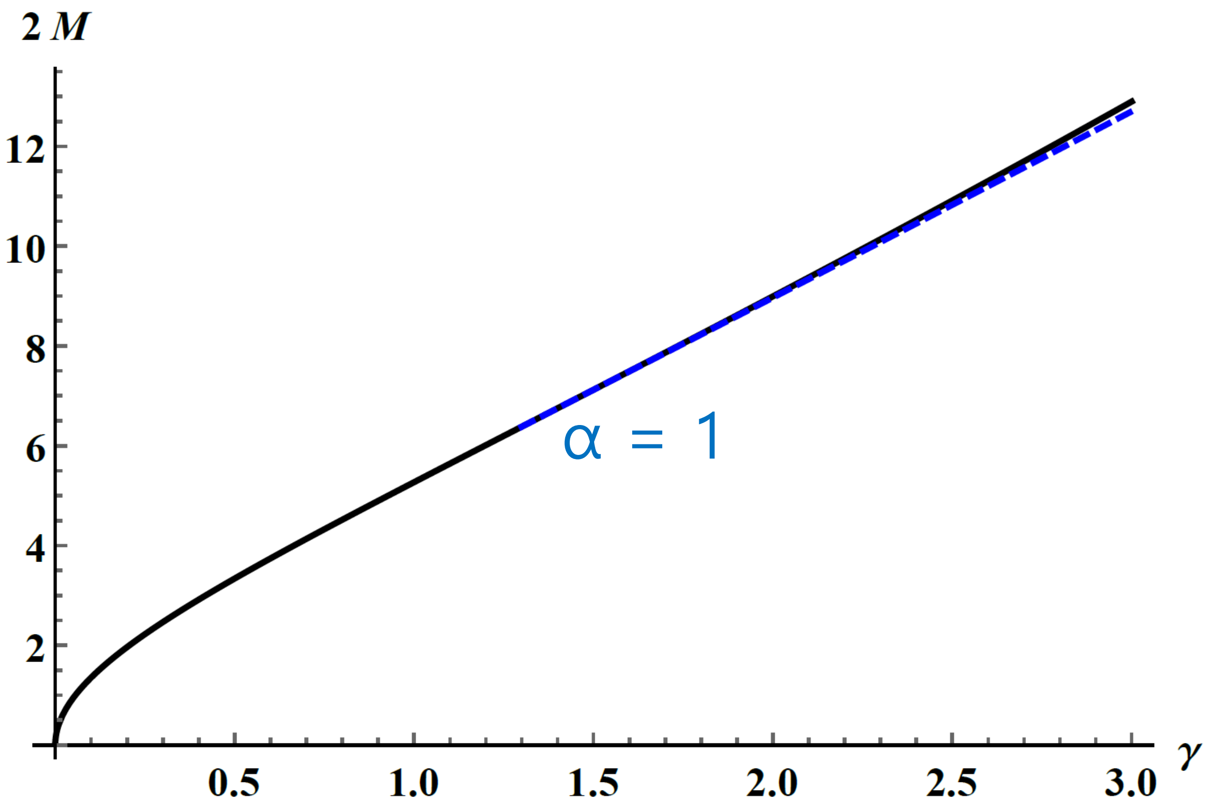
M_{min} first Increases then starts decreasing as γ increases.

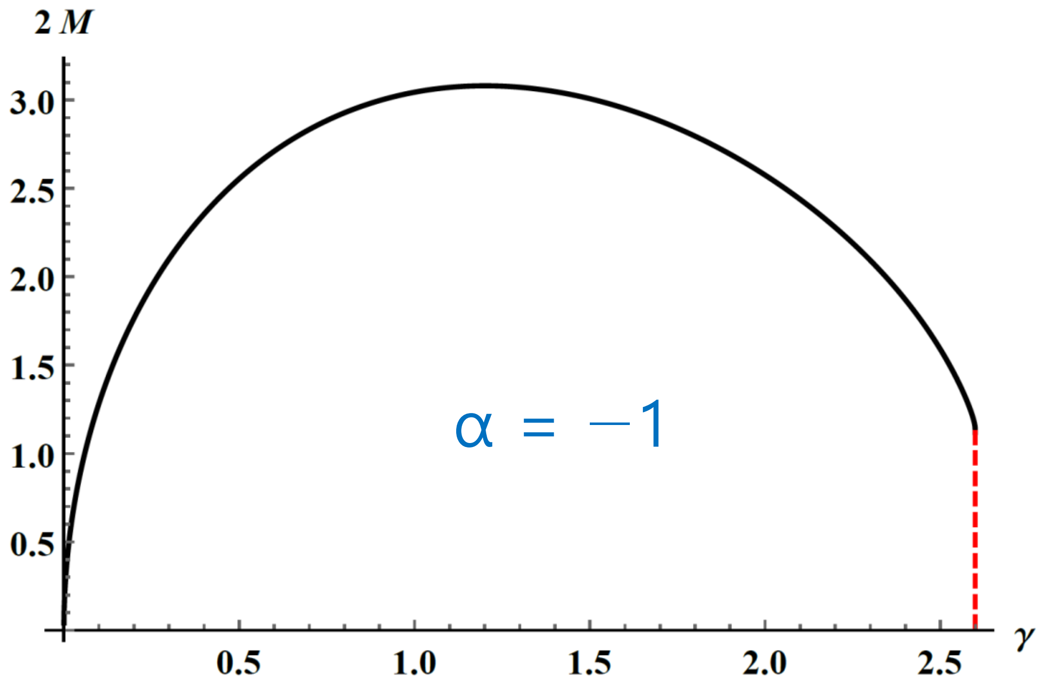
There exists max. value of M_{min} (at $\gamma=\gamma_{max}$) .

M_{min} increases as γ increases.

Remark : "relatively more attractive" "less attractive"

The lower bound for the black hole mass vs. γ





The blue dashed line represent the black holes having the minimum masses.

The red dashed line represents the maximized γ to get the black hole solution with the negative α.

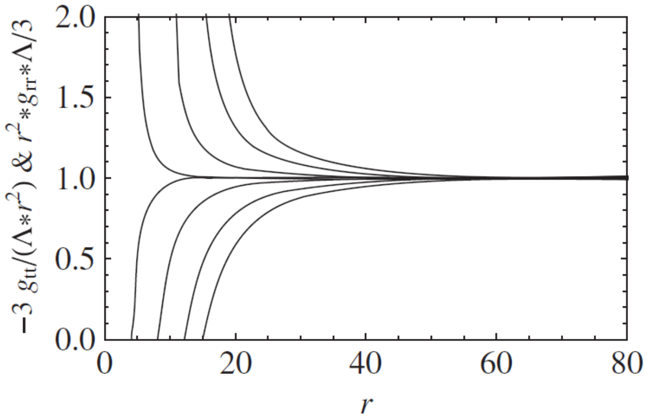
The black line represent the lower bound for black holes with maximum value of ϕ_h which have the minimum black hole radius r_h.

