# Open Quantum Systems for Quarkonia

**Xiaojun Yao**

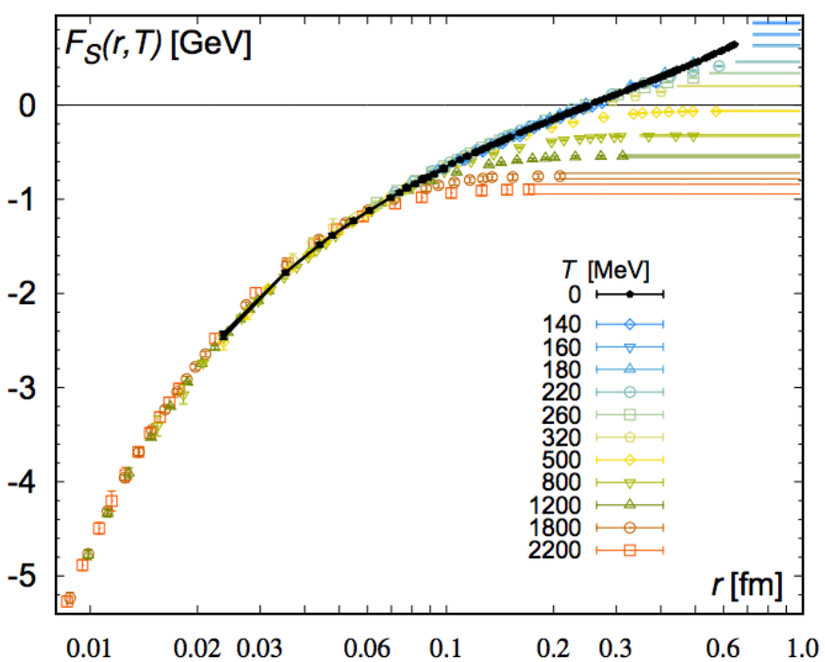
Quarkonia as Tools 2020
Jan. 17, 2020

Centre Paul Langevin, Aussois, France

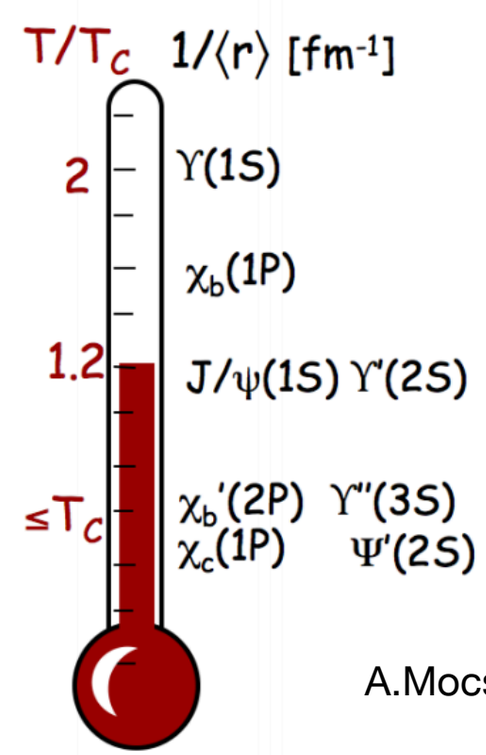
# Quarkonium as Probe of Quark-Gluon Plasma

- **Static screening**: suppression of color attraction —> melting at high T —> reduced production —> thermometer

$$T = 0 : V(r) = -\frac{A}{r} + Br \longrightarrow T \neq 0 :$$ Confining part flattened




