Quantum walks for constraint combinatorial optimisation

Sam Marsh & Jingbo Wang
The University of Western Australia

Combinatorial optimisation

(Backpack Problem, Drug Discovery, Traffic Design, Investment Portfolio, Graph Partition, Subset Sum, Lattice Paths, Travelling Salesman Problem, etc)

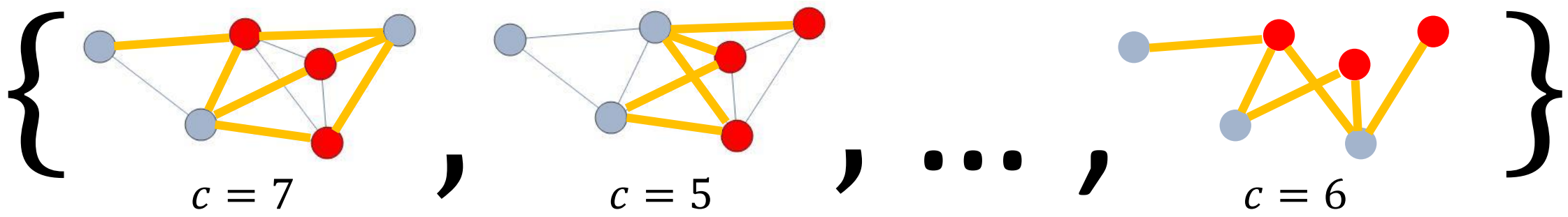
$40/100$ items $\rightarrow$ 10^{28}

$5\ pairs \rightarrow 10^{100}$

$50\ atoms \rightarrow 10^{48}$

$500\ stocks \rightarrow 10^{150}$

Graph Partition



$\min c(x)$ over all possible solutions x : $500\ nodes \rightarrow 10^{150} \overset{log}{\rightarrow}$ **$345\ qubits$**

Quantum Annealing

- Quantum adiabatic theorem

$$i\,\frac{d}{dt}\,|\psi(t)\rangle = \hat{H}(t)\,|\psi(t)\rangle$$

$|s\rangle$: min/max energy state of $\hat{H}(t_0) = \hat{B}$:

(known eigenvalues & eigenstates)

