

Unpolarized TMDs: phenomenological status

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The TMD new perspective

The data taking on Drell-Yan and SIDIS spectra started a long ago...
What has changed?

In the last years several new papers dedicated to unpolarized TMD phenomenology:

- ★ A. Bacchetta et al., arXiv:1912.07550 DY
- I.S., A. Vladimirov, arXiv:1912.06532 DY+SIDIS
- V. Bertone et al., JHEP 1906 (2019) 028 DY
- S. Camarda et al. arXiv:1910.07049 DY
- Bermudez Martinez et al., Phys.Rev. D99 (2019) no.7, 074008, Phys.Rev. D99 (2019) no.7, 074008 DY
- A. Bacchetta et al., Phys.Lett. B788 (2019) 542-545
- P. Sung et al., Int.J.Mod.Phys. A33 (2018) no.11, 1841006 DY
- I. Scimemi, A. Vladimirov Eur.Phys.J. C78 (2018) no.2, 89 DY
- A. Bacchetta et al. JHEP 1706 (2017) 081, Erratum: JHEP 1906 (2019) 051 DY+SIDIS
- M. Boglione et al. JHEP 1502 (2015) 095 SIDIS
- U. D' Alesio et al., JHEP 1411 (2014) 098 DY
- M. Anselmino et al., JHEP 1404 (2014) 005 SIDIS

★ TMD universality proven phenomenologically

Some notation

SIDIS

$$\ell(l) + H(P) \rightarrow \ell(l') + h(p_h) + X$$

$$d\sigma = \frac{2}{s - M^2} \frac{\alpha_{\text{em}}^2}{(q^2)^2} L_{\mu\nu} W^{\mu\nu} \frac{d^3 l'}{2E'} \frac{d^3 p_h}{2E_h}$$

DY

$$h_1(P_1) + h_2(P_2) \rightarrow l(l) + l'(l') + X$$

$$d\sigma = \frac{2\alpha_{\text{em}}^2}{s} d^4 q \sum_{GG'} \hat{L}_{\mu\nu}^{GG'} W_{GG'}^{\mu\nu} \Delta_G(q) \Delta_{G'}^*(q)$$

$$\Delta_G(q) = \frac{1}{q^2 + i0} \delta_{G\gamma} + \frac{1}{q^2 - M_Z^2 + i\Gamma_Z M_Z} \delta_{GZ}$$

Factorization for $q_T \ll Q$

SIDIS $W^{\mu\nu} = -2z_S \sum_f e_f^2 |C_V(Q^2, \mu^2)|^2 \int \frac{d^2b}{(2\pi)^2} e^{-i(bq_T)} \left[g_T^{\mu\nu} f_{1,f \leftarrow H}(x_S, b; \mu, \zeta_1) D_{1,f \rightarrow h}(z_S, b; \mu, \zeta_2) \right. \\ \left. + (g_T^{\mu\nu} b^2 - 2b^\mu b^\nu) \frac{m M}{4} h_{1,f \leftarrow H}^\perp(x_S, b; \mu, \zeta_1) H_{1,f \rightarrow h}^\perp(z_S, b; \mu, \zeta_2) \right] + O\left(\frac{q_T^2}{Q^2}\right)$

DY $W_{GG'}^{\mu\nu} = \sum_f |C_V(-Q^2, \mu^2)|^2 \int \frac{d^2b}{(2\pi)^2} e^{-i(bq_T)} \left[-2g_T^{\mu\nu} (g_R^G g_R^{G'} + g_L^G g_L^{G'}) (f_{1,f \leftarrow h_1} f_{1,\bar{f} \leftarrow h_2} + f_{1,\bar{f} \leftarrow h_1} f_{1,f \leftarrow h_2}) \right. \\ \left. - \frac{g_T^{\mu\nu} b^2 - 2b^\mu b^\nu}{2} M_1 M_2 (g_R^G g_R^{G'} + g_L^G g_L^{G'}) (h_{1,f \leftarrow h_1}^\perp h_{1,\bar{f} \leftarrow h_2}^\perp + h_{1,\bar{f} \leftarrow h_1}^\perp h_{1,f \leftarrow h_2}^\perp) \right] + O\left(\frac{q_T^2}{Q^2}\right)$

Currently power corrections are included only in kinematical factors (Bjorken variables)

Evolution: definition and perturbative/non-perturbative aspects

TMD evolution

$$\mu^2 \frac{d}{d\mu^2} F(x, b; \mu, \zeta) = \frac{\gamma_F(\mu, \zeta)}{2} F(x, b; \mu, \zeta)$$

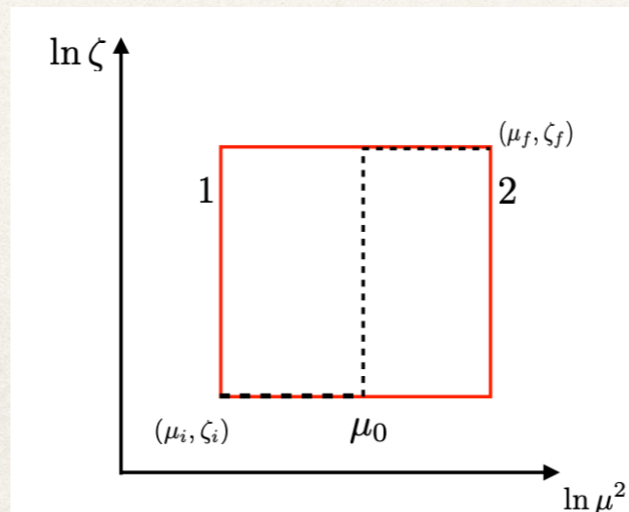
Perturbative series truncation leads to path dependence of the solution

$$\zeta \frac{d}{d\zeta} F(x, b; \mu, \zeta) = -\mathcal{D}(\mu, b) F(x, b; \mu, \zeta)$$

$$F(x, b; \mu_f, \zeta_f) = \exp \left[\int_P \left(\gamma_F(\mu, \zeta) \frac{d\mu}{\mu} - \mathcal{D}(\mu, b) \frac{d\zeta}{\zeta} \right) \right] F(x, b; \mu_i, \zeta_i)$$

CSS1981 $R(Q, b, C_1, C_2) = \exp \left[\int_{(C_1/b)^2}^{(C_2 Q)^2} \frac{d\mu}{\mu} A(\alpha_s(\mu), C_1) \ln \frac{C_2^2 Q^2}{\mu^2} + B(\alpha_s(\mu), C_1, C_2) \right]$

Collins 2011 $R(\mu, \zeta; \mu_0, \zeta_0) = \exp \left[-\frac{\mathcal{D}}{2} \ln \sqrt{\frac{\zeta}{\zeta_0}} + \int_{\mu_0}^{\mu} \frac{d\nu}{\nu} \gamma_F(\alpha_s(\nu)) - 2\Gamma_{cusp}(\alpha_s(\nu)) \ln \sqrt{\frac{\zeta}{\nu^2}} \right]$



The transition to the non-perturbative regime is fixed at μ_0

Evolution: definition and perturbative/non-perturbative aspects

TMD evolution

$$\mu^2 \frac{d}{d\mu^2} F(x, b; \mu, \zeta) = \frac{\gamma_F(\mu, \zeta)}{2} F(x, b; \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} F(x, b; \mu, \zeta) = -\mathcal{D}(\mu, b) F(x, b; \mu, \zeta)$$

$$F(x, b; \mu_f, \zeta_f) = \exp \left[\int_P \left(\gamma_F(\mu, \zeta) \frac{d\mu}{\mu} - \mathcal{D}(\mu, b) \frac{d\zeta}{\zeta} \right) \right] F(x, b; \mu_i, \zeta_i)$$