1804.10600, A.Bazavov, N.Brambilla
P.Petreczky, A.Vairo, J.H.Weber

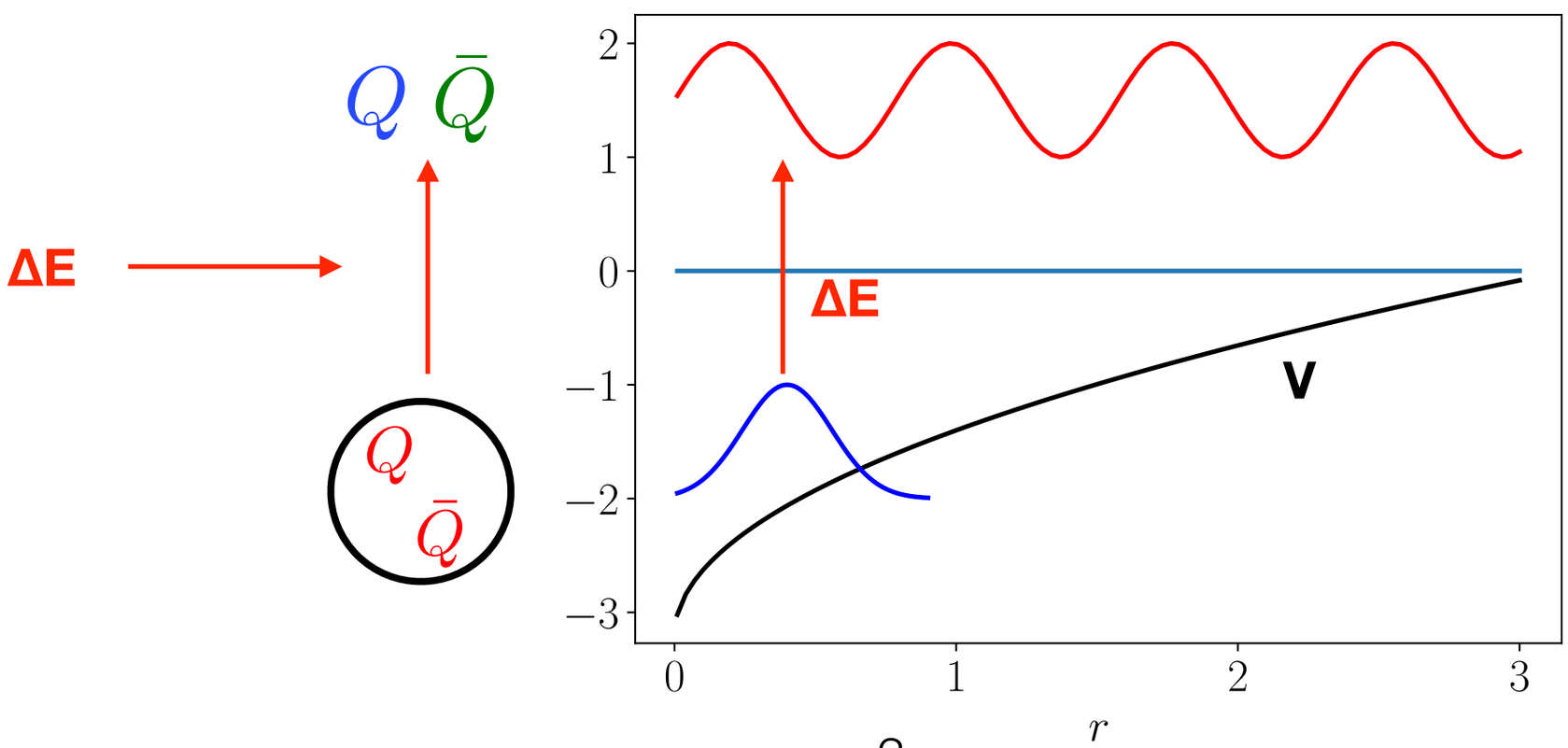



A.Mocsy,0811.0337

# Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer

- **Dynamical screening**: dissociation induced by dynamical process, imaginary potential





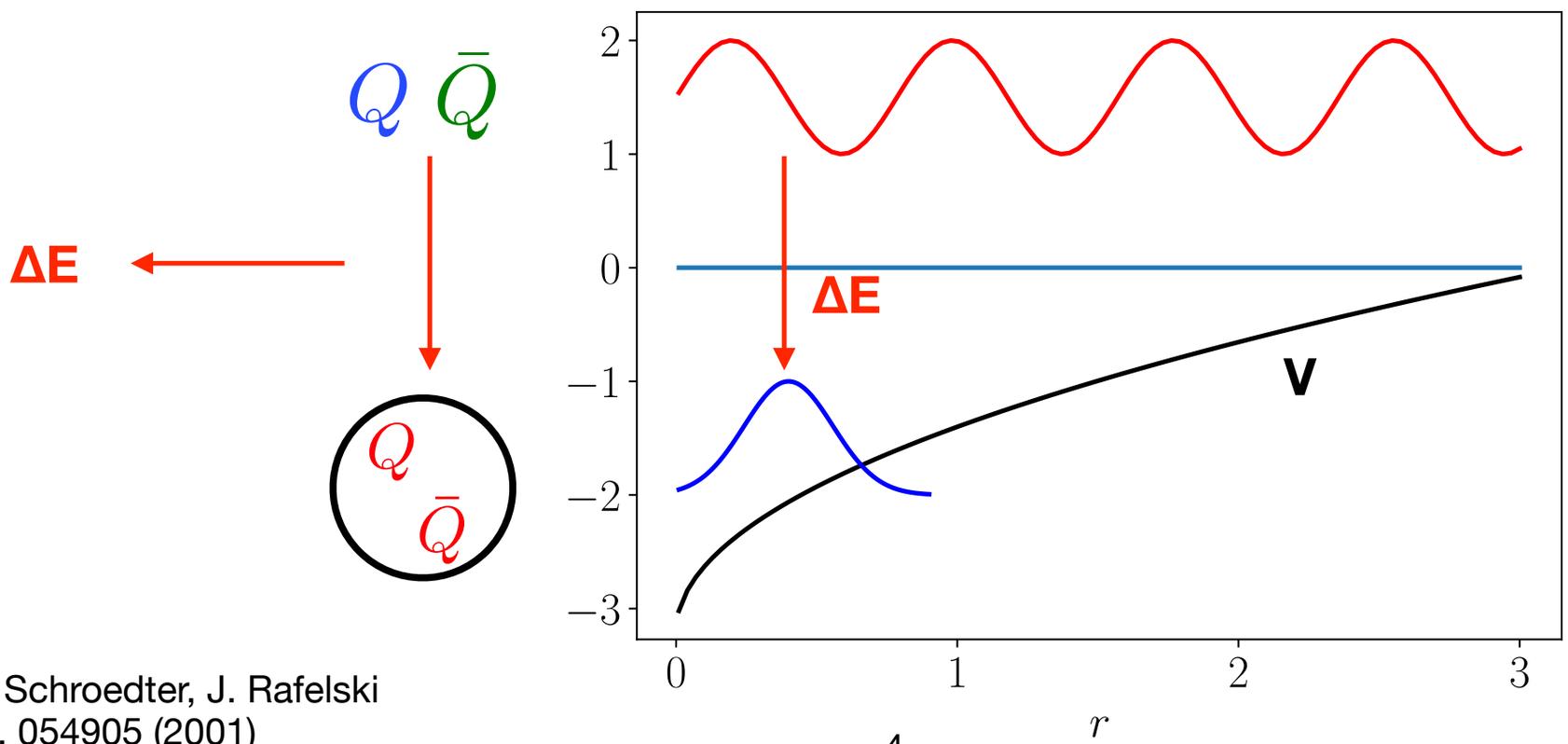

Xiaojun Yao (MIT)

# Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer

- Dynamical screening: dissociation induced by dynamical process, imaginary potential

- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting T



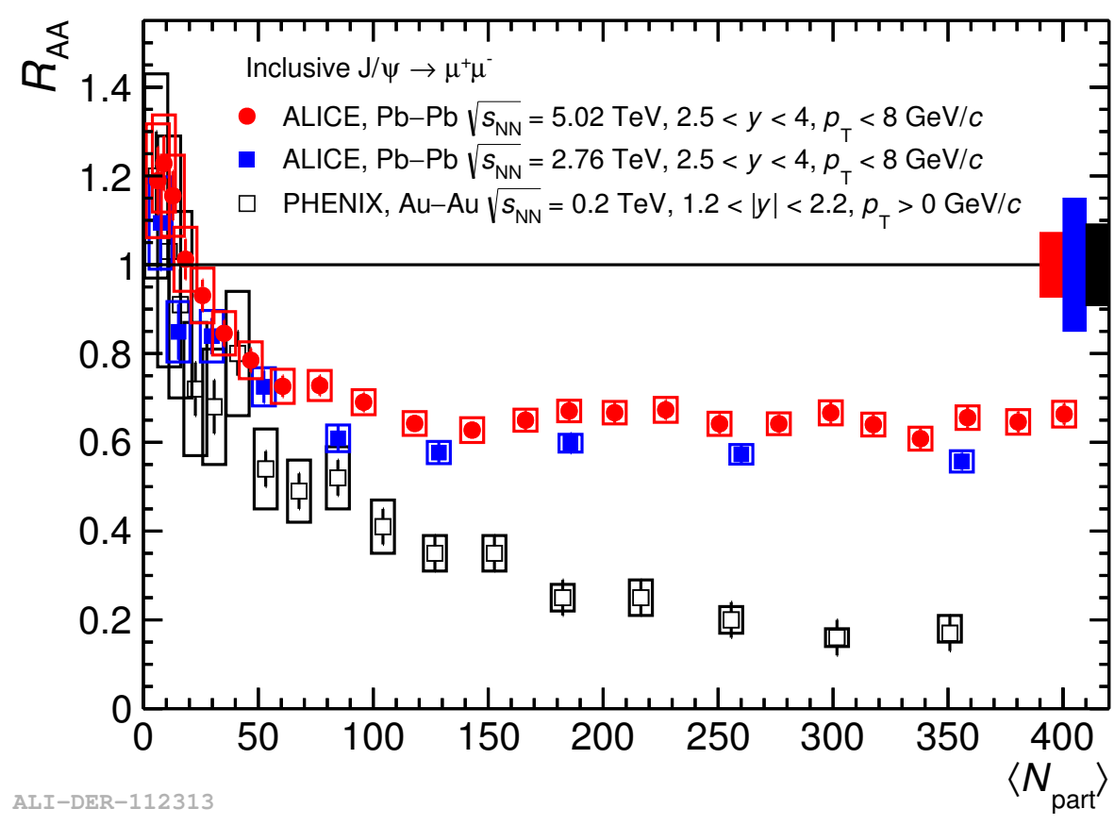

RL. Thews, M. Schroedter, J. Rafelski
Phys.Rev.C 63, 054905 (2001)

# Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> thermometer

- Dynamical screening: dissociation induced by in-medium scattering, imaginary potential

- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting T, **crucial for phenomenology** and theory consistency