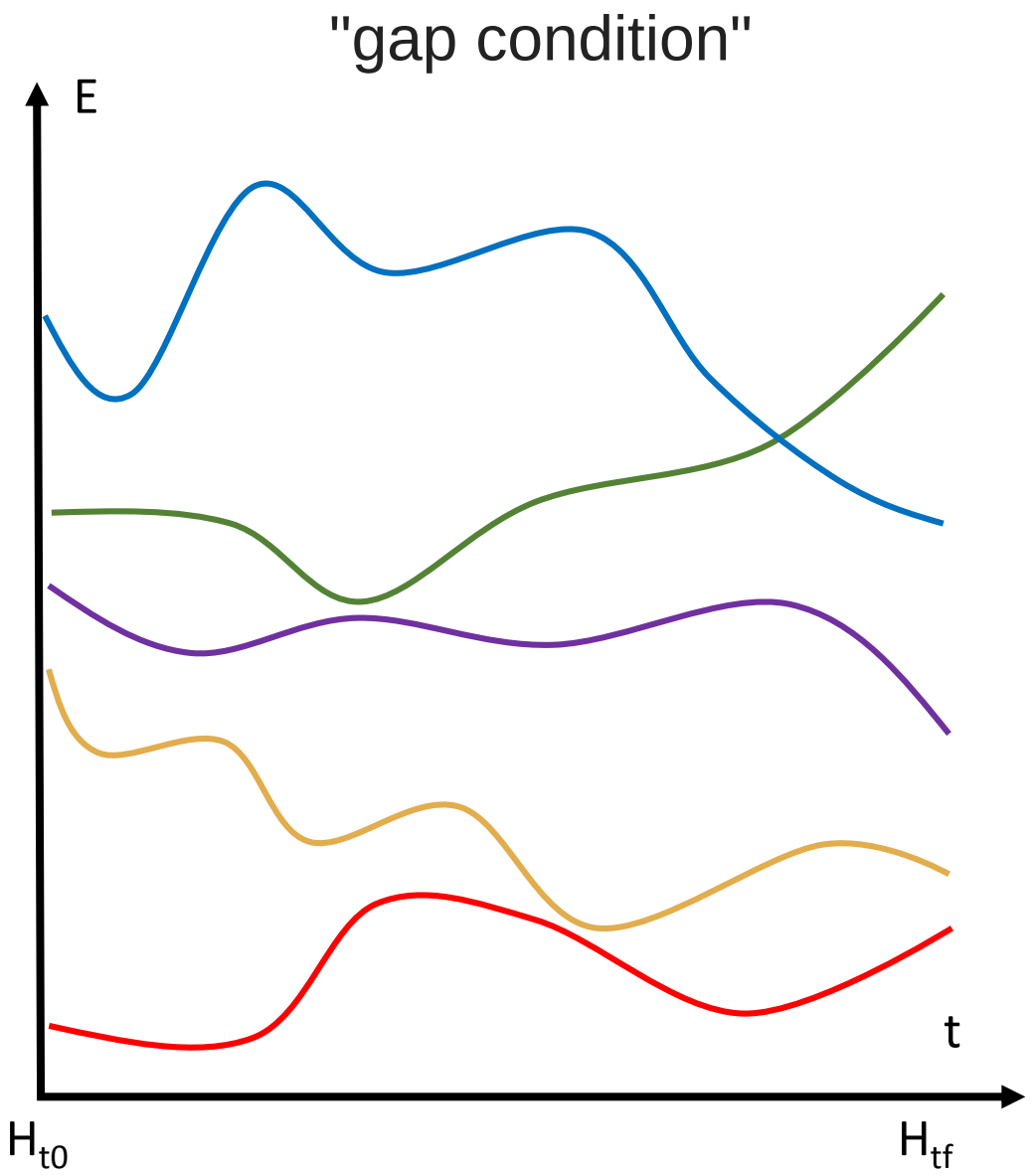
adiabatic

$$\hat{\boldsymbol{H}}(\boldsymbol{t}) = \left(\mathbf{1} - \frac{\boldsymbol{t}}{\boldsymbol{T}}\right)\hat{\boldsymbol{B}} + \frac{\boldsymbol{t}}{\boldsymbol{T}}\hat{\boldsymbol{Q}}$$

$$t = [0, T]$$

$|\psi\rangle$: min/max energy state of $\hat{H}(t_f) = \hat{Q}$:
(encodes combinortorial optimisation solution)

Quantum Adiabatic Evolution Algorithm, Farhi et al 2001



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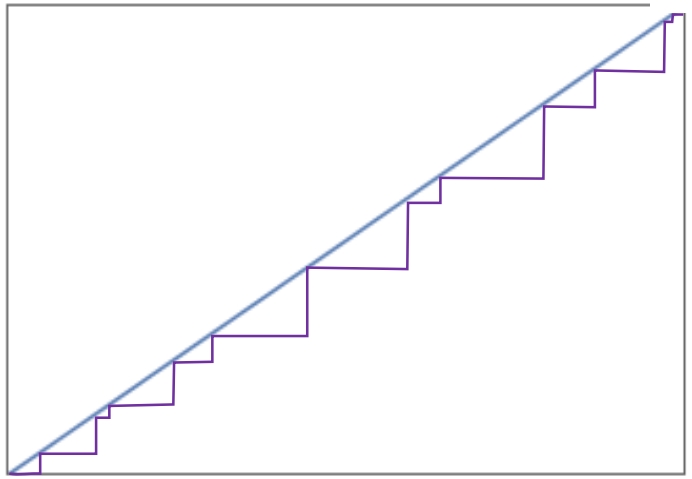
Quantum Adiabatic Evolution Algorithm, Farhi et al 2001

Quantum Approximate Optimisation Algorithm, Farhi et al 2016

Quantum-walk-assisted optimisation algorithm, Marsh & Wang 2019

QAOA



$$\hat{\boldsymbol{H}}_{\boldsymbol{\alpha}} = \left(\mathbf{1} - \frac{\boldsymbol{\alpha}}{\boldsymbol{p}}\right)\hat{\boldsymbol{B}} + \frac{\boldsymbol{\alpha}}{\boldsymbol{p}}\hat{\boldsymbol{Q}}$$
$$\boldsymbol{\alpha} = \mathbf{0}, \mathbf{1}, \ldots (\boldsymbol{p} - \mathbf{1})$$

$$\hat{\boldsymbol{U}}_{\boldsymbol{\alpha}} = e^{-i\,\Delta t\,\hat{\boldsymbol{H}}_{\boldsymbol{\alpha}}}$$
$$\approx \underbrace{e^{-i\,\beta\,\hat{B}}}_{\hat{U}_B(\beta)}\ \underbrace{e^{-i\,\gamma\,\hat{Q}}}_{\hat{U}_Q(\gamma)}$$

$$|\vec{\beta},\vec{\gamma}\rangle = \overbrace{\hat{U}_B(\beta_p)\,\hat{U}_Q(\gamma_p)}^{\approx \hat{\boldsymbol{U}}_{\boldsymbol{p}}} \ldots \overbrace{\hat{U}_B(\beta_1)\,\hat{U}_Q(\gamma_1)}^{\approx \hat{\boldsymbol{U}}_{\mathbf{1}}}\,|s\rangle$$