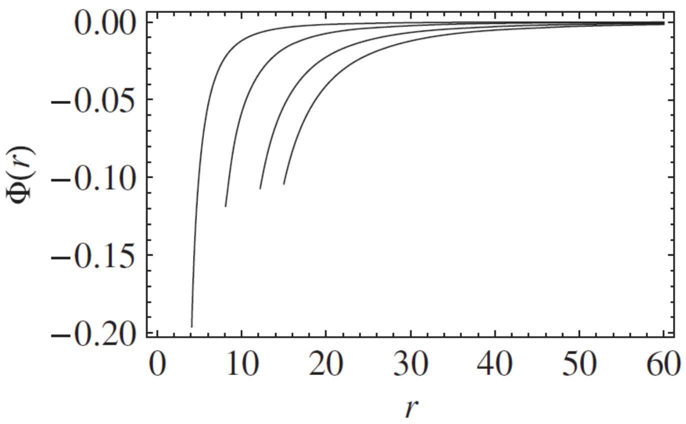
Negative cosmological constant with Gauss-Bonnet term : Phases

S. KHIMPHUN, BHL, W. LEE PRD(2016)

Solutions

α = 1.0 γ=1/2, Λ=1/2, κ=1





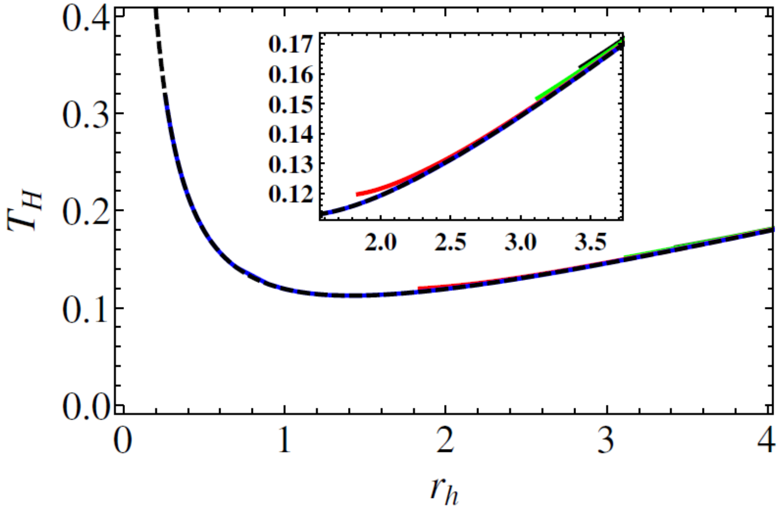
r_h = 4.09, 8.1, 12.2, and 15, respectively

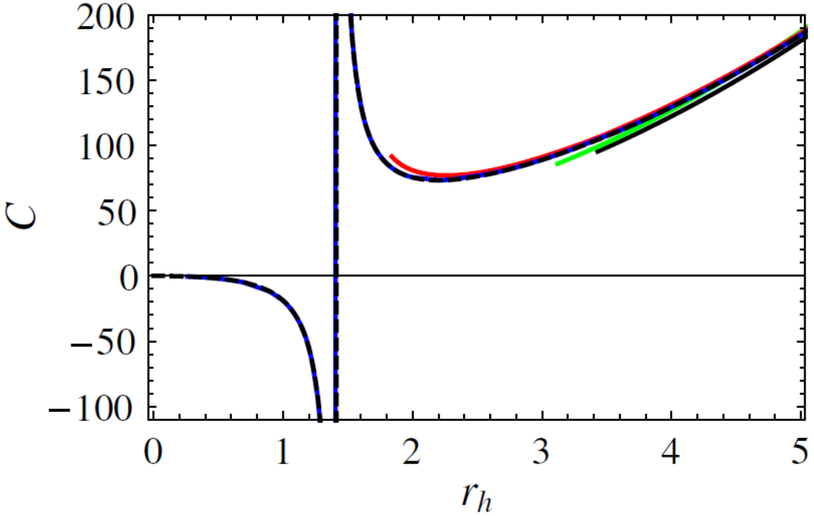
There exists the minimum mass of a black hole.

If the black hole horizon r_h becomes larger, the magnitude of the scalar hair becomes smaller.

Thermodynamics

γ=1/2, Λ=1/2, κ=1





α = 0 for the dashed line

α = 0.005 for the blue line,

α = 0.4 for the red line,

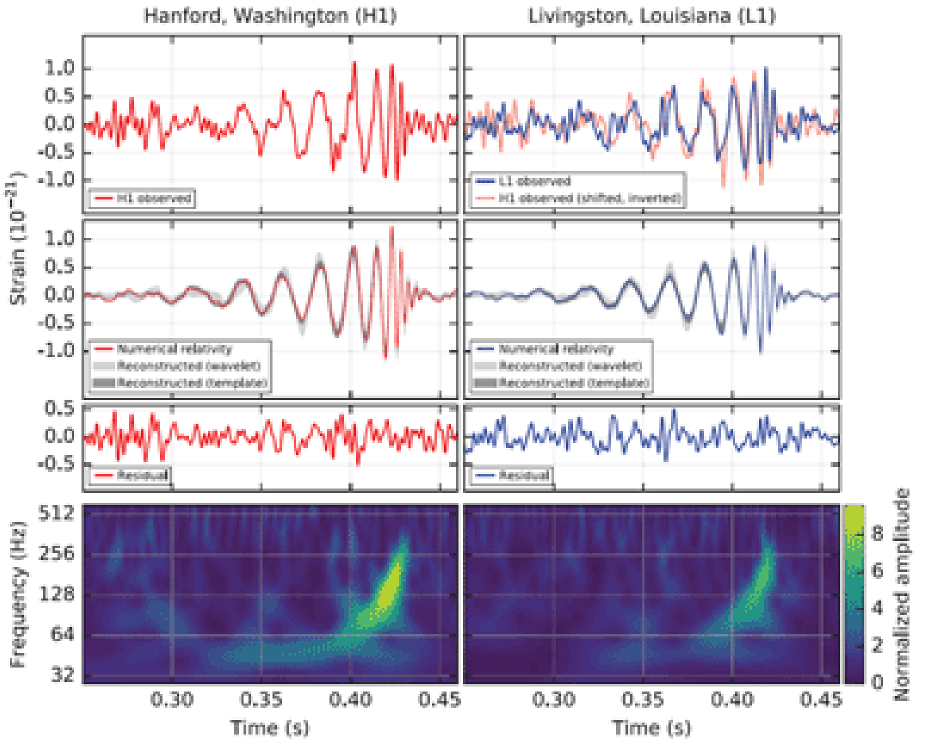
α = 0.8 for the green line,

α = 1.0 for the black line

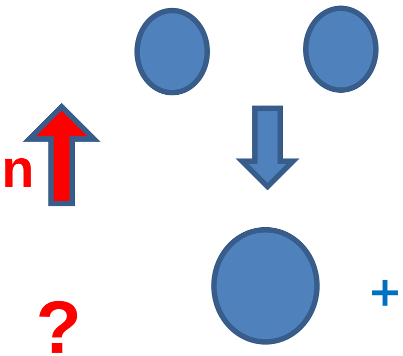
Black Hole Stability (nonperturbative)

