# Black holes in holographic quantum gravity

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# Black hole thermodynamics & AdS

- "Thermodynamic" behavior of black holes is one of the mysteries of GR.
- One can ask various interesting questions:

- precise statistical origin: entropy, Hawking radiation, phase transitions, …
- information loss & recovery, scrambling, chaos, complexity, …

- However, general thermodynamics of quantum gravity couldn't be studied well:

- Does not know how to define quantum gravity at general (especially high) temperature.
- Other conceptual subtleties to study full-fledged gravitational thermodynamics

- Some of these subtleties led Hawking & Page to study BH's in AdS spacetime.

- Global AdS is a gravitationally confining box.

$$ds^2_{D+1} = d\rho^2 - \cosh^2\frac{\rho}{\ell}\,dt^2 + \ell^2\sinh^2\frac{\rho}{\ell}\,ds^2(S^{D-1})$$

- Microscopic definition of quantum gravity, via CFT at the boundary $S^{D-1} \times R$: "AdS/CFT"
- Full-fledged study of BH thermodynamics is in principle possible.

# High temperature limit of CFT

- This leads us to study the thermodynamics of CFT on $S^{D-1} \times R$.
- It is easier to extract the physics at high temperature: This will be today's topic.

\- Euclidean CFT on $S_r^{D-1} \times S_\beta^1$, at $\beta \equiv T^{-1} \ll r^{-1}$.

- Kaluza-Klein (KK) modes on small $S^1$:

\- Divergent contributions to $\log Z$ come from these KK modes at $\beta \to 0$.

\- Encoded in the effective action of background fields (chemical potentials) on $S^{D-1}$.

\- Universal background fields: metric, gravi-photon & dilaton:

$$ds_D^2 = ds_{D-1}^2 + e^{-2\Phi}(d\tau + a)^2 \qquad \tau \sim \tau + \beta \ , \quad \beta e^{-\Phi} \sim \text{radius of } S^1$$

\- E.g. on $S^3$ from temperature $\beta^{-1}$ and angular velocities $\omega_{1,2}/\beta$ :

$$ds^2 = r^2 \left[ d\theta^2 + \sum_{i=1}^{2} n_i^2 \left( d\phi_i - \frac{i\omega_i}{\beta} d\tau \right)^2 \right] + d\tau^2 = r^2 \left[ d\theta^2 + \sum_i n_i^2 d\phi_i^2 + \frac{r^2 (\sum_i \omega_i n_i^2 d\phi_i)^2}{\beta^2 (1 - r^2 \sum_i \frac{n_i^2 \omega_i^2}{\beta^2})} \right] + e^{-2\Phi} (d\tau + a)^2$$

$$e^{-2\Phi} = 1 - r^2 \sum_i \frac{n_i^2 \omega_i^2}{\beta^2} \ , \quad a = -i \frac{r^2 \sum_i \omega_i n_i^2 d\phi_i}{\beta (1 - r^2 \sum_i \frac{n_i^2 \omega_i^2}{\beta^2})} \qquad (n_1, n_2) = (\cos\theta, \sin\theta)$$

# Effective action & derivative expansion

- In CFT, small $\beta$ expansion = $r^{-1}$ expansion = derivative expansion on $S^{D-1}$.

- ∞-tower of derivative expansion. Most coefficients depend on coupling constants.

$$\frac{A_1}{\beta^3}\int d^3x e^{3\Phi}\sqrt{g} + \frac{A_2}{\beta}\int d^3x\sqrt{g}e^{\Phi}R + A_3\int d^3x\sqrt{g}(\beta e^{-\Phi})(\beta^{-1}da)^2 + \cdots$$

cosmological constant　　Einstein-Hilbert　　gravi-photon kinetic

- However, Chern-Simons terms on $S^{D-1}$ (for even $D$) are coupling independent.

- 't Hooft anomalies of global symmetries determine them. (More later)
- They are sub-leading terms in the normal thermal free energy.

$$k\beta^{-2}\int a\wedge da + k_I\beta^{-1}\int \mathcal{A}^I\wedge da + k_{IJ}\int \mathcal{A}^I\wedge d\mathcal{A}^J + \cdots$$

background gauge fields (~ chemical potential) for global symmetries

- CFT w/ AdS dual: The expansion constrains AdS black hole's thermodynamics.