Path in-dependence is restored
changing higher orders in γ_F

The optimal initial condition is identified when $\mathcal{D}(\mu_{saddle}, b) = 0$

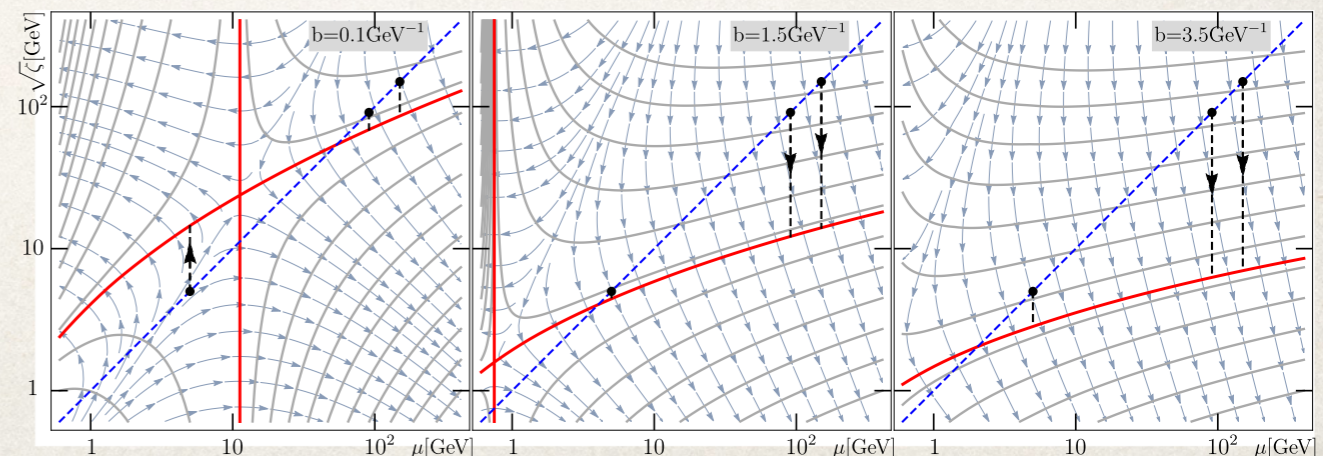
$$\gamma_F(\mu_{saddle}, \zeta_{saddle}) = 0$$

ζ -prescription

$$R(\mu_f, \zeta_f; \mu_{saddle}, \zeta_{saddle}) \equiv R(\mu_f, \zeta_f) = \left(\frac{\zeta_f}{\zeta_{\mu_f}} \right)^{-\mathcal{D}}$$

$$F(x, b, \mu_f, \zeta_f) = R(\mu_f, \zeta_f) F_{optimal}(x, b)$$

The optimal TMD is scaleless



Message: One cannot achieve a description of the whole TMD spectrum with a unique effective theory

For the spectrum we have two factorization theorems:

$$Q \sim q_T \gg \Lambda_{had} : \quad d\sigma \sim \int H(Q, q_T, \mu) f_1(x_1, \mu) f_2(x_2, \mu)$$

$$Q \gg q_T \sim \Lambda_{had} : \quad d\sigma \sim \int e^{-i\mathbf{q}_T \cdot \mathbf{b}} H(Q, \mu) f_1(x_1, \mathbf{b}, \mu, \zeta_1) f_2(x_2, \mathbf{b}, \mu, \zeta_2)$$

For the intermediate regions we have just phenomenologically based approaches (Y-terms, \mathbf{b}_{min} , interpolations..).

A different understanding is required (operators, eft, ...)

The intermediate region does not provide clear details on PDF and TMDPDF.

TMDPDF: Lattice

There was a set of works on measurement of CS kernel on lattice

- ▶ Ji's group [1801.05930][1910.00800][1911.03840] ..most enthusiastic
- ▶ MIT group [1811.00026][1901.03685][1910.08569]
- ▶ Regensburg⁺⁺ group [1111.4249][1506.07826][2001.in prep] ..most conservative

Lattice restrictions

- ▶ Equal-time correlators only
- ▶ **Very** small energies ($P^+ \sim 3\text{GeV}$, is an absolute maximum nowadays)
- ▶ Not too small distances (lattice artifacts)
- ▶ Not too large distances (lattice sizes)

Slide from A. Vladimirov

TMDPDF: Extraction with modeling (ζ -prescription)

$$\lim_{b \rightarrow 0} f_{1,f \leftarrow h}(x, b) = \sum_{f'} \int_x^1 \frac{dy}{y} C_{f \leftarrow f'} \left(\frac{x}{y}, \mathbf{L}_{\mu_{\text{OPE}}}, a_s(\mu_{\text{OPE}}) \right) f_{1,f' \leftarrow h}(y, \mu_{\text{OPE}}),$$

$$\lim_{b \rightarrow 0} D_{1,f \rightarrow h}(z, b) = \sum_{f'} \int_z^1 \frac{dy}{y} C_{f \rightarrow f'} \left(\frac{z}{y}, \mathbf{L}_{\mu_{\text{OPE}}}, a_s(\mu_{\text{OPE}}) \right) \frac{d_{1,f' \rightarrow h}(y, \mu_{\text{OPE}})}{y^2},$$

