DeepMind

Deep Dive on Graph Networks for Learning Simulation

4th Inter-experiment Machine Learning Workshop
22 October 2020
Alvaro Sanchez-Gonzalez - DeepMind

Victor Bapst, Peter Battaglia, Kyle Cranmer, Miles Cranmer, Meire Fortunato, Jonathan Godwin, Jessica Hamrick, Shirley Ho, Jure Leskovec, Tobias Pfaff, Rex Ying,



Simulation is fundamental to science and technology

<u>Largest supercomputers in the world (Nov 2019)</u>

#1. "Summit" @ Oak Ridge: "A Sneak Peek at 19 Science Simulations for the Summit Supercomputer in 2019"

1.	Evol	lution	of the	universe

- 2. Whole-cell simulation
- Inside a nuclear reactor
- 4. Post-Moore's Law graphene circuits
- 5. Formation of matter
- 6. Cell's molecular machine
- 7. Unpacking the nucleus
- 8. Mars landing
- 9. Deep learning for microscopy
- 10. Elements from star explosions

- 11. Cancer data
- 12. Earthquake resilience for cities
- 13. Nature of elusive neutrinos
- 14. Extreme weather with deep learning
- 15. Flexible, lightweight solar cells
- 16. Virtual fusion reactor
- 17. Unpredictable material properties
- 18. Genetic clues in the opioid crisis
- 19. Turbulent environments



Simulation is fundamental to science and technology

<u>Largest supercomputers in the world (Nov 2019)</u>

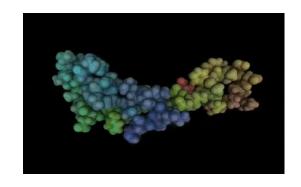
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#1. "Summit" @ Oak Ridge: "A Sneak Peek at 19 Science Simulations for the Summit Supercomputer in 2019"
#2 "Sierra" @ Lawrence Livermore: "[nuclear] simulation in lieu of underground testing"
#3 "Sunway TaihuLight" @ NSC, Wuxi: "simulated the Universe with 10 trillion digital particles"
#4 "Tianhe-2A" @ NSC, Guangzhou: "main application ... is for computational fluid dynamics (CFD) ... aircraft simulations"
#5 "Frontera" @ TACC: "high-resolution climate simulations, molecular dynamics models with millions of atoms"
#6 "Piz Daint" @ CSCS: "simulate processes for projects in geophysics, materials science, chemistry, ... climate modeling"
#7 "Trinity" @ Los Alamos: "A trillion-particle simulation? No sweat for the Trinity supercomputer at Los Alamos"
(#8 "ABCI" @ AIST, Japan: not simulation, but deep learning)
#9 "SuperMUC-NG" @ Leibniz Supercomputing Centre: "Researchers Visualize the Largest Turbulence Simulation Ever"
#10 "Lassen" @ Lawrence Livermore: "The system is designated for unclassified simulation and analysis"
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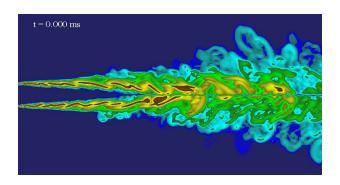


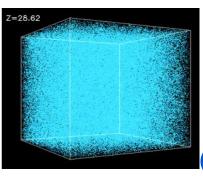
Why *learn* simulation?

Engineered simulators:

- 1. Substantial effort to build
- 2. Substantial resources to run
- 3. Only as accurate as the designer
- Not always suitable for solving inverse problems





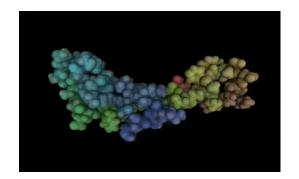


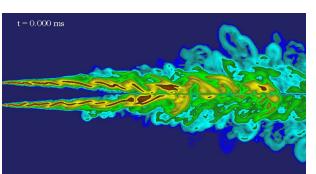


Why learn simulation?

Engineered simulators:

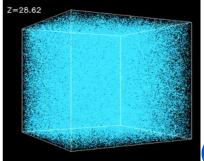
- Substantial effort to build
- 2. Substantial resources to run
- 3. Only as accurate as the designer
- Not always suitable for solving inverse problems





Learned simulators:

- 1. Shared architectures
- 2. Accuracy-efficiency trade off
 - 3. As accurate as the available data
 - 4. Gradient-based planning
 - 5. Interpretable models!*



Graph Networks for Learning Physical Simulation

Deep Dive on our most recent models:

"Learning to Simulate Complex Physics with Graph Networks" (ICML 2020)

Alvaro Sanchez-Gonzalez*, Jonathan Godwin*, Tobias Pfaff*, et al.

Arxiv: arxiv.org/abs/2002.09405

Video page: <u>sites.google.com/view/learning-to-simulate</u>

"Learning Mesh-Based Simulation with Graph Networks" (arXiv, under review)

Tobias Pfaff*, Meire Fortunato*, Alvaro Sanchez-Gonzalez*, Peter Battaglia

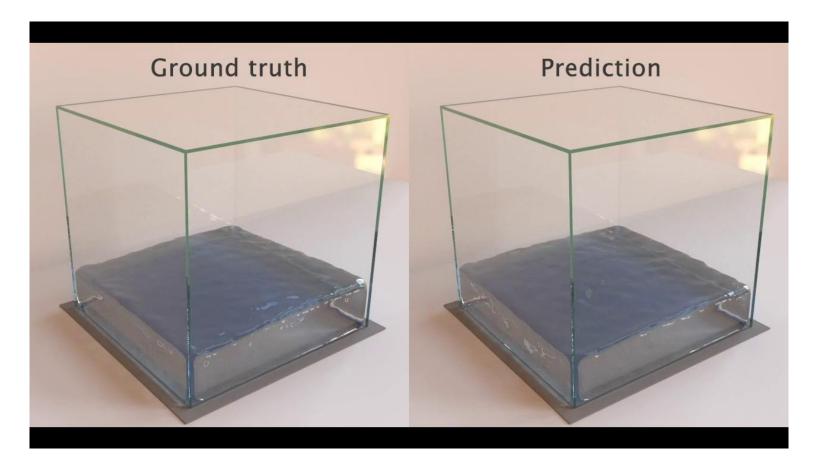
Arxiv: arxiv.org/abs/2010.03409

Video page: sites.google.com/view/meshgraphnets

- Focus on general principles behind the design
 - Applicable to other domains

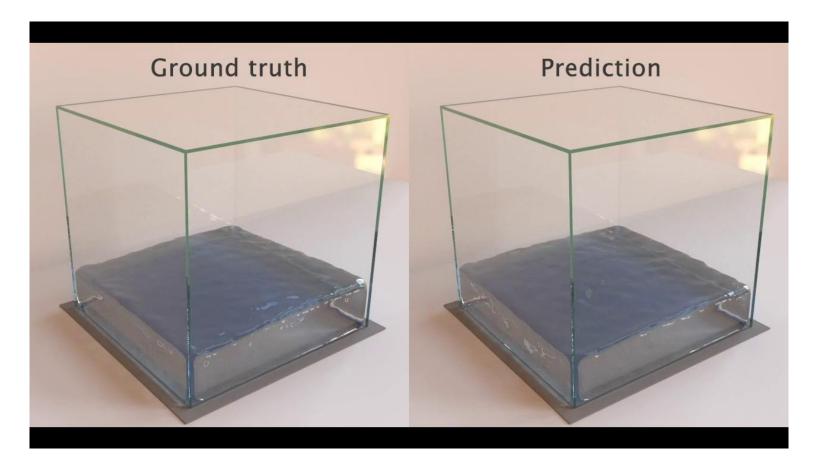


Water simulation (SPH)





Water simulation (SPH)