Colliding Black Holes : A Black Hole Merger + Gravitational Wave

GW150914



fragmentation



Merging

\+ Gravitational Wave

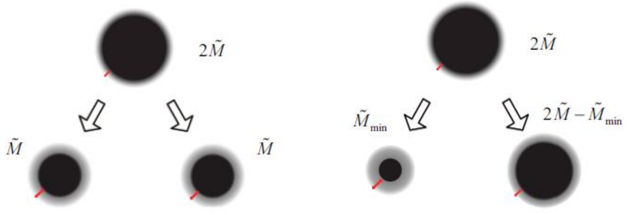
?

Observed!

Q : Can Black Holes be splitted into fragmentation?

Reverse Process

Can a Black Hole be unstable splitting into two Black Holes ?



(a) (b)

For Schwarzschild black hole

If the Black Hole splits into two black holes with mass fractions of $(1\text{-}\delta, \delta)$, then

$$\frac{S_f}{S_i} = \frac{(\delta r_h)^2 + ((1-\delta) r_h)^2}{r_h^2} = \delta^2 + (1-\delta)^2 \leq 1$$

Schwarzschild black holes are always stable under the fragmentation.

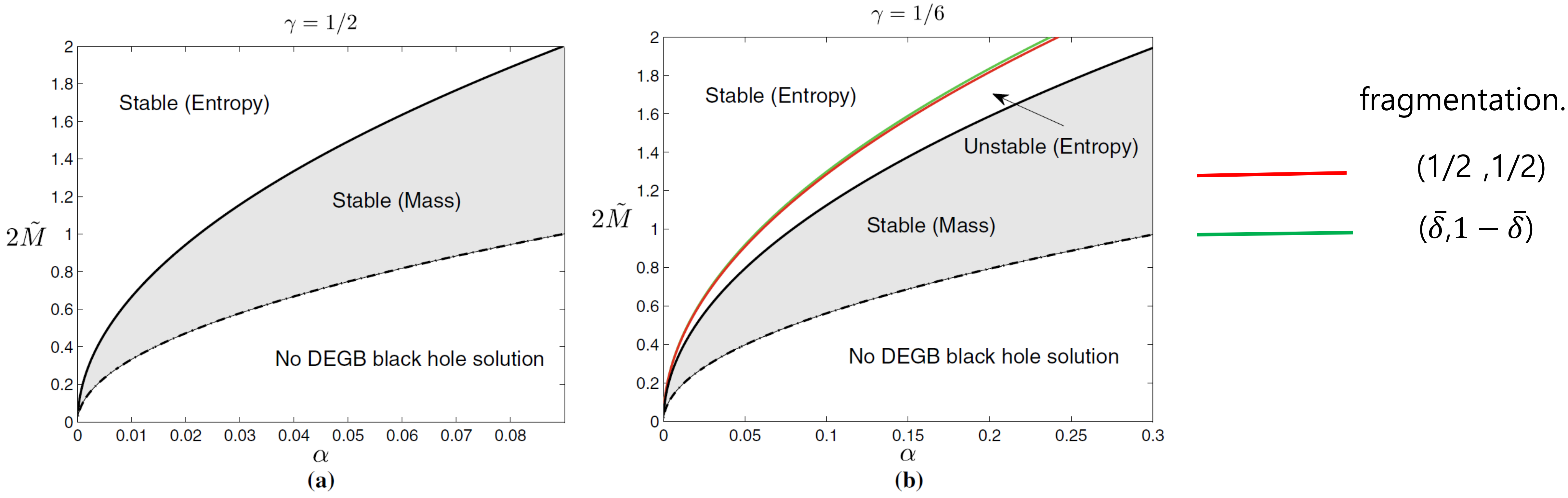
Note : The entropy ratio approaches 1 as $\delta \to 0$.

In other words,
Marginally stable under the fragmentation shooting off the BH with infinitesimal mass.

These phenomena become different in the theory with the higher order of curvature term.

Fragmentation Instability for DEGB Black Holes

DEGB black hole with mass M decaying into two daughter BHs with mass fraction (1-δ, δ).