PDF are part of the model

NNLO

T. Gehrmann et al. JHEP 06 (2014) 155,

M.G. Echevarria et al. Phys. Rev. D93 (2016) 011502, JHEP 09 (2016) 004

NNNLO

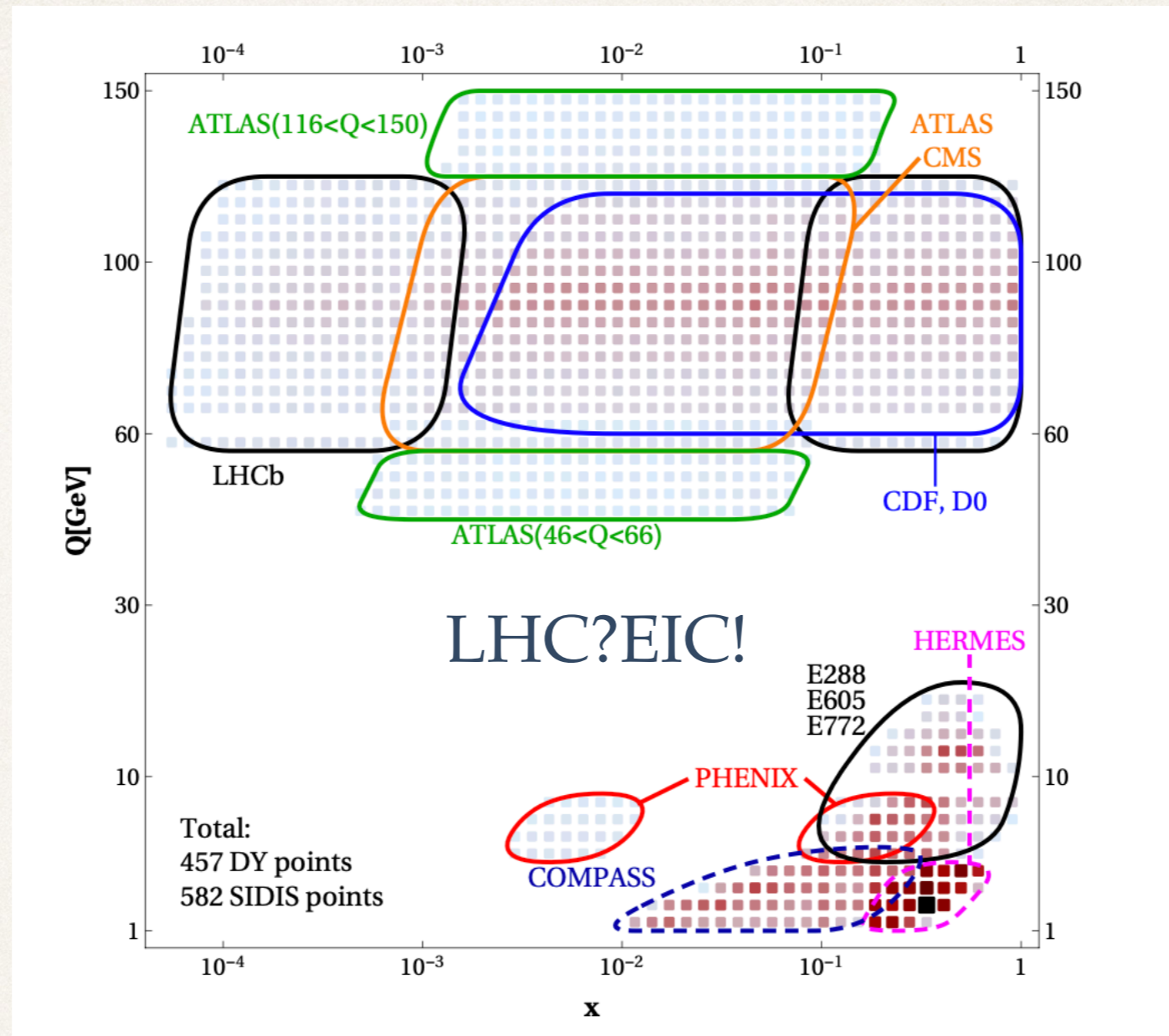
M. X. Luo et al. arXiv:1912.05778

$$f_{1,f \leftarrow h}(x, b) = \int_x^1 \frac{dy}{y} \sum_{f'} C_{f \leftarrow f'}(y, \mathbf{L}_{\mu_{\text{OPE}}}, a_s(\mu_{\text{OPE}})) f_{1,f' \leftarrow h} \left(\frac{x}{y}, \mu_{\text{OPE}} \right) f_{\text{NP}}(x, b),$$

$$D_{1,f \rightarrow h}(z, b) = \frac{1}{z^2} \int_z^1 \frac{dy}{y} \sum_{f'} y^2 C_{f \rightarrow f'}(y, \mathbf{L}_{\mu_{\text{OPE}}}, a_s(\mu_{\text{OPE}})) d_{1,f' \rightarrow h} \left(\frac{z}{y}, \mu_{\text{OPE}} \right) D_{\text{NP}}(z, b)$$

$$f_{\text{NP}}(x, b) = \exp \left(- \frac{\lambda_1(1-x) + \lambda_2 x + x(1-x)\lambda_5}{\sqrt{1 + \lambda_3 x^{\lambda_4}}} b^2 \right)$$

$$D_{\text{NP}}(x, b) = \exp \left(- \frac{\eta_1 z + \eta_2(1-z)}{\sqrt{1 + \eta_3 (b/z)^2}} \frac{b^2}{z^2} \right) \left(1 + \eta_4 \frac{b^2}{z^2} \right),$$

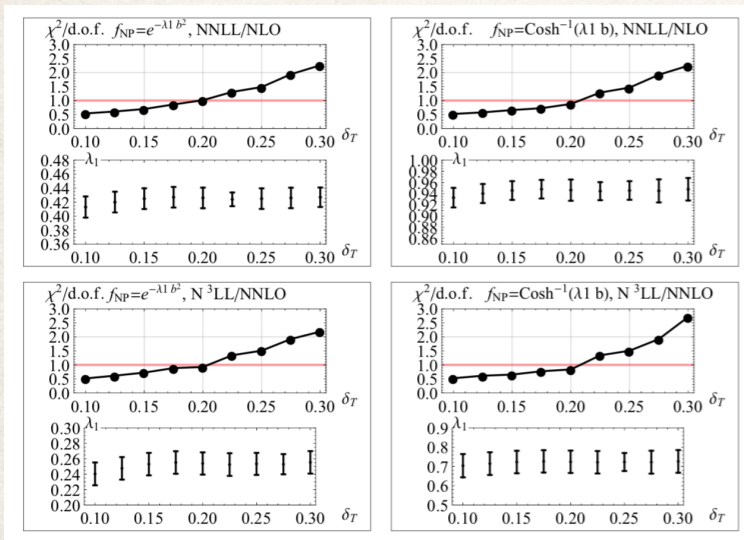


LHC can fill the empty space with an improved triggering at low q_T !

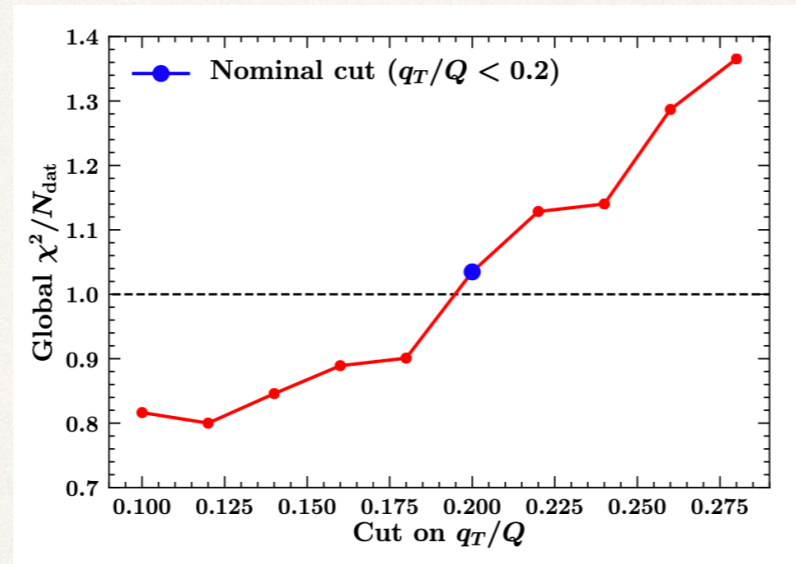
EIC is expected to fill the empty space.

Both LHC and EIC are necessary to check the universality of TMD!

