The Odderons and the Sivers function

Renaud Boussarie

Quarkonia as Tools 2020

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[Altinoluk, RB, Kotko], JHEP 1905 (2019) 156

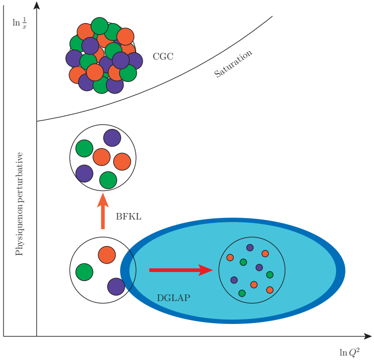
[Altinoluk, RB], JHEP 1910 (2019) 208

[RB, Mehtar-Tani]

[RB, Hatta, Szymanowski, Wallon] 1912.08182[hep-ph]

QCD at moderate x: Parton Distributions

$$Q^2 \sim s$$



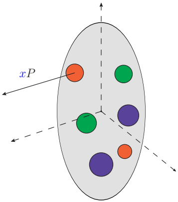
Operator definition for parton distributions

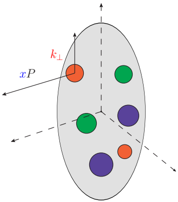
Parton distribution function

$$\mathcal{F}(x) \propto \int dz^+ e^{ixP^- z^+} \left\langle P \left| F^{-i}(z^+) \left[z^+, 0^+\right] F^{-i}(0) \left[0^+, z^+\right] \right| P \right\rangle$$

Transverse Momentum Dependent distribution

$$\mathcal{F}(x, k_\perp) \propto \int d^4z \delta(z^-) e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P \left| F^{-i}(z) \mathcal{U}_{z,0} F^{-i}(0) \mathcal{U}_{0,z} \right| P \right\rangle$$





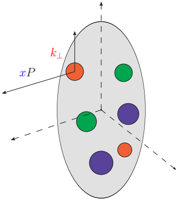
Operator definition for parton distributions

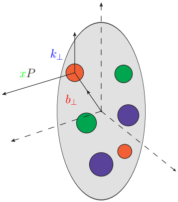
TMD distribution

$$\mathcal{F}(x, k_\perp) \propto \int d^4z\,\delta(z^-)\,e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P \left| F^{-i}(z)\,\mathcal{U}_{z,0}\,F^{-i}(0)\,\mathcal{U}_{0,z} \right| P \right\rangle$$

Generalized TMD distribution

$$\mathcal{F}(x, k_\perp, \Delta) \propto \int d^4z\,\delta(z^-)\,e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P+\Delta \left| F^{-i}(z)\,\mathcal{U}_{z,0}\,F^{-i}(0)\,\mathcal{U}_{0,z} \right| P \right\rangle$$

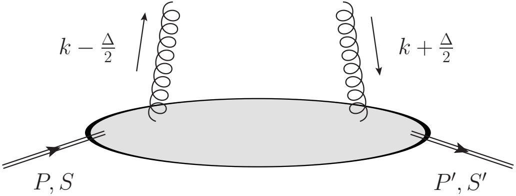




GTMD

Parametrization and coupling to the target hadron

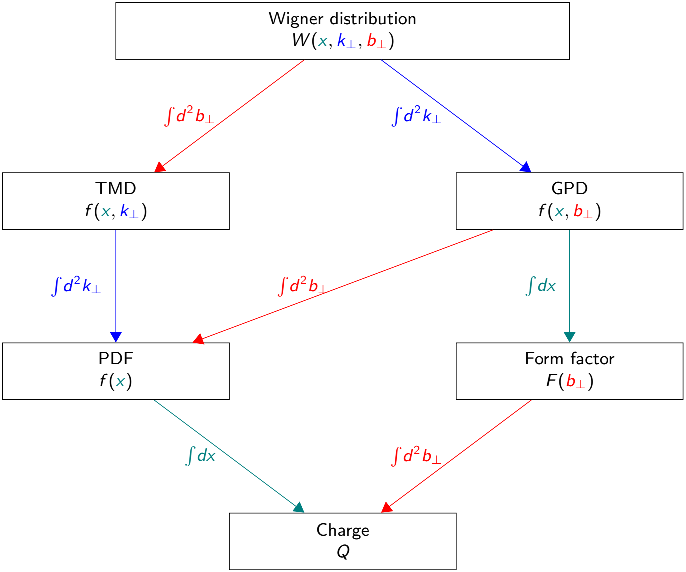
[Lorcé, Pasquini], [Bhattacharya, Metz, Ojha, Tsai, Zhou]



$$\int d^4v\,\delta(v^-)e^{ix\bar{P}^- v^+ - i(\mathbf{k}\cdot\mathbf{v})}\langle P'S'|\mathrm{Tr}\left[F^{i-}\left(-\frac{v}{2}\right)\mathcal{U}^{[+]}_{\frac{v}{2},-\frac{v}{2}}F^{i-}\left(\frac{v}{2}\right)\mathcal{U}^{[-]}_{-\frac{v}{2},\frac{v}{2}}\right]|PS\rangle$$

$$= (2\pi)^3\frac{\bar{P}^-}{2M}\bar{u}_{P'S'}\left[F^g_{1,1} + i\frac{\sigma^{i-}}{\bar{P}^-}\left(\mathbf{k}^i F^g_{1,2} + \mathbf{\Delta}^i F^g_{1,3}\right) + i\frac{\sigma^{ij}\mathbf{k}^i\mathbf{\Delta}^j}{M^2}F^g_{1,4}\right]u_{PS}$$

The family tree of parton distributions



(So-called) non-universality of TMD distributions:

The importance of gauge links

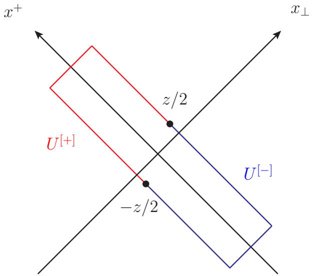
[Collins, Soper, Sterman], [Brodsky, Hwang, Schmidt], [Belitsky, Ji, Yuan], [Bomhof, Mulders, Pijlman], [Boer, Mulders, Pijlman]

[Kharzeev, Kovchegov, Tuchin]

TMD gauge links

"Non-universality" of quark TMD distributions

Gauge links can be future-pointing or past-pointing



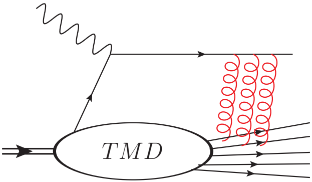
$$q^{[+]}(x, k_\perp) \propto \left\langle P, S \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[+]}_{\frac{z}{2}, -\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P, S \right\rangle$$

$$q^{[-]}(x, k_\perp) \propto \left\langle P, S \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[-]}_{\frac{z}{2}, -\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P, S \right\rangle$$

For naive T-odd distributions, $q^{[+]} = -q^{[-]}$: Sivers sign flip

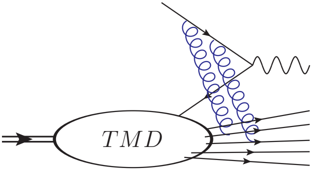
The Sivers effect

SIDIS



Final state interactions: $q^{[+]}$

Drell-Yan

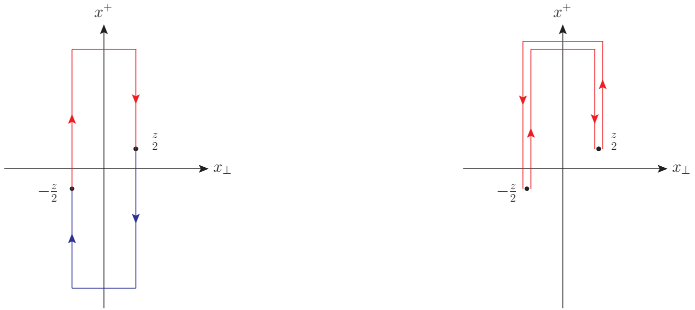


Initial state interactions: $q^{[-]}$

The Sivers distribution comes with a relative $-$ sign between SIDIS and DY: different gauge links for a naive T-odd quantity!