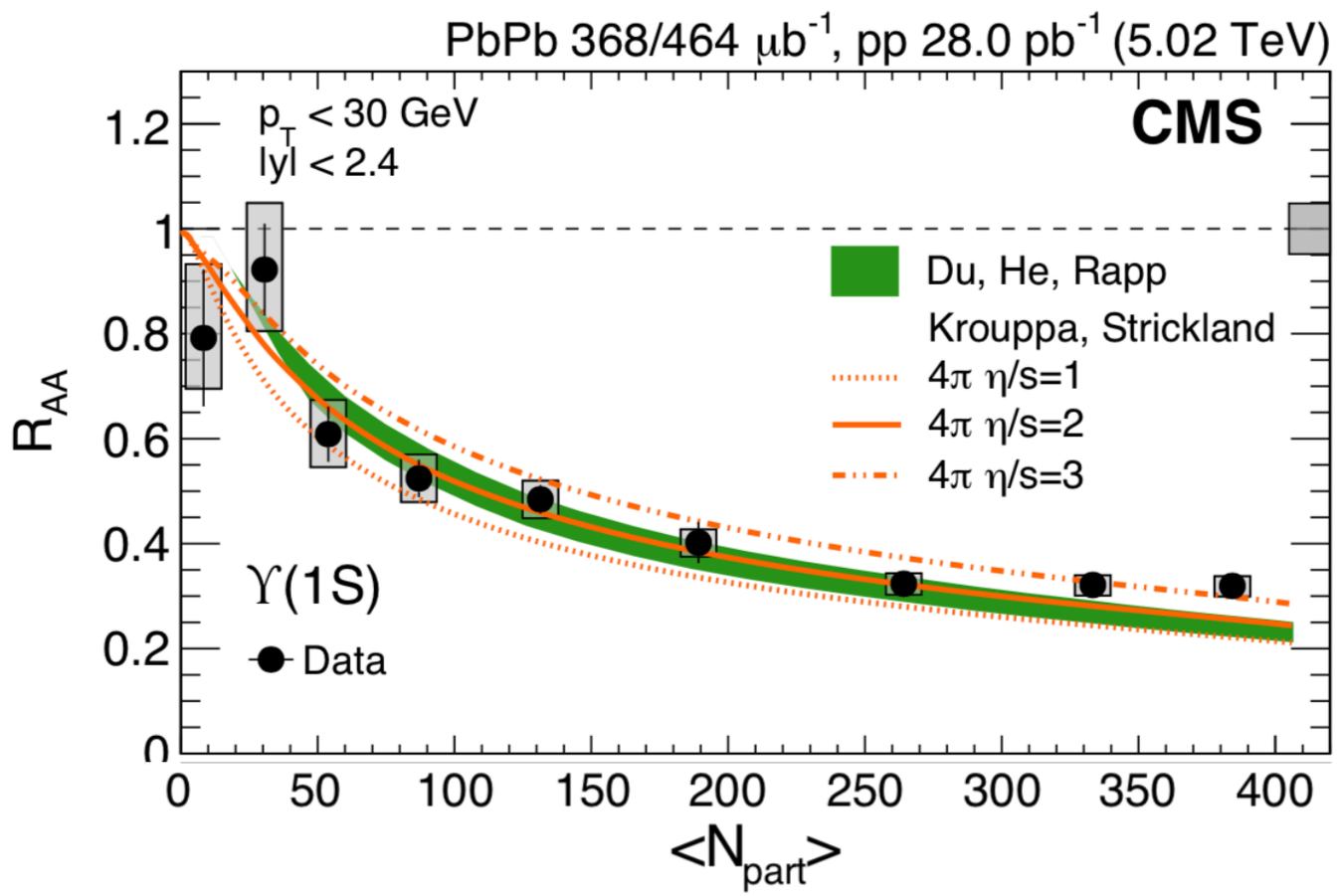

Xiaojun Yao (MIT)

# Phenomenological Success of Transport Theory

## Evolution of distribution in phase space

$$(\partial_t + \boldsymbol{v} \cdot \nabla) f(\boldsymbol{x}, \boldsymbol{p}, t) = -C^{(-)}(\boldsymbol{x}, \boldsymbol{p}, t) + C^{(+)}(\boldsymbol{x}, \boldsymbol{p}, t)$$

**Dissociation** **Recombination**




X.Du, R.Rapp, M.He,1706.08670

B.Krouppa, M.Strickland,1605.03561

See talk by JaeBeom Park

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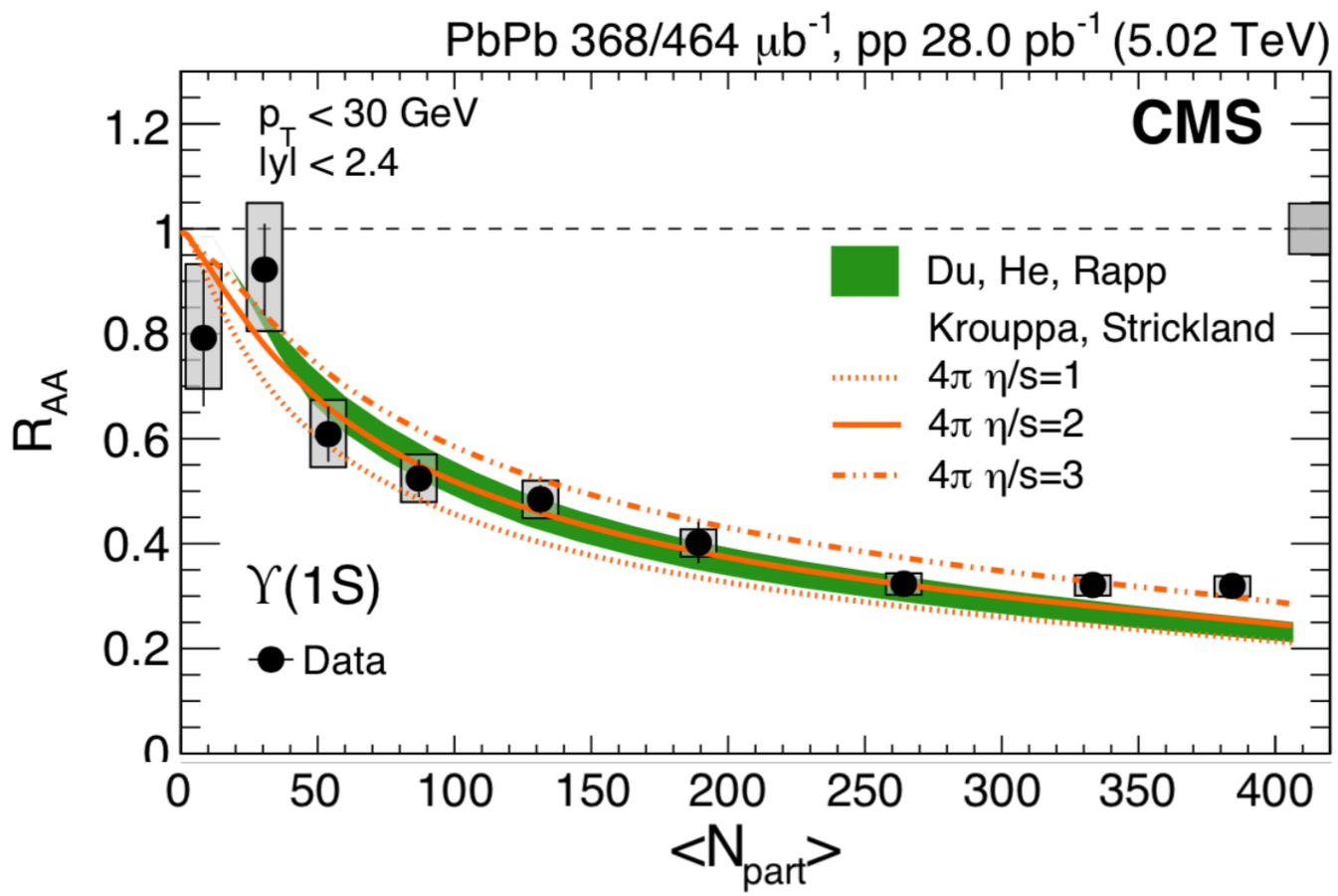