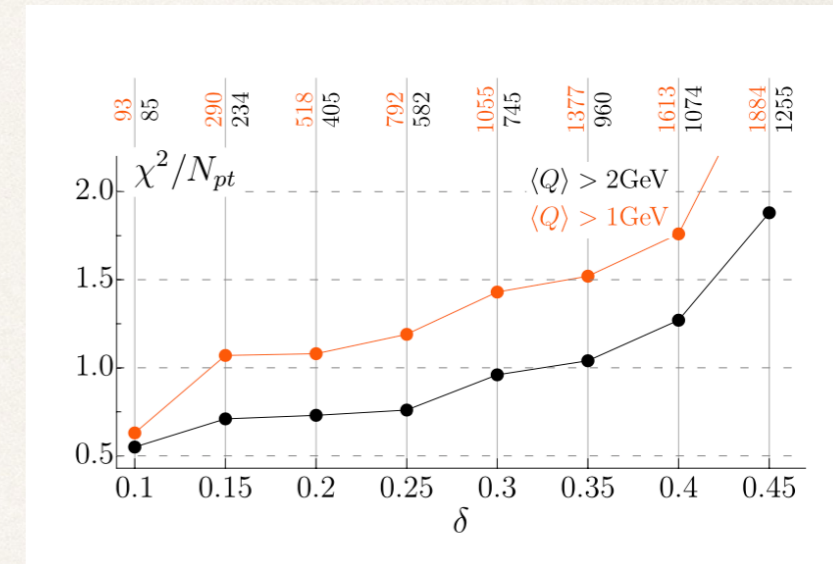
TMD validity range



SV17, DY



Pavia19, DY



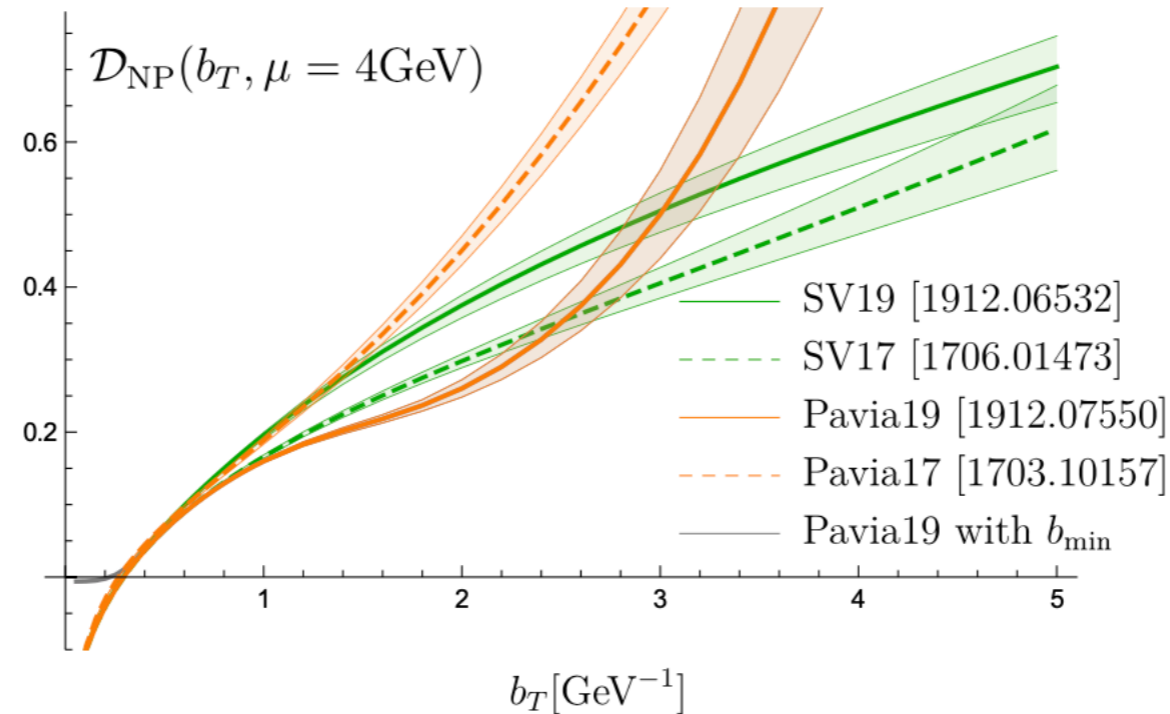
SV19, SIDIS

The range of validity of TMD is independent of models, implementation, perturbative order, experiment type

Evolution kernel (differences in the schemes, lattice,..)

Recent extractions

Name	Order	Data set	Model	Comm.
Pavia17	LO	DY+SIDIS	$\mathcal{D}_{\text{pert}}(\mu, b) + g_2 \frac{b^2}{2}$	$b_{\text{min}}^{\text{max}}$ -prescription
SV17	NNLO	DY	$\mathcal{D}_{\text{pert}}(\mu, b) + c_0 b^2$	ζ -prescription
SV19	NNLO/N ³ LO	DY+SIDIS	$\mathcal{D}_{\text{resum}}(\mu, b^*) + c_0 b b^*$	ζ -prescription
Pavia19	NNNLO	DY	$\mathcal{D}_{\text{pert}}(\mu, b) + g_2 \frac{b^2}{4} + g_{2b} \frac{b^4}{4}$	$b_{\text{min}}^{\text{max}}$ -prescription



Slide from A. Vladimirov

Preliminary results on lattice do not predict a steep raise ..

Evolution kernel (differences in the schemes, lattice,..)

Lattice Results for Rapidity Anomalous Dimension

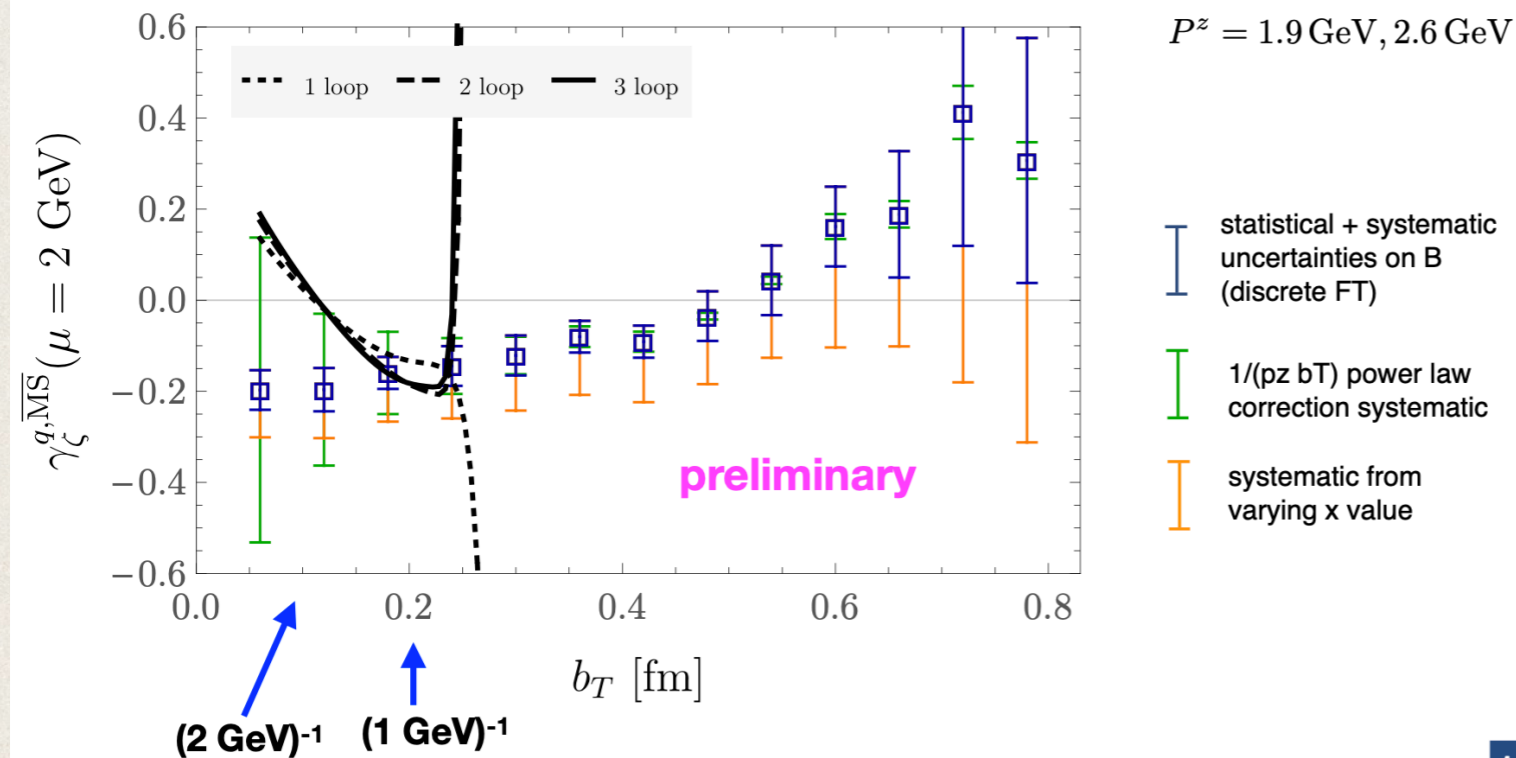
Ongoing work by P. Shanahan, M. Wagman, Y. Zhao