# Kerr black holes in AdS$_5$

- Plug in our $S^3$ background fields to:

$$\frac{A_1}{\beta^3}\int d^3x e^{3\Phi}\sqrt{g} + \frac{A_2}{\beta}\int d^3x\sqrt{g}e^{\Phi}R + A_3\int d^3x\sqrt{g}(\beta e^{-\Phi})(\beta^{-1}da)^2 + \cdots$$

- Leading terms in derivative expansion. (Take $\Omega \equiv \omega_1 = \omega_2$ for simplicity.)

$$S = \log Z + \beta E + 2\Omega J = A_1\frac{\beta}{(\beta^2-\Omega^2)^2} + A_2\frac{3\beta^3 - 4\beta\Omega^2}{(\beta^2-\Omega^2)^2} + A_3\frac{\beta\Omega^2}{(\beta^2-\Omega^2)^2} + \beta E + 2\Omega J + \mathcal{O}(\beta^0)$$

- Extremize in $\beta, \Omega$ & solve for $E, J, S$. 2 more eqns.
- Expressions for $S, E, J$ in terms of $\beta, \Omega$, and the unknowns $A_i$.

- Fitting three $A_i$'s, one reproduces all divergent parts of $E, J, S$ carried by Kerr BH's. (Here, $\Omega = -a\beta + \cdots$.) Although 3 parameters are fitted by hand, still quite nontrivial constraints.

$$E = \frac{\pi l^2}{G}\left(\frac{3+a^2}{8(1-a^2)^3}\frac{l^4\pi^4}{\beta^4} - \frac{3+3a^2-2a^4}{8(1-a^2)^3}\frac{l^2\pi^2}{\beta^2} - \frac{3-11a^2+6a^4-2a^6}{16(1-a^2)^3} + \mathcal{O}(\beta^2)\right)$$

$$J = \frac{\pi l^2}{G}\left(\frac{a}{4(1-a^2)^3}\frac{l^4\pi^4}{\beta^4} - \frac{2a-a^3}{4(1-a^2)^3}\frac{l^2\pi^2}{\beta^2} + \frac{a}{8(1-a^2)^3} + \mathcal{O}(\beta^2)\right)$$

$$S = \frac{\pi^2 l^3}{G}\left(\frac{1}{2(1-a^2)}\frac{l^3\pi^3}{\beta^3} - \frac{3-2a^2}{4(1-a^2)^2}\frac{l\pi}{\beta} + \mathcal{O}(\beta)\right)$$

# High T expansion of indices

- AdS/CFT models w/ SUSY: "BPS states" w/ coupling-independent spectrum.
- "Witten index": coupling independent partition function. For 4d SCFTs on $S^3 \times S^1$,

$$Z(\omega_1, \omega_2) \equiv \mathrm{Tr}\left[e^{-\beta(E-\frac{3}{2}R-J_1-J_2)}e^{-\frac{1}{2}\Delta R}e^{-\omega_1 J_1 - \omega_2 J_2}\right]$$

$$\Delta = \omega_1 + \omega_2 + 2\pi i \qquad E - \frac{3}{2}R - J_1 - J_2 \sim \{\mathcal{Q}, \mathcal{Q}^\dagger\}$$

- On $S^3$, coupling-independent CS terms determine the "high T" or large charge asymptotics.

- $\beta$ is merely a regulator of the index. Easy to compute at $\beta \to 0^+$.
- Since the leading term is $O(\beta^0)$, $\beta \to 0$ is a fake thermal circle parameter.
- True derivative expansion parameters are $|\omega_i| \ll 1$.

$$e^{-2\Phi} = 1 + \boxed{r^2 \sum_i \frac{n_i^2 \omega_i^2}{\beta^2}} \; , \quad a = -i \frac{r^2 \sum_i \omega_i n_i^2 d\phi_i}{\beta(1 - r^2 \sum_i \frac{n_i^2 \omega_i^2}{\beta^2})}$$

- Background fields: same metric, dilaton, gravi-photon (up to small shifts of $\omega_i$ by $\beta$)
- Background $U(1)_R$ & other flavor symmetries' gauge fields:

$$A^I = -\frac{i\Delta_I}{\beta} d\tau \xrightarrow{\text{rearrange to 3d fields}} A^I = A_4^I(d\tau + a) + \mathcal{A}^I \qquad A_4^I = -\frac{i\Delta^I}{\beta} \qquad \mathcal{A}^I = -A_4^I a$$

# CS coefficients from anomalies

- Knowing these CS coefficients, one can determine the leading free energy.
- There turn out to be two types of CS terms.

