Lattice gauge theory applications for Quantum Computing

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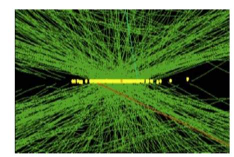
- Variational quantum eigensolver

- Lattice gauge theory example
  - model
  - code
  - results

- Conclusion

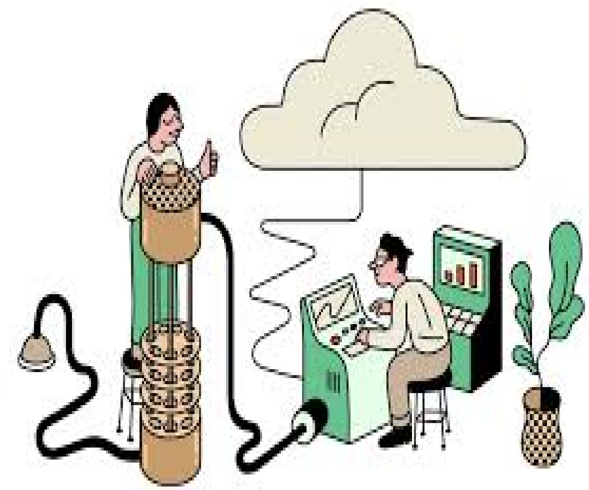
DESY and Quantum Computing

- Member of European project
  Quantentechnologies for Lattice Gauge Theorie (QTFLAG)
  (within European Quantum Flagship)
- DESY-Triumf cooperation (QC one pillar)
- Cooperation with Institut for quantum computing, Waterloo, Canada
- Heibrids graduate research school in Berlin
- Dashh graduate research school in Hamburg
- Common project between DESY and CERN Open Lab
  $\rightarrow$ PhD funded through the Gentner-Programme
  $\rightarrow$ project: fast, precise simulations with Neural Network methods like GANs



Quantum Computing: how?

- python programming language
  - → company provides quantum libraries
- very convenient setup
  - → simulator runs on your local machine
  - → hardware usable through quantum cloud service
  - → build on reservation system
- documentation, tutorials and examples availabe on website
  - → you can start now



Variational quantum simulation

- start with some initial state $|\Psi_{\text{init}}\rangle$
- apply succesive gate operations $\equiv$ unitary operations $e^{-iS\theta}$
- examples for S: Pauli matrices σ_x, σ_y, σ_z, parametric CNOT

$$|\Psi(\vec{\theta})\rangle = e^{-iS_{(n)}\theta_n} \dots e^{-iS_{(1)}\theta_1} |\psi_{\text{init}}\rangle$$

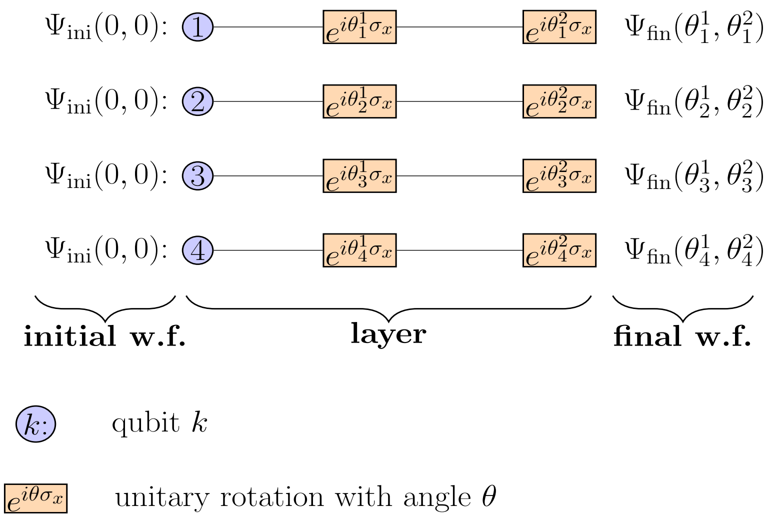
- with $R_j := e^{-iS_{(j)}\theta_j}$ cost function evaluated on quantum computer

$$C := \left\langle \psi_{\text{init}} \middle| \left(\prod_{j=1}^{n} R_j\right)^{\dagger} H \prod_{j=1}^{n} R_j \middle| \psi_{\text{init}} \right\rangle$$