I. Stewart at REF2019

Exploratory quenched ($n_f = 0$) simulation

Exploits universality: uses 1.2 GeV pseudoscalar meson

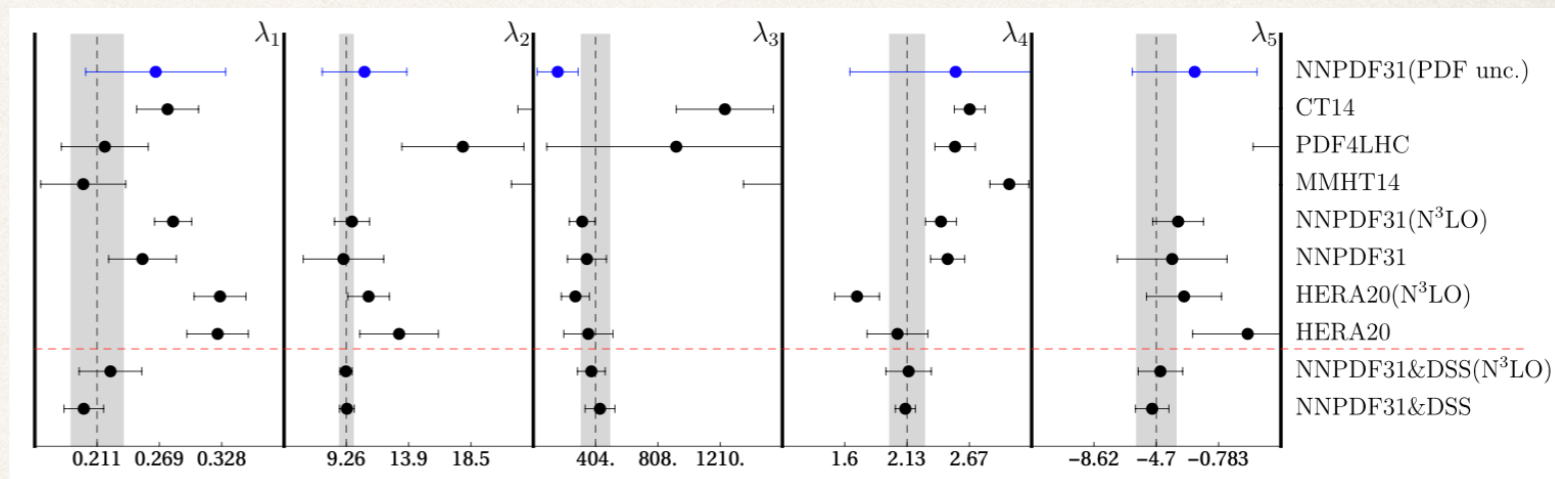
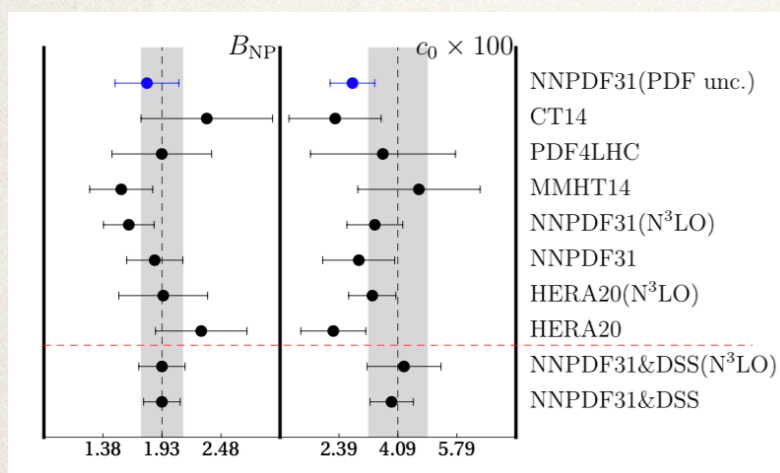
Includes renormalization



Preliminary results on lattice do not predict a steep raise ..

Fit results 2019

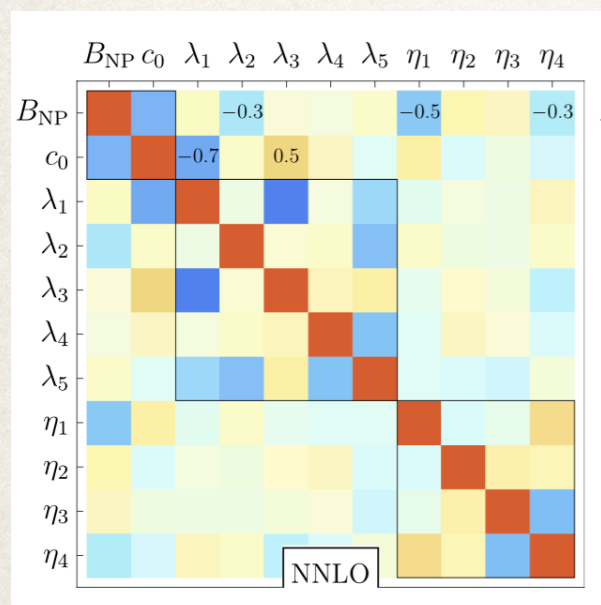
PDF	$\chi^2/\text{d.o.f.}$
NNPDF3.1 [47]	1.168
HERAPDF2.0 [48]	0.95
CT14 [49]	1.63
MMHT14[50]	1.36
PDF4LHC [51]	1.52



Fit results 2019

THE PARAMETERS OF THE EVOLUTION KERNEL SHOULD BE UNCORRELATED WITH THE REST

DY+SIDIS:1912.06532

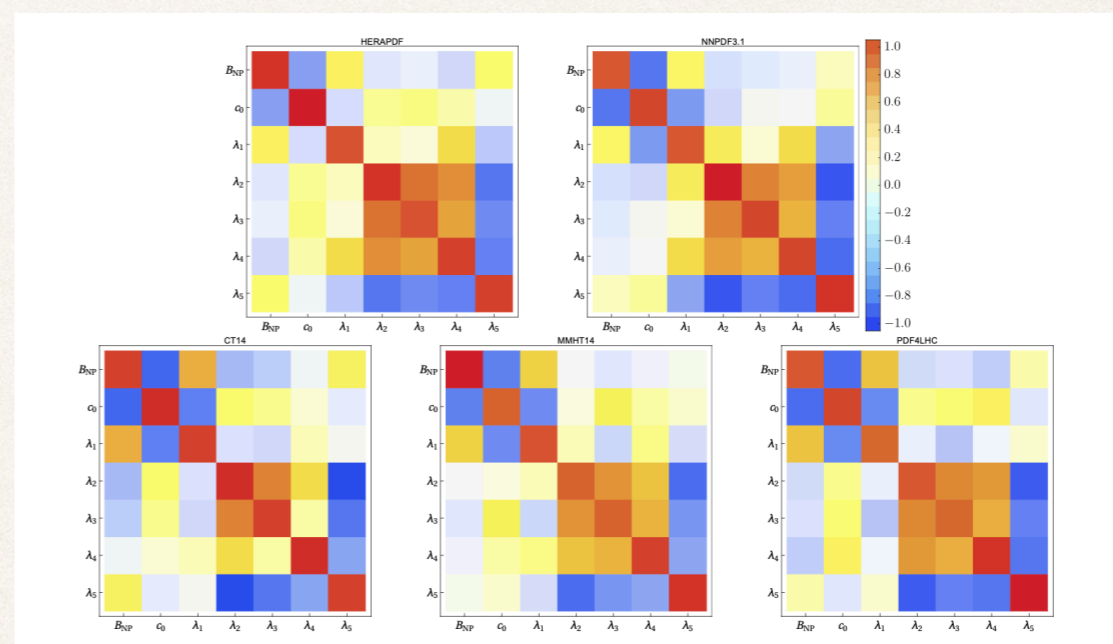


ζ -prescription

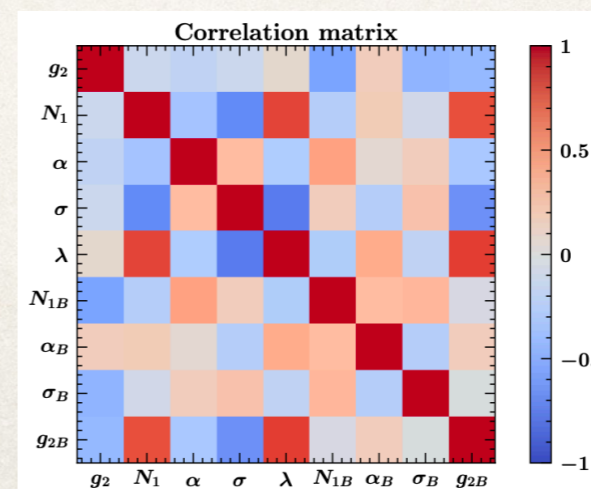
Better the PDF set better the uncorrelation!