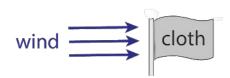
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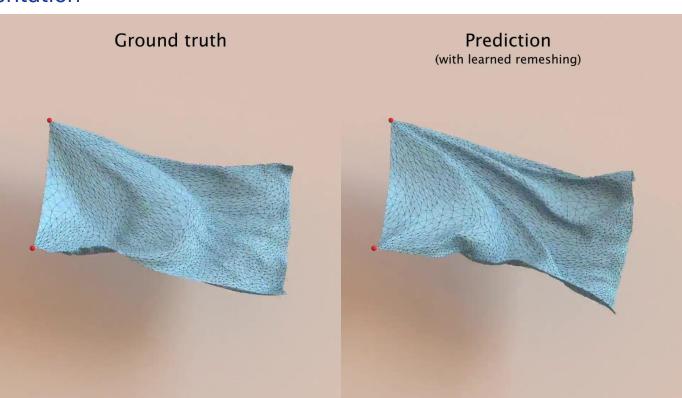
Multiple materials



Cloth simulation (ArcSim)

- Triangular dynamic mesh
- Lagrangian representation

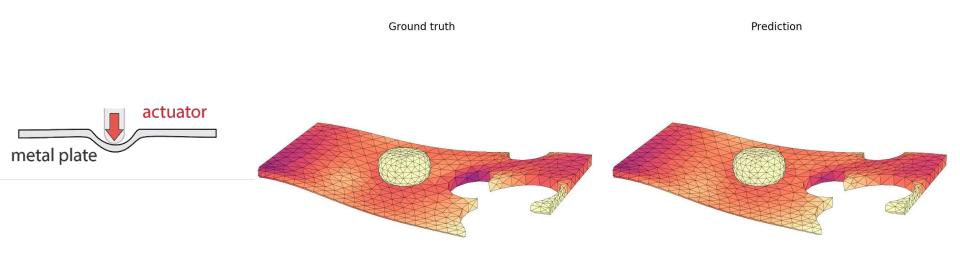




Video page: <u>sites.google.com/view/meshgraphnets</u>

Structural dynamics (COMSOL)

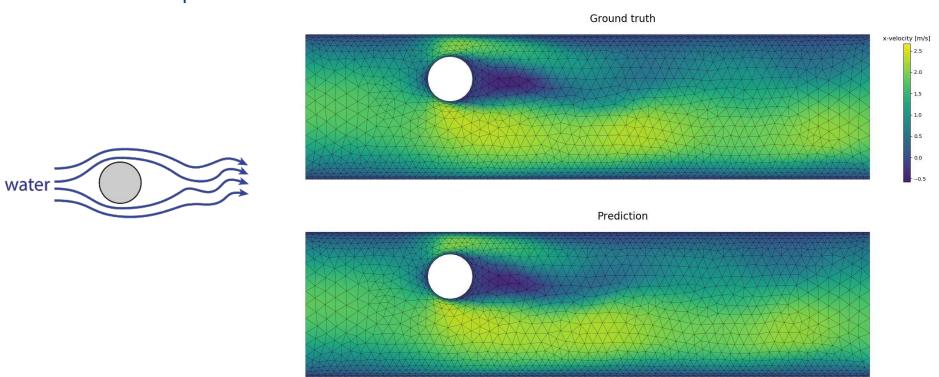
- Tetrahedral mesh
- Lagrangian representation
- Quasi-static simulation



Video page: sites.google.com/view/meshgraphnets

Incompressible fluids (COMSOL)

- Navier-Stokes
- Eulerian representation



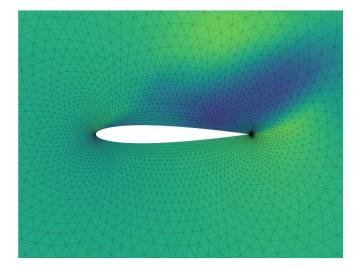
Video page: sites.google.com/view/meshgraphnets

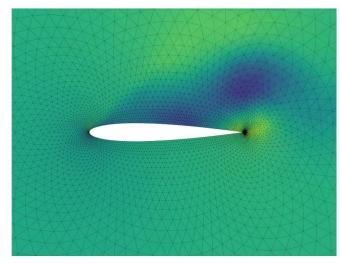
Aerodynamics (SU2)

- Navier-Stokes
- Eulerian representation

Ground truth Prediction

mach number 0.58 angle of attack 21.9







airfoil

x-velocity [m/s]

Why Graph Network based simulators?

- Adaptability
 - Same model → Vastly different materials and domains

- Data efficiency
 - < 1000 training trajectories</p>

- Performance
 - Latest model: ~10 to 100 times faster than ground truth simulator