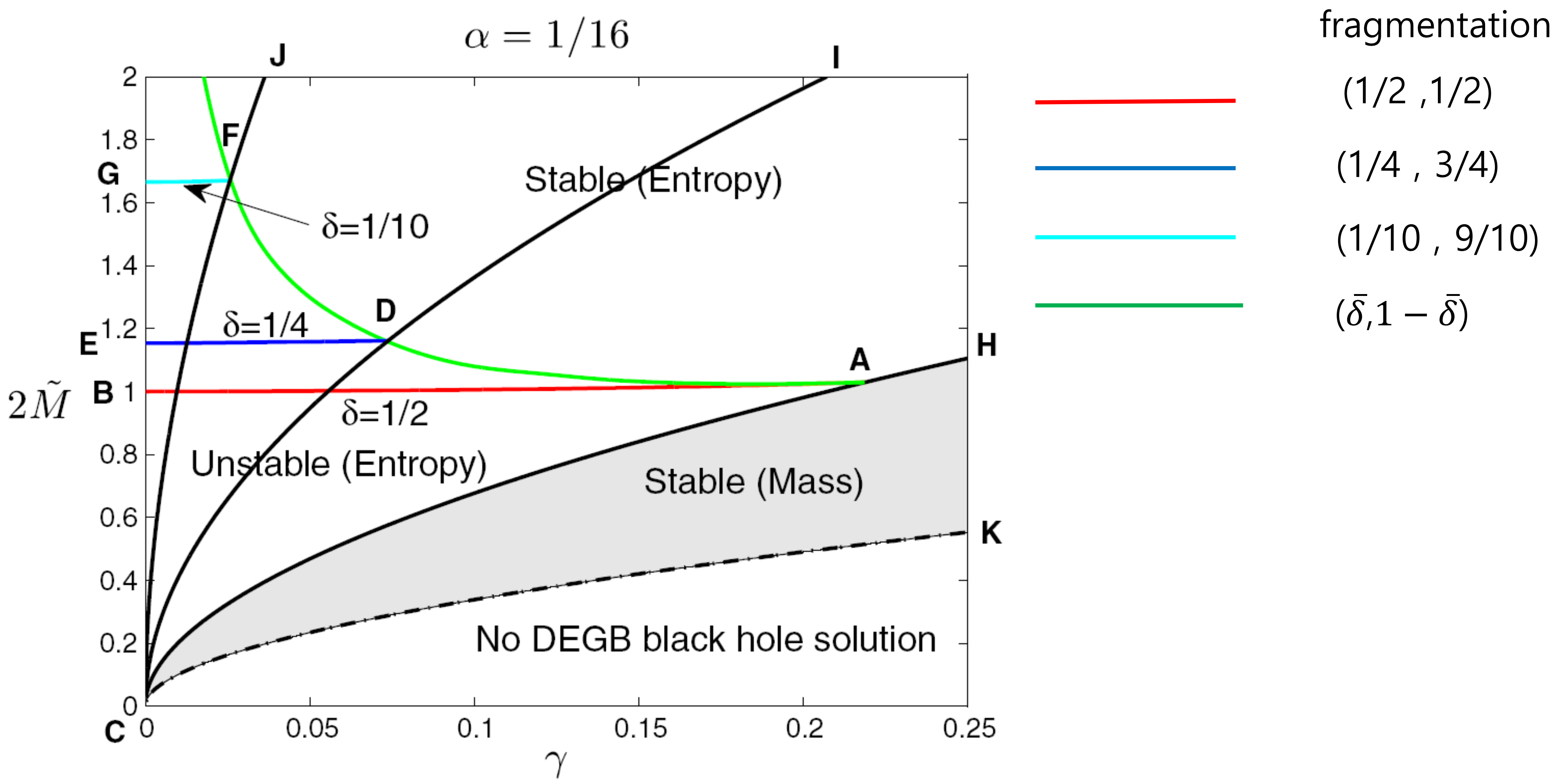
Note : 1) It cannot decay into black holes with mass smaller than the minimum mass M_{min}.

Hence, $\delta_m \leq \delta \leq 1/2$, $\delta_m = M_{min}/M$.

2) The BHs with $M < 2M_{min}$ are absolutely stable.

The black hole can be fragmented only when its mass exceeds twice of minimum mass.

The phase diagrams with respect to γ and $\tilde{M}$



For δ=1/4 the stable (mass) region is the region ICK

the unstable (mass) region is the region ECD

the stable (entropy) region is above the line EDI .

3.Cosmological Effects of the Gauss-Bonnet term - Inflation

- An action with a Gauss−Bonnet term:

$$S = \int d^4x\,\sqrt{-g}\left[\frac{1}{2\kappa^2}\,R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\; - V(\phi) - \frac{1}{2}\xi(\phi)R^2_{GB}\right]$$

Gauss-Bonnet term

$$R^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$$G_{\mu\nu}=\kappa^2\left(T_{\mu\nu} + T^{GB}_{\mu\nu}\right)$$

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi + V(\phi) - \frac{1}{2}g_{\mu\nu}\left(g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi + 2V\right)$$

$\kappa^2 = 8\pi G$ $\quad G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,R$

$$T^{GB}_{\mu\nu} = 4\left(\partial^\rho\partial^\sigma\xi R_{\mu\rho\nu\sigma} - \Box\xi R_{\mu\nu} + 2\partial_\rho\partial_{(\mu}\xi R^{\rho}_{\nu)} - \frac{1}{2}\partial_\mu\partial_\nu\xi R\right) - 2\left(2\partial_\rho\partial_\sigma\xi R^{\rho\sigma} - \Box\xi R\right)g_{\mu\nu}$$

$$\Box\phi - V_{,\phi}(\phi) - \frac{1}{2}T^{GB} = 0 \qquad T^{GB} = \xi_{,\phi}(\phi)R^2_{GB}$$

- FLRW Universe metric: $ds^2 = -\,dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right)$

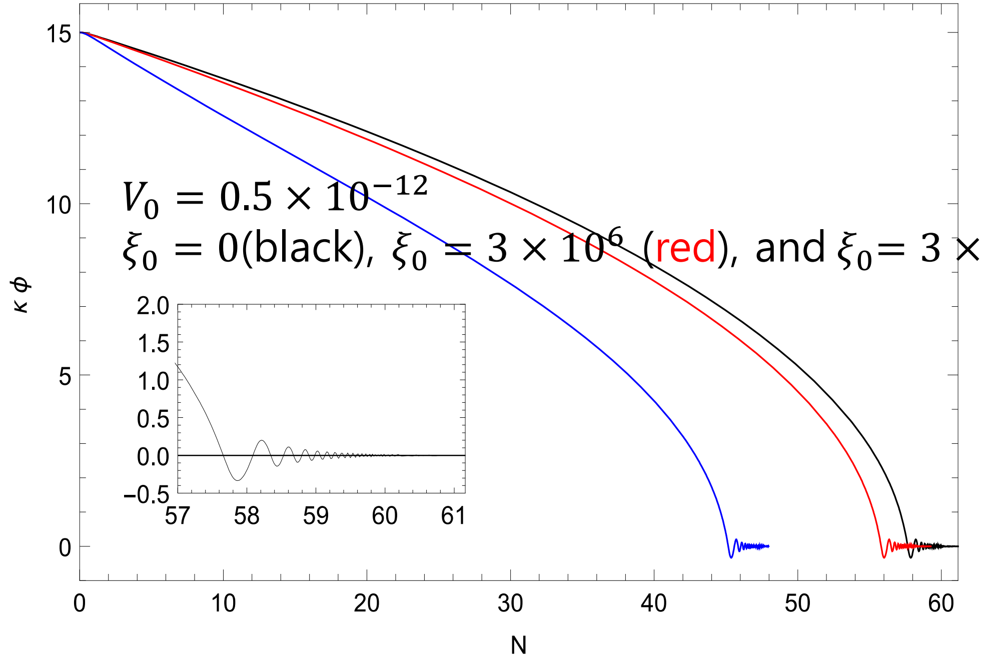
Einstein and Field equations yield:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot\phi^2 + V - \frac{3K}{\kappa^2a^2} + 12\dot\xi H\left(H^2 + \frac{K}{a^2}\right)\right)$$

$$\dot H = -\frac{\kappa^2}{2}\left(\dot\phi^2 - \frac{2K}{\kappa^2a^2} - 4\ddot\xi\left(H^2 + \frac{K}{a^2}\right) - 4\dot\xi H\left(2\dot H - H^2 - \frac{3K}{a^2}\right)\right)$$

$$\ddot\phi + 3H\dot\phi + V_{,\phi} + 12\xi_{,\phi}\left(H^2 + \frac{K}{a^2}\right)\left(\dot H + H^2\right) = 0$$