F. Hautmann, I.S., A. Vladimirov, Preliminary

DY

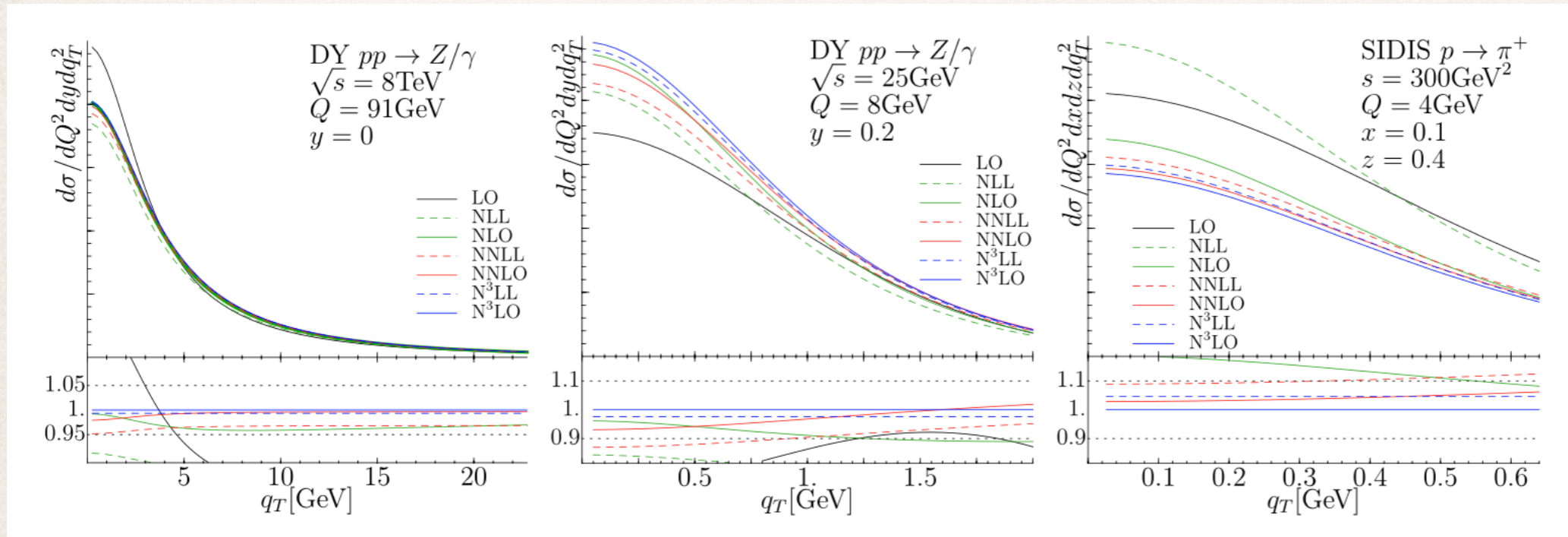


ζ -prescription



Pavia2019:1912.07550

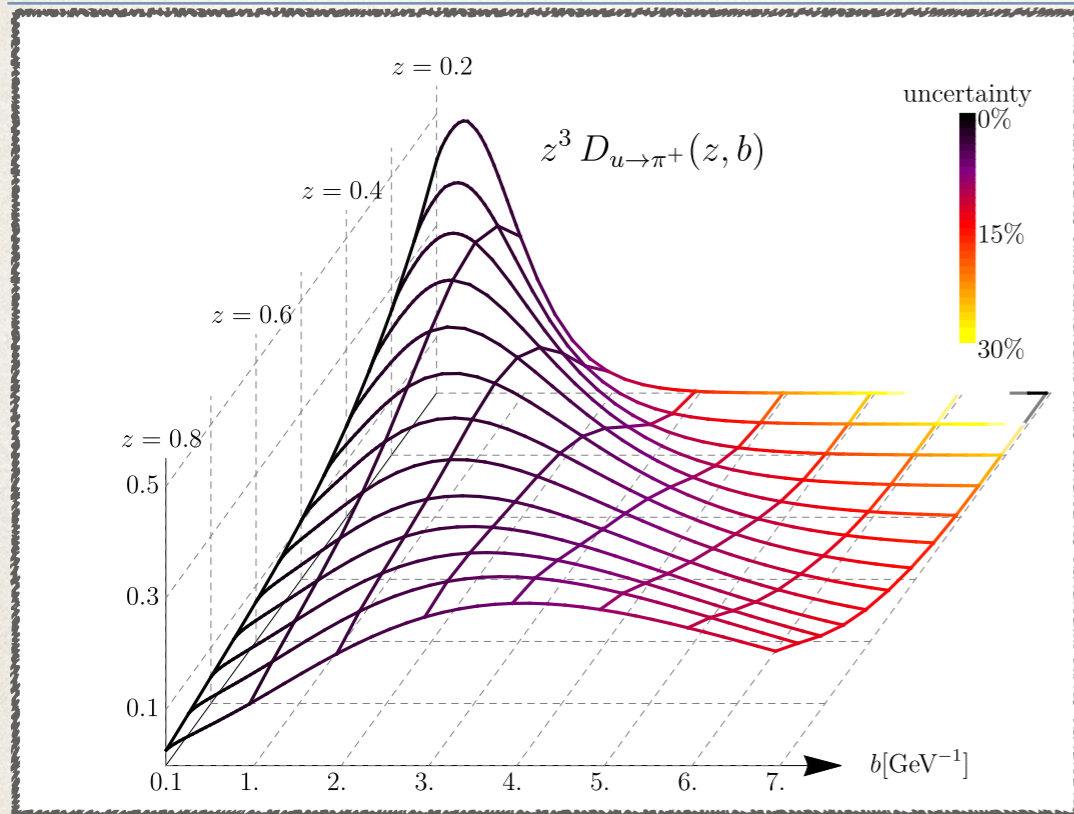
Convergence of the perturbative series



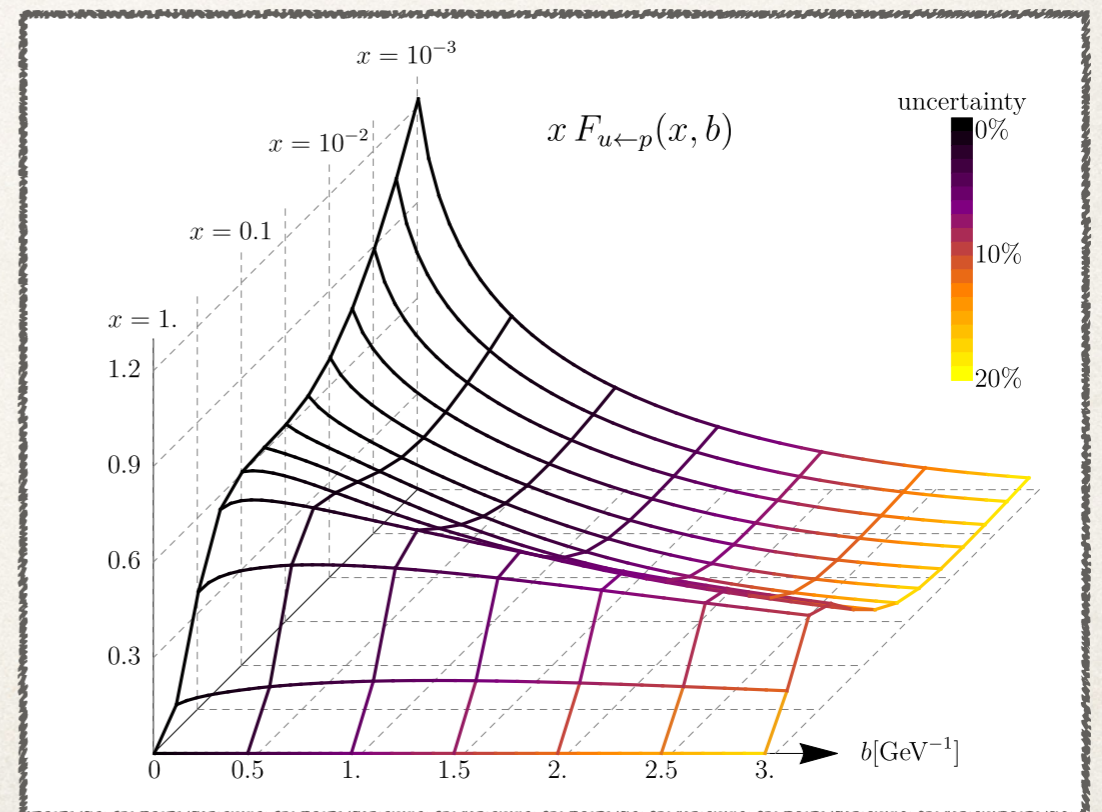
ζ -prescription

The legend of the perturbative orders means that $N^k\text{LO}$ ($N^k\text{LL}$) incorporates a_s^k -order (a_s^{k-1} -order) of the coefficient function, a_s^k -order of anomalous dimensions with a_s^{k+1} -order of Γ_{cusp} . The TMD distributions and the NP part of the evolution is the same for all cases.

“Dancing like a quark in a hadron”



Fragmentation



TMDPDF

Conclusions

We have reached a unified understanding of DY and SIDIS spectra (universality) using TMD both in theory and data: we are ready for EIC era!

Many constraints from LHC data: Triggering should be changed to prepare EIC (low transverse momenta should be studied!)

PDF extraction can strongly benefit from TMD analysis: joined fits? N3LO perturbative calculations for TMD are ready.

Gluon TMD are still very elusive in data: theory well developed (NNLO and NNNLO)