TMD gauge links

"Non-universality" of gluon TMD distributions

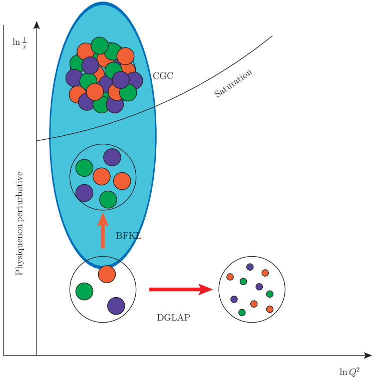


$$\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[-]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right] \qquad \mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[+]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]$$

Even more possibilities for gluon TMD distributions!

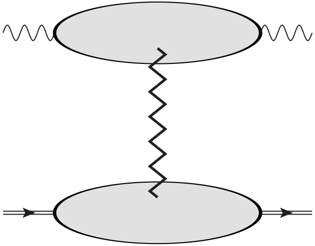
QCD at small x

$$Q^2 \ll s$$



The Pomeron

Regge theory: for asymptotic values of s, an effective particle with the quantum numbers of the vacuum is exchanged

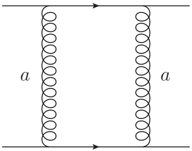


Positive C-parity: Pomeron exchange, negative C-parity: Odderon exchange

- How can we understand the Pomeron and the Odderon in perturbative QCD?
- How does it couple to hadrons?

Naive perturbative Pomeron and Odderon

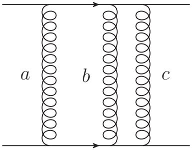
Naive perturbative description of the target hadron



Two gluons on a color singlet state

$$\mathrm{tr}(t^a t^a)$$

Leading Pomeron



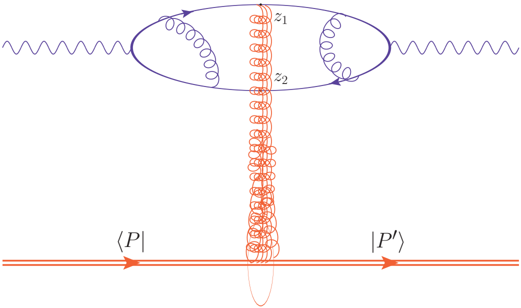
Three gluons on a color singlet state

$$\mathrm{tr}(t^a t^b t^c) = \frac{1}{4}(d^{abc} + i f^{abc})$$

f^{abc}: subleading Pomeron
d^{abc}: leading Odderon

More involved but still for perturbative targets: BFKL, BKP, BLV...

Semi-classical small x physics (shockwaves, CGC)



$$\mathcal{A}^{Y_c} = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2 \, \Phi^{Y_c}(\vec{z}_1, \vec{z}_2) \, \langle P'|[\mathrm{Tr}(U^{Y_c}_{\vec{z}_1} U^{Y_c\dagger}_{\vec{z}_2}) - N_c]|P\rangle$$

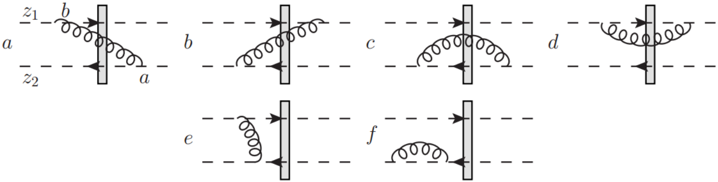
Dipole operator $\mathcal{U}^{Y_c}_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^{Y_c}_{\vec{z}_i} U^{Y_c\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation!

Y_c independence: B-JIMWLK hierarchy of equations
[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

Evolution for the dipole operator

$$\mathcal{U}_{12}^{Y_c+\delta Y} - \mathcal{U}_{12}^{Y_c}$$



B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

$$\frac{\partial \mathcal{U}_{12}^{Y_c}}{\partial Y_c} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{Y_c} + \mathcal{U}_{32}^{Y_c} - \mathcal{U}_{12}^{Y_c} + \mathcal{U}_{13}^{Y_c}\mathcal{U}_{32}^{Y_c} \right]$$

$$\frac{\partial \mathcal{U}_{13}^{Y_c}\mathcal{U}_{32}^{Y_c}}{\partial Y_c} = \ldots$$

Evolves a dipole into a double dipole

The seemingly incompatible nature of the distributions

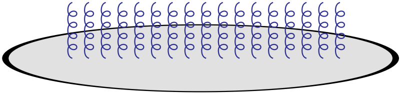
Two different kinds of gluon distributions

Moderate x distributions	Low x distributions
GTMD, GPD, TMD, PDF...	Dipole scattering amplitude
$\langle P^{(\prime)} \vert F^{-i} W F^{-j} W \vert P \rangle$	$\langle P^{(\prime)} \vert \mathrm{tr}(U_1 U_2^\dagger) \vert P \rangle$

(G)TMD distributions from QCD shockwaves

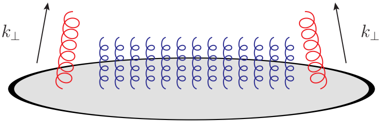
From the CGC to a TMD

From Wilson lines...



$$\left\langle P \left| \mathrm{Tr}\left(U_{\frac{r}{2}} U^{\dagger}_{-\frac{r}{2}} \right) \right| P \right\rangle$$

To a parton distribution



$$\left\langle P \left| \mathrm{Tr}\left(\partial^i U_{\frac{r}{2}} \partial^i U^{\dagger}_{-\frac{r}{2}} \right) \right| P \right\rangle$$

From the CGC to a TMD

Staple gauge links from a Wilson line operator

[Dominguez, Marquet, Xiao, Yuan]

Consider the derivative of a path-ordered Wilson line, denoting

$$[x_1^+, x_2^+]_{\vec{x}} \equiv \mathcal{P} \exp\left[ig \int_{x_1^+}^{x_2^+} dx^+ b^-(x^+, \vec{x}) \right]$$

For a given shockwave operator $U_{\vec{x}} = [-\infty, +\infty]_{\vec{x}}$

$$\partial^i U_{\vec{x}} = ig \int dx^+ [-\infty, x^+]_{\vec{x}} F^{-i}(x^+, \vec{x}) [x^+, +\infty]_{\vec{x}}$$

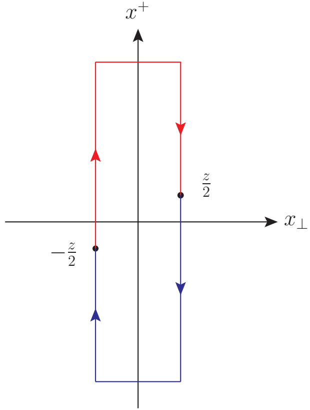
$$\partial^j U_{\vec{x}}^\dagger = -ig \int dx^+ [+\infty, x^+]_{\vec{x}} F^{-j}(x^+, \vec{x}) [x^+, -\infty]_{\vec{x}}$$

$$(\partial^i U_{\vec{x}}^\dagger) U_{\vec{x}} = -ig \int dx^+ [+\infty, x^+]_{\vec{x}} F^{-i}(x^+, \vec{x}) [x^+, +\infty]_{\vec{x}}$$

Taking the derivative of a shockwave operator allows to extract a physical gluon

From the CGC to a TMD

The dipole TMD

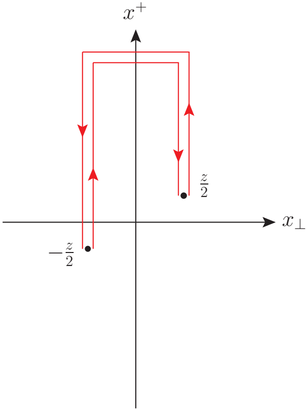


$$\mathcal{F}_{qg}^{(1)}(x, k_\perp) \propto \int d^4z\delta\left(z^+\right) e^{ix(P\cdot z)+i(k_\perp\cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[-]\dagger} F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]\right| P\right\rangle$$

$$\to \int d^2z_\perp e^{i(k_\perp\cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[\left(\partial^i U^\dagger_{\frac{z}{2}}\right)\left(\partial^i U_{-\frac{z}{2}}\right)\right]\right| P\right\rangle$$