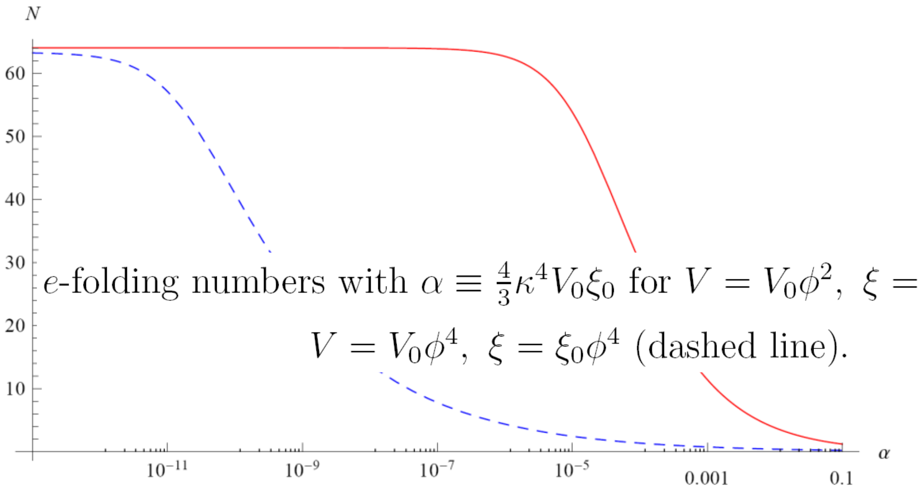
S. Koh, BHL, W. Lee, Tumurtushaa
PRD90 (2014) no.6, 063527



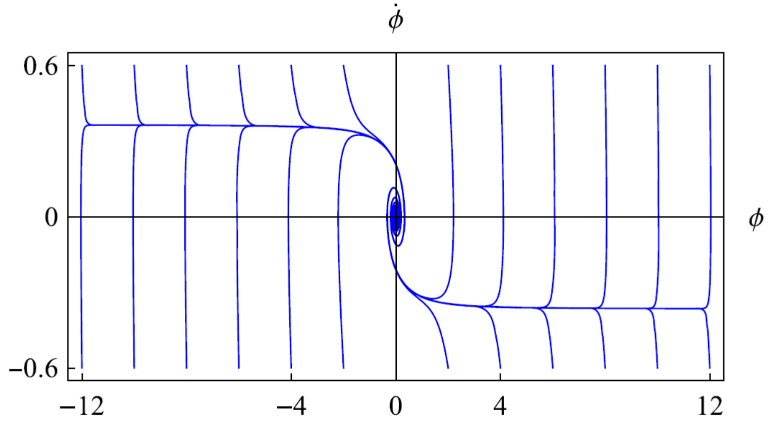
$V_0 = 0.5 \times 10^{-12}$

$\xi_0 = 0$(black), $\xi_0 = 3 \times 10^6$ (red), and $\xi_0 = 3 \times 10^7$ (blue).

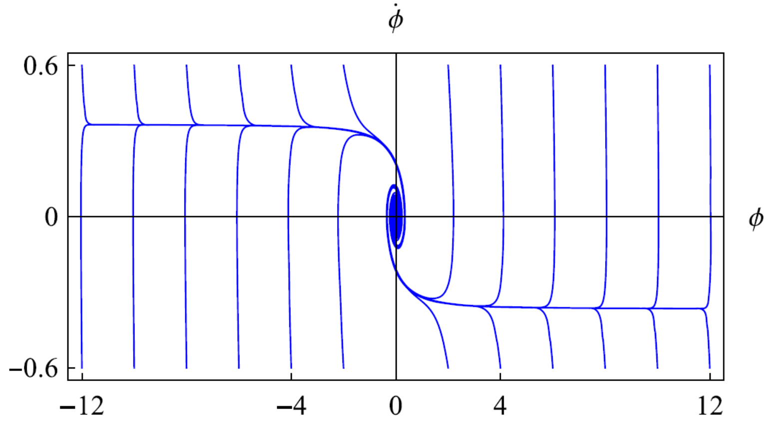
The duration of inflation gets shorter
as the Gauss-Bonnet coupling constant increases.
(making the effective potential steeper)



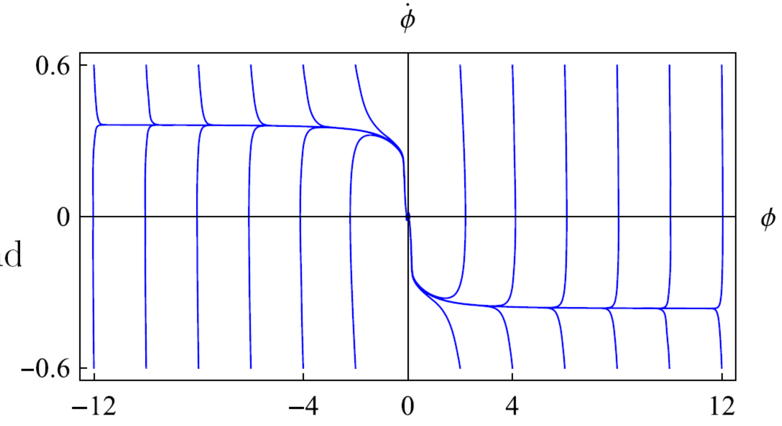
e-folding numbers with $\alpha \equiv \frac{4}{3}\kappa^4 V_0 \xi_0$ for $V = V_0\phi^2,\ \xi = \xi_0\phi^2$ (solid line) and $V = V_0\phi^4,\ \xi = \xi_0\phi^4$ (dashed line).