Lattice preliminary results are starting to be available

Back up

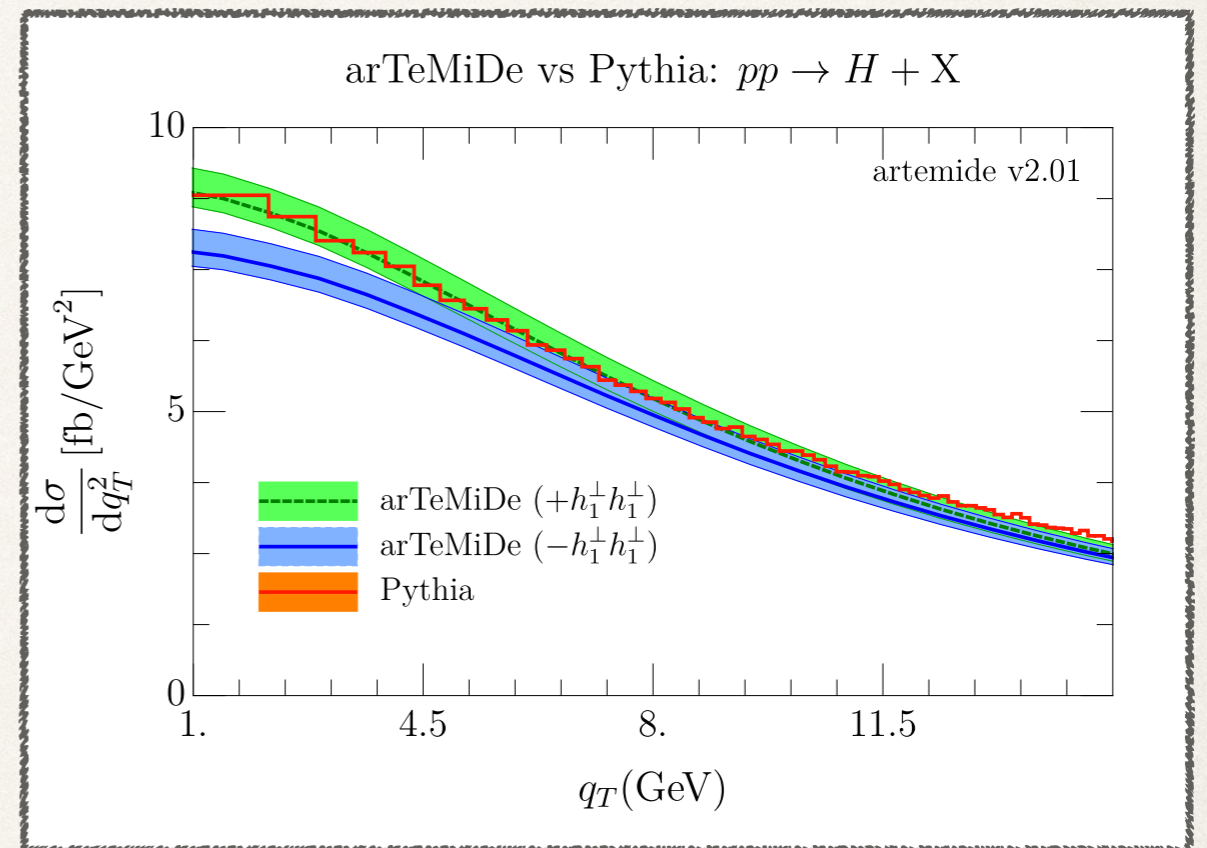
Higgs spectrum

The cross section depends on unpolarized and
Linearly polarized gluons

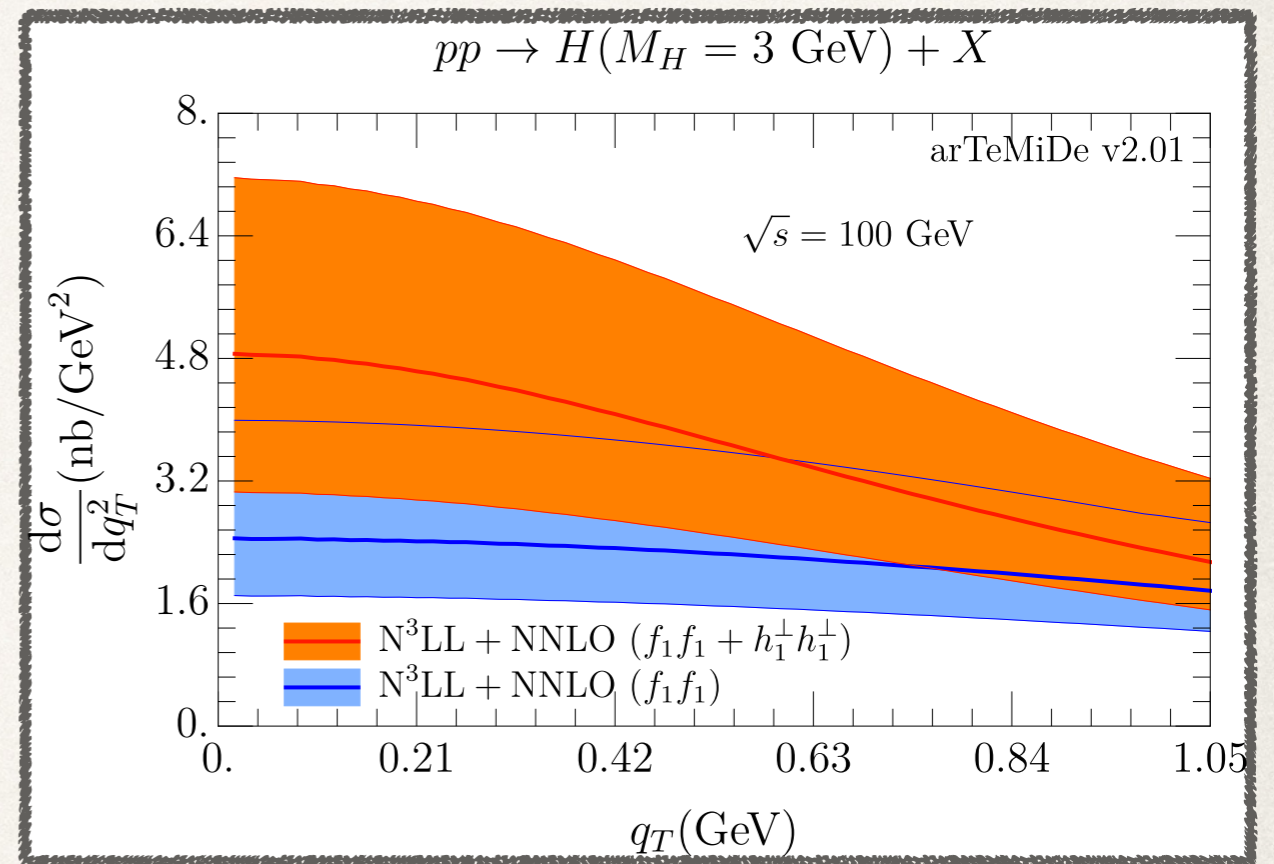
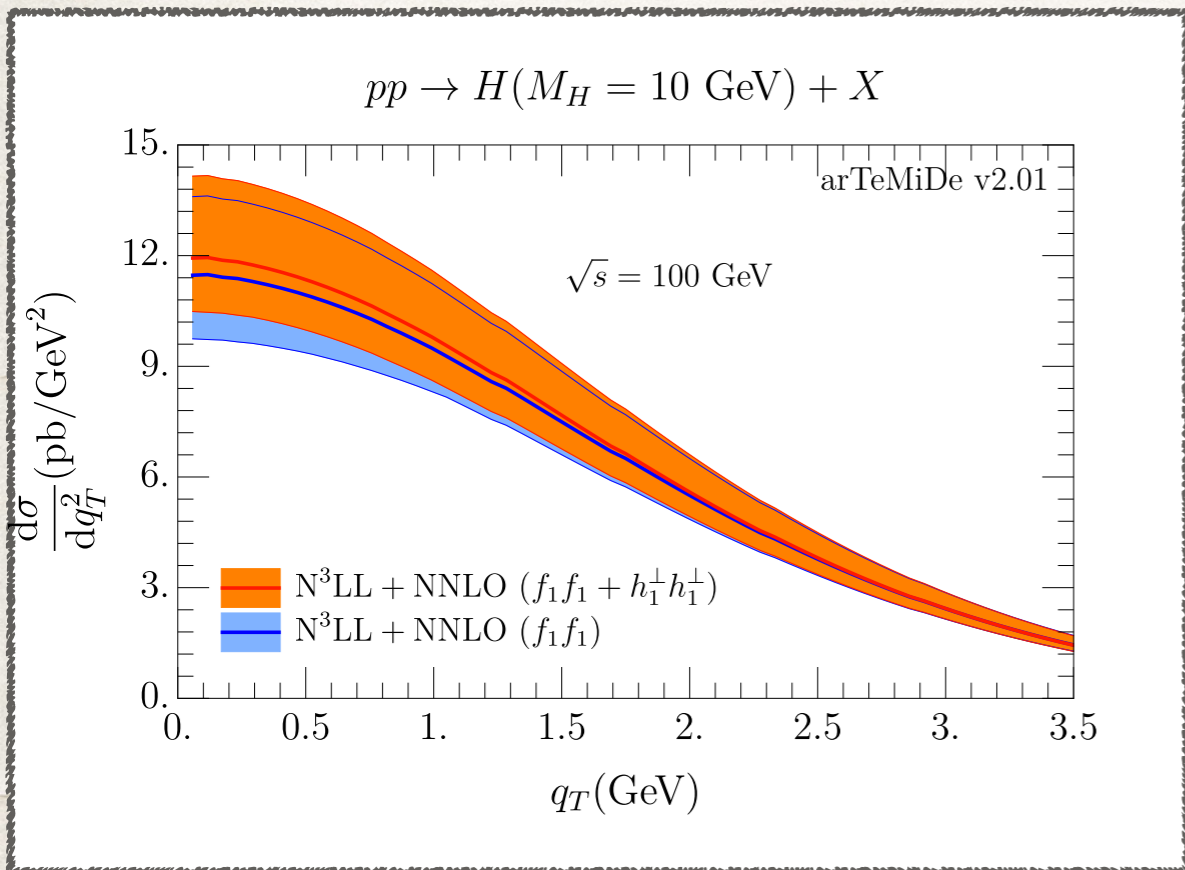
NNLO for linearly polarized gluons
Leal-Gomez et al. 1907.05896
Luo et al. 1908.03831

$$\frac{d\sigma}{dyd^2q_T} = \frac{2\sigma_0(\mu)}{\pi} C_t^2(\mu) U(\mu, -\mu) |C_H(-m_H^2, -\mu^2)|^2 \int \frac{d^2\mathbf{b}}{4\pi} e^{i(\mathbf{b}q_T)} \Phi_{g\leftarrow h_1}^{\mu\nu}(x_1, \mathbf{b}; \mu, \zeta_1) \Phi_{g\leftarrow h_2}^{\mu\nu}(x_2, \mathbf{b}; \mu, \zeta_2).$$

