Quarkonia As Tools 2020

Centre Paul Langevin

Exclusive double quarkonium production & Generalized TMDs of gluons

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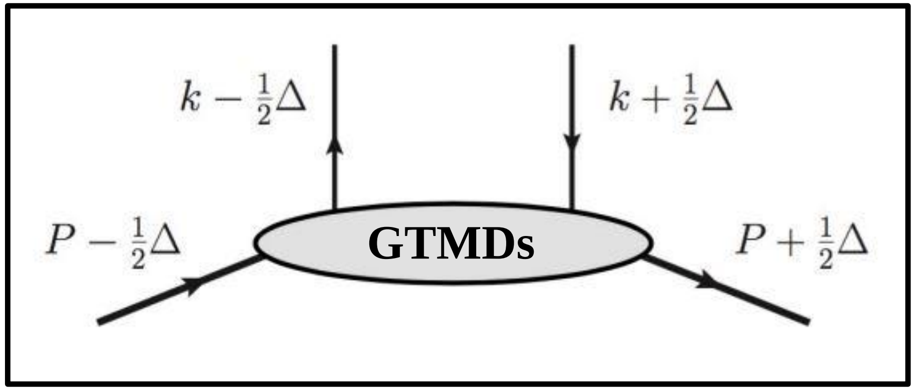
Outline

- Generalized TMDs (GTMDs)
- Wigner Functions
- Observables for GTMDs : The state of art
- Summary

Generalized Transverse Momentum-dependent Distributions

- **GTMD matrix element – a graphical depiction :**



Symmetric frame

$$P = \frac{p+p'}{2} \qquad \vec{P}_{\perp} = 0 \qquad \Delta = p' - p$$

P: **average nucleon momentum**

Δ: **momentum transfer to nucleon**

- **GTMD correlator – definition :**

Quark polarization → $[\Gamma]$

$$W^{q[\Gamma]}_{\lambda,\lambda'} = \frac{1}{2}\int \frac{dz^- d^2\vec{z}_{\perp}}{(2\pi)^3}\, e^{ik.z} < p',\lambda'|\bar{\psi}^q(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi^q(\tfrac{z}{2})|p,\lambda > \Big|_{z^+=0}$$

- **Parameterization of correlator through GTMDs :** $X^q(x,\xi,\vec{k}^2_{\perp},\vec{\Delta}^2_{\perp},\vec{k}_{\perp}\cdot\vec{\Delta}_{\perp})$

x **: "average" longitudinal momentum fraction of quark**

$\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$ **: longitudinal momentum transfer to nucleon**

$\vec{k}_{\perp}$ **: "average" transverse momentum of quark**

Generalized Transverse Momentum-dependent Distributions

- **Twist-2 projections :** $\Gamma = \gamma^+ \quad \gamma^+\gamma_5 \quad i\sigma^{j+}\gamma_5$

- **Leading-twist chiral even case : (Meissner, Metz, Schlegel, 0906.5323)**

$$W^{q[\gamma^+]}_{\lambda,\lambda'} = \frac{1}{2M}\bar{u}(p',\lambda')\left[\boldsymbol{F^q_{1,1}} + \frac{i\sigma^{i+}k^i_\perp}{P^+}\boldsymbol{F^q_{1,2}} + \frac{i\sigma^{i+}\Delta^i_\perp}{P^+}\boldsymbol{F^q_{1,3}} + \frac{i\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}\boldsymbol{F^q_{1,4}}\right]u(p,\lambda)$$

$$W^{q[\gamma^+\gamma_5]}_{\lambda,\lambda'} = \frac{1}{2M}\bar{u}(p',\lambda')\left[-\frac{i\epsilon^{ij}k^i_\perp\Delta^j_\perp}{M^2}\boldsymbol{G^q_{1,1}} + \frac{i\sigma^{i+}\gamma_5 k^i_\perp}{P^+}\boldsymbol{G^q_{1,2}} + \frac{i\sigma^{i+}\gamma_5\Delta^i_\perp}{P^+}\boldsymbol{G^q_{1,3}} + i\sigma^{+-}\gamma_5\boldsymbol{G^q_{1,4}}\right]u(p,\lambda)$$

- **General results :**

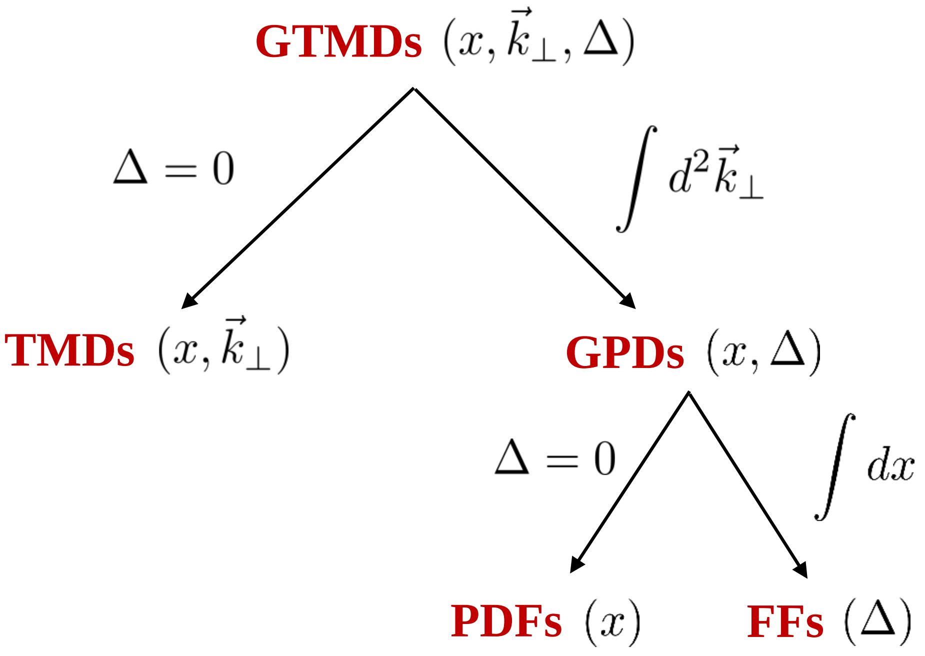
  i. **16 leading-twist GTMDs for quarks**

  ii. **16 leading-twist GTMDs for gluons (Lorcé, Pasquini, 1307.4497)**

  iii. **GTMDs are complex functions**

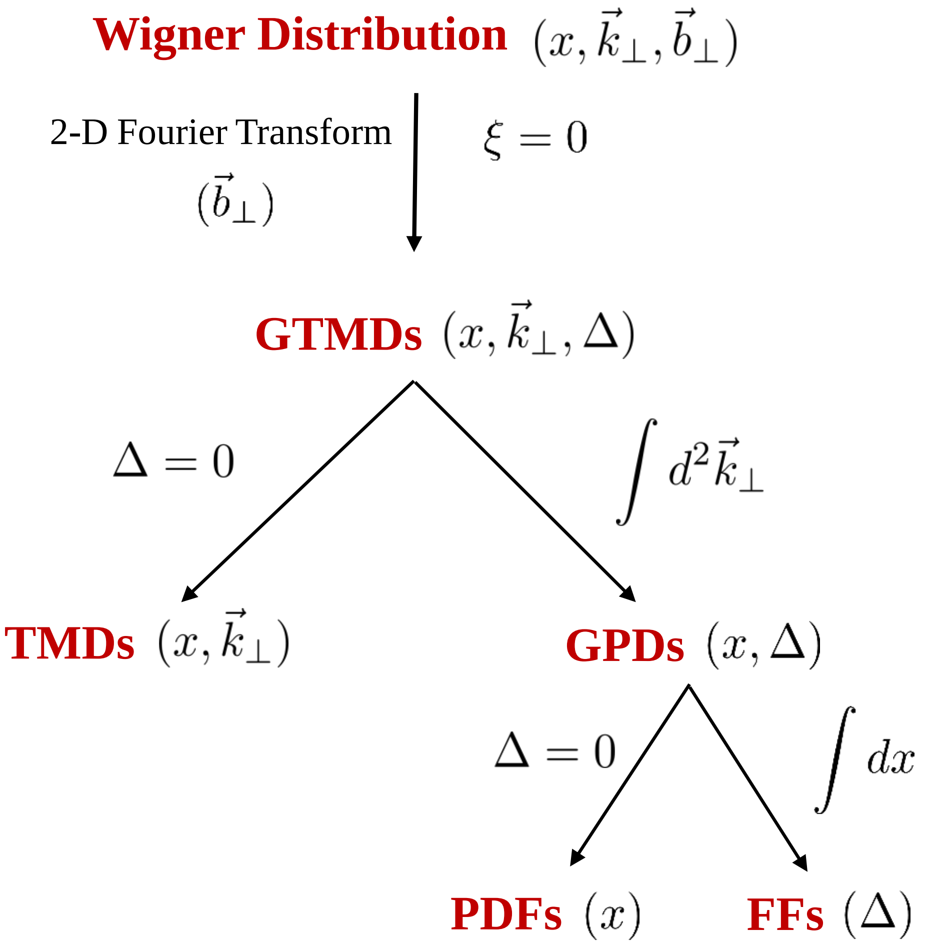
Why are GTMDs interesting?