- Gauge non-invariant CS terms

- 4d effective action $S_{\text{eff}} = -\log Z$ respects 't Hooft anomaly: $\delta_\epsilon S_{\text{eff}} \sim \epsilon\, F \wedge F + \ldots$
- This should be reflected in the 3d background fields' effective action.
- It demands the existence of certain gauge non-invariant CS terms.

[Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla, Sharma] (2012)

$$S_{\text{non-inv}} = -i\frac{\beta(5a-3c)}{8\pi^2}\int_{S^3} C_{IJK}\left(A_4^I \mathcal{A}^J \wedge d\mathcal{A}^K + A_4^I A_4^J \mathcal{A}^K \wedge da + \frac{1}{3}A_4^I A_4^J A_4^K a \wedge da\right)$$

- Gauge invariant CS terms: More elaborate arguments by [Jensen, Loganayagam, Yarom] (2013) determine them all from anomalies. (See also [Di Pietro, Komargodski] (2014).)

$$S_{\text{inv}} \sim -i(a-c)\int_{S^3} k_I \mathcal{A}^I \wedge da$$

[$C_{IJK}$, $k_I$ are coefficients of the cubic anomaly polynomial in a suitable normalization.]

# Free energy & BH's

- Plugging in our background fields, one obtains the leading free energy.

$$\log Z \sim (5a-3c)\frac{C_{IJK}\Delta^I\Delta^J\Delta^K}{6\omega_1\omega_2}+\frac{4\pi^2(a-c)k_I\Delta^I}{\omega_1\omega_2}$$

- If one only turns on the chemical potential for the U(1) superconformal R-symmetry, one obtains a universal formula in terms of two central charges.

$$\log Z \sim -\frac{16i}{27}\frac{3c-2a}{\omega_1\omega_2} \qquad C_{111}=6\ ,\quad k_1=1\ ,\quad \Delta^1=\frac{2\pi i}{3}$$

- It is known that $3c-2a>0$ is always met by interacting SCFTs. [Hofman, Maldacena]
- Its Legendre transformation is subtle. The proper interpretations established only recently.
- As a result, at large $J_1, J_2 \gg a, c$, one obtains a macroscopic entropy.

$$S=\sqrt{3}\left[2(3c-2a)(J_1+\tfrac{R}{2})(J_2+\tfrac{R}{2})\right]^{\frac{1}{3}}$$

- This entropy precisely agrees with the Bekenstein-Hawking entropy of BPS black holes in $AdS_5$, accounting for their microstates from CFT.

# Conclusion & remarks

- Implications to CFT:

- What I explained generalize the Cardy formula of 2d CFT, to all even dimensional SCFTs.

$$Z(\tau) \sim \mathrm{Tr}\left[e^{2\pi i \tau L_0}\right] \sim \exp\left[\frac{\pi i c}{12\tau}\right] \quad \text{at } \tau \to i0^+$$

- 2d conformal symmetry is very strong. Universal high T asymptotics.
- For SCFTs, continues to find universal asymptotics in terms of $3c - 2a$.

- Implications to black hole physics:

- Black holes in AdS from dual CFT: deconfined quark-gluon plasma [Witten] (1998)
- However, only recently we have been able to confirm and study this quantitatively.
- Today, I explained to you the simplest route to understand the recent developments.

- Today's talk is only a small part of the recent advances in BPS AdS black holes.

- We counted the microstates of BPS $AdS_{D+1}$ black holes from $SCFT_D$ Cardy indices for all $D = 3,4,5,6$. [Choi, J. Kim, SK, Nahmgoong] [Choi, SK] [J. Kim, SK, Song] [Nahmgoong] [Choi, Hwang, SK] [Choi, Hwang] ……