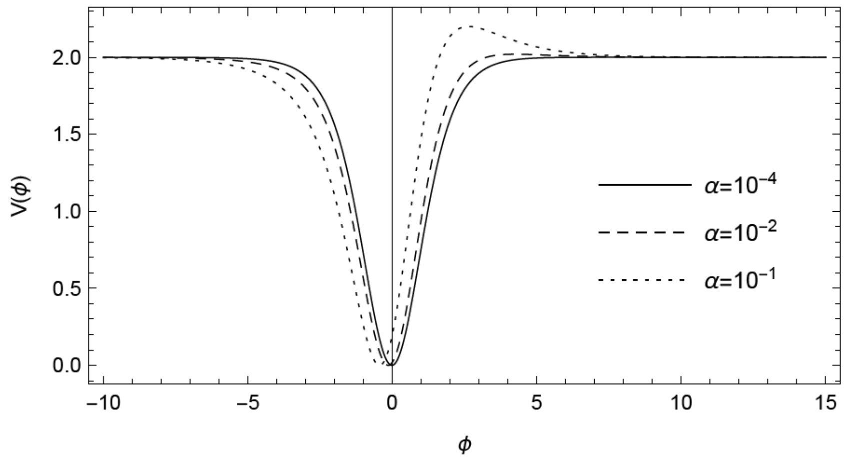
standard inflation without the Gauss-Bonnet term

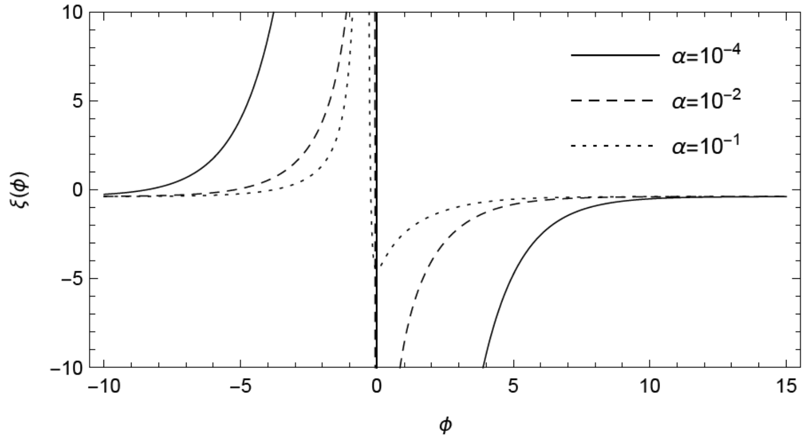


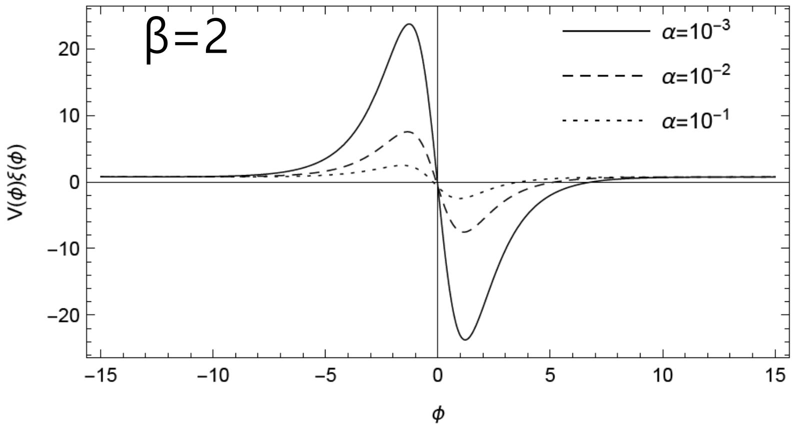
chaotic inflation with the monomial GaussBonnet coupling,



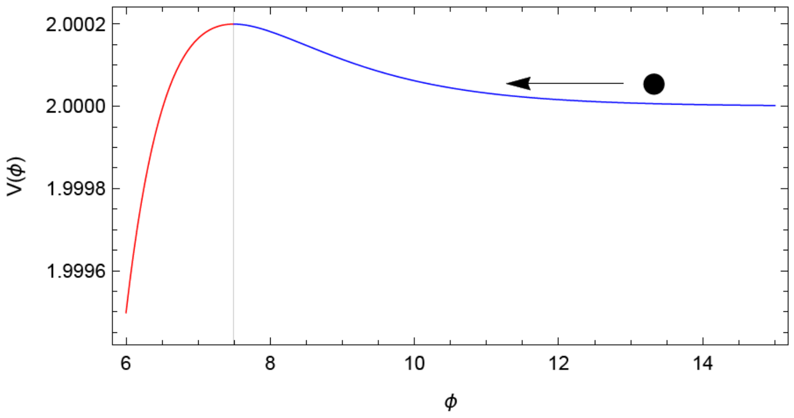
chaotic inflation with the inverse monomial Gauss-Bonnet coupling



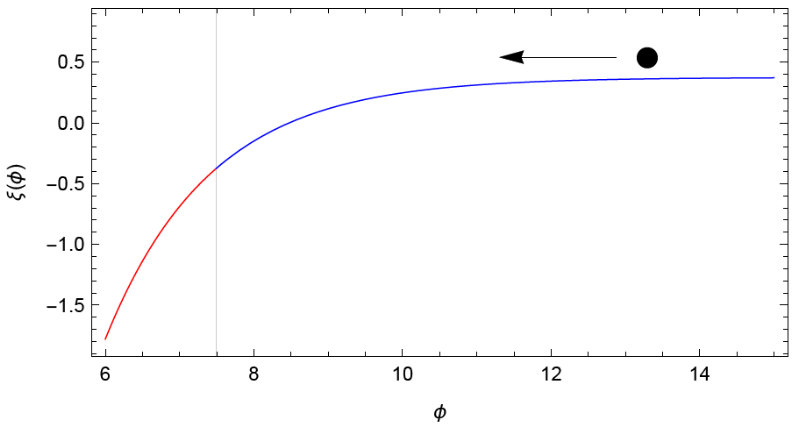




At the early stage, the Gauss-Bonnet coupling function ξ makes φ climb up the potential slope. At the late stage, φ rolls down as usual.



(a)



(b)

The blue-tilt of the spectrum for the tensor modes would be realized when the scalar field is initially released at large value $\varphi_0 > \varphi_*$.

If the scalar field is initially released otherwise $\varphi_0 < \varphi_*$, the spectrum would be red-tilted.

The spectrum is scale invariant at the boundary $\varphi_0 = \varphi_*$.