GTMDs as "Mother Functions"



GTMDs contain new physics (beyond TMDs & GPDs)

GTMDs as "Mother Functions"



GTMDs contain new physics (beyond TMDs & GPDs)

GTMDs as "Mother Functions"

Wigner Distribution $(x, \vec{k}_\perp, \vec{b}_\perp)$ → **Going 5-dimensional!**

2-D Fourier Transform $(\vec{b}_\perp)$ ↓ $\xi = 0$

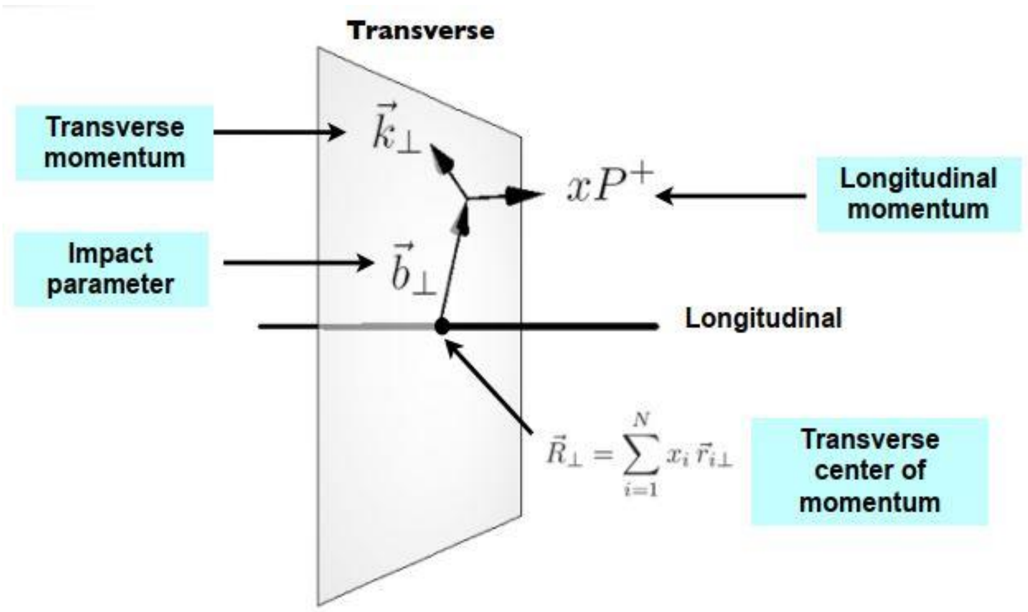
GTMDs $(x, \vec{k}_\perp, \Delta)$

$\Delta = 0$ ↙ ↘ $\int d^2\vec{k}_\perp$

TMDs $(x, \vec{k}_\perp)$ **GPDs** (x, Δ)

$\Delta = 0$ ↙ ↘ $\int dx$

PDFs (x) **FFs** (Δ)



Barbara Pasquini, talk at INT 2018

GTMDs contain new physics (beyond TMDs & GPDs)

First numerical calculation of Wigner distributions in a light-cone constituent model (Lorcé, Pasquini, 1106.0139)

Wigner distributions

Wigner distributions in NRQM
(Wigner, 1932)

- **Calculate from wave-functions :**

$$W(x,k) = \int \frac{dx'}{2\pi} e^{-ikx'} \psi(x + \frac{x'}{2})\, \psi^*(x - \frac{x'}{2})$$

- **Connection with probability densities & observables :**

- **Position-space probability :** $$|\psi(x)|^2 = \int dk\, W(x,k)$$
- **Momentum-space probability :** $$|\psi(k)|^2 = \int dx\, W(x,k)$$
- **Expectation value of observables :**

$$< O > = \int dx \int dk\ O(x,k)\, W(x,k)$$

Wigner distributions in parton physics
(Belitsky, Ji, Yuan, 2003)

- **Defined through F.T. of GTMD correlator :**

$$W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} W^{[\Gamma]}(x, \vec{k}_\perp, \vec{\Delta}_\perp)\Big|_{\xi=0}$$

- **Application :**

Orbital Angular Momentum (OAM)

- **Other definitions for OAM available**
  (Ji & Jaffe-Manohar definitions)
- **Most intuitive definition via Wigner functions**

$$< L_z^q > =$$
$$\int dx \int d^2\vec{k}_\perp \int d^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\Big|_z W_L^{q[\gamma^+]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

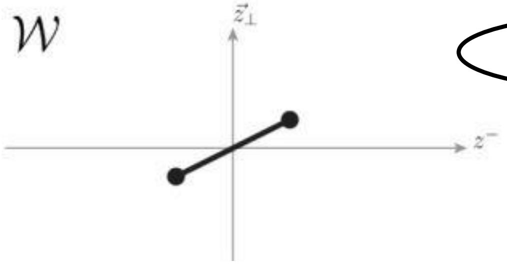
GTMDs & Parton orbital motion

GTMD & parton OAM (Lorcé, Pasquini, 2011/ Hatta, 2011)

$$L_z = \int dx \int d^2\vec{k}_\perp \int d^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\Big|_z W_L^{[\gamma^+]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

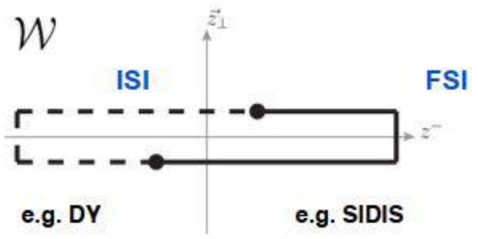
$$= -\int dx \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} \boxed{Re.} \boldsymbol{F_{1,4}(x, \vec{k}_\perp^2)}\Big|_{\xi=0}$$