X.Du, R.Rapp, M.He,1706.08670

B.Krouppa, M.Strickland,1605.03561

See talk by JaeBeom Park

**Why transport equation successful? Connection to QCD?**

**Recombination is modeled, calculate from QCD consistently with dissociation?**

# Open Quantum System

$$H = H_S + H_E + H_I$$

$\rho(t=0) = \rho_S \otimes \rho_E$

**System & environment**

**(Heavy quark pairs & QGP)**

**Unitary evolution**

**Time reversible**

$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$

**System & environment**

**Trace out (integrate out) environment**

$\rho_S(t=0)$

**System(heavy quark pairs)**

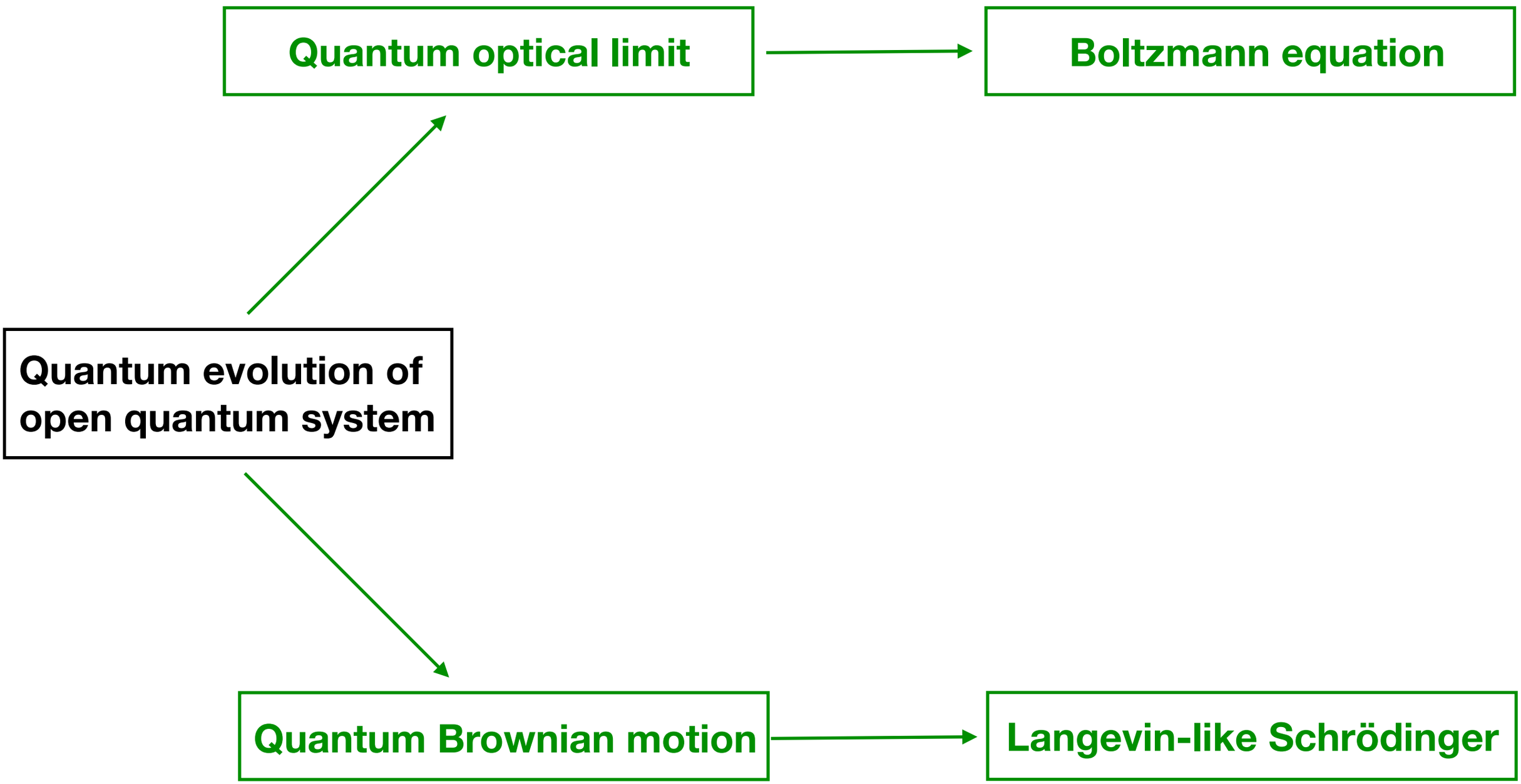
**Non-unitary**

**Time irreversible**

$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$

**Monotonicity of relative entropy under partial trace**

# Two Limits of Open Quantum System Evolution



# Two Limits of Open Quantum System Evolution

**Quantum optical limit** → **Boltzmann equation**

$\tau_R \gg \tau_E$

$\tau_R \gg \tau_S$

$\tau_S$ **System intrinsic time scale**

**Quantum evolution of open quantum system**

$\tau_E$ **Environment correlation time**

$\tau_R \gg \tau_E$

$\tau_S \gg \tau_E$

$\tau_R$ **System relaxation time**

**Quantum Brownian motion** → **Langevin-like Schrödinger**

# Two Limits of Open Quantum System Evolution

**Quantum optical limit**

$\tau_R \gg \tau_E$

$\tau_R \gg \tau_S$

**Quantum evolution of open quantum system**

$\tau_R \gg \tau_E$

$\tau_S \gg \tau_E$

**Quantum Brownian motion**

$$\tau_R \gg \tau_E$$

**During system relaxation, environment correlation has lost —> Markovian process**

$$\tau_R \gg \tau_S$$

**During system relaxation, heavy quark pair has revolved many periods, then it makes sense to use the concept of a well-defined bound state in the calculation**

$$\tau_S \gg \tau_E$$

**System only feels low frequency part of environment correlation**



Xiaojun Yao (MIT)