Generalization



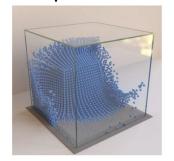
Generalization to more time-steps

Video page: sites.google.com/view/meshgraphnets

Train time

Pairs of states

Input state

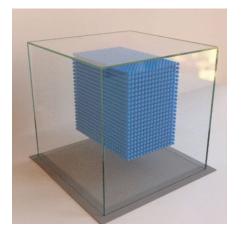


Target state



Test time

1000s of steps





Generalization to many more particles

Training

1 x 1 domain 2k particles 600 steps





Generalization to many more particles

Training

1 x 1 domain 2k particles 600 steps



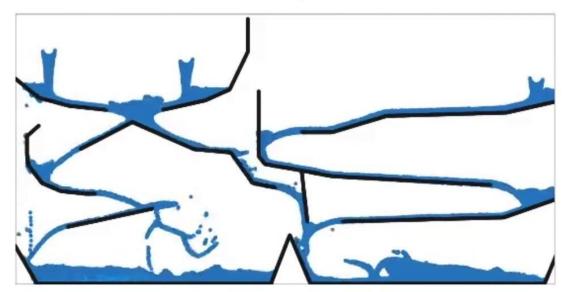
Generalization

2 x 2 domain 28k particles 2500 steps



8 x 4 domain 85k particles 5000 steps

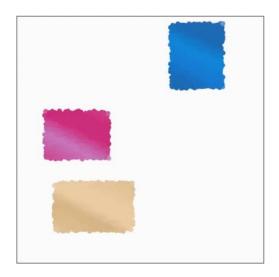






Generalization to initial conditions

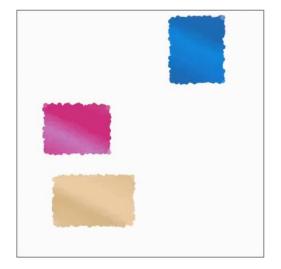
Training



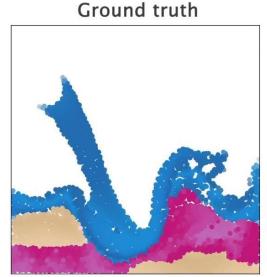


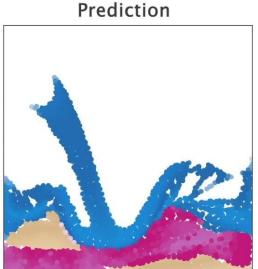
Generalization to initial conditions

Training



Generalization



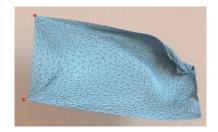


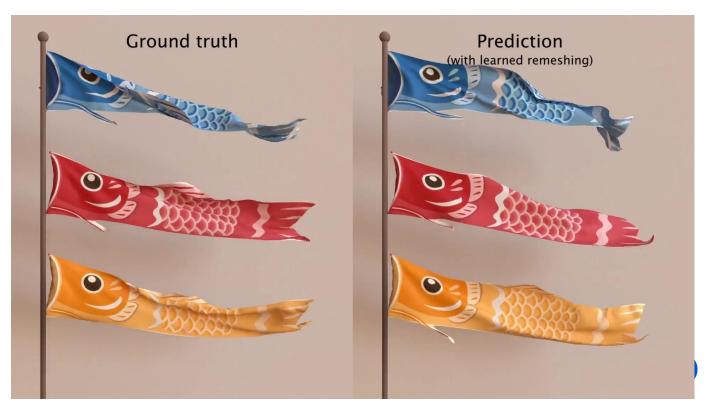
slow motion

Generalization to different meshes

Generalization

Training

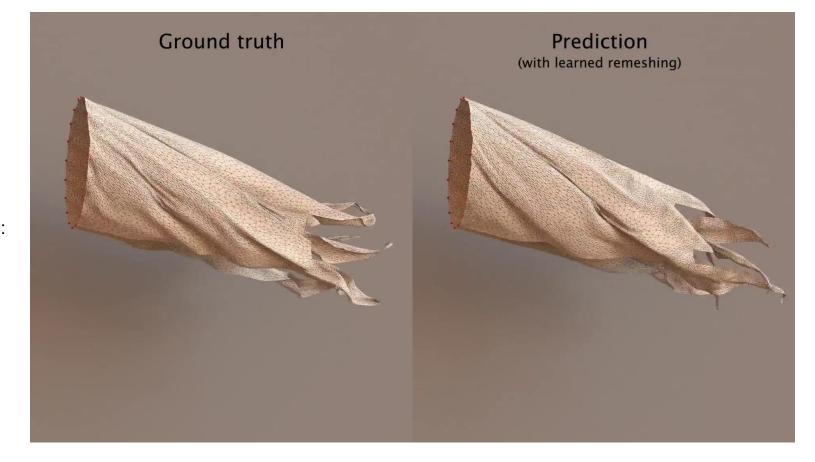




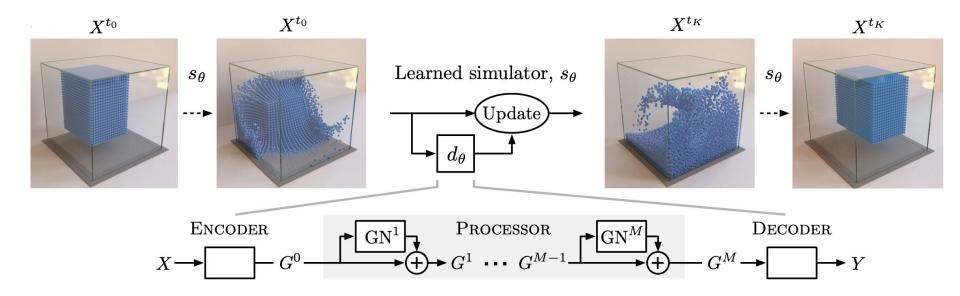
Generalization to larger meshes

Training: 2k nodes

Generalization: >20k nodes



Graph Network-Based Simulators



Main design principle: neural networks are dumb, let's make their life easy



"If I have seen further it is by standing on the shoulders of giants."

-Sir Isaac Newton-

Our Neural Networks should also *have* the knowledge of giants!

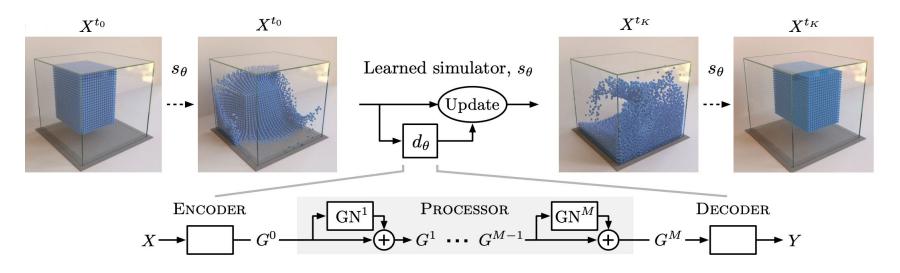


Inductive Biases

Physics-inspired inductive biases

"An **inductive bias** allows a learning algorithm to prioritize one solution (or interpretation) over another."

Mitchell, T. M.. The need for biases in learning generalizations. (1980)



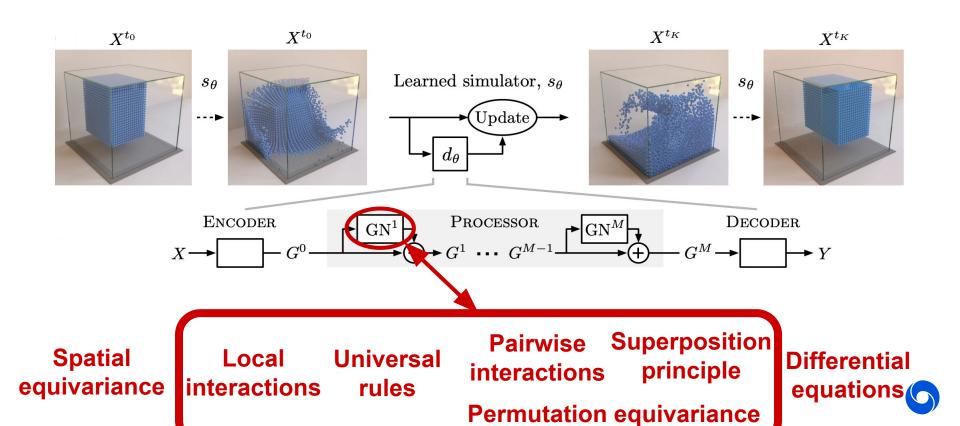
Spatial Local Universal equivariance interactions rules

Pairwise Superposition interactions principle

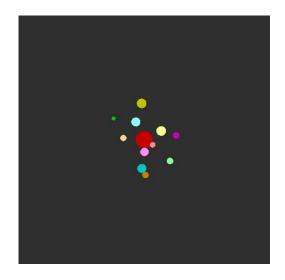
Permutation equivariance

Differential equations

Physics-inspired inductive biases



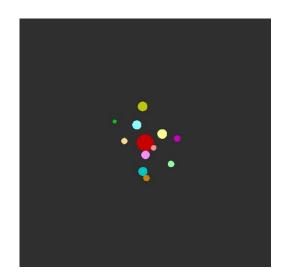
• MLPs operate over vectors

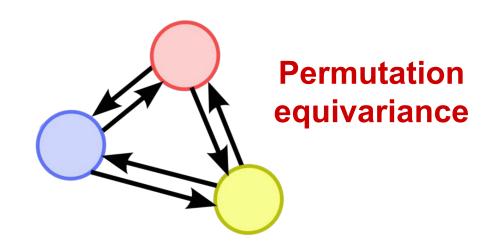






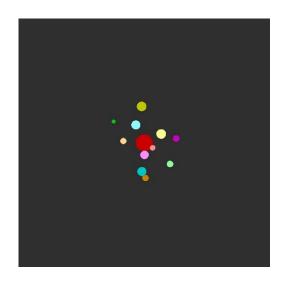
Neural networks that operate over graphs

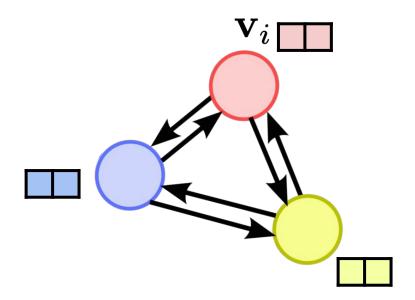






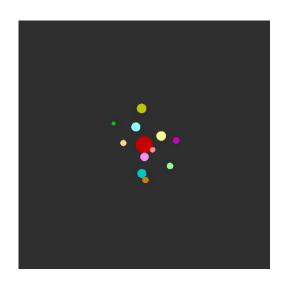
- Neural networks that operate over graphs
 - Node features

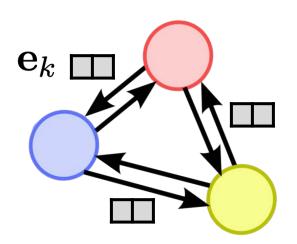






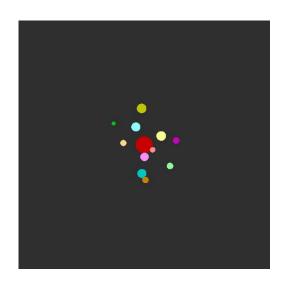
- Neural networks that operate over graphs
 - Edge features