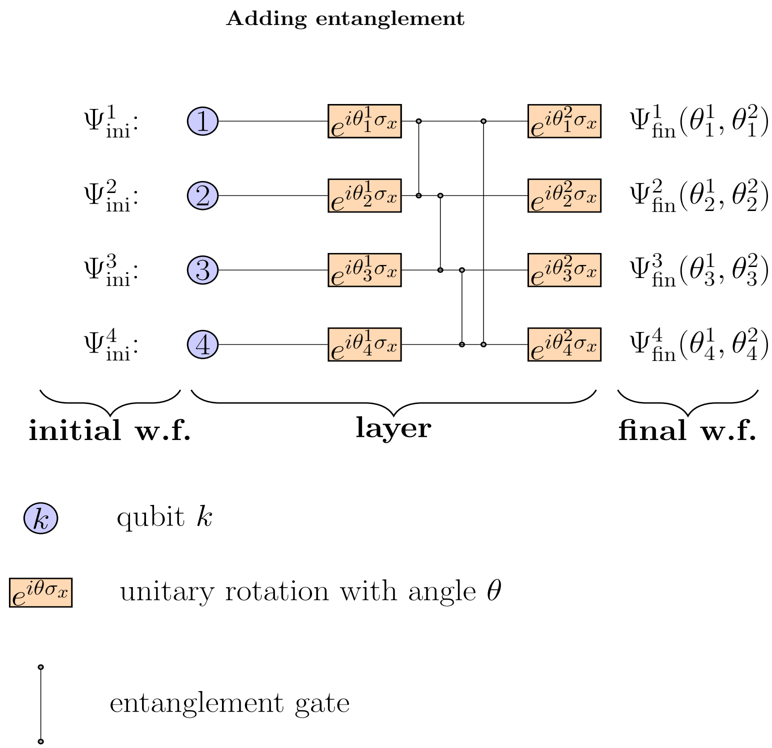
- goal: minimize C over the angles $\vec{\theta}$
$\rightarrow$ obtain minimal energy, i.e. ground state

- goal: minimization performed classically (hybrid classical-quantum approach)
$\leftarrow$ also possible on quantum computer itself

Example: only qubit rotations



Adding entanglement



A typical model structure

- 1-dimensional Heisenberg model

$$\hat{H} = -\frac{1}{2}\sum_{j=1}^{N}\left(J_x\sigma_j^x\sigma_{j+1}^x + J_y\sigma_j^y\sigma_{j+1}^y + J_z\sigma_j^z\sigma_{j+1}^z + h\sigma_j^z\right)$$

- Pauli matrices

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

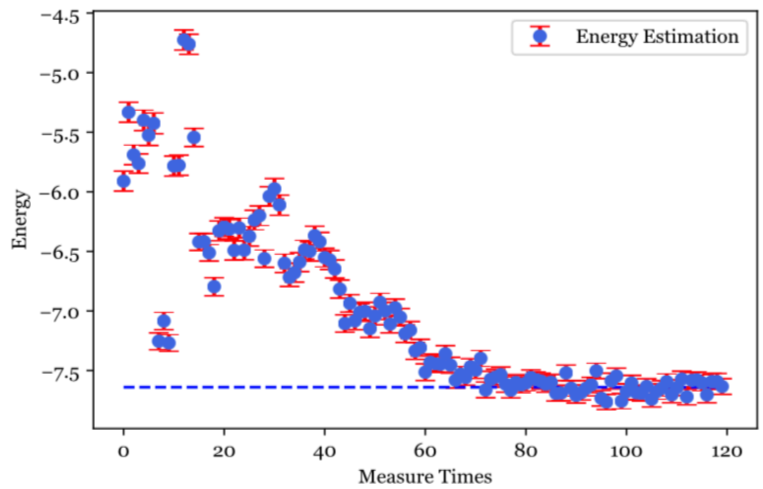
- nearest neighbour interaction, tensor products
- Hamiltonian expressed in Pauli matrices $\rightarrow$ suitable for quantum computer
- shows phase transitions, critical behaviour, non-trivial spectrum

$$H = \sum_{i=1}^{N} \beta \left[\sigma_x(i)\sigma_x(i+1) + \sigma_y(i)\sigma_y(i+1) + \sigma_z(i)\sigma_z(i+1)\right] + J\sigma_z(i)$$

```
from pyquil.quil import Program
import pyquil.api as api
from pyquil.gates import *
qvm = api.QVMConnection()              [hardware → qvm = api.QPUConnection()]
import numpy as np
from pyquil.api import QVMConnection
from scipy.optimize import minimize
from grove.pyvqe.vqe import VQE

from pyquil.paulis import ID, sX, sY, sZ

def smallansatz(params):
return Program(RX(params[0], 0))

beta=0.12578,J=1.87
hamiltonian=0
for k in range(3):
l = (k+1)%3
hamiltonian += beta*(sX(k)*sX(l)+sY(k)*sY(l) + sZ(k)*sZ(l)) +J*sZ(k)
print(hamiltonian)

initialangle = [0.0]

vqeinst = VQE(minimizer=minimize,minimizerkwargs='method': 'nelder-mead')

angle = 2.0
vqeinst.expectation(smallansatz([angle]), hamiltonian, None, qvm)
result = vqeinst.vqerun(smallansatz, hamiltonian, initialangle, None, qvm=qvm)
print(result)
```