$$\Phi_{g\leftarrow h_1}^{\mu\nu}(x_1, \mathbf{b}) \Phi_{g\leftarrow h_2}^{\mu\nu}(x_2, \mathbf{b}) = \frac{1}{2} (f_{1,g\leftarrow h_1}(x_1, \mathbf{b}) f_{1,g\leftarrow h_2}(x_2, \mathbf{b}) + h_{1,g\leftarrow h_1}^\perp(x_1, \mathbf{b}) h_{1,g\leftarrow h_2}^\perp(x_2, \mathbf{b}))$$



Toy-Higgs (preliminary tests)



Back up: SIDIS

$$\frac{d\sigma}{dx dz dQ^2 d\vec{p}_\perp^2} = \frac{\pi}{\sqrt{1-\zeta_\perp^2}} \frac{\alpha_{\text{em}}^2}{Q^4} \frac{y^2}{1-\varepsilon} \frac{z_S}{z} \times \sum_f e_f^2 \left[\left(1 + \frac{\vec{q}_T^2}{Q^2} \frac{\varepsilon - \frac{\gamma^2}{2}}{1 + \gamma^2} \right) W_{f_1 D_1}^f(Q, \sqrt{\vec{q}_T^2}, x_S, z_S) + \frac{\vec{q}_T^2}{Q^2} \frac{\varepsilon - \frac{\gamma^2}{2}}{1 + \gamma^2} W_{h_1^\perp H_1^\perp}^f(Q, \sqrt{\vec{q}_T^2}, x_S, z_S) \right]$$