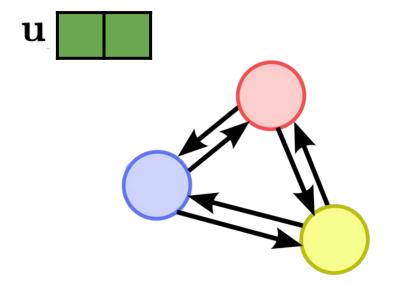






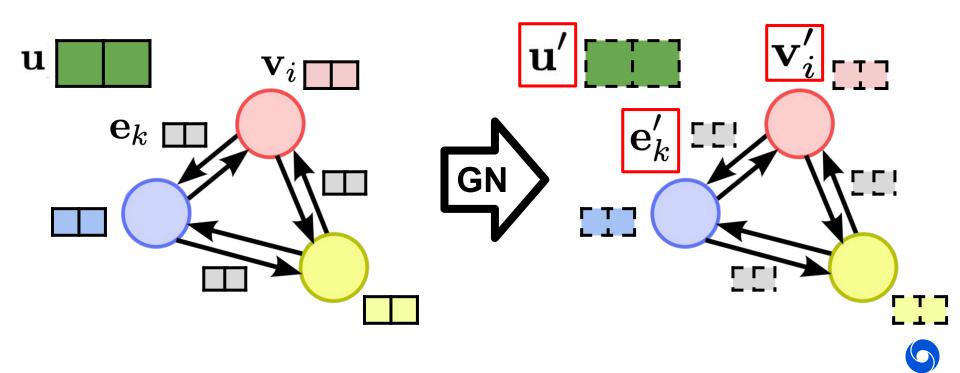
- Neural networks that operate over graphs
 - Global features







Update edge, node and global embeddings



Graph Networks (Battaglia et al., 2016, arXiV)

Edge (message) function (for every edge)

Edge features and e features ciobal features ciobal features $\phi^e\left(\mathbf{e}_k,\mathbf{v}_{r_k},\mathbf{v}_{s_k},\mathbf{u}
ight)\coloneqq \mathrm{NN}_e$ (

Pairwise interactions Universal rules



Receiver edge aggregation (Message pooling) (for every node)

$$\bar{\mathbf{e}}_i' \leftarrow \sum_{r_k=i} \mathbf{e}_k'$$

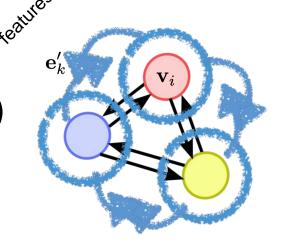
Superposition principle

Node function (for every node)

$$\mathbf{v}_i' \leftarrow \phi^v\left(\mathbf{ar{e}}_i', \mathbf{v}_i, \mathbf{u}\right) \coloneqq \mathrm{NN}_v\left(\mathbf{ar{e}}_i', \mathbf{v}_i, \mathbf{u}\right)$$

Local interactions

Universal rules



Graph Networks (Battaglia et al., 2016, arXiV)

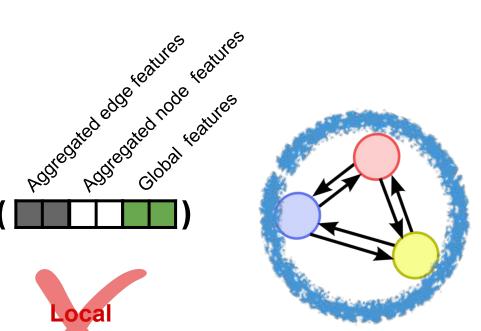
Global node and edge aggregation

$$\bar{\mathbf{v}}' \leftarrow \sum_i \mathbf{v}_i' \quad \bar{\mathbf{e}}' \leftarrow \sum_k \mathbf{e}_k'$$

Global function

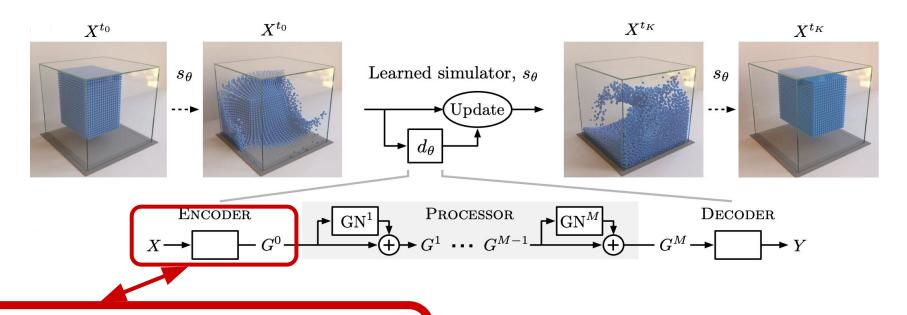
$$\mathbf{u}' \leftarrow \phi^u\left(\mathbf{ar{e}}', \mathbf{ar{v}}', \mathbf{u}
ight) \coloneqq \mathrm{NN}_u$$
 ($\mathbf{ar{v}}'$







Physics-inspired inductive biases



Spatial Local Universal equivariance interactions rules

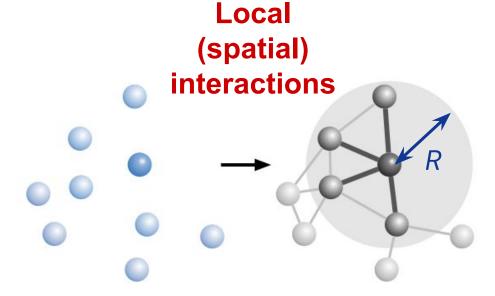
Pairwise Superposition interactions principle Permutation equivariance



Encoder

ENCODER $X \longrightarrow \boxed{ G^0}$

- Transform the inputs into a graph
 - Add connectivity with a certain radius R

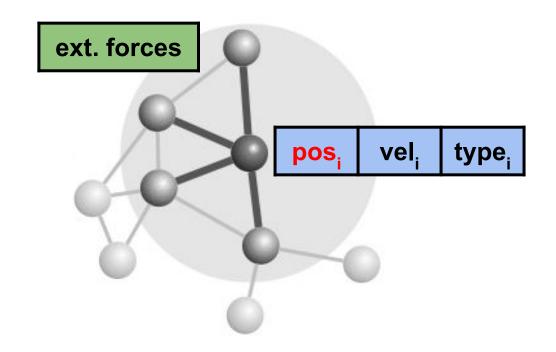




Encoder

ENCODER $X \longrightarrow \boxed{ G^0}$

Add features to the graph (naive approach)