Using the simulator

- Simulator with no noise
- IBM's *Simultaneous Perturbation Stochastic Approximation* (SPSA) algorithm
- dashed line exact result

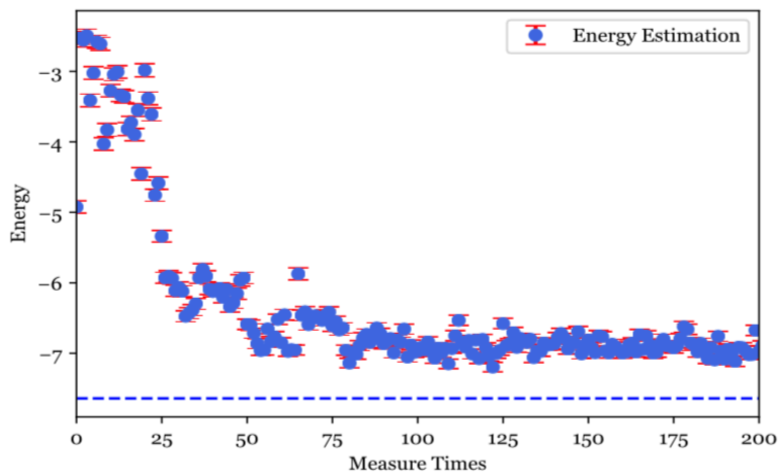


(figure by Xiaoyang Wang)

- 3 qubits

Switching on noise

- Simulator with noise

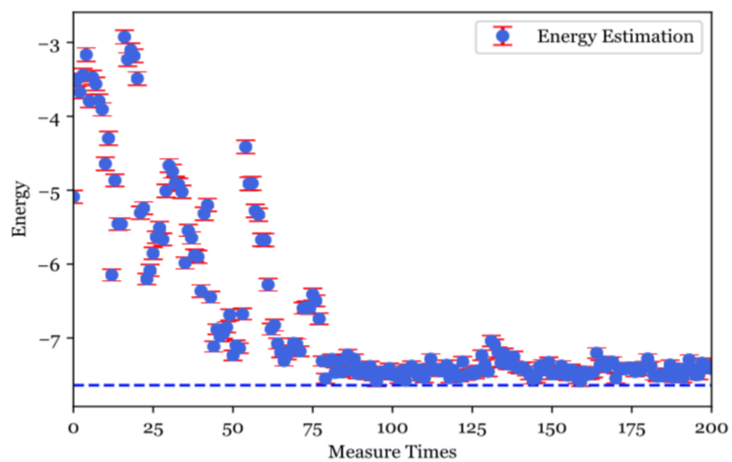


(figure by Xiaoyang Wang)

- 3 qubits
- measurement error $\rightarrow$ cannot find ground state
- perspective for larger number of qubits?

Error mitigation

- Simulator with no noise and error mitigation
- Companies offer error correction (mitigation)



(figure by Xiaoyang Wang)

- 3 qubits
- works well $\rightarrow$ find ground state
- perspective for larger number of qubits

Quantum Computing applications for VQE

- Theoretical particle physics
  → variational quantum simulations of models in high energy physics
  (e.g.: T. Hartung, K.J., quantum computing ζ-regularized vacuum expectation values, J.Math.Phys. 60 (2019) no.9, 093504)

- Experimental particle physics[1]
  → quantum annealing
  (e.g.: F. Babst et.al., A pattern recognition algorithm for quantum annealers, arXiv:1902.08324)

- Astroparticle physics
  → quantum networks
  (e.g.: E. T. Khabiboulline et.al., Quantum-Assisted Telescope Arrays arXiv:1809.03396, Phys.Rev.A100, 022316 (2019))
  → air showers

- Aerospace
  → Flight gate assignment
  (T. Stollenwerk et.al., arxiv:1811.09465)

[1] CERN Workshop: https://indico.cern.ch/event/719844/timetable/

Summary

- presently: Noisy Intermediate-Scale Quantum devices
  $\rightarrow$ lattice gauge theory models prime candidates
  - develop algorithms
  - test quantum devices
  - error mitigation of great help

  $\Rightarrow$ prepare for quantum devices with large number of qubits

- algorithms developed can be used
  - particle tracking in HEP
  - flight gate assignements in aerospace problem
  - airshower in astroparticle physics

- need to explore potential now by working together