i. **Gauge-invariant extension :**



Ji's OAM

Ji, Xiong, Yuan, 2012/ Lorcé, 2013

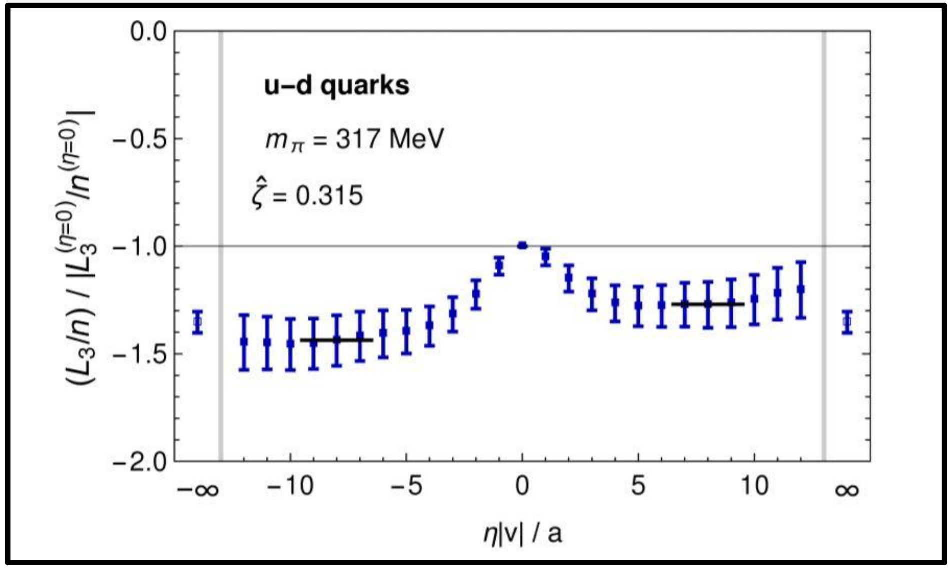


Jaffe-Manohar's OAM

Hatta, 2011

ii. **Same equation for gluons**

GTMDs & Parton orbital motion

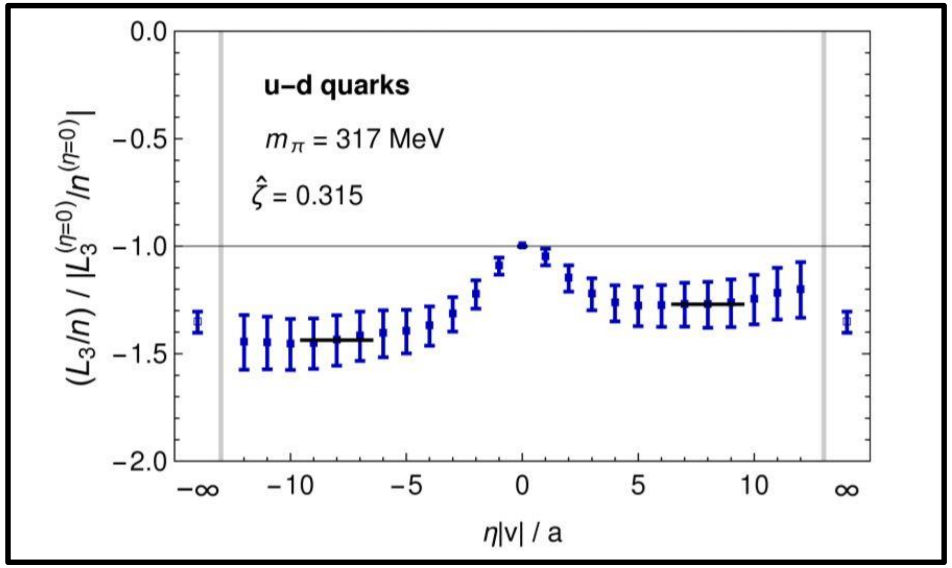


First lattice calculation of L_{JM} vs. L_{Ji}
(Engelhardt, 1701.01536)

i. **Figure shows** $L_{\mathrm{JM}}^{u-d}/L_{\mathrm{Ji}}^{u-d}$

ii. **Significant numerical differences between** L_{JM} **&** L_{Ji}

GTMDs & Parton orbital motion



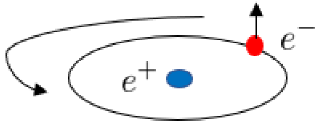
First lattice calculation of $L_{\rm JM}$ vs. $L_{\rm Ji}$
(Engelhardt, 1701.01536)

i. **Figure shows $L^{u-d}_{\rm JM}/L^{u-d}_{\rm Ji}$**

ii. **Significant numerical differences between $L_{\rm JM}$ & $L_{\rm Ji}$**

GTMDs related to strength of spin-orbit correlation
(Lorcé, Pasquini, 2011 / Lorcé, 2014)

Recall Spin-Orbit coupling in H atom!

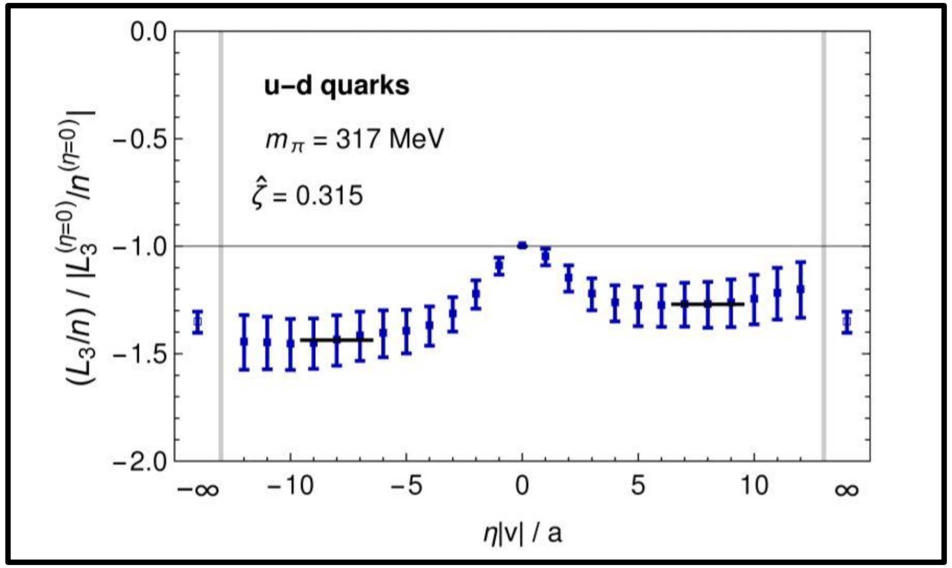


$F_{1,4}^{q/g}$	$\vec{L}_{q/g} \cdot \vec{S}_N$
$G_{1,1}^{q/g}$	$\vec{L}_{q/g} \cdot \vec{S}_{q/g}$

Nucleon

Quark & Gluon

GTMDs & Parton orbital motion



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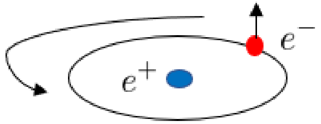
GTMDs related to strength of spin-orbit correlation
(Lorcé, Pasquini, 2011 / Lorcé, 2014)

GTMD & Sivers function
(talk by R. Bousarrie on Wednesday)

Recall Spin-Orbit coupling in H atom!



$F^{q/g}_{1,4}$ $\quad \vec{L}_{q/g} \cdot \vec{S}_N$ → Nucleon

$G^{q/g}_{1,1}$ $\quad \vec{L}_{q/g} \cdot \vec{S}_{q/g}$ → Quark & Gluon

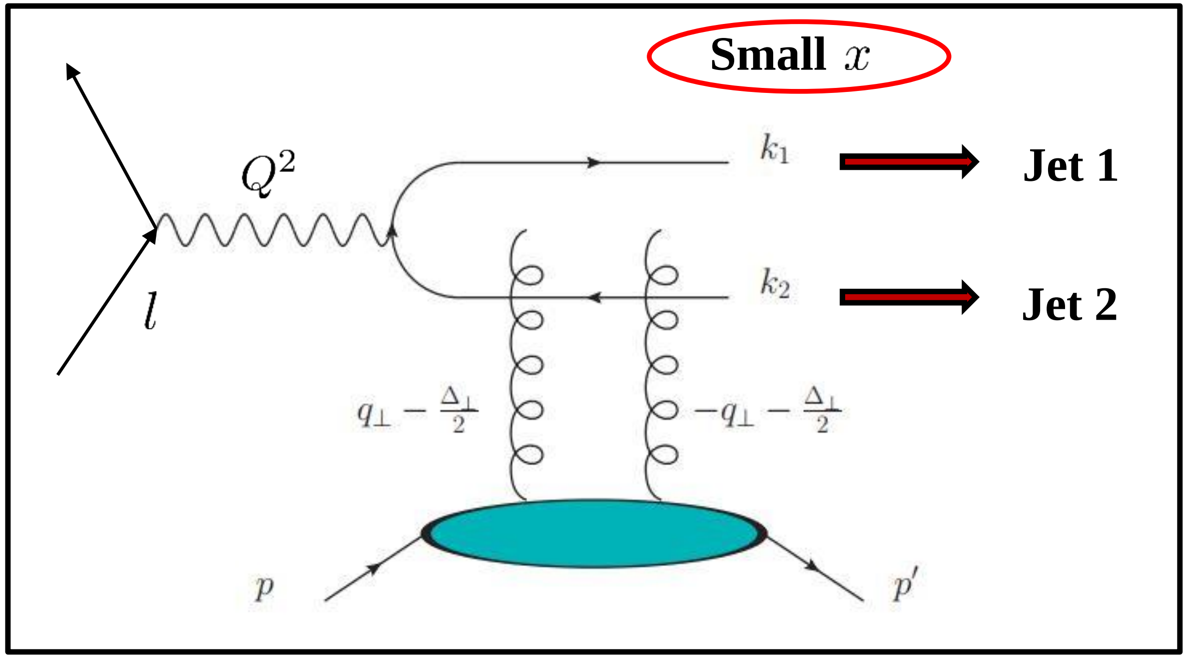
$$Im.\, F_{1,2}\Big|_{\Delta=0} = -f^{\perp}_{1T}$$

Probe Sivers function through unpolarized target!