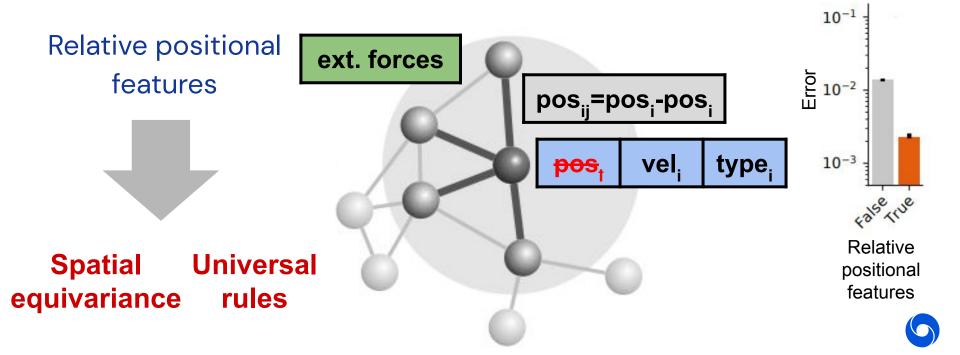




Encoder

ENCODER $X \longrightarrow G^0$

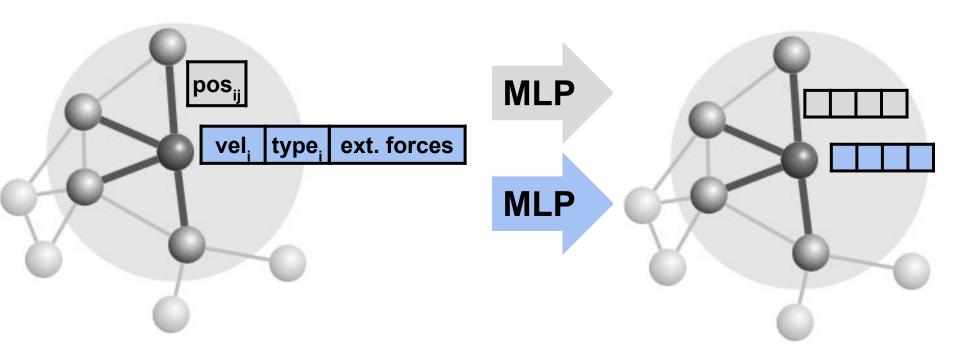
Add features to the graph



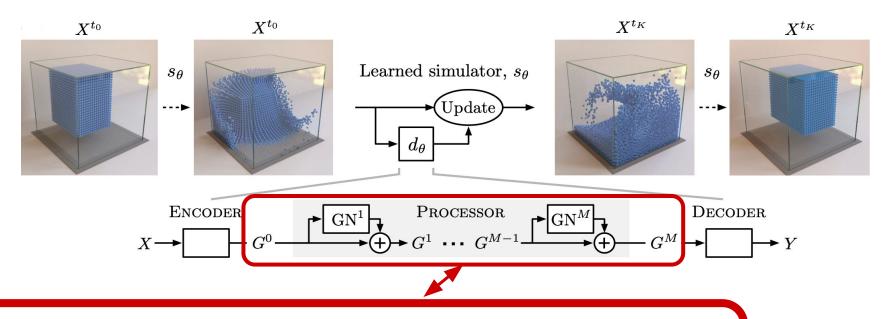
Encoder

ENCODER $X \longrightarrow \boxed{ G^0.}$

Embed graph features



Physics-inspired inductive biases



Spatial Local Universal equivariance interactions rules

Pairwise Superposition interactions principle Permutation equivariance

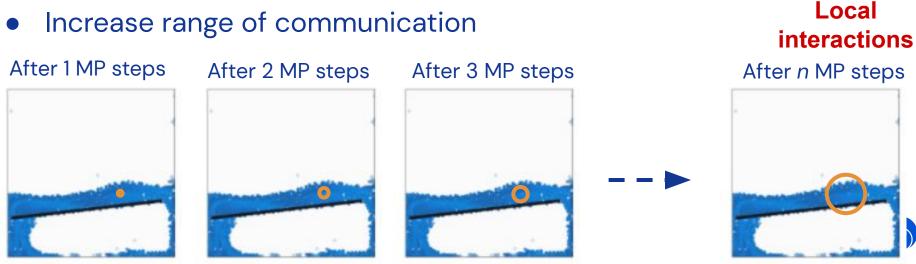
Differential equations

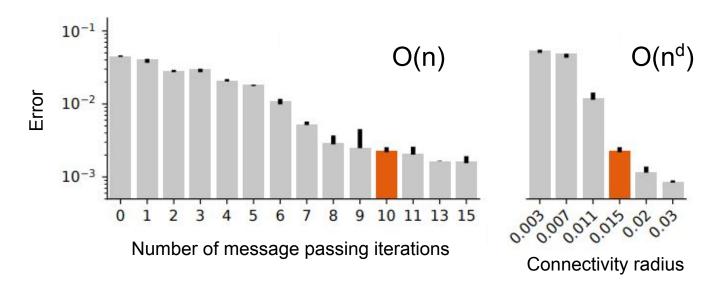
Processor: Graph Network stack

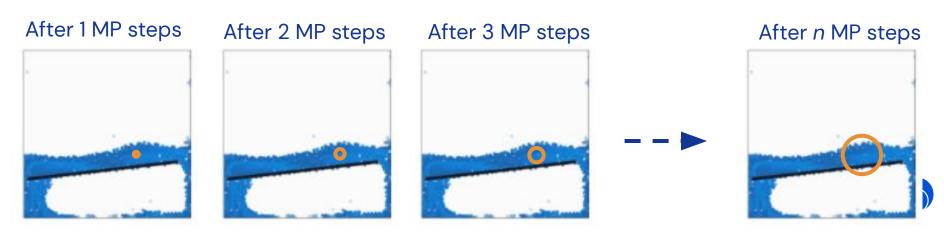
Iterative message passing (MP) without global updates



Increase range of communication







How does generalization work?

Training



Generalization to significantly more particles

Training



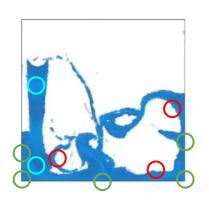
Interactions with obstacles

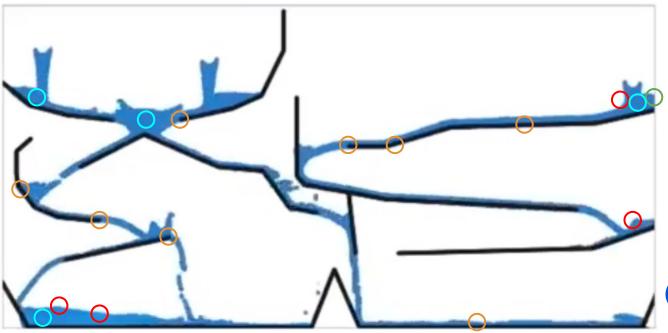
Water surface dynamics

Interaction with box boundaries

Dense blocks of water

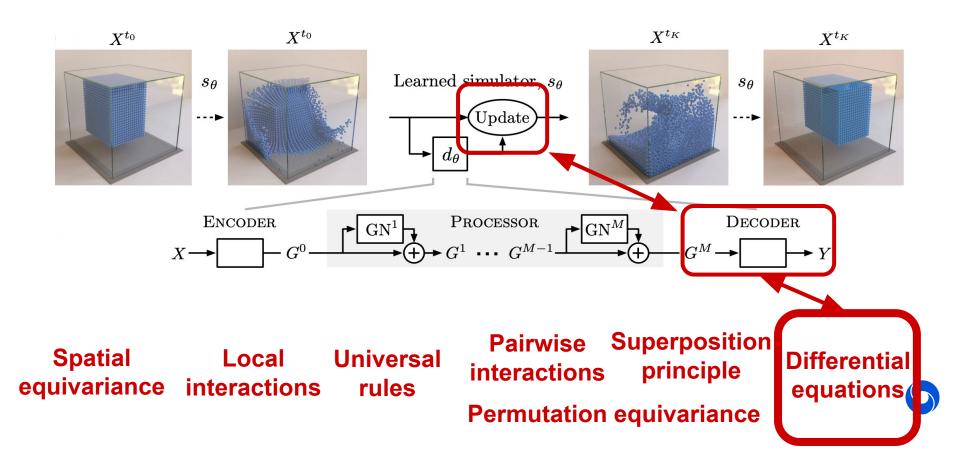
Generalization



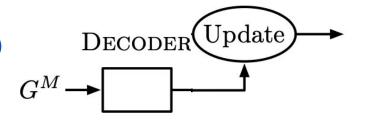




Physics-inspired inductive biases



Decoder and update (Newtonian system)

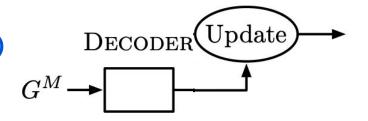


- Naive approach
 - \circ pos^{t+1} = NN(pos^t, vel^t)

Hard to predict static dynamics → Wrong prior



Decoder and update (Newtonian system)



Less naive approach

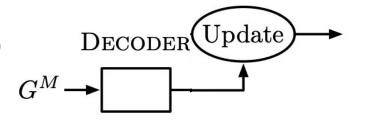
$$\circ$$
 pos^{t+1} = pos^t + NN(pos^t, vel^t)

Easy to predict static dynamics

Hard to predict inertial dynamics → Wrong prior



Decoder and update (Newtonian system)



- Better approach
 - \circ vel^{t+1} = vel^t + NN(pos^t, vel^t)
 - \circ pos^{t+1} = pos^t + vel^{t+1}

Easy to predict static and inertial dynamics!