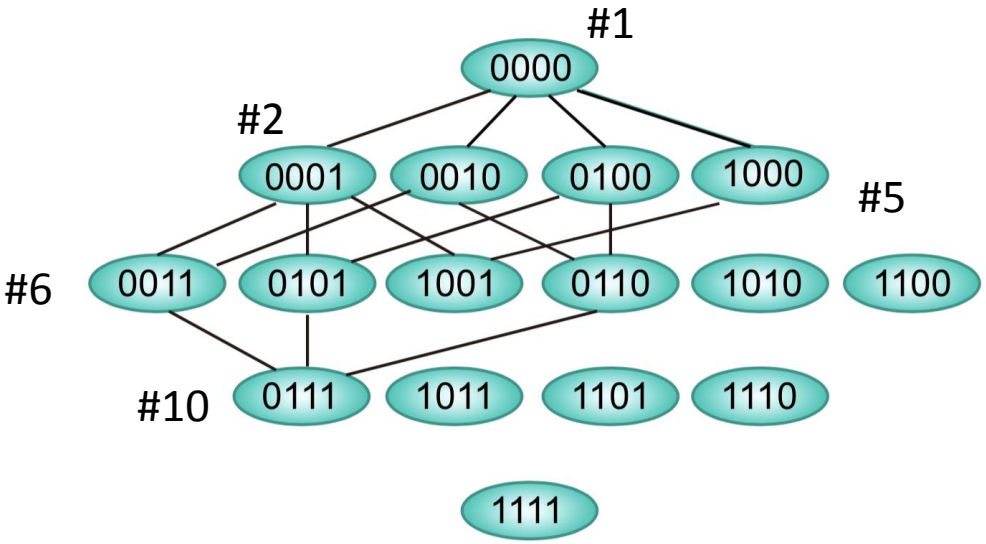
The **state evolution** using p timesteps

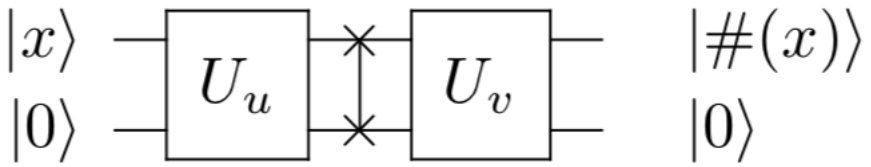
Vary $\{\vec{\beta},\vec{\gamma}\}$ to minimise $\langle\vec{\beta},\vec{\gamma}|\hat{Q}|\vec{\beta},\vec{\gamma}\rangle$

QWOA

$$|\vec{t},\vec{\gamma}\rangle = \hat{U}_W(t_p)\,\hat{U}_Q(\gamma_p)\ldots\hat{U}_w(t_1)\,\hat{U}_Q(\gamma_1)\,|s\rangle$$

QWOA $|\vec{t},\vec{\gamma}\rangle = \hat{U}_W(t_p)\,\hat{U}_Q(\gamma_p)\,\ldots\,\hat{U}_w(t_1)\,\hat{U}_Q(\gamma_1)\,|s\rangle$



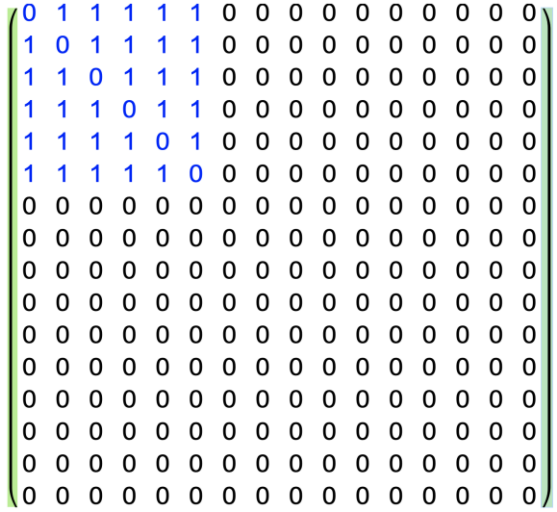


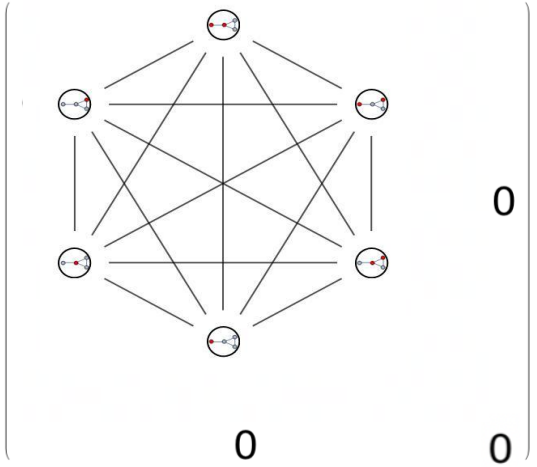
$$U_v\,|\#(x)\rangle\,|z\rangle = |\#(x)\rangle\,|z \oplus x\rangle$$