From the CGC to a TMD

The Weizsäcker-Williams TMD



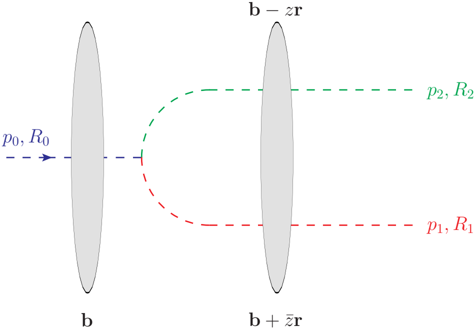
$$\mathcal{F}_{gg}^{(3)}(x,k_\perp) \propto \int d^4z\delta\left(z^+\right) e^{ix(P\cdot z)+i(k_\perp\cdot z_\perp)}\left\langle P\left|\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[+]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]\right|P\right\rangle$$

$$\to \int dz_\perp e^{i(k_\perp\cdot z_\perp)}\left\langle P\left|\mathrm{Tr}\left[\left(\partial^i U_{\frac{z}{2}}\right)U^\dagger_{-\frac{z}{2}}\left(\partial^i U_{-\frac{z}{2}}\right)U^\dagger_{\frac{z}{2}}\right]\right|P\right\rangle$$

General $1 \to 2$ process in the shockwave framework

Splitting of a particle into two particles in the external shockwave field

$$\mathcal{A} = (2\pi)\,\delta\left(p_1^+ + p_2^+ - p_0^+\right) \int d^2\boldsymbol{b}\, d^2\boldsymbol{r}\, e^{-i(\boldsymbol{q}\cdot\boldsymbol{r})-i(\boldsymbol{k}\cdot\boldsymbol{b})}\mathcal{H}(\boldsymbol{r})$$
$$\times \left[\left(U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}}\, T^{R_0}\, U^{R_2}_{\boldsymbol{b}-z\boldsymbol{r}}\right) - \left(U^{R_1}_{\boldsymbol{b}}\, T^{R_0}\, U^{R_2}_{\boldsymbol{b}}\right)\right],$$



Matching shockwave amplitudes and TMD amplitudes

[Altinoluk, RB], [RB, Mehtar-Tani]

We can cast the shockwave amplitude into a 1-body amplitude

$$\mathcal{A}_1 = (2\pi)\,\delta\left(p_1^+ + p_2^+ - p_0^+\right)\int d^2\boldsymbol{b}\, e^{-i(\boldsymbol{k}\cdot\boldsymbol{b})}(-i)\int d^2\boldsymbol{r}\, e^{-i(\boldsymbol{q}\cdot\boldsymbol{r})} r_\perp^\alpha \mathcal{H}(\boldsymbol{r})$$
$$\times\left[\left(\frac{e^{i\bar{z}(\boldsymbol{k}\cdot\boldsymbol{r})}-1}{(\boldsymbol{k}\cdot\boldsymbol{r})}\right)\left(\partial_\alpha U_{\boldsymbol{b}}^{R_1}\right) T^{R_0} U_{\boldsymbol{b}}^{R_2} + \left(\frac{e^{-iz(\boldsymbol{k}\cdot\boldsymbol{r})}-1}{(\boldsymbol{k}\cdot\boldsymbol{r})}\right) U_{\boldsymbol{b}}^{R_1} T^{R_0}\left(\partial_\alpha U_{\boldsymbol{b}}^{R_2}\right)\right]$$

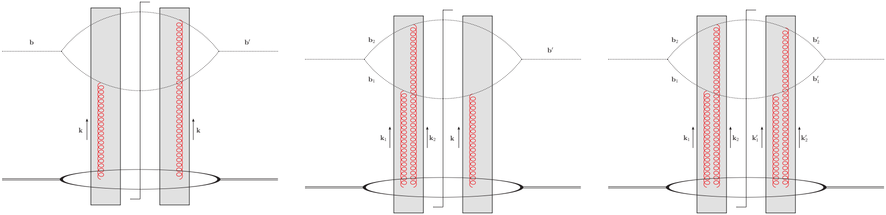
and a 2-body amplitude

$$\mathcal{A}_2 = (2\pi)\,\delta\left(p_1^+ + p_2^+ - p_0^+\right)\int \frac{d^2\boldsymbol{k}_1}{(2\pi)^2}\frac{d^2\boldsymbol{k}_2}{(2\pi)^2}(2\pi)^2\,\delta^2\left(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}\right)$$
$$\times\int d^2\boldsymbol{b}_1 d^2\boldsymbol{b}_2 e^{-i(\boldsymbol{k}_1\cdot\boldsymbol{b}_1)-i(\boldsymbol{k}_2\cdot\boldsymbol{b}_2)}\left(\partial^i U_{\boldsymbol{b}_1}^{R_1}\right) T^{R_0}\left(\partial^j U_{\boldsymbol{b}_2}^{R_2}\right)$$
$$\times\left[-\int d^2\boldsymbol{r} e^{-i(\boldsymbol{q}\cdot\boldsymbol{r})}\boldsymbol{r}^i\boldsymbol{r}^j\mathcal{H}(\boldsymbol{r})\left(\frac{e^{-iz(\boldsymbol{k}\cdot\boldsymbol{r})}}{(\boldsymbol{k}\cdot\boldsymbol{r})}\frac{e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r})}-1}{(\boldsymbol{k}_1\cdot\boldsymbol{r})}+\frac{e^{i\bar{z}(\boldsymbol{k}\cdot\boldsymbol{r})}}{(\boldsymbol{k}\cdot\boldsymbol{r})}\frac{e^{-i(\boldsymbol{k}_2\cdot\boldsymbol{r})}-1}{(\boldsymbol{k}_2\cdot\boldsymbol{r})}\right)\right]$$

Inclusive low x cross section

Inclusive low x cross section = TMD cross section

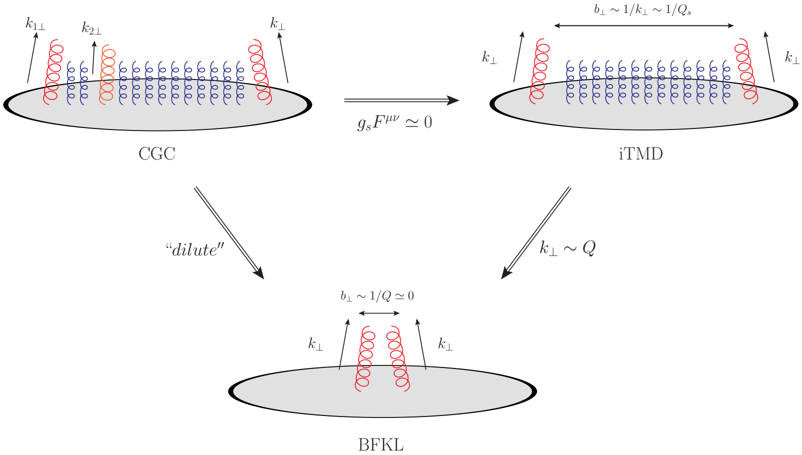
[Altinoluk, RB, Kotko], [Altinoluk, RB]



$$\sigma = \mathcal{H}_2^{ij}(k_\perp) \otimes \left\langle P \left| F^{-i} W F^{-j} W \right| P \right\rangle$$
$$+ \mathcal{H}_3^{ijk}(k_\perp, k_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W F^{-k} W \right| P \right\rangle$$
$$+ \mathcal{H}_4^{ijkl}(k_\perp, k_{1\perp}, k'_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W g_s F^{-k} W F^{-l} W \right| P \right\rangle$$

The dilute limit

The dilute limit in terms of TMD distributions



Two kinds of multiple scattering effects: higher genuine twists and higher kinematic twists

Small x formalisms vs. TMD factorization

Semiclassical (shockwaves, CGC) $k_\perp, Q \ll s$

$$\sum_i \mathcal{H}_2^{(i)}(\boldsymbol{k}) \otimes \mathcal{F}_2^{(i)}(\boldsymbol{k}) + \mathcal{H}_3^{(i)}(\boldsymbol{k}_1, \boldsymbol{k}_2) \otimes \mathcal{F}_3^{(i)}(\boldsymbol{k}_1, \boldsymbol{k}_2) + \mathcal{H}_4^{(i)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \otimes \mathcal{F}_4^{(i)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$

Small x Improved TMD $k_\perp, Q \ll s$?

$$\sum_i \mathcal{H}_2^{(i)}(\boldsymbol{k}) \otimes \mathcal{F}_2^{(i)}(\boldsymbol{k})$$

TMD at small x $k_\perp \ll Q \ll s$

$$\sum_i \mathcal{H}_2^{(i)}(\boldsymbol{0}) \otimes \mathcal{F}_2^{(i)}(\boldsymbol{k})$$

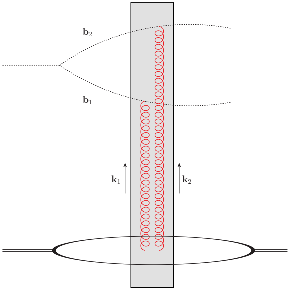
k_t factorization (BFKL, HEF...) $k_\perp \lesssim Q \ll s$

$$\mathcal{H}_2(\boldsymbol{k}) \otimes \mathcal{F}_2(\boldsymbol{k})$$

Exclusive low x cross section

Exclusive low x amplitude = GTMD amplitude

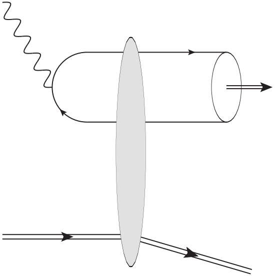
[Altinoluk, RB], [RB, Mehtar-Tani]



$$\mathcal{H}^{ij}(k_{1\perp}, k_{2\perp}) \otimes \langle P' | F^{-i} W F^{-j} W | P \rangle$$

Every exclusive low x process probes
a Wigner distribution!