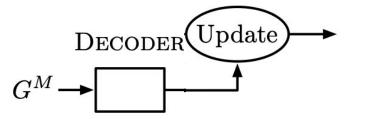
Decoder and update

$$G^{M} \longrightarrow \boxed{\begin{array}{c} \\ \\ \\ \end{array}}$$

$$\mathbf{F} = m \, rac{\mathrm{d} \mathbf{v}}{\mathrm{d} t} = m \mathbf{a}$$
 $\sum \mathbf{F} = 0 \Leftrightarrow rac{\mathrm{d} \mathbf{v}}{\mathrm{d} t} = 0$



Decoder and update



- \circ vel^{t+1} = vel^t + NN(pos^t, vel^t) · dt
- \circ pos^{t+1} = pos^t + vel^{t+1} · dt

Euler integrator!

Differential equations (Newton)

- Reformulate as an ODE:
 - Choose integrator

$$\frac{d \text{ vel}}{dt} = f(\text{pos, vel})$$

$$\frac{d \text{ pos}}{dt} = \text{ve}$$



Decoder and update

Newtonian

Cloth simulation

Quasistatic

Structural mechanics

Eulerian Navier Stokes

Aerodynamics

Incompressible fluid

$$\frac{d\dot{\mathbf{x}}}{dt} = f(\mathbf{x}, \dot{\mathbf{x}})$$

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x})$$

$$\frac{1}{dt} = f(\mathbf{v}, \mathbf{v})$$

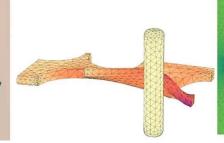
$$\frac{d \rho}{dt} = f(\mathbf{v}, \rho)$$

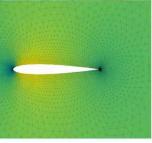
$$\frac{d\,\dot{\mathbf{v}}}{dt} = f(\mathbf{v})$$

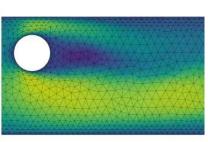


Particle-based fluids





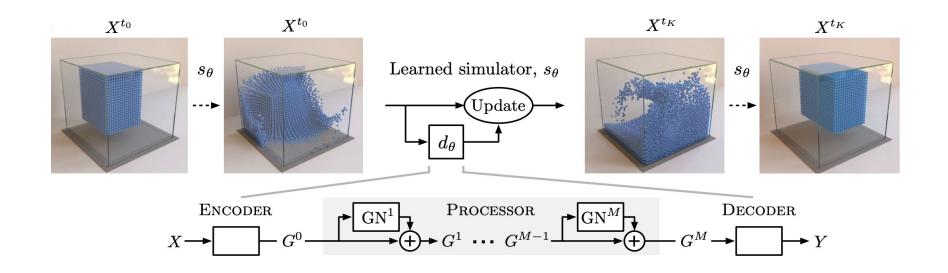




Differential equations



Training tricks



Training noise

Optimization tricks



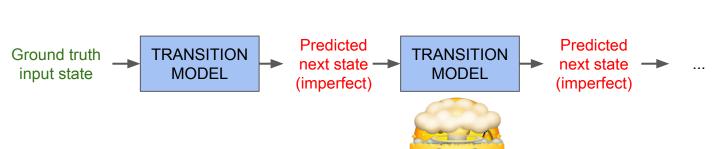
Generalizing to more training steps: training with noise

Train time (one step)



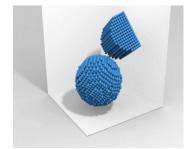
Test time

Model has never seen imperfect inputs



video credit:

Ummenhofer et al. 2020, Lagrangian Fluid Simulation with Continuous Convolutions

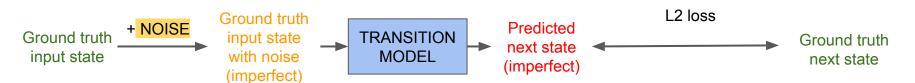




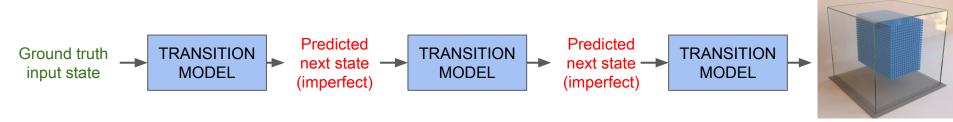
Generalizing to more training steps: training with noise

Train time (with noise)

Models learns to compensate imperfect inputs during training



Test time (error stays bounded)





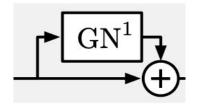
Optimization tricks

- Normalization of representations:
 - Weight initialization: $MLP(\sim N(0, 1)) \rightarrow \sim N(0, 1)$
 - Normalization of MLP inputs: μ =0, σ =1
 - \circ Normalization of MLP outputs: μ =0, σ =1
 - \circ LayerNorm: μ =0, σ =1 everywhere else!



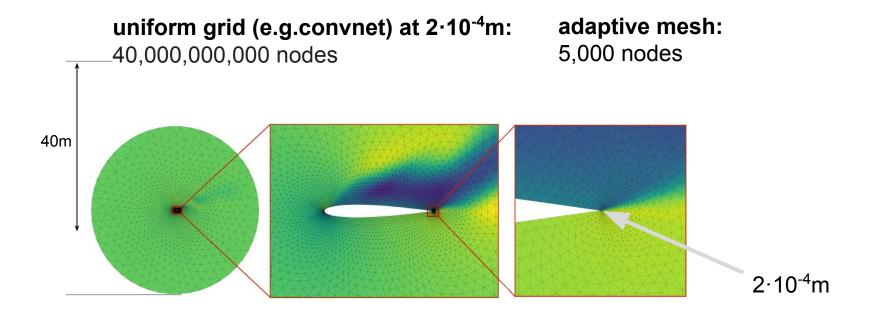
Optimization tricks

- Normalization of representations:
 - Weight initialization: $MLP(\sim N(O, 1)) \rightarrow \sim N(O, 1)$
 - \circ Normalization of MLP inputs: μ =0, σ =1
 - \circ Normalization of MLP outputs: μ =0, σ =1
 - \circ LayerNorm: μ =0, σ =1 everywhere else!
- Residual connections
 - At every message passing step