$$U_u\,|x\rangle\,|z\rangle = |x\rangle\,|z \oplus \#(x)\rangle$$

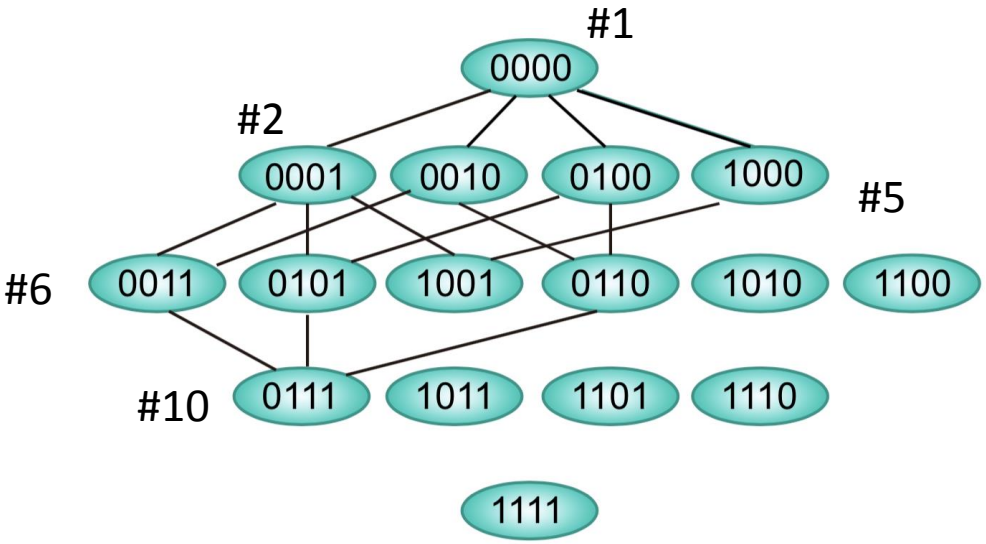
without sequence algorithm

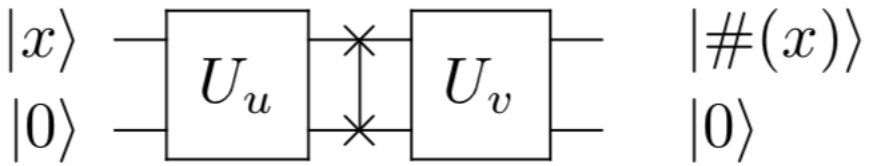
with sequence algorithm





QWOA $|\vec{t},\vec{\gamma}\rangle = \hat{U}_W(t_p)\,\hat{U}_Q(\gamma_p)\ldots\hat{U}_w(t_1)\,\hat{U}_Q(\gamma_1)\,|s\rangle$





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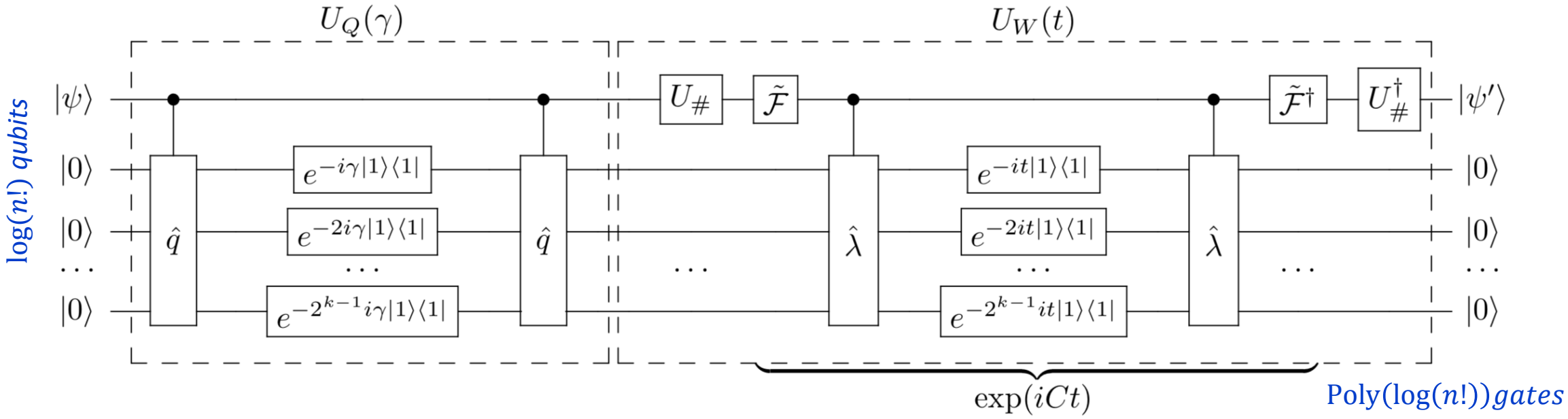
- **Highly efficient Circuit**



$$|\vec{t},\vec{\gamma}\rangle = \hat{U}_W(t_p)\,\hat{U}_Q(\gamma_p)\,\ldots\,\hat{U}_w(t_1)\,\hat{U}_Q(\gamma_1)\,|s\rangle$$