Observables for GTMDs

Exclusive Dijet Production in lN/lA collisions

Hatta, Xiao, Yuan, 1601.01585/ Altinoluk, Armesto, Beuf, Rezaeian, 1511.07452



Independent transverse vectors

$$\vec{P}_\perp = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$$

$$\vec{\Delta}_\perp = -(\vec{k}_{1\perp} + \vec{k}_{2\perp})$$

- **Parameterization of angular dependence of Wigner distribution :**

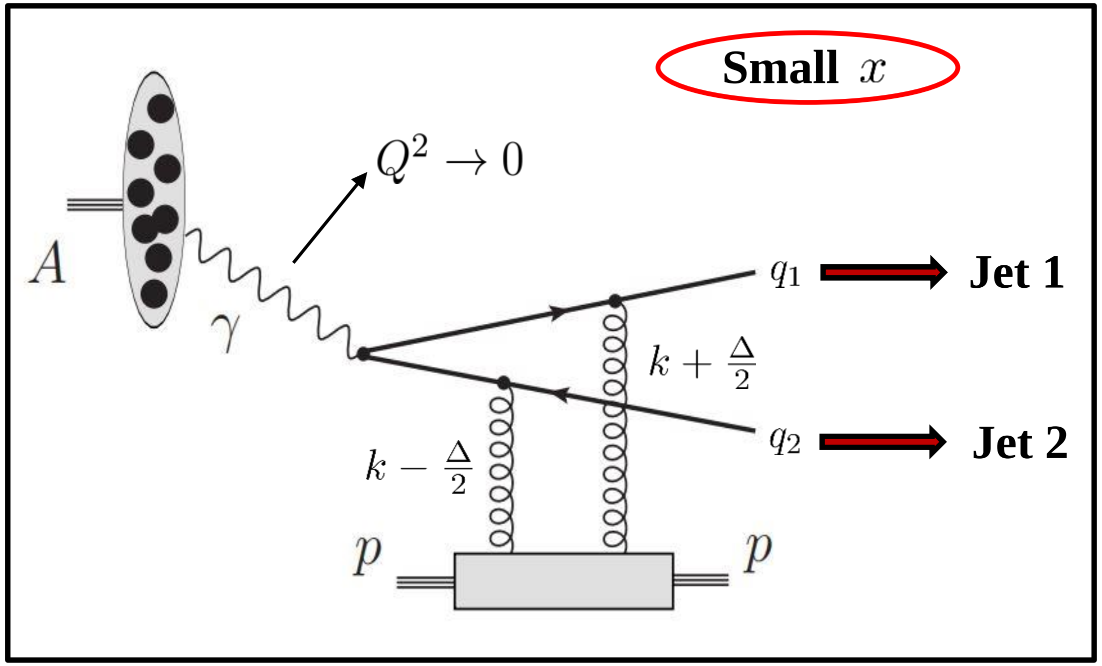
$$W(x, \vec{k}_\perp, \vec{b}_\perp) \approx W_0(x, |\vec{k}_\perp|, |\vec{b}_\perp|) + 2\cos 2\,(\phi_k - \phi_b)\, W_1(x, |\vec{k}_\perp|, |\vec{b}_\perp|) \longleftarrow$$ **"Elliptic Wigner Distribution"**

- **Cross-section :** $d\sigma \approx d\sigma_0 + 2\cos 2\,(\phi_{P_\perp} - \phi_{\Delta_\perp}) d\tilde{\sigma}$

Cosine angular modulation

Exclusive Dijet Production in Ultra-Peripheral pA/AA collisions

Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev, 1706.01765



- **Cross-section :** $d\sigma \approx A^2 + 2\cos 2\,(\phi_{q_\perp} - \phi_{\Delta_\perp})AB$

Cosine angular modulation

- **"A" & "B" related to azimuthally symmetric & elliptic components of Wigner function**

Independent transverse vectors

$$\vec{q}_\perp = \frac{1}{2}(\vec{q}_{2\perp} - \vec{q}_{1\perp})$$

$$\vec{\Delta}_\perp = -(\vec{q}_{1\perp} + \vec{q}_{2\perp})$$

Exclusive Dijet Production in lN collisions

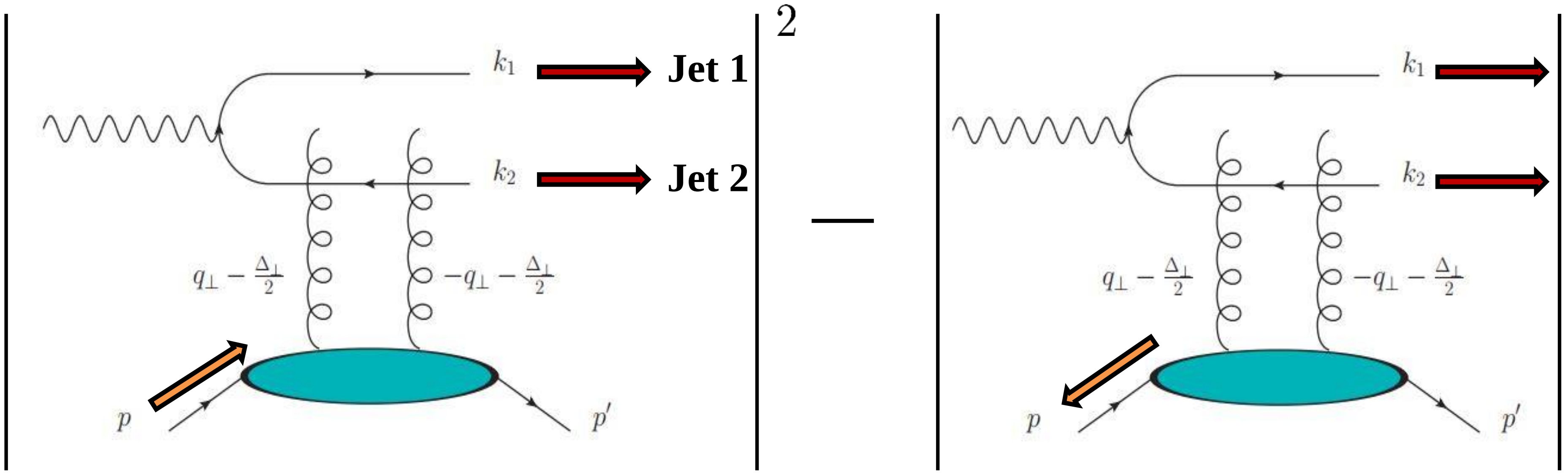
Hatta, Nakagawa, Xiao, Yuan, Zhao, 1612.02445/ Ji, Yuan, Zhao, 1612.02438

Access to gluon Orbital Angular Momentum at small / medium x through longitudinal spin-asymmetry

Example:

Small x

$d\sigma =$



$$\approx \sin(\phi_{P_\perp} - \phi_{\Delta_\perp}) L_g(x) \int d^2q_\perp\, q_\perp^2\, O(x, q_\perp)$$