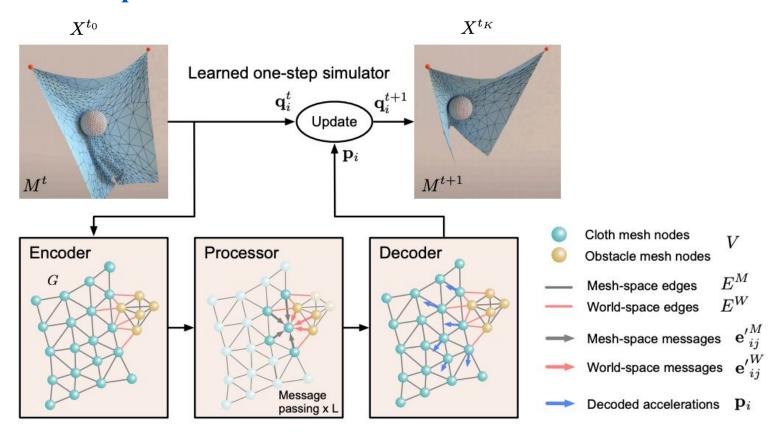


MeshGraphNets: Extension to meshes



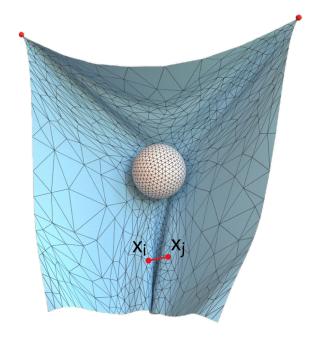


MeshGraphNets: Extension to meshes



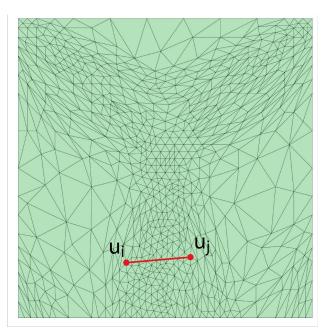


Dual space message passing



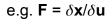
world space **x**

external dynamics: computing e.g. collision and contact



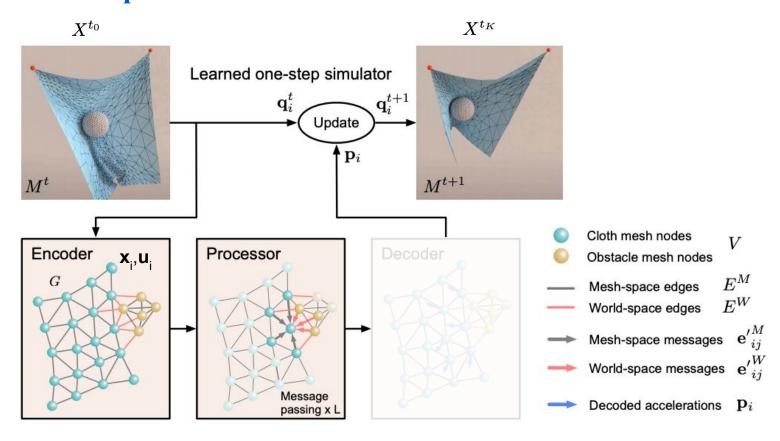
mesh space u

internal dynamics: estimating differential operators on the simulation manifold



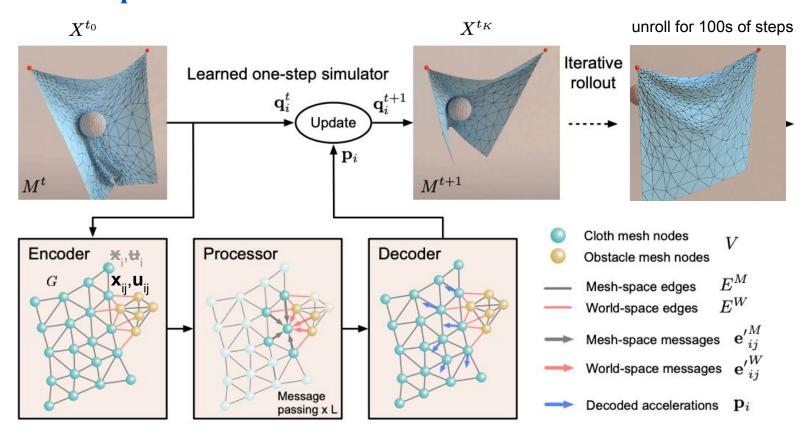


MeshGraphNets: Extension to meshes





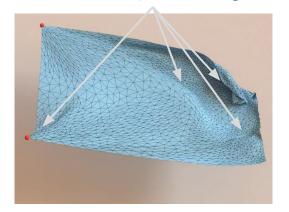
MeshGraphNets: Extension to meshes



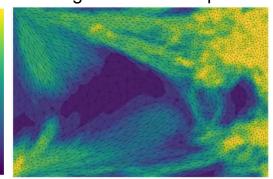


Learned adaptive remeshing

Fine-scale dynamics regions



Sizing field in mesh space

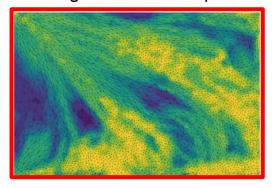


Resolution

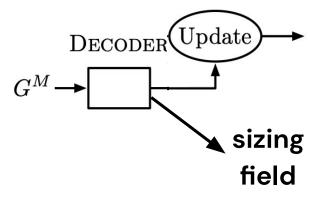
Fine-scale dynamics regions at later step



Sizing field in mesh space

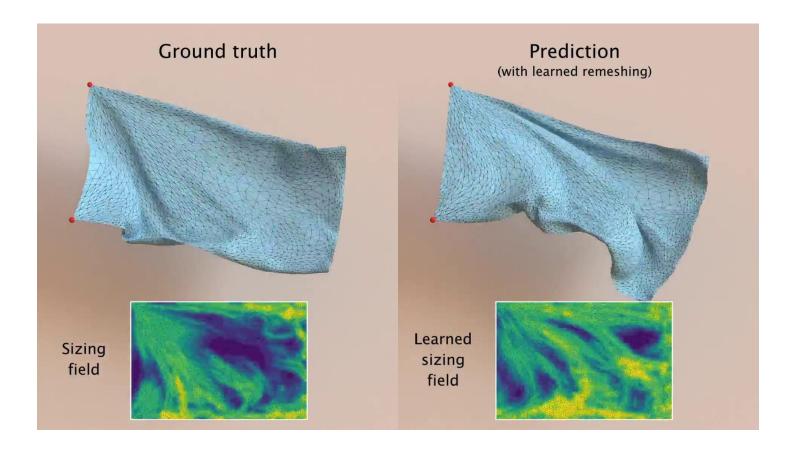


Predict sizing field and remesh!





Learned adaptive remeshing





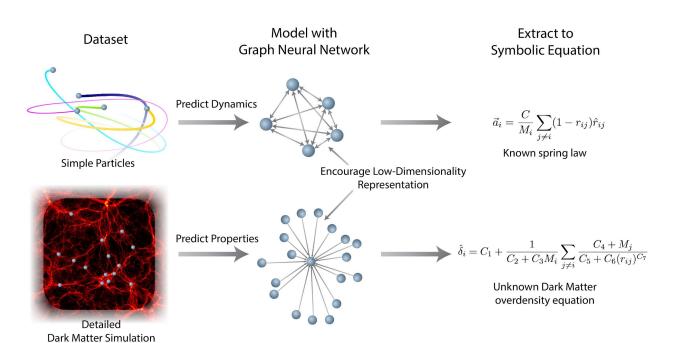
Bonus slide: additional inductive biases

- Hamiltonian inductive bias with Graph Networks
 - "Hamiltonian Graph Networks with ODE Integrators"
 Sanchez-Gonzalez et al., 2019, arXiv/NeurIPS 2019 workshop
 - Makes use of the full Graph Network (including global update).
 - Explores additional integrators
- Lagrangian inductive bias with Graph Networks
 - "Lagrangian Neural Networks"
 Cranmer et al., 2020, arXiv/ICLR 2020 workshop



Bonus slide: interpretable graph networks

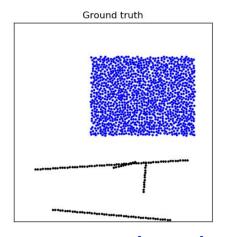
- "Discovering Symbolic Models from Deep Learning with Inductive Biases"
 Cranmer et al., NeurIPS 2020
- Extract symbolic models from edge and node functions of a GraphNet

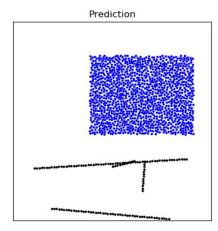




Datasets and source code

- "Learning to Simulate Complex Physics with Graph Networks"
 - o github.com/deepmind/deepmind-research/tree/master/learning_to_simulate