QWAO procedure

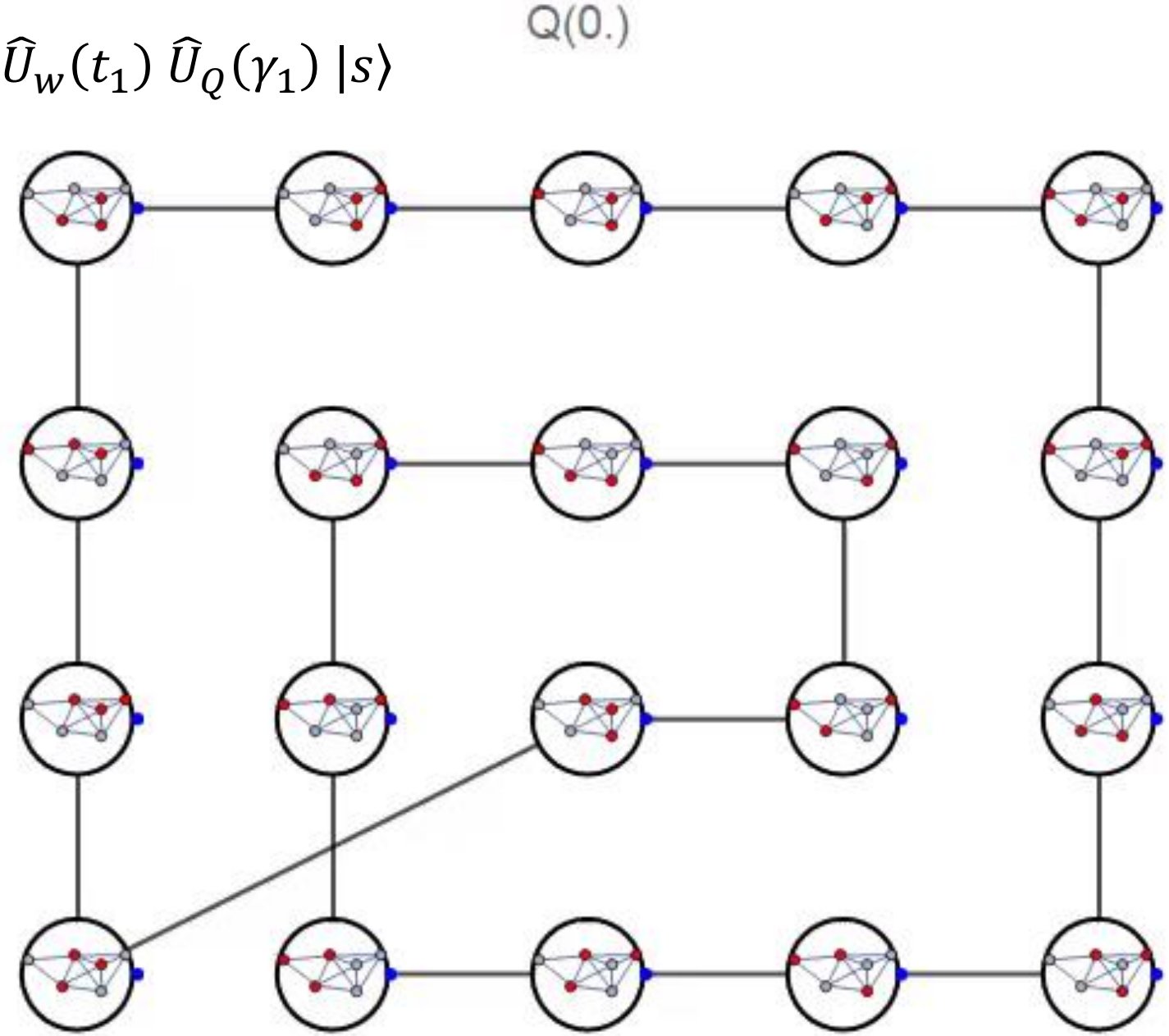
1. Start with equal superposition

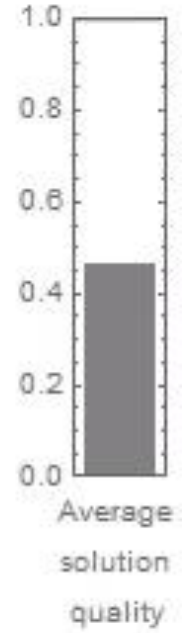
2. Repeat:
   - *change phase*
   - *quantum walk*