Deeply Virtual Meson Production



DVMP, the Pomeron and the Odderon

Exclusive low x cross section

Low x-GTMD equivalence for DVMP

[Altinoluk, RB], [RB,Mehtar-Tani]

$$\left\langle P', S' \left| \mathrm{Tr}\left(U_{\boldsymbol{x}_1} U^\dagger_{\boldsymbol{x}_2}\right) - N_c \right| P, S \right\rangle$$

$$= \frac{\alpha_s \bar{P}^-}{M} e^{-i\boldsymbol{\Delta}\cdot\left(\frac{\boldsymbol{x}_1+\boldsymbol{x}_2}{2}\right)} \delta\left(\Delta^-\right) \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}}$$

$$\times \left[e^{-i(\boldsymbol{k}\cdot\boldsymbol{r})} - \frac{1}{2}\left(e^{i\left(\boldsymbol{\Delta}\cdot\frac{\boldsymbol{r}}{2}\right)} + e^{-i\left(\boldsymbol{\Delta}\cdot\frac{\boldsymbol{r}}{2}\right)}\right) + \frac{(\boldsymbol{k}\cdot\boldsymbol{r})}{(\boldsymbol{\Delta}\cdot\boldsymbol{r})}\left(e^{i\left(\boldsymbol{\Delta}\cdot\frac{\boldsymbol{r}}{2}\right)} - e^{-i\left(\boldsymbol{\Delta}\cdot\frac{\boldsymbol{r}}{2}\right)}\right)\right]$$

$$\times \bar{u}_{P',S'} \left[F^g_{1,1} + i\frac{\sigma^{i-}}{\bar{P}^-}\left(\boldsymbol{k}^i F^g_{1,2} + \boldsymbol{\Delta}^i F^g_{1,3}\right) + i\frac{\sigma^{ij}\boldsymbol{k}^i\boldsymbol{\Delta}^j}{M^2} F^g_{1,4}\right] u_{P,S}$$

Gluon GTMDs

The dipole-type gluon GTMDs

$$\left\langle P', S' \left| \mathrm{Tr}\left(U_{\boldsymbol{x}_1} U^{\dagger}_{\boldsymbol{x}_2} \right) - N_c \right| P, S \right\rangle$$
$$= \frac{\alpha_s \bar{P}^-}{M} e^{-i \boldsymbol{\Delta} \cdot \left(\frac{\boldsymbol{x}_1 + \boldsymbol{x}_2}{2} \right)} \delta\left(\Delta^-\right) \int \frac{d^2 \boldsymbol{k}}{\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}}$$
$$\times\, e^{-i(\boldsymbol{k} \cdot \boldsymbol{r})} \bar{u}_{P',S'} \left[F^g_{1,1} + i \frac{\sigma^{i-}}{\bar{P}^-} \left(\boldsymbol{k}^i F^g_{1,2} + \boldsymbol{\Delta}^i F^g_{1,3} \right) \right] u_{P,S}$$

PT symmetry

$$F^{g*}_{1,(1,3)}(x, \xi, \boldsymbol{k}^2, \boldsymbol{k} \cdot \boldsymbol{\Delta}, \boldsymbol{\Delta}^2) = F^g_{1,(1,3)}(x, -\xi, \boldsymbol{k}^2, -\boldsymbol{k} \cdot \boldsymbol{\Delta}, \boldsymbol{\Delta}^2)$$

$$F^{g*}_{1,2}(x, \xi, \boldsymbol{k}^2, \boldsymbol{k} \cdot \boldsymbol{\Delta}, \boldsymbol{\Delta}^2) = -F^g_{1,2}(x, -\xi, \boldsymbol{k}^2, -\boldsymbol{k} \cdot \boldsymbol{\Delta}, \boldsymbol{\Delta}^2)$$

Gluon GTMDs

The dipole-type gluon GTMDs

$$\left\langle P', S' \left| \mathrm{Tr}\left(U_{\mathbf{x}_1} U^{\dagger}_{\mathbf{x}_2}\right) - N_c \right| P, S \right\rangle$$
$$= \frac{\alpha_s \bar{P}^-}{M} e^{-i\boldsymbol{\Delta}\cdot\left(\frac{\mathbf{x}_1+\mathbf{x}_2}{2}\right)} \delta\left(\Delta^-\right) \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}}$$
$$\times\, e^{-i(\boldsymbol{k}\cdot\boldsymbol{r})} \bar{u}_{P',S'} \left[F^g_{1,1} + i\frac{\sigma^{i-}}{\bar{P}^-}\left(\boldsymbol{k}^i F^g_{1,2} + \boldsymbol{\Delta}^i F^g_{1,3}\right)\right] u_{P,S}$$

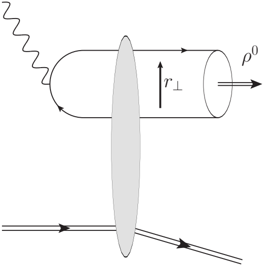
Small ξ limit

$$F^g_{1,(1,3)} = f_{1,(1,3)}(x, \xi, \boldsymbol{k}^2, |\boldsymbol{k}\cdot\boldsymbol{\Delta}|, \boldsymbol{\Delta}^2) + i\frac{(\boldsymbol{k}\cdot\boldsymbol{\Delta})}{M^2} g_{1,(1,3)}(x, \xi, \boldsymbol{k}^2, |\boldsymbol{k}\cdot\boldsymbol{\Delta}|, \boldsymbol{\Delta}^2)$$

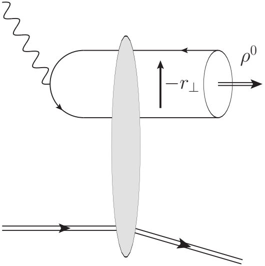
$$F^g_{1,2} = \frac{(\boldsymbol{k}\cdot\boldsymbol{\Delta})}{M^2} f_{1,2}(x, \xi, \boldsymbol{k}^2, |\boldsymbol{k}\cdot\boldsymbol{\Delta}|, \boldsymbol{\Delta}^2) + i g_{1,2}(x, \xi, \boldsymbol{k}^2, |\boldsymbol{k}\cdot\boldsymbol{\Delta}|, \boldsymbol{\Delta}^2)$$

DVMP and the Pomeron(s)

Pomeron exchange: C odd meson production



$+$



$$\frac{1}{2}\left[\mathrm{Tr}\left(U_{b+\frac{r}{2}}U^\dagger_{b-\frac{r}{2}}\right) + \mathrm{Tr}\left(U_{b-\frac{r}{2}}U^\dagger_{b+\frac{r}{2}}\right)\right] - N_c$$

The Pomerons

Fourier transform of the $(r_\perp \leftrightarrow -r_\perp)$-symmetric dipole (Pomeron)

$$\bar{u}_{P',S'}\left[\gamma^+ f_{1,1} + i\sigma^{i+}\frac{\boldsymbol{\Delta}^j}{M}\left(\frac{\boldsymbol{k}^i\boldsymbol{k}^j}{M^2}f_{1,2} + \delta^{ij}(f_{1,3} - \frac{1}{2}f_{1,1})\right)\right]u_{P,S}.$$