Reconstruction of the Scalar Field Potential in Inflationary Models with a Gauss-Bonnet term

S. Koh, BHL, G. Tumurtushaa
PRD95(2017)

We obtain the scalar field potential in terms of ns and r

V $\rightleftarrows$ n_s, r

$$V(N) = \frac{1}{8c_1} r(N) e^{-\int [n_s(N)-1]dN} \,,$$

and then we find ξ(N)

$$\xi(N) = \frac{3}{4\kappa^4}\left[\frac{1}{V(N)} + \int \frac{r(N)}{8V(N)} dN + c_2\right] \,,$$

the relation between the number of e-folding N and the scalar field

$$\int_{\phi_e}^{\phi} d\phi = \int \sqrt{\frac{r(N)}{8\kappa^2}} dN \,.$$

Example model

$$n_s(N) - 1 = -\frac{\beta}{N+\alpha},$$
$$r(N) = \frac{q}{N^p+\alpha}.$$

gives

$$\xi(N) = \frac{3}{4\kappa^4}\left[\left(\frac{8}{q}\frac{N^p+\alpha}{(N+\alpha)^\beta} + \frac{(N+\alpha)^{1-\beta}}{1-\beta}\right)c_1 + c_2\right],$$

$$V(N) = \frac{q}{8c_1}\frac{(N+\alpha)^\beta}{N^p+\alpha}.$$

$$\phi - \phi_e = N\sqrt{\frac{q}{8\kappa^2\alpha}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{p};1+\frac{1}{p};-\frac{N^p}{\alpha}\right),$$

the case of p=2

$$V(\phi) = \frac{q}{8c_1\alpha}\text{sech}^2\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\left[\alpha + \sqrt{\alpha}\sinh\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\right]^\beta,$$

$$\xi(\phi) = \frac{3}{4\kappa^4}\left[\frac{q\left(\alpha+\sqrt{\alpha}\sinh\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\right) + 8(1-\beta)\alpha\cosh^2\left(\sqrt{\frac{8}{q}}\kappa\phi\right)}{q(1-\beta)\left(\alpha+\sqrt{\alpha}\sinh\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\right)^\beta}c_1 + c_2\right],$$

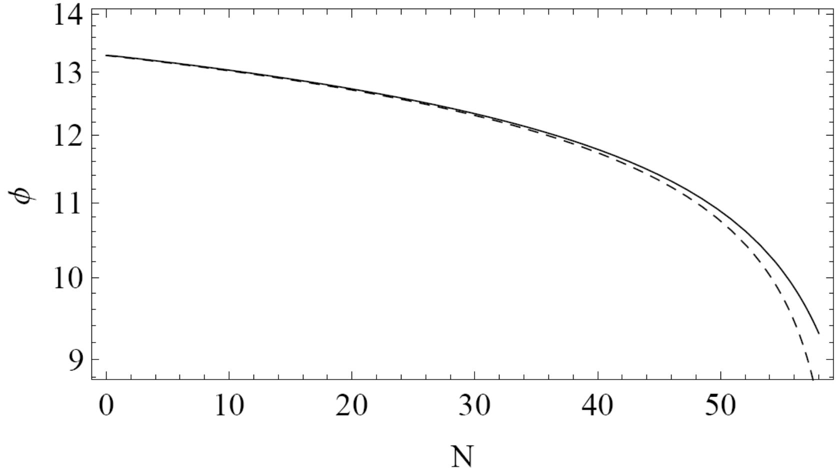
In α→0 limit,

$$V(\phi) \sim \tanh^2\left(\sqrt{\frac{8}{\alpha}}\kappa\phi\right),$$
$$\xi(\phi) \sim -\frac{3c_1}{4\sqrt{\alpha}\kappa^4}\text{csch}\left(\sqrt{\frac{8\kappa^2}{q}}\phi\right)$$

$$N = \sqrt{\alpha}\sinh\left(\sqrt{\frac{8}{q}}\kappa\phi\right),$$

or

$$\phi(N) = -\sqrt{\frac{q}{8\kappa^2}}\text{arcsinh}\left(\frac{N}{\sqrt{\alpha}} - \sqrt{\frac{8\kappa^2}{q}}C\right),$$

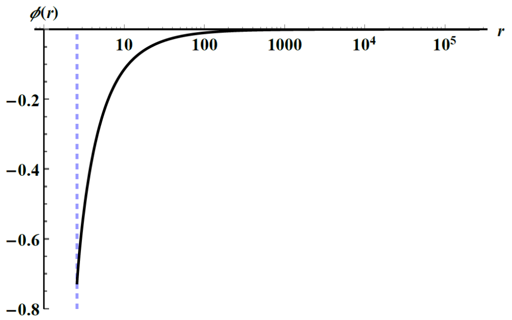


4.Summary

We have studied the Black Hole with Gauss-Bonnet term

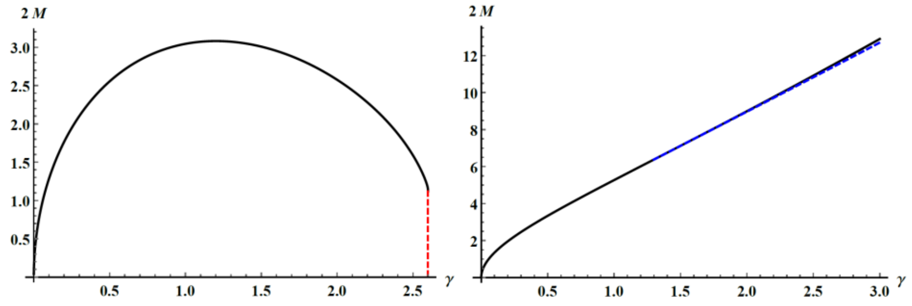
Numerically constructed the static DEGB **hairy** black hole in asymptotically flat spacetime (& asymptotically AdS).

- There exists **minimum mass**.
- BHs have hairs.



(b) $\phi(r)$ vs. r

When the scalar field on the horizon is the maximum, the DGB black hole solution has the minimum horizon size.

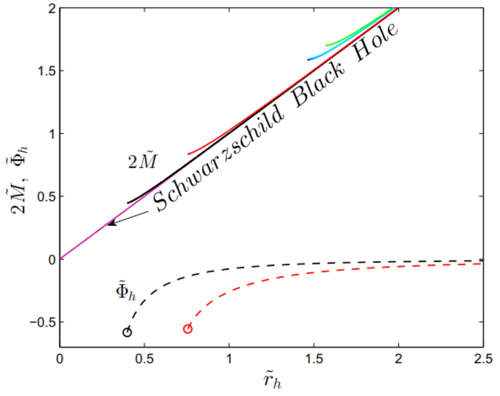


(a) The black hole mass vs. γ with $\alpha = -1$. (b) The black hole mass vs. γ with $\alpha = 1$.