# Separation of Scales

**Separation of scales in vacuum** $M \gg Mv \gg Mv^2$

**QCD**

$M$ — perturbative matching

**HQET/NRQCD**

Heavy quark physics, A.Manohar, M.Wise
hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

$Mv$ — perturbative matching / non-perturbative matching

**potential NRQCD**

$Mv^2 \sim 500$ MeV

hep-ph/9907240, N.Brambilla
A.Pineda, J.Soto, A.Vairo

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**Inside QGP: thermal scales: T**

$$\tau_E \sim \frac{1}{T} \qquad \tau_S \sim \frac{1}{Mv^2}$$

**Case 1** $Mv \gg T$ **Quantum optical limit**

**Case 2** $T \gg Mv^2$ **Quantum Brownian motion**

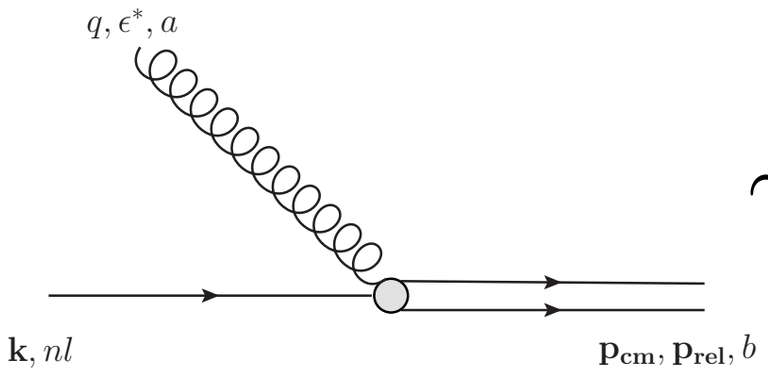
# Quantum Optical Limit

**Separation of scales** $M \gg Mv \gg Mv^2 \gtrsim T$

**NR & multipole expansions**

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r\, {\rm Tr}\Big( {\rm S}^\dagger (i\partial_0 - H_s){\rm S} + {\rm O}^\dagger (iD_0 - H_o){\rm O} + \boxed{V_A({\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}{\rm S} + {\rm h.c.})} + \frac{V_B}{2}{\rm O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, {\rm O}\} + \cdots \Big)$$

**Dipole interaction** $r \sim \dfrac{1}{Mv}$




$$\sim grT \sim g\frac{T}{Mv}$$

**Weak coupling**

Relaxation rate $(grT)^2 T \sim T\dfrac{\alpha_s T^2}{(Mv)^2} \lesssim \alpha_s v^2 T$

**So** $\tau_R \gg \tau_E$

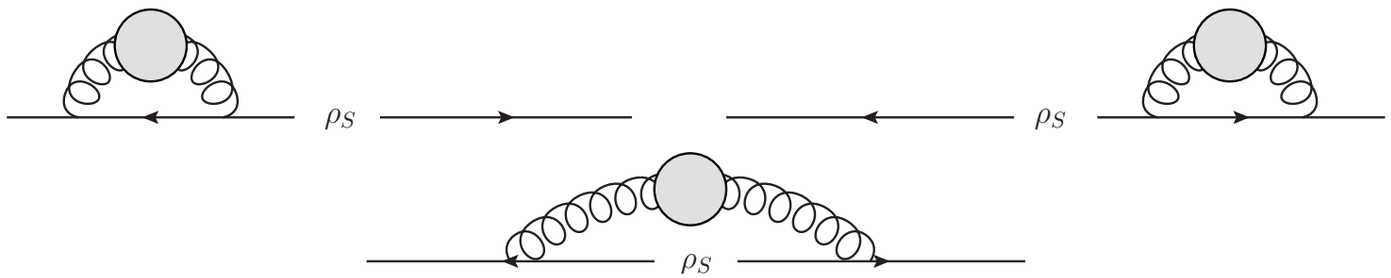
**Markovian**

**Arguments breakdown if (1) large log: Mv —> T, VA has no running at one loop**
**(2) large pT: medium boosted in rest frame of quarkonium, constrain to low pT**

# From Open Quantum System to Transport Equation

**Lindblad equation:**

$$\rho_S(t) = \rho_S(0) - i\Big[tH_S + \sum_{a,b} \sigma_{ab}(t) L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d} \gamma_{ab,cd}\Big(L_{ab}\rho_S(0)L^\dagger_{cd} - \frac{1}{2}\{L^\dagger_{cd}L_{ab}, \rho_S\}\Big)$$



# From Open Quantum System to Transport Equation

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**Markovian approximation**

**Wigner transform (smearing for positivity)**

$$f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \langle \boldsymbol{k} + \frac{\boldsymbol{k}'}{2}, nl, 1|\rho_S(t)|\boldsymbol{k} - \frac{\boldsymbol{k}'}{2}, nl, 1\rangle$$

**Boltzmann transport equation**

$$\frac{\partial}{\partial t} f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^{(+)}_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^{(-)}_{nls}(\boldsymbol{x}, \boldsymbol{k}, t)$$