In the forward limit, we recover the unpolarized TMD

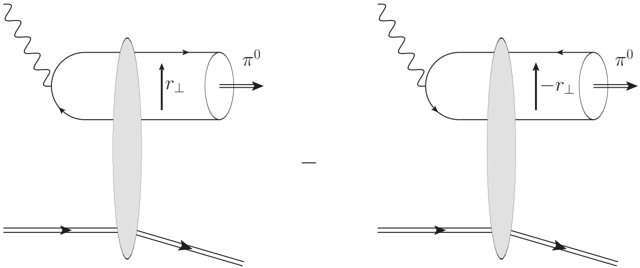
$$xf(x, \boldsymbol{k}^2) = \mathrm{Re}\, F^g_{1,1}(x, 0, k^2, 0, 0)$$

The forward Pomeron is the unpolarized TMD

$$\int d^2\boldsymbol{v} e^{-i(\boldsymbol{k}\cdot\boldsymbol{v})}\boldsymbol{k}^2\langle P', S'|\mathcal{P}(\boldsymbol{v})|P, S\rangle = \frac{g_s^2}{2}N_c(2\pi)^2\delta(P'^+ - P^+)\left(\bar{u}_{P',S'}\gamma^+ u_{P,S}\right) xf(x, \boldsymbol{k}^2)$$

DVMP and the Odderon(s)

Odderon exchange: C even meson production



$$\frac{1}{2}\left[\mathrm{Tr}\left(U_{b+\frac{r}{2}}U^\dagger_{b-\frac{r}{2}}\right) - \mathrm{Tr}\left(U_{b-\frac{r}{2}}U^\dagger_{b+\frac{r}{2}}\right)\right]$$

The Odderons

Fourier transform of the $(r_\perp \leftrightarrow -r_\perp)$-antisymmetric dipole (Odderon)

$$i\frac{\boldsymbol{k}^j}{M}\bar{u}_{P',S'}\left[\frac{\boldsymbol{\Delta}^j}{M}\gamma^+ g_{1,1} + i\sigma^{i+}\left(\delta^{ij} g_{1,2} + \frac{\boldsymbol{\Delta}^i\boldsymbol{\Delta}^j}{M^2}\left(g_{1,3} - \frac{1}{2}g_{1,1}\right)\right)\right] u_{P,S}.$$

With explicit spinors, we see 3 types of coupling to the target:

- The Vector Odderon $i(\boldsymbol{k}\cdot\boldsymbol{\Delta})g_{1,1}$
- The Spin Odderon $(\boldsymbol{k}\times\boldsymbol{S})^z g_{1,2}$
  Appears in semi-inclusive transverse SSA [Zhou], [Szymanowski, Zhou], [Boer, Echevarria, Mulders, Zhou], [Dong, Zheng, Zhou]
- A new type of Odderon $(\boldsymbol{k}\cdot\boldsymbol{\Delta})(\boldsymbol{\Delta}\times\boldsymbol{S})^z g_{1,3}$

The Odderons

Fourier transform of the $(r_\perp \leftrightarrow -r_\perp)$-antisymmetric dipole (Odderon)

$$i\frac{\boldsymbol{k}^j}{M}\bar{u}_{P',S'}\left[\frac{\boldsymbol{\Delta}^j}{M}\gamma^+ g_{1,1} + i\sigma^{i+}\left(\delta^{ij} g_{1,2} + \frac{\boldsymbol{\Delta}^i\boldsymbol{\Delta}^j}{M^2}(g_{1,3} - \frac{1}{2}g_{1,1})\right)\right]u_{P,S}.$$

In the forward limit, it reduces to the Sivers TMD:

$$xf_{1T}^\perp(x,\boldsymbol{k}^2) = -\frac{1}{2}\mathrm{Im}\,F_{1,2}^g(x,0,\boldsymbol{k}^2,0,0)$$

The forward Odderon is the gluon Sivers function

$$\int d^2\boldsymbol{v} e^{-i(\boldsymbol{k}\cdot\boldsymbol{v})}\boldsymbol{k}^2\langle P',S'|\mathcal{O}(\boldsymbol{v})|P,S\rangle$$
$$= -\frac{g_s^2}{4}N_c(2\pi)^2\delta(P'^+ - P^+)\frac{\boldsymbol{k}^j}{M}\left(\bar{u}_{P',S'}\sigma^{+j}u_{P,S}\right)xf_{1T}^\perp(x,\boldsymbol{k}^2).$$

Probing the Sivers function

Thanks to the Odderon/GTMD equivalence, the cross section for exclusive π^0 electroproduction at small x and small t with unpolarized lepton and proton beams is a direct probe for the gluon Sivers function

$$\frac{d\sigma}{dx_B dQ^2 d|t|} \simeq \frac{\pi^5 \alpha_{\rm em}^2 \alpha_s^2 f_\pi^2}{8 x_B N_c M^2 Q^2}\left(1 - y + \frac{y^2}{2}\right) \times \left[\int_0^1 dz \frac{\phi_\pi(z)}{z\bar{z}Q^2} \int d\boldsymbol{k}^2 \frac{\boldsymbol{k}^2}{\boldsymbol{k}^2 + z\bar{z}Q^2} x f_{1T}^{\perp}(x, \boldsymbol{k}^2)\right]^2 .$$

We can thus understand the gluonic content of the transversely polarized protons without polarizing the proton beam.

The Odderon at HERA

Why did HERA not see the Odderon in DVMP?

- Historically, small x physics has long been assumed to not contain spin effects
- The exchange of an Odderon is mostly a spin flip effect!
- Neglecting spin flip effects means that Odderon-induced DVMP cancels in the small t limit
- [Berger, Donnachie, Dosch, Kilian, Nachtmann, Rueter] assumed the leading contribution would come with an outgoing excited proton state
- Hence experimentalists looked for the Odderon either in processes with no hard scale, or by trying to tag outgoing excited proton states

Conclusions

- GTMD distributions are what allows to match standard parton distributions and semi-classical descriptions of small x physics
- Inclusive physics at small x are a great place to study the so-called non-universality of TMD distributions, and gluon saturation effects
- Exclusive physics at small x is particularly interesting to understand spin physics without going beyond the eikonal approximation
- Processes with Odderon exchange constitute a direct probe for the gluonic content of transversely polarized protons, with unpolarized proton beams

Backup

Operator product expansion (OPE)

- Moderate x OPE: factorization

$$\mathcal{O}(z) \to \sum_n C_n(z,\mu)\mathcal{O}_n(\mu)$$

  - Operators are ordered in twists (dimension − spin)
  - Divergences in C_n are canceled via renormalization of $\mathcal{O}_n$
  - Easy task: resumming powers of s and logarithms of Q^2. Difficulty: including twist corrections and logarithms of s

- Low x OPE:

$$\mathcal{O}(z) \to C_0(z,Y)\mathcal{O}_0(Y) + \alpha_s C_1(z,Y)\mathcal{O}_1(Y) + ...$$

  - Operators are sorted by representations of $SU(N_c)$, order by order in α_s
  - Built order by order in α_s. The spurious pole in $C_n(z,Y)$ is canceled via the B/JIMWLK RGE of $\mathcal{O}_{n-1}(Y)$
  - Easy task: resumming twists and logarithms of s. Difficulty: including subeikonal corrections and logarithms of Q^2

Small dipole "correlation" expansion

Taylor expansion of the Wilson line operators

$$U^{R_1}_{\boldsymbol{b}+\frac{\boldsymbol{r}}{2}}\, T^{R_0}\, U^{R_2}_{\boldsymbol{b}-\frac{\boldsymbol{r}}{2}} - U^{R_1}_{\boldsymbol{b}}\, T^{R_0}\, U^{R_2}_{\boldsymbol{b}} = \frac{\boldsymbol{r}^i}{2}\left[\left(\partial^i U^{R_1}_{\boldsymbol{b}}\right) T^{R_0}\, U^{R_2}_{\boldsymbol{b}} - U^{R_1}_{\boldsymbol{b}}\, T^{R_0}\left(\partial^i U^{R_2}_{\boldsymbol{b}}\right)\right] + O\left(\boldsymbol{r}^2\right)$$

allows for a match at leading twist

$$d\sigma = \mathcal{H}\left(b, r\right) \otimes \left[U^{R_1}_{\boldsymbol{b}+\frac{\boldsymbol{r}}{2}}\, T^{R_0}\, U^{R_2}_{\boldsymbol{b}-\frac{\boldsymbol{r}}{2}} - U^{R_1}_{\boldsymbol{b}}\, T^{R_0}\, U^{R_2}_{\boldsymbol{b}}\right]$$

$$\times\, \mathcal{H}^*\left(b', r'\right) \otimes \left[U^{R_2\dagger}_{\boldsymbol{b'}-\frac{\boldsymbol{r'}}{2}}\, T^{R_0\dagger}\, U^{R_1\dagger}_{\boldsymbol{b'}+\frac{\boldsymbol{r'}}{2}} - U^{R_2\dagger}_{\boldsymbol{b'}}\, T^{R_0\dagger}\, U^{R_1\dagger}_{\boldsymbol{b'}}\right]$$

$$\rightarrow d\sigma^{(i)}_{\boldsymbol{k}=\boldsymbol{0}} \otimes \Phi^{(i)}\left(x, \boldsymbol{k}\right) + O\left(\boldsymbol{r}^2\right)$$

How to extend this to higher twist corrections?