Sinusoidal angular modulation

OAM

"Odderon"

Independent transverse vectors

$$\vec{P}_\perp = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$$

$$\vec{\Delta}_\perp = -(\vec{k}_{1\perp} + \vec{k}_{2\perp})$$

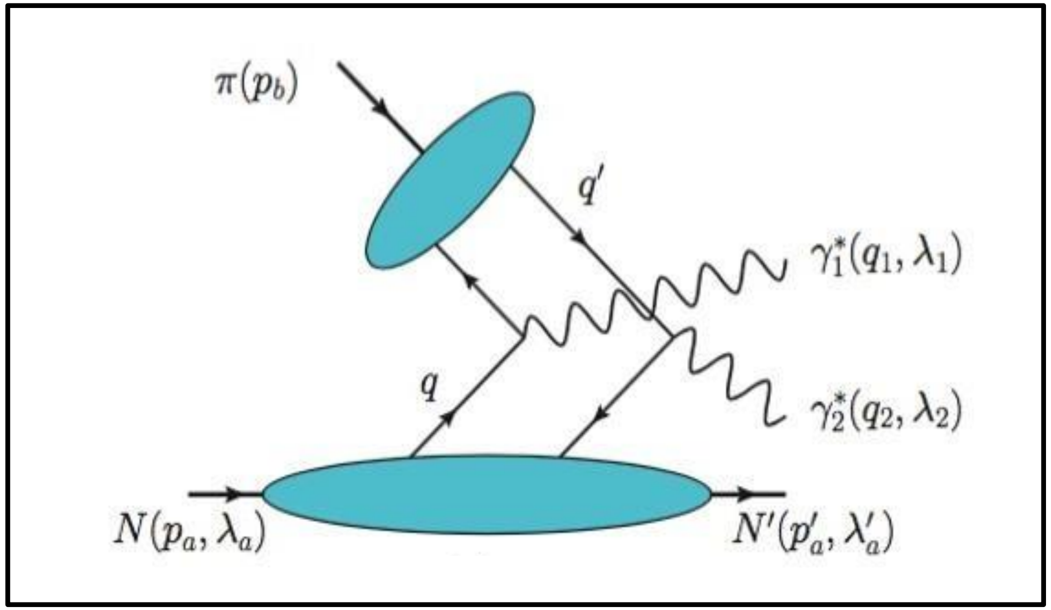
Exclusive Double Drell -Yan Process

SB, Metz, Zhou, 1702.04387

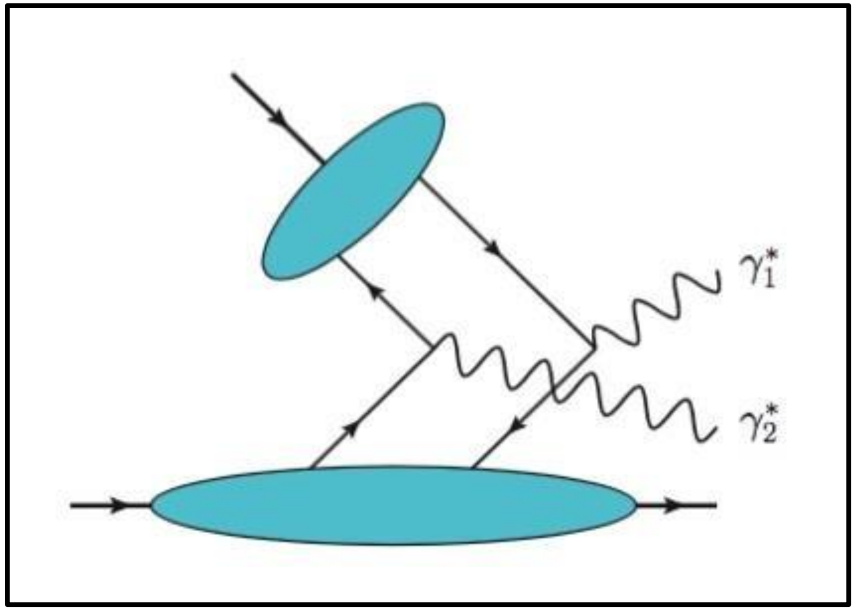
Sensitivity to quark GTMDs

Process $\pi(p_b)\, N(p_a, \lambda_a) \to (l_1^- l_1^+)(l_2^- l_2^+) N'(p_a, \lambda_a)$

Leading-order diagrams



Graph (a)

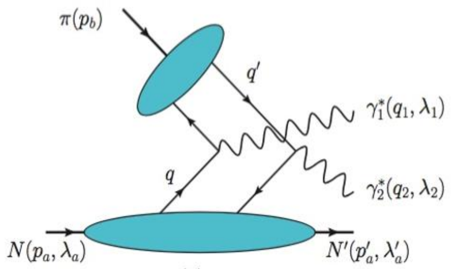


Graph (b)

Scattering Amplitude

$$\mathcal{T}^{\mu\nu}_{\lambda_a,\lambda_a'} = i\sum_{q,q'} e_q e_q' e^2 \frac{1}{N_c}\int d^2\vec{k}_{a\perp}\int d^2\vec{k}_{b\perp}\delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2}-\vec{k}_{a\perp}-\vec{k}_{b\perp}\right)\Phi_\pi^{q'q}(x_b,\vec{k}^2_{b\perp})$$
$$\Big[-i\varepsilon_\perp^{\mu\nu}\Big(W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda_a'}(x_a,\vec{k}_{a\perp}) - W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda_a'}(-x_a,-\vec{k}_{a\perp})\Big)$$
$$-g_\perp^{\mu\nu}\Big(W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda_a'}(x_a,\vec{k}_{a\perp}) + W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda_a'}(-x_a,-\vec{k}_{a\perp})\Big)\Big]$$

Graph (a)

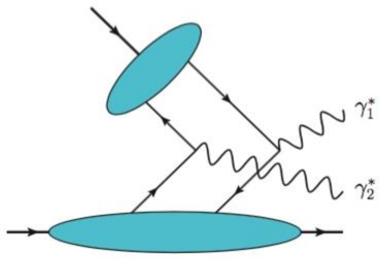


Longitudinal parton momenta fixed as :

$$x_a = \frac{(q_1^+ - q_2^+)}{2P_a^+}$$

$$x_b = 1 - \frac{q_1^-}{p_b^-} = \frac{q_2^-}{p_b^-}$$

Graph (b)



Longitudinal parton momenta fixed as :

$$x_a = -\frac{(q_1^+ - q_2^+)}{2P_a^+}$$

$$x_b = \frac{q_1^-}{p_b^-}$$

ERBL region

$$\xi_a = \frac{(q_1^+ + q_2^+)}{2P_a^+}$$

$$-\xi_a \le x_a \le \xi_a$$

Independent transverse vectors

$$\vec{\Delta}_{a\perp} = -(\vec{q}_{1\perp} + \vec{q}_{2\perp})$$

$$\Delta\vec{q}_\perp = (\vec{q}_{1\perp} - \vec{q}_{2\perp})$$

Polarization Observables

Summing over photon polarization $(\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2})$

↓

Interference between

F & ***G***

drops out!

Unpolarized

$$\tau_{UU} = \frac{1}{2}\sum_{\lambda,\lambda'} |\mathcal{T}_{\lambda,\lambda'}|^2$$

SSA

$$\tau_{LU} = \frac{1}{2}\sum_{\lambda'} \left(|\mathcal{T}_{+,\lambda'}|^2 - |\mathcal{T}_{-,\lambda'}|^2\right) \quad \tau_{XU} \quad \tau_{YU} \cdots$$

DSA

$$\tau_{LL} = \frac{1}{2}\Big(\left(|\mathcal{T}_{+,+}|^2 - |\mathcal{T}_{+,-}|^2\right) - \left(|\mathcal{T}_{-,+}|^2 - |\mathcal{T}_{-,-}|^2\right)\Big) \quad \tau_{XX} \quad \tau_{YY} \cdots$$

- **All of the GTMDs can be “disentangled” through suitable linear combination of Polarization Observables**

Polarization Observables

Summing over photon polarization ($\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2}$)

↓

Interference between **F** & **G** drops out!