3. Measure

$500\ nodes \rightarrow 10^{150}$

$\xrightarrow{log}$ **$345\ qubits$**

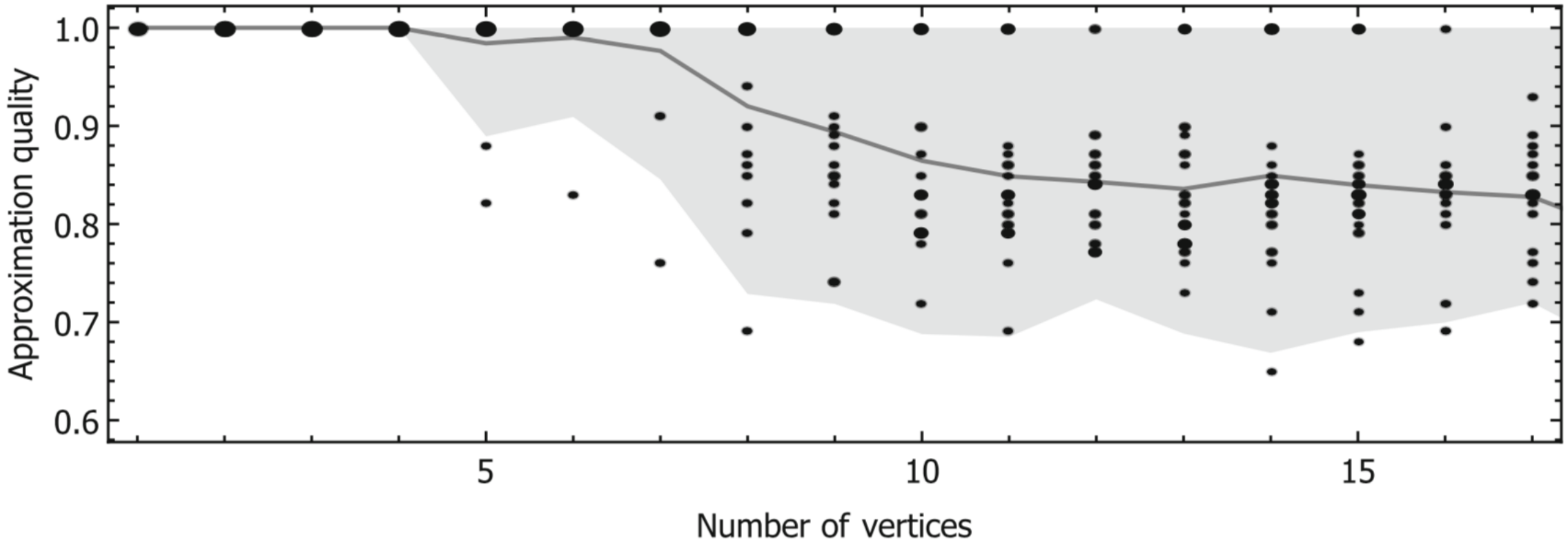




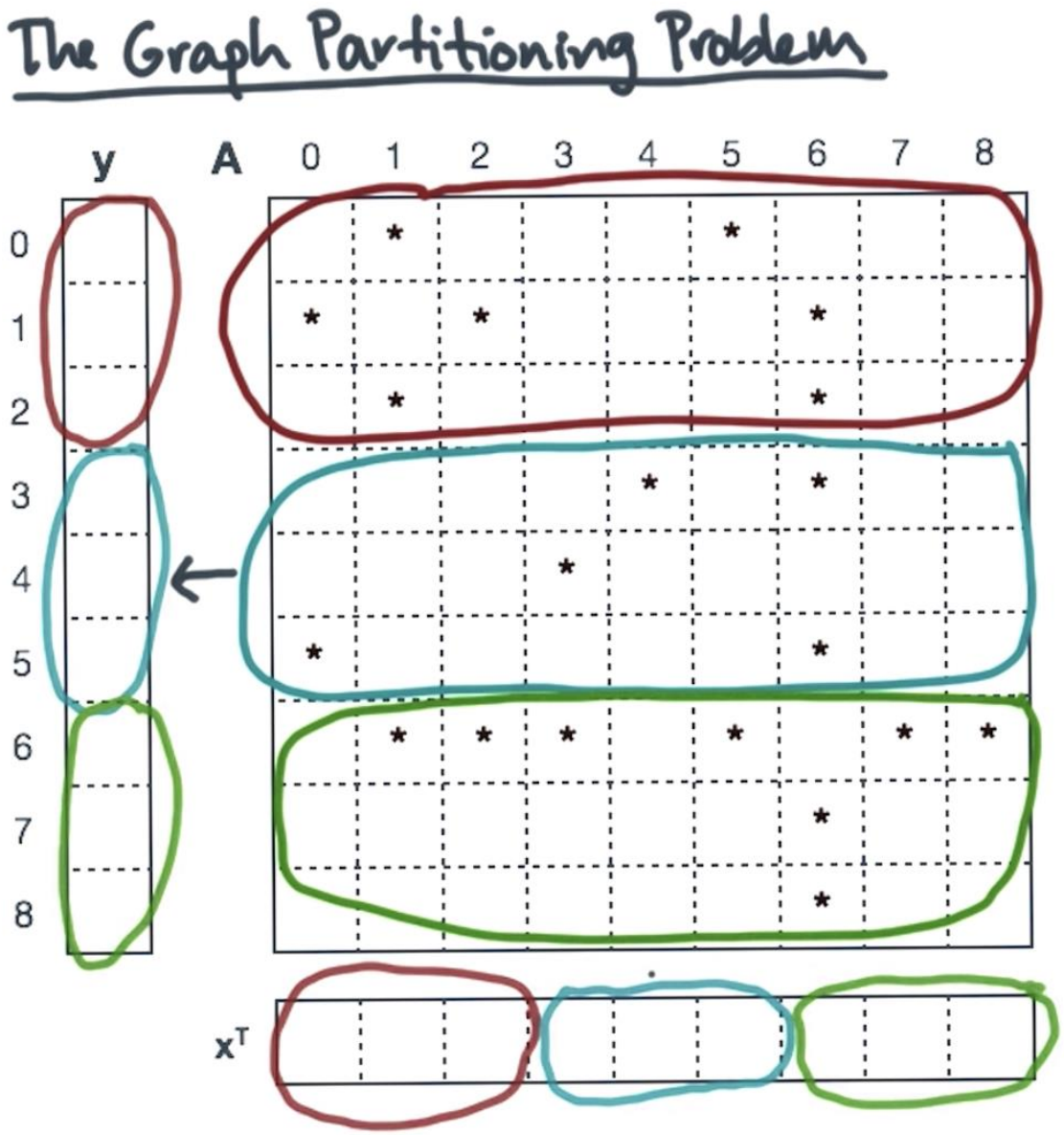
Conclusion

- *Highly efficient* quantum walk over exponentially large set of combinatorial objects with *adaptable connectivity*;
- Substantially reduced search space using *sequence function*;
- Quantum circuit design independent of *variational parameters;*
- Applications over a wide range of NP-hard problems.