Power expansion for TMD observables: dealing with powers of $k_\perp/Q$

Consider (hypothetical) hard subamplitudes with non-zero transverse momenta in the t channel. The amplitude would read:

$$\mathcal{H}_1^i(\boldsymbol{k}) \otimes \int d^2\boldsymbol{x}_1 e^{-i(\boldsymbol{k}\cdot\boldsymbol{x}_1)} [\pm\infty, x_1] F^{i-}(x_1) [x_1, \pm\infty]$$

$$+ \mathcal{H}_2^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2) \otimes \int d^2\boldsymbol{x}_1 d^2\boldsymbol{x}_2 e^{-i(\boldsymbol{k}_1\cdot\boldsymbol{x}_1)-i(\boldsymbol{k}_2\cdot\boldsymbol{x}_2)} [\pm\infty, x_1] F^{i-}(x_1) [x_1, x_2] F^{j-}(x_2) [x_2, \pm\infty]$$

$$+ \ldots$$

$$= \mathcal{H}_1^i(\boldsymbol{k}) \otimes \mathcal{O}_1^i(\boldsymbol{k}) + \mathcal{H}_2^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2) \otimes \mathcal{O}_2^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2) + \ldots$$

Power expansion for TMD amplitudes:
dealing with powers of $k_\perp/Q$

Leading twist amplitude

$$\mathcal{A}_{LT} = \mathcal{H}_1^i(\mathbf{0}) \otimes \mathcal{O}_1^i(\boldsymbol{k})$$

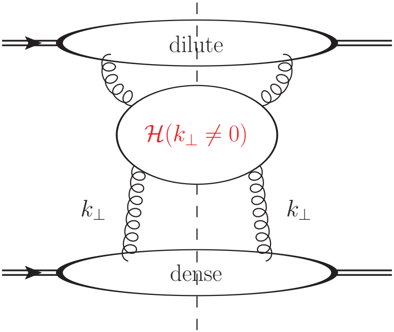
Next-to-leading twist amplitude

$$\mathcal{A}_{NLT} = \boldsymbol{k} \cdot \left(\partial_{\boldsymbol{k}} \mathcal{H}_1^i\right)(\mathbf{0}) \otimes \mathcal{O}_1^i(\boldsymbol{k}) + \mathcal{H}_2^{ij}(\mathbf{0}, \mathbf{0}) \otimes \mathcal{O}_2^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2)$$

First term: kinematic twist correction, second term: genuine twist corrections

Small x Improved TMD framework (ITMD)

A hybrid framework with off-shell gluons from the target
[Kotko, Kutak, Marquet, Petreska, Sapeta, Van Hameren]



- QCD gauge invariance for multileg amplitudes with an off-shell leg restored with target counterterms [Kotko]
- TMD gauge links are built from the [Bomhof, Mulders, Pijlman] techniques
- Eventually, looks like BFKL, but with distinct TMD distributions for different color flow structures. Interpolates between the TMD regime $|k_\perp| \ll Q$ and the BFKL regime $|k_\perp| \sim Q$

QCD shockwaves $k_\perp, Q \ll s$

$$\sum_i \mathcal{H}_2^{(i)}(\boldsymbol{k}) \otimes \mathcal{F}_2^{(i)}(\boldsymbol{k}) + \mathcal{H}_3^{(i)}(\boldsymbol{k}_1, \boldsymbol{k}_2) \otimes \mathcal{F}_3^{(i)}(\boldsymbol{k}_1, \boldsymbol{k}_2) + \mathcal{H}_4^{(i)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \otimes \mathcal{F}_4^{(i)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$

TMD at small x $k_\perp \ll Q \ll s$

$$\sum_i \mathcal{H}_2^{(i)}(\boldsymbol{0}) \otimes \mathcal{F}_2^{(i)}(\boldsymbol{k})$$

BFKL $k_\perp \lesssim Q \ll s$

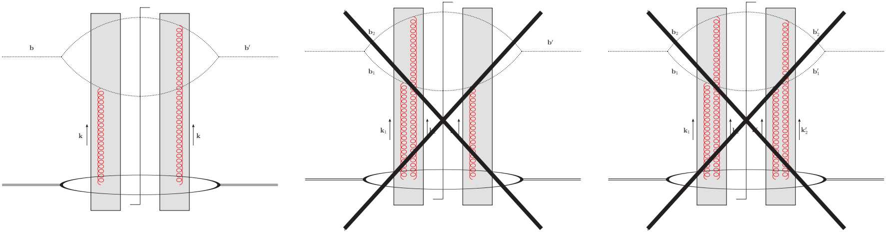
$$\mathcal{H}_2(\boldsymbol{k}) \otimes \mathcal{F}_2(\boldsymbol{k})$$

Small x Improved TMD $k_\perp, Q \ll s$?

$$\sum_i \mathcal{H}_2^{(i)}(\boldsymbol{k}) \otimes \mathcal{F}_2^{(i)}(\boldsymbol{k})$$

First, take the Wandzura-Wilczek approximation

[Altinoluk, RB, Kotko]: matches ITMD cross sections



$$\sigma = \mathcal{H}_2^{ij}(k_\perp) \otimes \left\langle P \left| F^{-i} W F^{-j} W \right| P \right\rangle$$

$$+\mathcal{H}_3^{ijk}(k_\perp, k_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W F^{-k} W \right| P \right\rangle$$

$$+\mathcal{H}_4^{ijkl}(k_\perp, k_{1\perp}, k'_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W g_s F^{-k} W F^{-l} W \right| P \right\rangle$$

WW approximation at large k_t: the BFKL limit

- At large transverse momentum transfer, no multiple scattering from the gauge links

TMD with staple gauge links

$$\int \frac{d^2\boldsymbol{k}}{(2\pi)^2} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \int dx^+ \left\langle P \left| F^{i-}(x) \left[x^+, \pm\infty\right]_{\boldsymbol{x}} \left[\pm\infty, 0^+\right]_{\boldsymbol{0}} F^{j-}(0) \left[0^+, \pm\infty\right]_{\boldsymbol{0}} \left[\pm\infty, x^+\right]_{\boldsymbol{x}} \right| P \right\rangle$$

Large $k_\perp \sim Q \Rightarrow$ small transverse distance $x_\perp$

$$\left[x^+, \pm\infty\right]_{\boldsymbol{x}} \left[\pm\infty, y^+\right]_{\boldsymbol{0}} \sim \left[x^+, y^+\right]_{\boldsymbol{x}\sim\boldsymbol{0}}.$$

All TMD distributions shrink into the unintegrated PDF

$$\int \frac{d^2\boldsymbol{k}}{(2\pi)^2} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \int dx^+ \left\langle P \left| F^{i-}(x) \left[x^+, 0^+\right]_{\boldsymbol{0}} F^{j-}(0) \left[0^+, x^+\right]_{\boldsymbol{0}} \right| P \right\rangle \Bigg|_{x^-=0}$$

and one recovers a BFKL cross section.