Unpolarized $$\tau_{UU} = \frac{1}{2}\sum_{\lambda,\lambda'} |\mathcal{T}_{\lambda,\lambda'}|^2$$

SSA $$\tau_{LU} = \frac{1}{2}\sum_{\lambda'} \left(|\mathcal{T}_{+,\lambda'}|^2 - |\mathcal{T}_{-,\lambda'}|^2\right) \quad \tau_{XU} \quad \tau_{YU}\cdots$$

DSA $$\tau_{LL} = \frac{1}{2}\left(\left(|\mathcal{T}_{+,+}|^2 - |\mathcal{T}_{+,-}|^2\right) - \left(|\mathcal{T}_{-,+}|^2 - |\mathcal{T}_{-,-}|^2\right)\right) \quad \tau_{XX} \quad \tau_{YY}\cdots$$

- **All of the GTMDs can be "disentangled" through suitable linear combination of Polarization Observables**
- **Generic GTMD :** $X = Re.(X) + i\, Im.(X)$
  - **i. "Direct Access" : Access** $|X|^2$
  - **ii. Access real/ imaginary parts :** $Re.(X)$ / $Im.(X)$
- $Re.\,\boldsymbol{F}_{1,1}$ & $Re.\boldsymbol{G}_{1,4}$ **treated as `key' candidates to get access to real and imaginary part of other GTMDs since they are presumably large :** $Re.\,\boldsymbol{F}_{1,1}\Big|_{\Delta=0} = f_1(x, \vec{k}_\perp^{\,2})$ $Re.\,\boldsymbol{G}_{1,4}\Big|_{\Delta=0} = g_1(x, \vec{k}_\perp^{\,2})$

Access through Interference

$$\frac{1}{2}(\tau_{XY}-\tau_{YX}) = \frac{4}{M_a^2}(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\, Re.\Big\{C^{(-)}\big[F_{1,1}\phi_\pi\big]C^{(+)}\big[\vec{\beta}_\perp\cdot\vec{k}_{a\perp}\, \boldsymbol{F^*_{1,4}}\,\phi_\pi^*\big] - C^{(+)}\big[G_{1,4}\phi_\pi\big]C^{(-)}\big[\vec{\beta}_\perp\cdot\vec{p}_{a\perp}\,\boldsymbol{G^*_{1,1}}\,\phi_\pi^*\big]\Big\}$$

This linear combination is sensitive to strength of spin-orbit correlation

Switch on / off the effect you want by exploiting tensor structures →

$$\varepsilon_\perp^{\mu\nu}\epsilon_\mu^*\epsilon_\nu^* W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda_a'}(x_a,\vec{k}_{a\perp})$$

$$g_\perp^{\mu\nu}\epsilon_\mu^*\epsilon_\nu^* W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda_a'}(x_a,\vec{k}_{a\perp})$$

F-type GTMDs

G-type GTMDs

Convolution Integral

$$C^{(\pm)}\Big[w(\vec{k}_{a\perp},\vec{k}_{b\perp})\,X\,\Phi_\pi\Big] = \frac{e^2}{\sqrt{1-\xi_a^2}N_c}\sum_{q,q'}e_q e_q'\int d^2\vec{k}_{a\perp}\int d^2\vec{k}_{b\perp}\,\delta^{(2)}\Big(\frac{\Delta\vec{q}_\perp}{2}-\vec{k}_{a\perp}-\vec{k}_{b\perp}\Big)w(\vec{k}_{a\perp},\vec{k}_{b\perp})$$
$$\times\Big[X^{qq'}(x_a,\vec{k}_{a\perp})\pm X^{qq'}(-x_a,-\vec{k}_{a\perp})\Big]\Phi_\pi^{q'q}(x_b,\vec{k}_{b\perp}^2)$$

weight factor

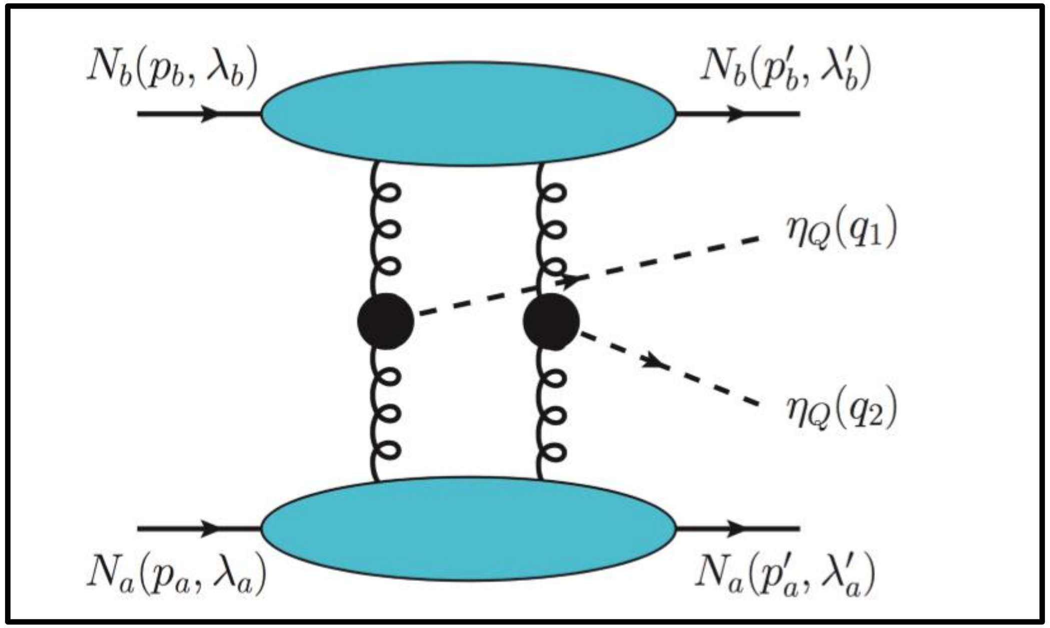
$$\vec{\beta}_\perp = \frac{\vec{\Delta}_{a\perp}^2\,\Delta\vec{q}_\perp-(\vec{\Delta}_{a\perp}\cdot\Delta\vec{q}_\perp)\,\vec{\Delta}_{a\perp}}{\vec{\Delta}_{a\perp}^2\,\Delta\vec{q}_\perp^{\,2}-(\vec{\Delta}_{a\perp}\cdot\Delta\vec{q}_\perp)^2}$$

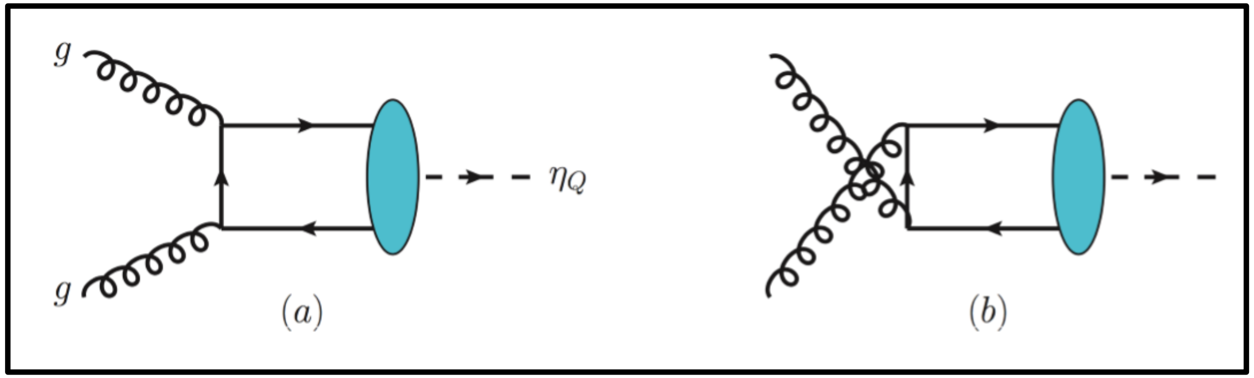
Exclusive Double Quarkonium Production in Hadronic Collisions