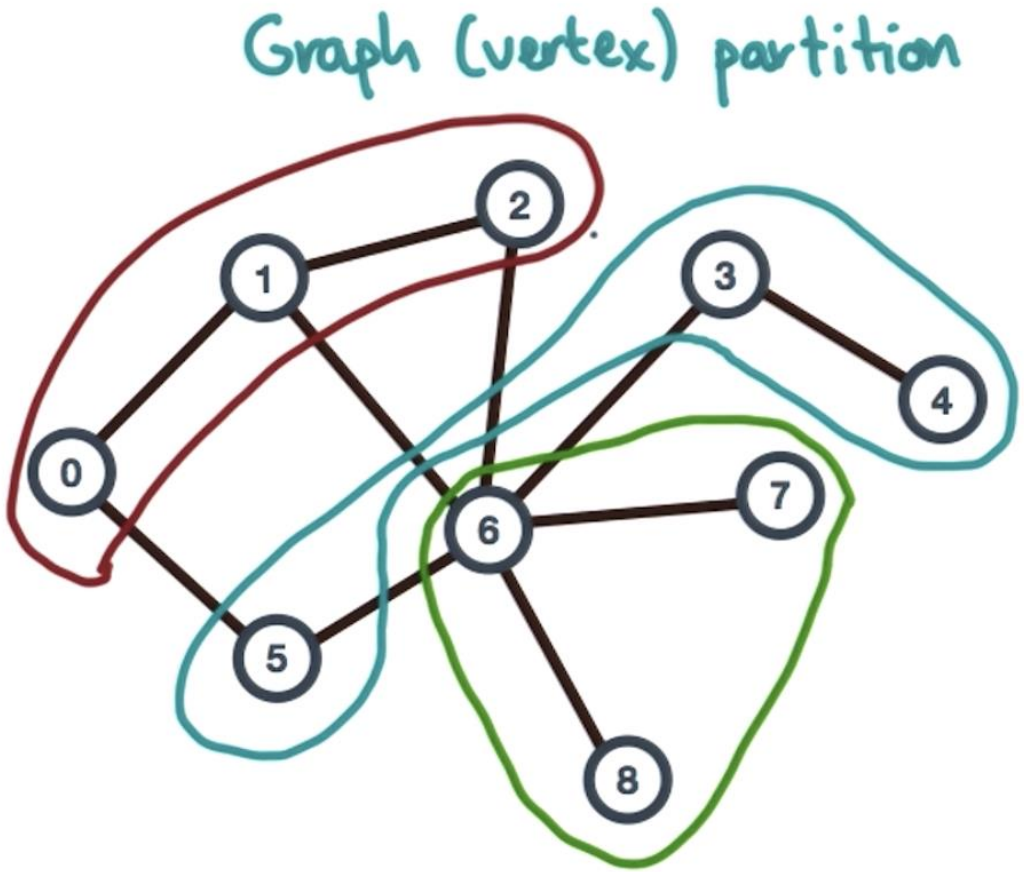
Preliminary results



eg: distributed computing

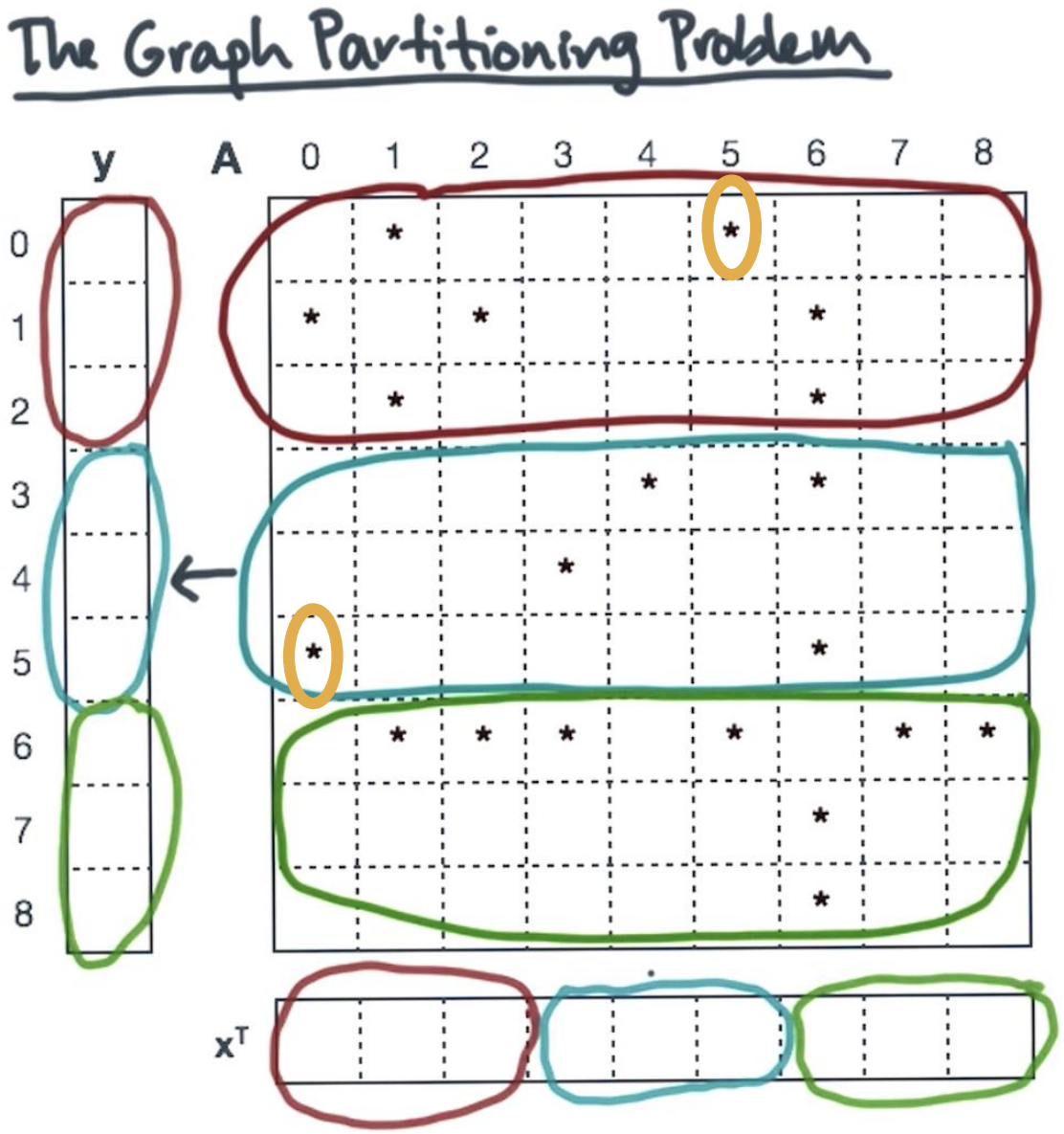


$y \leftarrow A \cdot x$