BFKL distributions and genuine twist corrections

Unintegrated PDF = 2-Reggeon matrix element

$$\int d^2\boldsymbol{x} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \int dx^+ \left\langle P \left| F^{i-}(x) \left[x^+, 0^+\right]_{\mathbf{0}} F^{j-}(0) \left[0^+, x^+\right]_{\mathbf{0}} \right| P \right\rangle \Big|_{x^-=0}$$

Integration by parts

$$\int dx^+ \int d^2\boldsymbol{x} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \boldsymbol{k}^i \boldsymbol{k}^j \langle P \left| \left[-\infty, x^+\right]_{\mathbf{0}} A^-(x) \left[x^+, +\infty\right]_{\mathbf{0}} \left[+\infty, 0^+\right]_{\mathbf{0}} A^-(0) \left[0^+, -\infty\right]_{\mathbf{0}} \right| P \rangle$$

We recognize the so-called nonsense polarizations in axial gauge. We could define a Reggeon operator:

$$R(x) = \int dx^+ \left[-\infty, x^+\right]_{\mathbf{0}} A^-(x) \left[x^+, +\infty\right]_{\mathbf{0}}$$

and rewrite the unintegrated PDF as

$$\int \frac{d^2\boldsymbol{k}}{(2\pi)^2} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \frac{\boldsymbol{k}^i \boldsymbol{k}^j}{\boldsymbol{k}^2} \boldsymbol{k}^2 \left\langle P \left| R(\boldsymbol{x}) R^\dagger(0) \right| P \right\rangle$$

BFKL distributions and genuine twist corrections

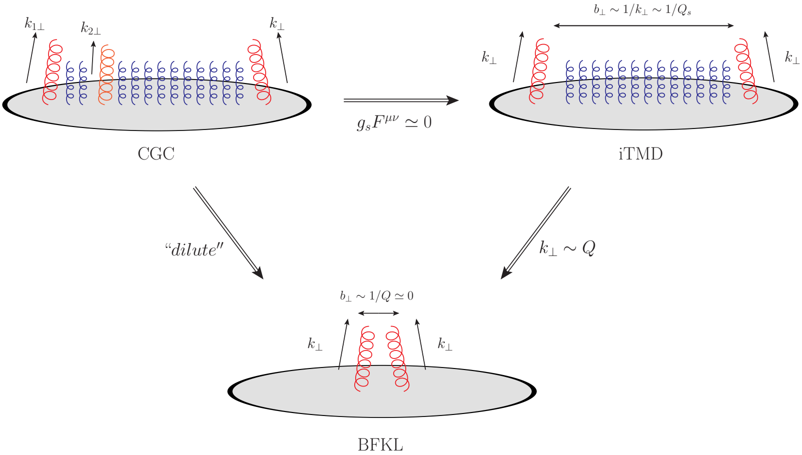
What is neglected in BFKL: 3- and 4-Reggeon matrix elements.

$$\langle P\,|RR|\,P\rangle\,,\quad \langle P\,|R(g_s R)R|\,P\rangle\,,\quad \langle P\,|R(g_s R)(g_s R)R|\,P\rangle$$

They are not perturbatively suppressed.

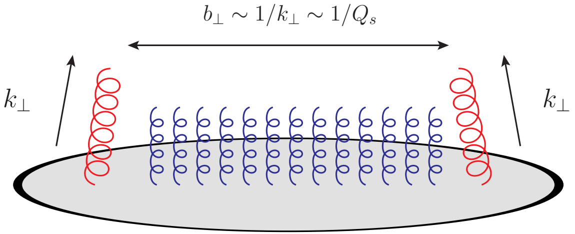
Suppression = WW approximation (unquantifiable)

The dilute limit in terms of TMD distributions



Two kinds of multiple scattering effects: higher genuine twists and higher kinematic twists

"Saturation" from a TMD gauge link

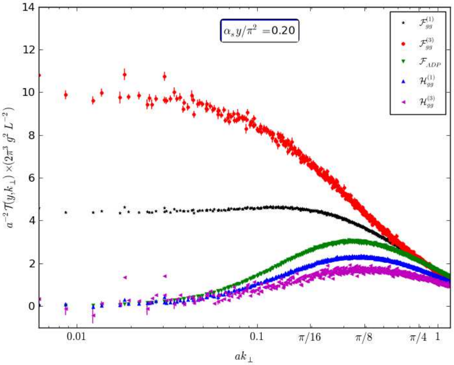


$$g_s^2 \int d^4 b\, \delta\left(b^-\right) e^{i(k\cdot b)} \left\langle P \left| F^{i-}(b)\, \mathcal{U}_{b,0}^{[\pm]} F^{j-}(0)\, \mathcal{U}_{0,b}^{[\pm]} \right| P \right\rangle$$

Expected at small $k_\perp/Q$

"Saturation" from a TMD gauge link

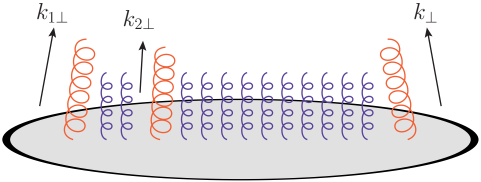
Link length $\sim 1/|k_\perp|$, hence effect suppressed at large $k_\perp$



[Marquet, Petreska, Roiesnel ; Marquet, Roiesnel, Taels]

"Saturation" as an enhancement of genuine twists

Large gluon occupancy $\Rightarrow g_s F \sim 1$



$$g_s^2 \int d^4 b_1 d^4 b_2 d^4 b' \delta\left(b_1^-\right) \delta\left(b_2^-\right) \delta\left(b'^-\right) e^{i(k_1 \cdot b_1) + i(k_2 \cdot b_2) - i\left(k \cdot b'\right)}$$
$$\times \frac{\left\langle P \left| F^{i-}(b_1) \mathcal{U}^{[\pm]}_{b_1, b_2} g_s F^{j-}(b_2) \mathcal{U}^{[\pm]}_{b_2, b'} F^{k-}(b') \mathcal{U}^{[\pm]}_{b', b_1} \right| P \right\rangle}{\langle P | P \rangle}$$

$k_\perp / Q$-suppressed: expected at large $k_\perp$?

Match without an expansion

Trick: rewrite operators in terms of their derivatives

$$U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}} - U^{R_1}_{\boldsymbol{b}} = -ir^{\mu}_{\perp}\int\frac{d^2\boldsymbol{k}_1}{(2\pi)^2}\int d^2\boldsymbol{b}_1 e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{b}_1-\boldsymbol{b})}\frac{e^{i\bar{z}(\boldsymbol{k}_1\cdot\boldsymbol{r})}-1}{(\boldsymbol{k}_1\cdot\boldsymbol{r})}\left(\partial_\mu U^{R_1}_{\boldsymbol{b}}\right)$$

Rewrite the amplitude

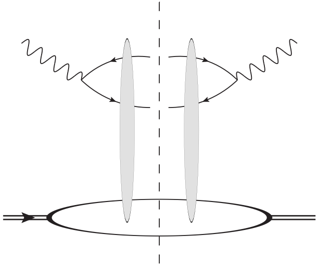
$$\mathcal{A} = (2\pi)\,\delta\left(p_1^+ + p_2^+ - p_0^+\right)\int d^2\boldsymbol{b}\, d^2\boldsymbol{r}\, e^{-i(\boldsymbol{q}\cdot\boldsymbol{r})-i(\boldsymbol{k}\cdot\boldsymbol{b})}\mathcal{H}(\boldsymbol{r})$$
$$\times\left[\left(U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}} - U^{R_1}_{\boldsymbol{b}}\right)T^{R_0}\left(U^{R_2}_{\boldsymbol{b}-z\boldsymbol{r}} - U^{R_2}_{\boldsymbol{b}}\right) + \left(U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}} - U^{R_1}_{\boldsymbol{b}}\right)T^{R_0}U^{R_2}_{\boldsymbol{b}} + U^{R_1}_{\boldsymbol{b}}T^{R_0}\left(U^{R_2}_{\boldsymbol{b}-z\boldsymbol{r}} - U^{R_2}_{\boldsymbol{b}}\right)\right]$$

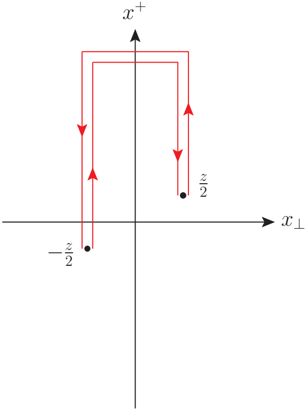
genuine twist — kinematic + genuine twists

Extracting genuine twists: Taylor, IbP, resummation.