- Tensorflow 1 & Sonnet 1 (GPU)
- DeepMind Graph Nets library (TF1 and TF2 compatible)
 - github.com/deepmind/graph_nets



learned_simulator.py (_build)

One step model interface:

```
def _build(self, position_sequence, n_particles_per_example,
           global_context=None, particle_types=None):
  input_graphs_tuple = self._encoder_preprocessor(
      position_sequence, n_particles_per_example, global_context,
      particle types)
  normalized_acceleration = self._graph_network(input_graphs_tuple)
  next_position = self._decoder_postprocessor(
      normalized acceleration, position sequence)
  return next_position
```



learned_simulator.py (_build)

One step model interface:

```
def _build(self, position_sequence, n_particles_per_example,
           global_context=None, particle_types=None):
 input graphs tuple = self. encoder preprocessor(
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      particle_types)
  normalized_acceleration = self._graph_network(input_graphs_tuple)
  next_position = self._decoder_postprocessor(
      normalized acceleration, position sequence)
  return next_position
```

Fixed radius connectivity:

```
# Extract important features from the position_sequence.
most_recent_position = position_sequence[:, -1]

# Get connectivity of the graph.
(senders, receivers, n_edge
) = connectivity_utils.compute_connectivity_for_batch_pyfunc(
    most_recent_position, n_node, self._connectivity_radius)
```



Normalized node features:

```
# Normalized velocity sequence, merging spatial an time axis.
velocity_sequence = time_diff(position_sequence) # Finite-difference.
velocity_stats = self._normalization_stats["velocity"]
normalized_velocity_sequence = (
    velocity_sequence - velocity_stats.mean) / velocity_stats.std
flat_velocity_sequence = snt.MergeDims(start=1, size=2)(
    normalized_velocity_sequence)
node features.append(flat_velocity_sequence)
# Particle type.
if self._num_particle_types > 1:
  particle_type_embeddings = tf.nn.embedding_lookup(
      self._particle_type_embedding, particle_types)
  node_features.append(particle_type_embeddings)
```



Normalized edge features:

```
# Collect edge features.
edge_features = []
# Relative displacement and distances normalized to radius
normalized_relative_displacements = (
    tf.gather(most_recent_position, senders) -
    tf.gather(most_recent_position, receivers)) / self._connectivity_radius
edge_features.append(normalized_relative_displacements)
normalized_relative_distances = tf.norm(
    normalized_relative_displacements, axis=-1, keepdims=True)
edge_features.append(normalized_relative_distances)
```



Normalized global features:

```
# Normalize the global context.
if global_context is not None:
   context_stats = self._normalization_stats["context"]
   # Context in some datasets are all zero, so add an epsilon for numerical
   # stability.
   global_context = (global_context - context_stats.mean) / tf.math.maximum(
        context_stats.std, STD_EPSILON)
```



Build a GraphsTuple with node, edge and global features:

```
return gn.graphs.GraphsTuple(
    nodes=tf.concat(node_features, axis=-1),
    edges=tf.concat(edge_features, axis=-1),
    globals=global_context,
    n_node=n_node, n_edge=n_edge,
    senders=senders, receivers=receivers,)
```

```
# Copy the globals to all of the nodes, if applicable.
if input_graph.globals is not None:
   broadcasted_globals = gn.blocks.broadcast_globals_to_nodes(input_graph)
   input_graph = input_graph.replace(
        nodes=tf.concat([input_graph.nodes, broadcasted_globals], axis=-1),
        globals=None)
```



learned_simulator.py (_build)

One step model interface:

```
def _build(self, position_sequence, n_particles_per_example,
           global_context=None, particle_types=None):
 input_graphs_tuple = self._encoder_preprocessor(
      position_sequence, n_particles_per_example, global_context,
      particle types)
 normalized_acceleration = self._graph_network(input_graphs_tuple)
 next_position = self._decoder_postprocessor(
      normalized_acceleration, position_sequence)
 return next_position
```

```
self._graph_network = graph_network.EncodeProcessDecode(
    output_size=num_dimensions, **graph_network_kwargs)
```



graph_network.py (_networks_builder)

Build off-the-shelf Graph Networks:

```
# The encoder graph network independently encodes edge and node features.
encoder_kwargs = dict(
    edge_model_fn=build_mlp_with_layer_norm,
    node model fn=build mlp with layer norm)
self._encoder_network = gn.modules.GraphIndependent **encoder_kwargs)
# Create `num_message_passing_steps` Interaction Networks with unshared
# parameters that update the node and edge latent features.
self._processor_networks = []
for _ in range(self._num_message_passing_steps):
  self._processor_networks.append(
      gn.modules.InteractionNetwork(
          edge_model_fn=build_mlp_with_layer_norm,
          node model fn=build mlp with layer norm))
# The decoder MLP decodes node latent features into the output size.
self._decoder_network = build_mlp(
    hidden_size=self._mlp_hidden_size,
    num_hidden_layers=self._mlp_num_hidden_layers,
    output_size=self._output_size)
```



graph_network.py (_build)

Embed features, run message passing and device node features:

```
def _build(self, input_graph: gn.graphs.GraphsTuple) -> tf.Tensor:
  """Forward pass of the learnable dynamics model."""
  # Encode the input graph.
  latent_graph_0 = self._encoder_network(input_graph)
  # Do `m` message passing steps in the latent graphs.
 latent_graph_m = self._process(latent_graph_0)
  # Decode from the last latent graph
  return self. decoder network(latent graph.nodes)
```



graph_network.py (_process)

Several steps of message passing with residual connections:

```
latent_graph_prev_k = latent_graph_0
# Do `m` message passing steps in the latent graphs.
for processor_network_k in self._processor_networks:
 # One step of message passing.
 latent graph k = processor_network_k(latent_graph_prev_k)
 # Add residuals with previous layer.
 latent_graph_k = latent_graph_k.replace(
      nodes=latent_graph_k.nodes+latent_graph_prev_k.nodes,
      edges=latent_graph_k.edges+latent_graph_prev_k.edges)
  latent_graph_prev_k = latent_graph_k
latent_graph_m = latent_graph_k
```



learned_simulator.py (_build)

One step model interface:

```
def _build(self, position_sequence, n_particles_per_example,
           global_context=None, particle_types=None):
  input_graphs_tuple = self._encoder_preprocessor(
      position_sequence, n_particles_per_example, global_context,
      particle_types)
  normalized_acceleration = self._graph_network(input_graphs_tuple)
 next_position = self._decoder_postprocessor(
      normalized acceleration, position sequence)
  return next_position
```

Inverse normalization and Euler Integrator:

```
def _decoder_postprocessor(self, normalized_acceleration, position_sequence):
  # The model produces the output in normalized space so we apply inverse
 # normalization.
  acceleration_stats = self._normalization_stats["acceleration"]
  acceleration = (
     normalized_acceleration * acceleration_stats.std
      ) + acceleration_stats.mean
 # Use an Euler integrator to go from acceleration to position, assuming
  # a dt=1 corresponding to the size of the finite difference.
 most_recent_position = position_sequence[:, -1]
 most_recent_velocity = most_recent_position - position_sequence[:, -2]
  new_velocity = most_recent_velocity + acceleration # * dt = 1
  new_position = most_recent_position + new_velocity # * dt = 1
  return new_position
```



Conclusions

- Graph Networks are powerful models for everyone!
- Architectures for simulation (mesh-based and-particle based)
- General principles to find well-matched inductive biases
- Effective model training tricks
- A reference implementation for one of these models at

https://github.com/deepmind/deepmind-research/tree/master/learning_to_simulate



DeepMind

Learningour attention!
Thanks for your attention!

a Question Ltime?

Battaglia, Kyle Cranmer, Miles Cranmer, rortunato, Jonathan Godwin, Jessica Hamrick, Shirley Ho, Jure Leskovec, Tobias Pfaff, Rex Ying,