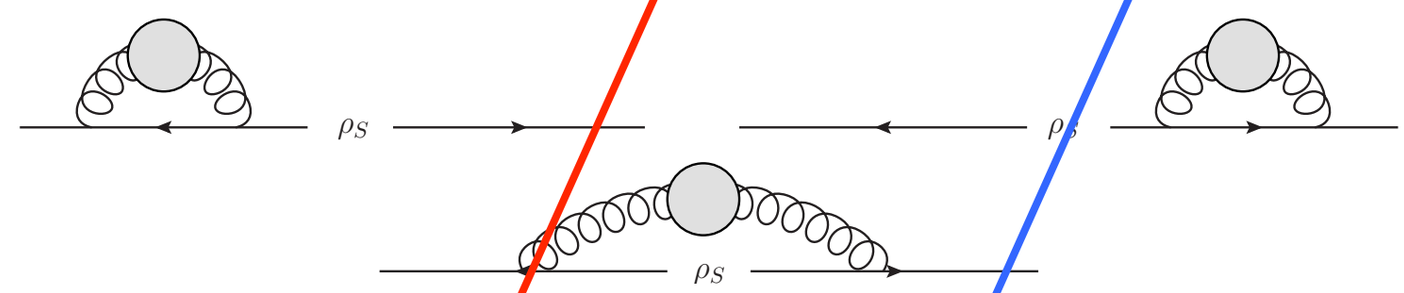


XY T.Mehen,1811.07027

# From Open Quantum System to Transport Equation

**Lindblad equation:** **Correction to Hamiltonian, static screening**

$$\rho_S(t) = \rho_S(0) - i\Big[tH_S + \sum_{a,b} \sigma_{ab}(t) L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d} \gamma_{ab,cd}\Big(L_{ab}\rho_S(0)L^\dagger_{cd} - \frac{1}{2}\{L^\dagger_{cd}L_{ab}, \rho_S\}\Big)$$




**Recombination** **Dissociation**

**Boltzmann transport equation**

$$\frac{\partial}{\partial t} f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^{(+)}_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^{(-)}_{nls}(\boldsymbol{x}, \boldsymbol{k}, t)$$



XY T.Mehen,1811.07027

# Detailed Balance and Phenomenology

**Demonstrate detailed balance dynamically**

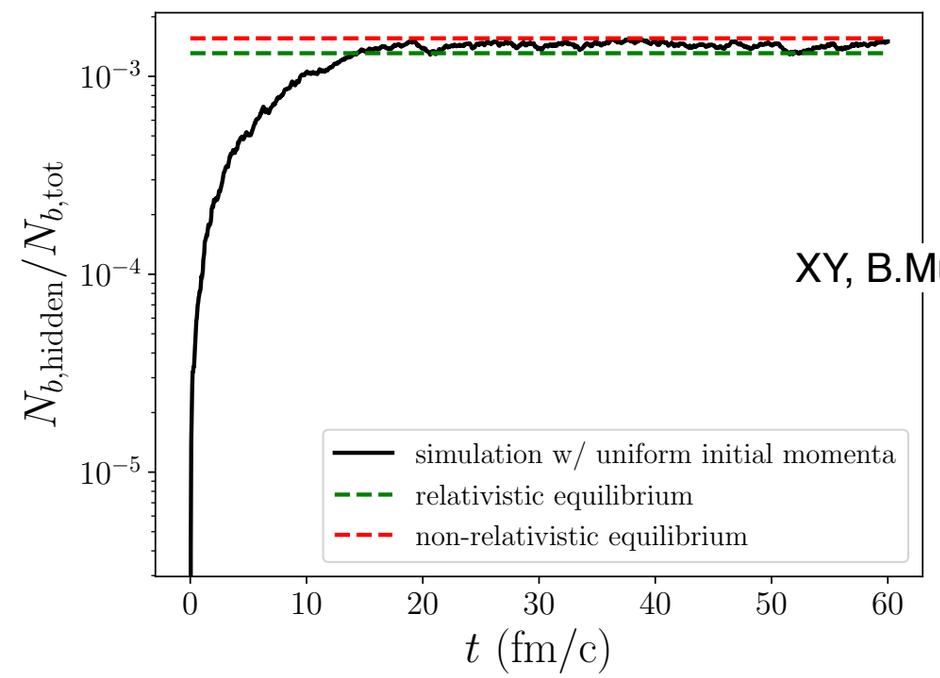
$$O^\dagger \boldsymbol{r} \cdot g\boldsymbol{E}S$$

↓

**"random"**

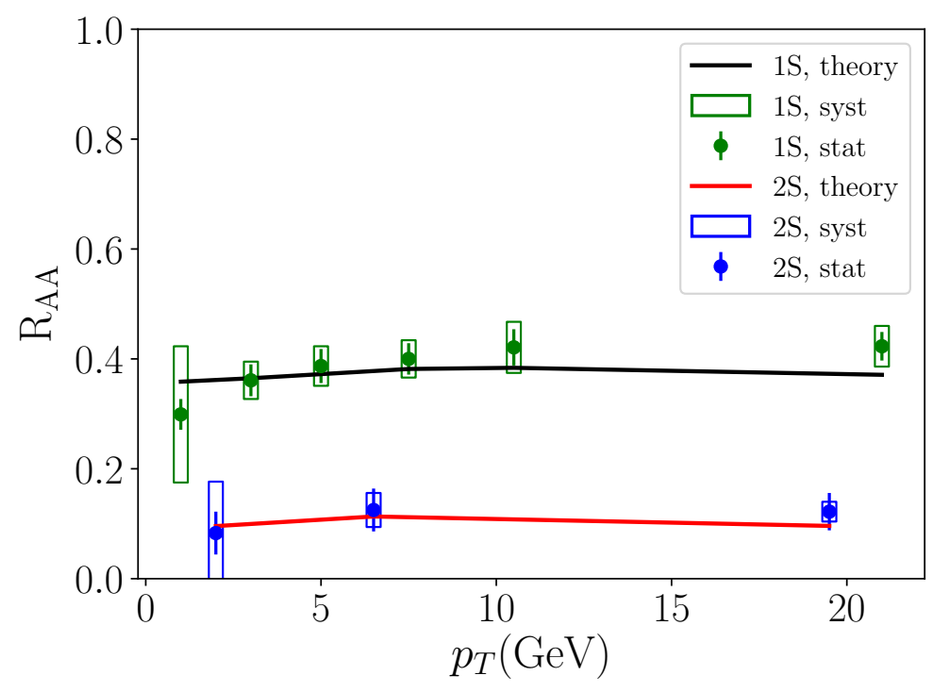
**Insight from random matrix theory**

Quarkonium percentage v.s. time




XY, B.Müller,1709.03529

**Phenomenology: coupled transports**






XY,W.Ke,Y.Xu,S.Bass,B.Müller, 1807.06199, 1812.02238

# Quantum Brownian Motion Limit

**Quantum evolution of heavy quark pair density matrix (not necessarily Lindblad)**

$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}\right] + \sum_{i=1}^{N_{LB}} \gamma_i \left(\hat{L}_i \rho_{Q\bar{Q}} \hat{L}_i^\dagger - \frac{1}{2}\hat{L}_i \hat{L}_i^\dagger \rho_{Q\bar{Q}} - \frac{1}{2}\rho_{Q\bar{Q}} \hat{L}_i \hat{L}_i^\dagger\right)$$