Weizsäcker-Williams TMD

$T^{R_0} = 1, U^{R_1} = U, U^{R_2} = U^\dagger$

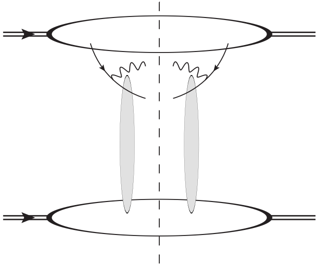


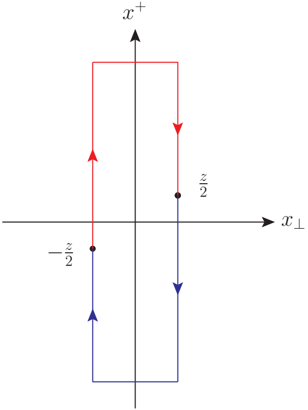


$$\mathcal{F}_{gg}^{(3)}(x \sim 0, k_\perp) \propto \int d^2 z_\perp e^{-i(k_\perp \cdot z_\perp)} \langle P | \mathrm{Tr}(\partial^i U^\dagger_{\frac{z}{2}}) U_{\frac{z}{2}} (\partial^i U^\dagger_{-\frac{z}{2}}) U_{-\frac{z}{2}} | P \rangle$$

Dipole TMD

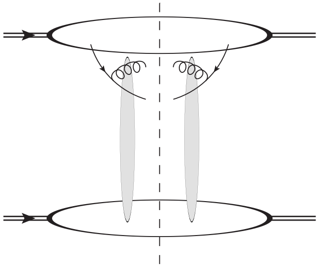
$T^{R_0} = 1, U^{R_1} = U, U^{R_2} = 1$

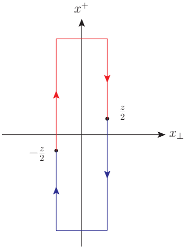




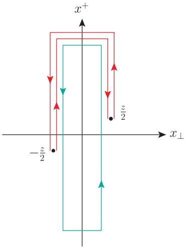
$$\mathcal{F}_{qg}^{(1)}(x \sim 0, k_\perp) \propto \int d^2 z_\perp e^{-i(k_\perp \cdot z_\perp)} \langle P | \mathrm{tr}(\partial^i U_{\frac{z}{2}})(\partial^i U^\dagger_{-\frac{z}{2}}) | P \rangle$$

Forward dijet production in pA collisions



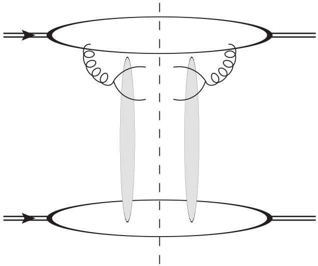


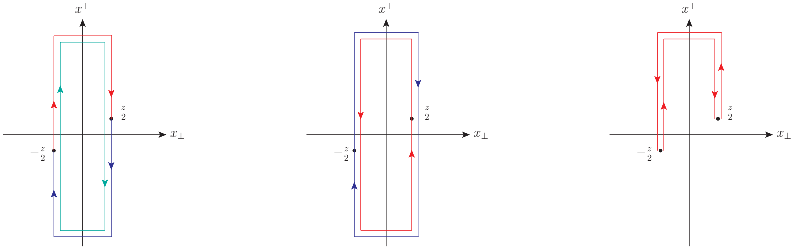
$\mathcal{F}_{qg}^{(1)}$



$\mathcal{F}_{qg}^{(2)}$

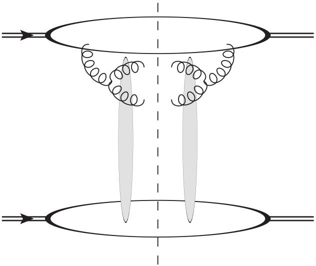
Forward dijet production in pA collisions

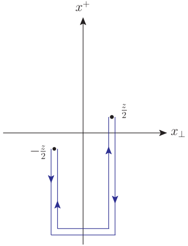




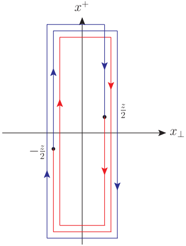
$\mathcal{F}_{gg}^{(1)}$ $\mathcal{F}_{gg}^{(2)}$ $\mathcal{F}_{gg}^{(3)}$

Forward dijet production in pA collisions

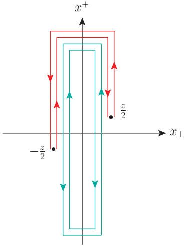




$\mathcal{F}_{gg}^{(4)}$



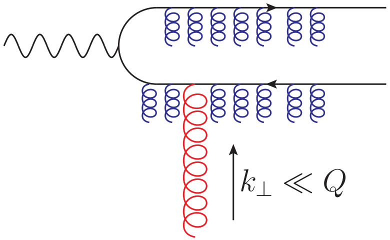
$\mathcal{F}_{gg}^{(5)}$



$\mathcal{F}_{gg}^{(6)}$

Genuine saturation effects at the EIC

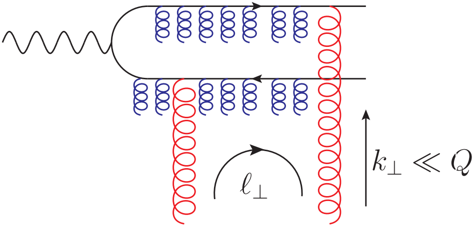
Back-to-back forward dijet/dihadron production



CGC in the correlation limit = leading twist TMD factorization

= leading power 1-body contribution

Back-to-back forward dijet/dihadron production



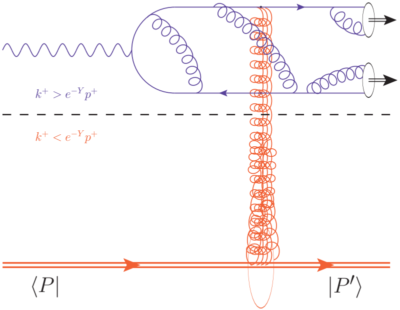
Even at leading power of $k_\perp/Q$, the genuine higher twist term contributes thanks to the loop transverse momentum

$\int d^2\ell_\perp/Q^2(...) \to Q_s^2/Q^2$?

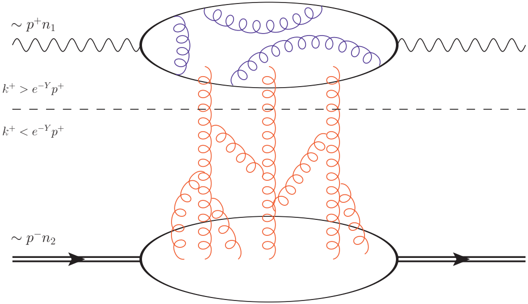
The discrepancy between correlated CGC or TMD and observation will be due to genuine saturation

Credits to [Mäntysaari,Müller,Salazar,Schenke] for their numerical observation

QCD at large s : semi-classical descriptions



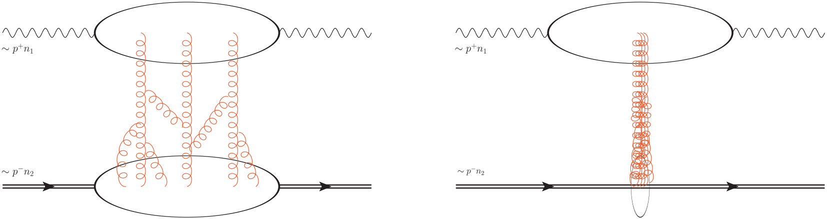
- Eikonal expansion
  $\sigma = \sigma_0 + \frac{1}{s}\sigma_1 + \ldots$
- Resummation of logarithms
  $\sigma_0 = \sum_n [A_n(\alpha_s \ln s)^n + \alpha_s B_n(\alpha_s \ln s)^n \ldots]$
- Renormalization group equation:
  Balitsky/Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner (B/JIMWLK) evolution for the shockwave operators.



Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A^{\mu a}_{Y_c}(|k^+| > e^{-Y_c} p^+, k^-, \vec{k}) + b^{\mu a}_{Y_c}(|k^+| < e^{-Y_c} p^+, k^-, \vec{k})$$

$e^{-Y_c} \ll 1$



$$b^+(x^+, x^-, \vec{x}) \longrightarrow \frac{1}{\Lambda} b^+(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$b^-(x^+, x^-, \vec{x}) \longrightarrow \Lambda b^-(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$b^k(x^+, x^-, \vec{x}) \longrightarrow b^k(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$\Lambda \sim \sqrt{\frac{s}{m_t^2}}$$

$$b^\mu(x) \to b^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu + O(\sqrt{\frac{m_t^2}{s}})$$

Shockwave approximation

The interactions with the background field can be exponentiated