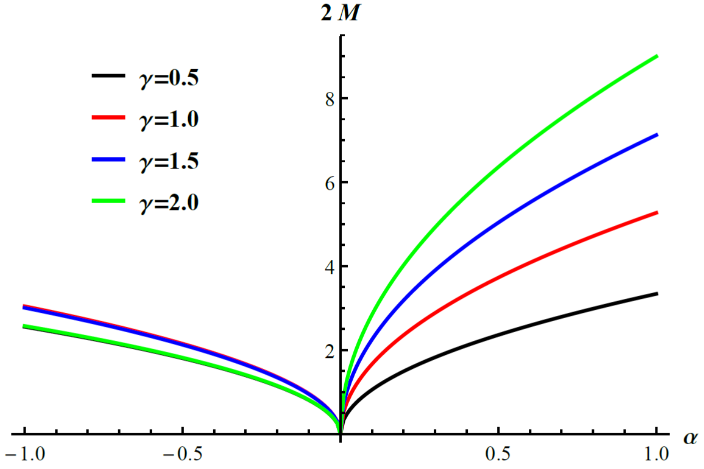
The BH solution & its properties are strongly dependent on the signature of the Gauss-Bonnet term

"more attractive" "less attractive"

If $\alpha > 0$,



the amount of black hole hair decreases as the DGB black hole mass increases. DGB black hole configurations go to EGB black hole cases for small α and γ.

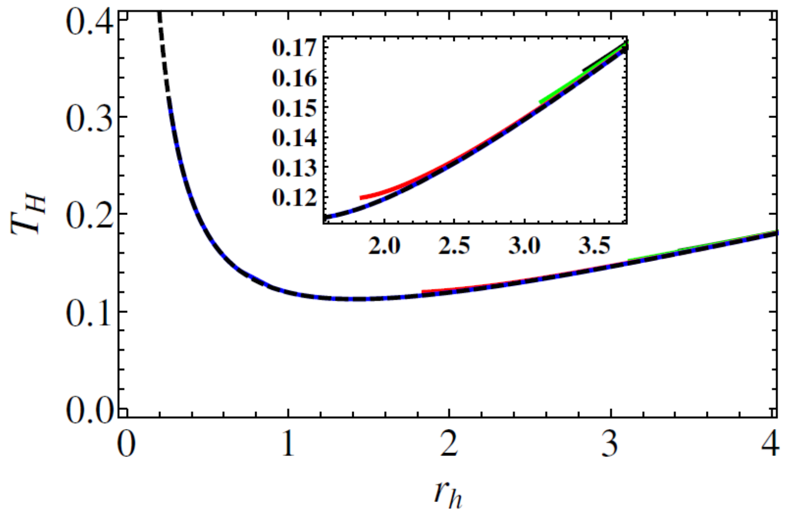


Summary - continued

- $\Lambda<0$ with Gauss-Bonnet term : Phases

There exists the minimum mass of a black hole.

If the black hole horizon r_h becomes larger, the magnitude of the scalar hair becomes smaller.

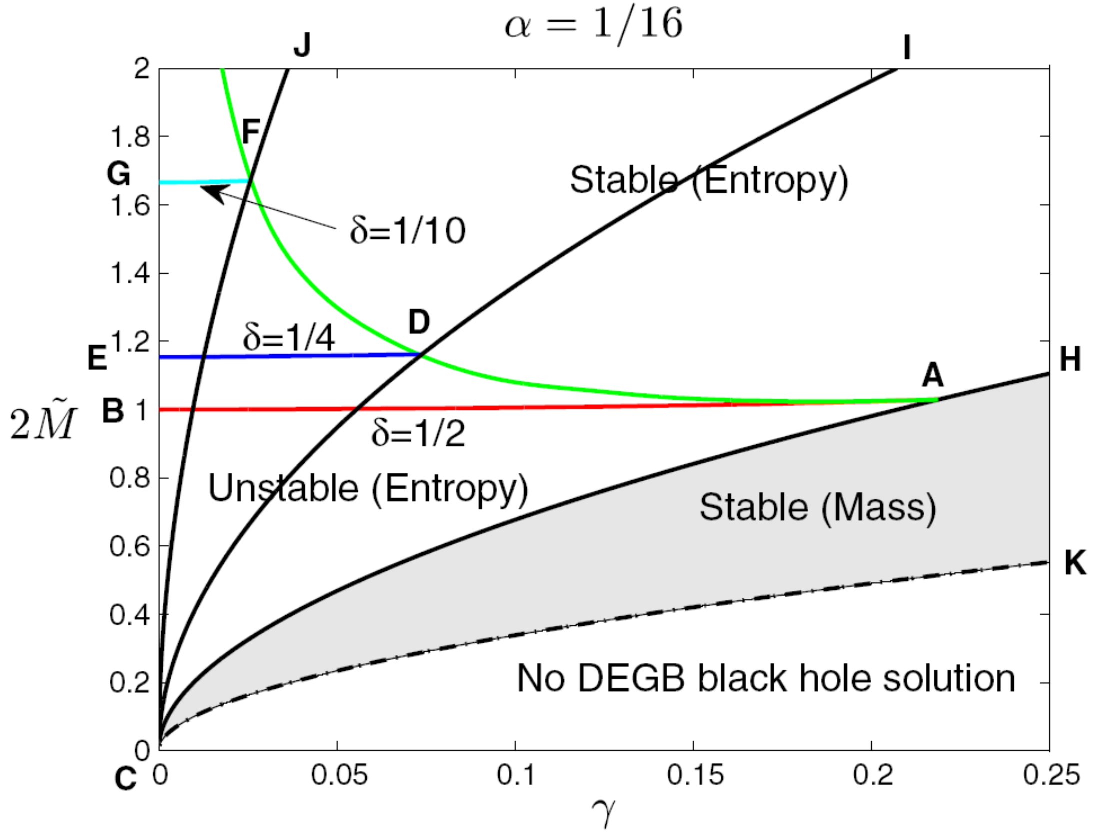


- (in)stability of black holes ($\Lambda=0$) :

DGB black holes (like most of the 4-dim. BHs) are perturbatively stable.

- Fragmentation instability of black holes:

There exists region unstable under fragmention.



Summary - continued

Cosmological implications

- GB term makes the e-folding smaller.
- For monomial potential and monomial coupling to GB term,
  r is enhanced for $\alpha > 0$ while it is suppressed for $\alpha < 0$.
- N≈60 condition requires that $\alpha \approx 10^{-6}$ for $V \sim \varphi^2$ $\alpha \approx 10^{-12}$ for $V \sim \varphi^4$.

- The model parameter must take values in interval

  $2.1276 \times 10^6 \le \xi_0 \le 3.7796 \times 10^6$

  to be consistent with accuracy of future observation in which

  $n_s = 0.9682 \pm 10^3$.

Summary - continued

The blue-tilt of the spectrum for the tensor modes would be realized when the scalar field is initially released at large value $\varphi_0 > \varphi_*$.

We showed the possibility of observational data be used to reconstruct the potential restricting viable models.

To summarize,

- we have studied the role of the Gauss-Bonnet term in the gravity theory through
  the Black Hole properties
  and the Cosmological implications.

- Gravity theory with Gauss-Bonnet term and the negative cosmological term may play some role via holography principle.
  Could provide the geometry towards more realistic holographic model.

Thank you!