Goal 1: Balance work! ($\frac{nnz}{proc}$ equal)

eg: distributed computing



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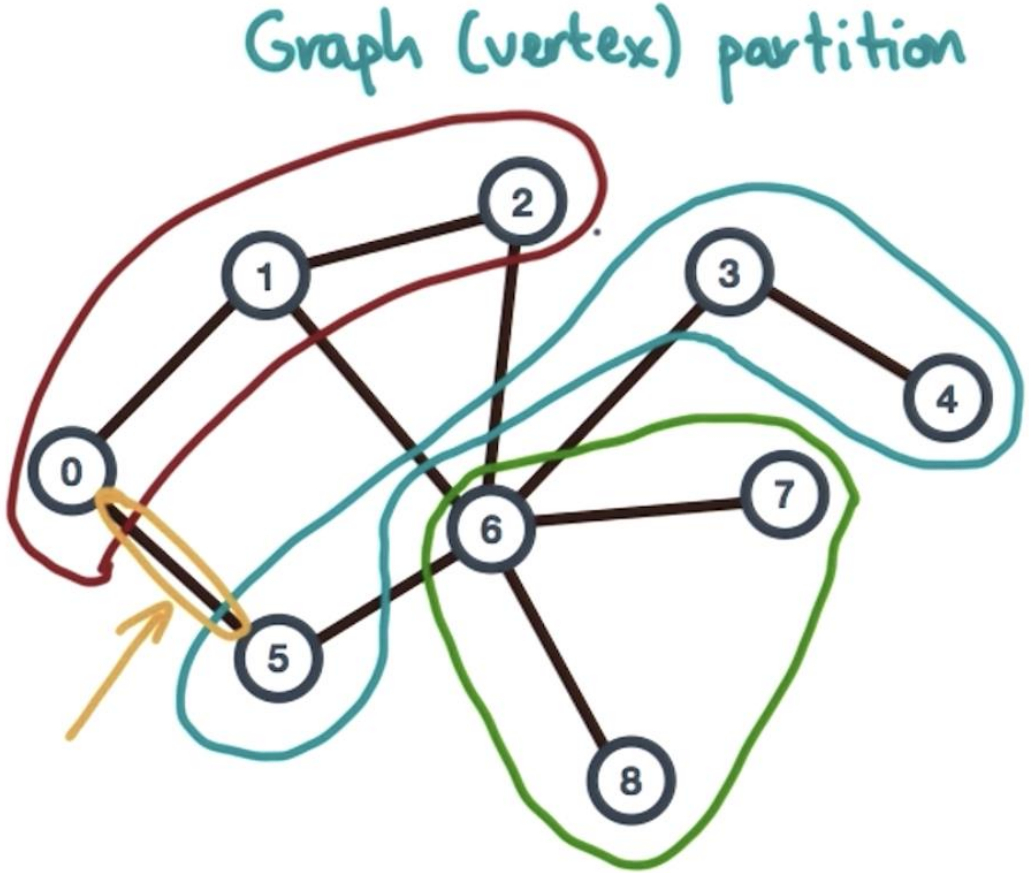

Goal 2: Minimize edge cuts!