SB, Metz, Ojha, Tsai, Zhou, 1802.10550

Process $N_a(p_a, \lambda_a) + N_b(p_b, \lambda_b) \to \eta_Q(q_1) + \eta_Q(q_2) + N_a(p'_a, \lambda'_a) + N_b(p'_b, \lambda'_b)$

Leading-order diagrams





Kinematics (TMD type)

$s = (p_a + p_b)^2 \approx 2p_a^+ p_b^- = \text{large}$ $\qquad q_1^2 = q_2^2 = m_\eta^2$ $\qquad |\vec{q}_{i\perp}| << m_\eta$

Scattering Amplitude

$\Delta\vec{q}_\perp = (\vec{q}_{1\perp} - \vec{q}_{2\perp})$

$$\mathcal{T}_{\lambda_a,\lambda'_a;\lambda_b,\lambda'_b} = -i\,A \int d^2\vec{k}_{a\perp} \int d^2\vec{k}_{b\perp}\, \delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2} - \vec{k}_{a\perp} - \vec{k}_{b\perp}\right)$$

$$\times \Big[\boldsymbol{W^g_{\lambda_a,\lambda'_a}(x_a,\vec{k}_{a\perp})}\, W^g_{\lambda_b,\lambda'_b}(x_b,\vec{k}_{b\perp}) + W^g_{\lambda_a,\lambda'_a}(-x_a,-\vec{k}_{a\perp})\, W^g_{\lambda_b,\lambda'_b}(-x_b,-\vec{k}_{b\perp})$$

$$+ \boldsymbol{\widetilde{W}^g_{\lambda_a,\lambda'_a}(x_a,\vec{k}_{a\perp})}\, \widetilde{W}^g_{\lambda_b,\lambda'_b}(x_b,\vec{k}_{b\perp}) + \widetilde{W}^g_{\lambda_a,\lambda'_a}(-x_a,-\vec{k}_{a\perp})\, \widetilde{W}^g_{\lambda_b,\lambda'_b}(-x_b,-\vec{k}_{b\perp}) \Big]$$

Radial wavefunction of η_Q at origin

$$A = \frac{g_s^4\, R_0^2(0)\, s}{N_c(N_c^2-1)\,\pi\, m_\eta^5\,(1+\xi_a)(1+\xi_b)}$$

Longitudinal parton momenta fixed

$$x_a = \frac{(q_1^+ - q_2^+)}{2P_a^+} \qquad x_b = \frac{(q_1^- - q_2^-)}{2P_b^-} \qquad -\xi \le x \le \xi$$

ERBL region

Gluon GTMDs

$$W^{g\,[ij]}_{\lambda,\lambda'}(P,\Delta,x,\vec{k}_\perp) = \frac{1}{P^+}\int \frac{dz^-\, d^2\vec{z}_\perp}{(2\pi)^3}\, e^{ik\cdot z}\, \langle p',\lambda'|\, F_a^{+i}(-\tfrac{z}{2})\, \mathcal{W}_{ab}(-\tfrac{z}{2},\tfrac{z}{2})\, F_b^{+j}(\tfrac{z}{2})\, |p,\lambda\rangle\Big|_{z^+=0}$$

$$W^g_{\lambda,\lambda'} = \delta_\perp^{ij} W^{g[ij]}_{\lambda,\lambda'} \to \boldsymbol{W^{q[\gamma^+]}_{\lambda,\lambda'}}$$

$$\widetilde{W}^g_{\lambda,\lambda'} = -i\epsilon_\perp^{ij} W^{g[ij]}_{\lambda,\lambda'} \to \boldsymbol{W^{q[\gamma^+\gamma^5]}_{\lambda,\lambda'}}$$

Polarization Observables

$$\tau_{UU} = \frac{1}{2}\sum_{\lambda_b,\lambda'_b}\frac{1}{2}\sum_{\lambda_a,\lambda'_a}|\mathcal{T}_{\lambda_a,\lambda'_a;\lambda_b,\lambda'_b}|^2$$

$$\tau_{LU} = \frac{1}{2}\sum_{\lambda_b,\lambda'_b}\frac{1}{2}\sum_{\lambda'_a}\left(|\mathcal{T}_{+,\lambda'_a;\lambda_b,\lambda'_b}|^2 - |\mathcal{T}_{-,\lambda'_a;\lambda_b,\lambda'_b}|^2\right)$$

$$\tau_{LL} = \frac{1}{2}\sum_{\lambda_b,\lambda'_b}\frac{1}{2}\left(\left(|\mathcal{T}_{+,+;\lambda_b,\lambda'_b}|^2 - |\mathcal{T}_{+,-;\lambda_b,\lambda'_b}|^2\right) - \left(|\mathcal{T}_{-,+;\lambda_b,\lambda'_b}|^2 - |\mathcal{T}_{-,-;\lambda_b,\lambda'_b}|^2\right)\right)$$

We sum / average over polarizations of Nucleon `b'

Accessing $F_{1,4}$ ($\vec{\Delta}_{a\perp} \neq 0, \vec{\Delta}_{b\perp} = 0$)

Assumed hierarchy of magnitude of GTMDs

$F_{1,1} > G_{1,4} > \text{others}$

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$$\frac{1}{4}(\tau_{UU} + \tau_{LL} - \tau_{XX} - \tau_{YY})$$

$$\approx \frac{1}{M^4}\left(\varepsilon_\perp^{ij} \Delta q_\perp^i \Delta_{a\perp}^j\right)^2 C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, \boldsymbol{F_{1,4}(x_a, \vec{k}_{a\perp})}\, F_{1,1}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[\vec{\beta}_\perp \cdot \vec{p}_{a\perp}\, \boldsymbol{F^*_{1,4}(x_a, \vec{p}_{a\perp})}\, F^*_{1,1}(x_b, \vec{p}_{b\perp})\Big]$$

$$+\; C\Big[G_{1,4}(x_a, \vec{k}_{a\perp})\, G_{1,4}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[G^*_{1,4}(x_a, \vec{p}_{a\perp})\, G^*_{1,4}(x_b, \vec{p}_{b\perp})\Big]$$

Convolution integral

$$C\Big[w(\vec{k}_{a\perp}, \vec{k}_{b\perp})\, X\, Y\Big] = \frac{2A}{\sqrt{1-\xi_a^2}\,\sqrt{1-\xi_b^2}} \int d^2\vec{k}_{a\perp} \int d^2\vec{k}_{b\perp}\, \delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2} - \vec{k}_{a\perp} - \vec{k}_{b\perp}\right) \times w(\vec{k}_{a\perp}, \vec{k}_{b\perp})\, X(x_a, \vec{k}_{a\perp})\, Y(x_b, \vec{k}_{b\perp})$$

Weight factor

$$\vec{\beta}_\perp = \frac{\vec{\Delta}^2_{a\perp}\, \Delta\vec{q}_\perp - (\vec{\Delta}_{a\perp} \cdot \Delta\vec{q}_\perp)\, \vec{\Delta}_{a\perp}}{\vec{\Delta}^2_{a\perp}\, \Delta\vec{q}^{\,2}_\perp - (\vec{\Delta}_{a\perp} \cdot \Delta\vec{q}_\perp)^2} = \frac{\vec{\Delta}_{a\perp} \times (\Delta\vec{q}_\perp \times \vec{\Delta}_{a\perp})}{\left(\varepsilon_\perp^{ij} \Delta q_\perp^i \Delta_{a\perp}^j\right)^2}$$

Accessing $G_{1,1}$ ($\vec{\Delta}_{a\perp} \neq 0, \vec{\Delta}_{b\perp} = 0$)

Assumed hierarchy of magnitude of GTMDs

$$F_{1,1} > G_{1,4} > \text{others}$$

Direct Access

$$\frac{1}{4}\left(\tau_{UU} + \tau_{LL} + \tau_{XX} + \tau_{YY}\right)$$

$$\approx \quad \frac{\varepsilon_\perp^{ij}\Delta_{a\perp}^j}{M}\frac{\varepsilon_\perp^{kl}\Delta_{a\perp}^l}{M}\, C\left[\frac{k_{a\perp}^i}{M}\boldsymbol{G_{1,1}(x_a,\vec{k}_{a\perp})}\, G_{1,4}(x_b,\vec{k}_{b\perp})\right] C\left[\frac{p_{a\perp}^k}{M}\boldsymbol{G_{1,1}^*(x_a,\vec{p}_{a\perp})}\, G_{1,4}^*(x_b,\vec{p}_{b\perp})\right]$$

$$+ \quad C\left[F_{1,1}(x_a,\vec{k}_{a\perp})\, F_{1,1}(x_b,\vec{k}_{b\perp})\right] C\left[F_{1,1}^*(x_a,\vec{p}_{a\perp})\, F_{1,1}^*(x_b,\vec{p}_{b\perp})\right]$$

Takes over $\left|G_{1,1}\right|^2$ contribution. Hence, $G_{1,1}$ cannot probably be accessed directly.