**Only real potential** — **Encoding imaginary potential**

**Quantum state diffusion method**

**Directly solve density matrix evolution**

**Saclay**: color singlet and octet

J-P Blaizot, M.A.Escobedo…

**TUM**: pNRQCD, color singlet and octet

N.Brambilla, M.A.Escobedo, A.Vairo…

See talks by Stéphane Delorme, Denys Yen Arrebato Villar

**Map onto Langevin-like Schrödinger**

$$i\frac{\partial}{\partial t}\psi_{Q\bar{Q}} = (H_{Q\bar{Q}} - i\gamma + \xi)\psi_{Q\bar{Q}}$$

**Dissipation** — **Fluctuation**

**Nantes** R.Katz, P-B Gossiaux…

**Stavanger/Osaka**

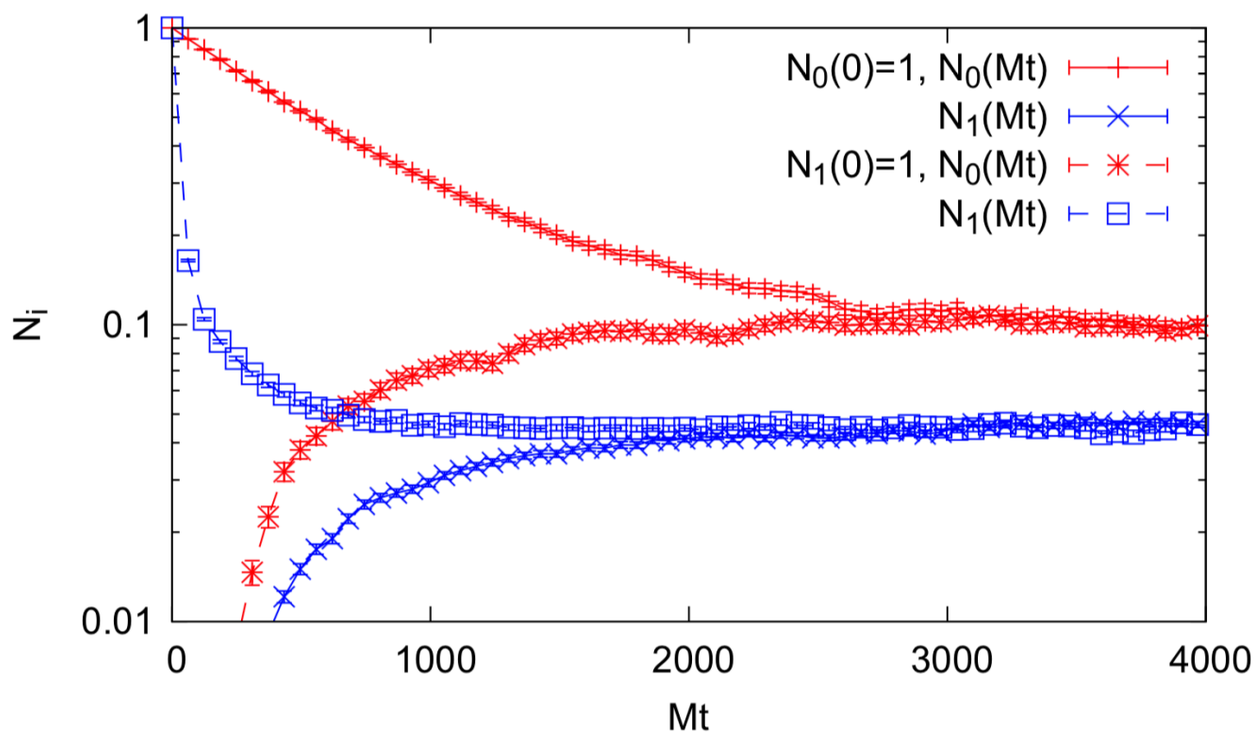
Y.Akamatsu, M.Asakawa, A.Rothkopf…

# Langevin-like (Stochastic) Schrödinger Equation

$$i\frac{\partial}{\partial t}\psi_{Q\bar{Q}} = (H_{Q\bar{Q}} - i\gamma + \xi)\psi_{Q\bar{Q}}$$

**Dissipation** **Fluctuation**

**Solve in position space, start with a specific state, compute** $|\langle\psi_{nls}|\psi_{Q\bar{Q}}(t)\rangle|^2$




T. Miura, Y.Akamatsu, M.Asakawa, A.Rothkopf, 1908.06293

**Stochastic term—>decoherence of wavefunction—>dissociation**

**Recombination is automatically included in this approach**

# Conclusion

- Open quantum system approach for quarkonium inside QGP
  - Quantum optical limit
    - Separation of scales justifies approximations

$$M \gg Mv \gg Mv^2 \gtrsim T$$

    - Lindblad equation —> Boltzmann transport equation
    - Explain why transport equation works
  - Quantum Brownian motion limit
    - Stochastic Schrödinger equation
    - Wavefunction decoherence