Access through Interference ($\vec{\Delta}_{a\perp} \neq 0, \vec{\Delta}_{b\perp} = 0$)

Assumed hierarchy of magnitude of GTMDs

$F_{1,1} > G_{1,4} >$ others

$$\frac{1}{2}(\tau_{UL} + \tau_{LU})$$

$$\approx \quad 2\,Im.\Big\{ -\frac{1}{M^2}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right) C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, \boldsymbol{F_{1,4}(x_a, \vec{k}_{a\perp})}\, F_{1,1}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[F_{1,1}^*(x_a, \vec{p}_{a\perp})\, F_{1,1}^*(x_b, \vec{p}_{b\perp})\Big]$$

$$+ \frac{1}{M^2}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right) C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, \boldsymbol{G_{1,1}(x_a, \vec{k}_{a\perp})}\, G_{1,4}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[G_{1,4}^*(x_a, \vec{p}_{a\perp})\, G_{1,4}^*(x_b, \vec{p}_{b\perp})\Big]\Big\}$$

$$\approx \quad 2\,Im.\Big\{ -\frac{1}{M^2}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right) C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, \boldsymbol{F_{1,4}(x_a, \vec{k}_{a\perp})}\, F_{1,1}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[F_{1,1}^*(x_a, \vec{p}_{a\perp})\, F_{1,1}^*(x_b, \vec{p}_{b\perp})\Big]\Big\}$$

$Re.\,\boldsymbol{G_{1,1}}$, $Im.\,\boldsymbol{G_{1,1}}$ **overwhelmed by three powers of** $\boldsymbol{F_{1,1}}$

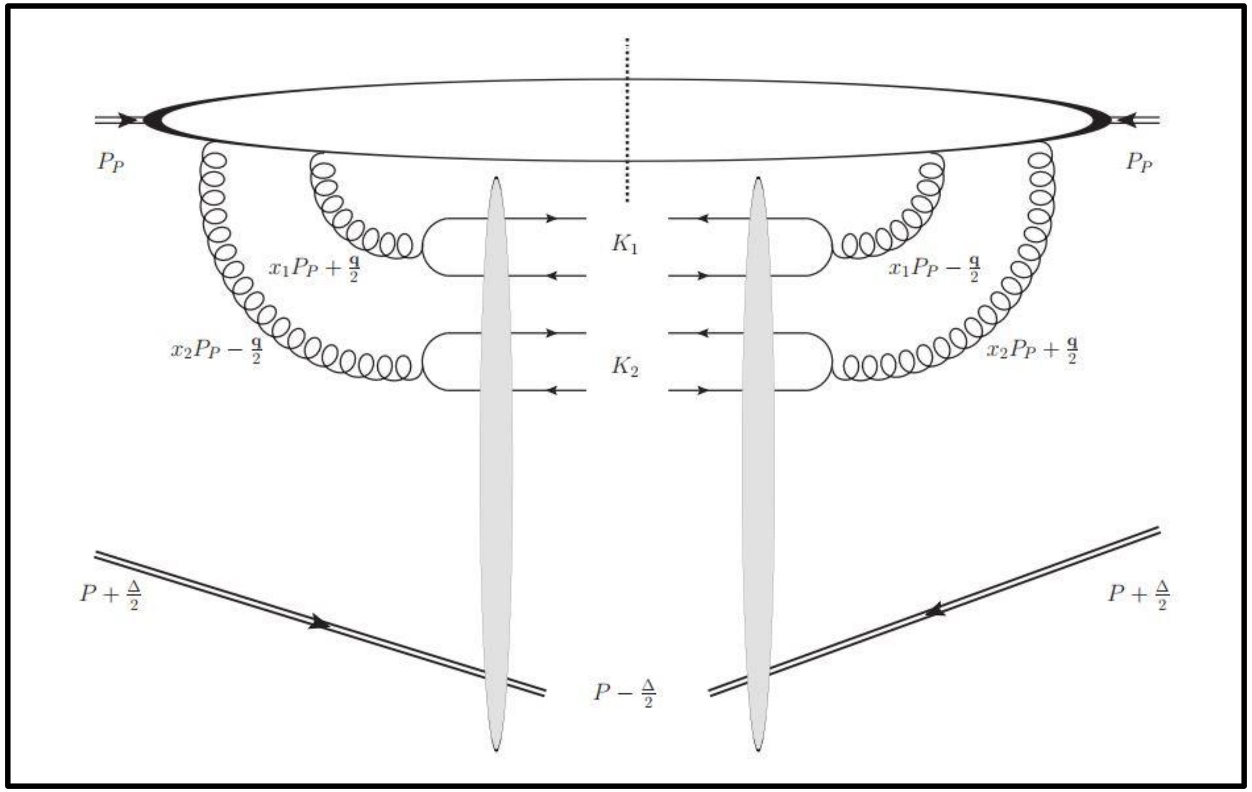
$$\frac{1}{2}(\tau_{XY} - \tau_{YX})$$

$$\approx \quad 2\,Re.\Big\{ -\frac{1}{M^2}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right) C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, \boldsymbol{F_{1,4}(x_a, \vec{k}_{a\perp})}\, F_{1,1}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[F_{1,1}^*(x_a, \vec{p}_{a\perp})\, F_{1,1}^*(x_b, \vec{p}_{b\perp})\Big]\Big\}$$

This linear combination is sensitive to gluon Orbital Angular Momentum

Single-exclusive NN scattering

Boussarie, Hatta, Xiao, Yuan, 1807.08697



- **<u>Example</u> : Result for $\chi_1\,\chi_1$ production**

$$d\sigma \approx F(x_1, x_2)\left(G_1(\vec{K}_\perp, \vec{\Delta}_\perp) + \frac{\vec{K}_\perp^2}{2M^2} G_2(\vec{K}_\perp, \vec{\Delta}_\perp)\right)^2$$

Gluon dPDF — **Unpol. gluon GTMD** — **Linearly pol. gluon GTMD**

Summary

i. GTMDs have attracted considerable interest

- “Mother distribution” character
- Link to Wigner function
- Direct link to parton OAM & spin-orbit correlations

ii. GTMDs can be accessed in “exclusive” processes

- Gluon GTMDs : Dijet production in UPCs might be most promising
- Quark GTMDs : Double DY has low count rate. Can we do better?

iii. Time to go 5-dimensional!

Back-Up Slides

GTMDs $(x, \vec{k}_\perp, \Delta)$

	U	L	T
U	$F_{1,1}$	$G_{1,1}$	$H_{1,1}$ $H_{1,2}$
L	$F_{1,4}$	$G_{1,4}$	$H_{1,7}$ $H_{1,8}$
T	$F_{1,2}$ $F_{1,3}$	$G_{1,2}$ $G_{1,3}$	$H_{1,3}$ $H_{1,4}$ $H_{1,5}$ $H_{1,6}$

$\Delta = 0$

$\int d^2\vec{k}_\perp$

TMDs $(x, \vec{k}_\perp)$

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_T^\perp$

GPDs (x, Δ)

	U	L	T
U	H		E_T
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	H_T $\tilde{H}_T$