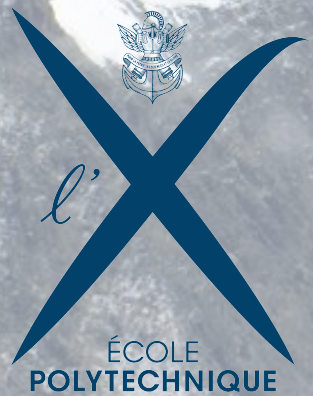


# $J/\psi$ and $J/\psi$ +jet leptonproduction: what to expect from the EIC

Quarkonia as Tools 2020, Aussois

Pieter Tael  
École polytechnique, Paris

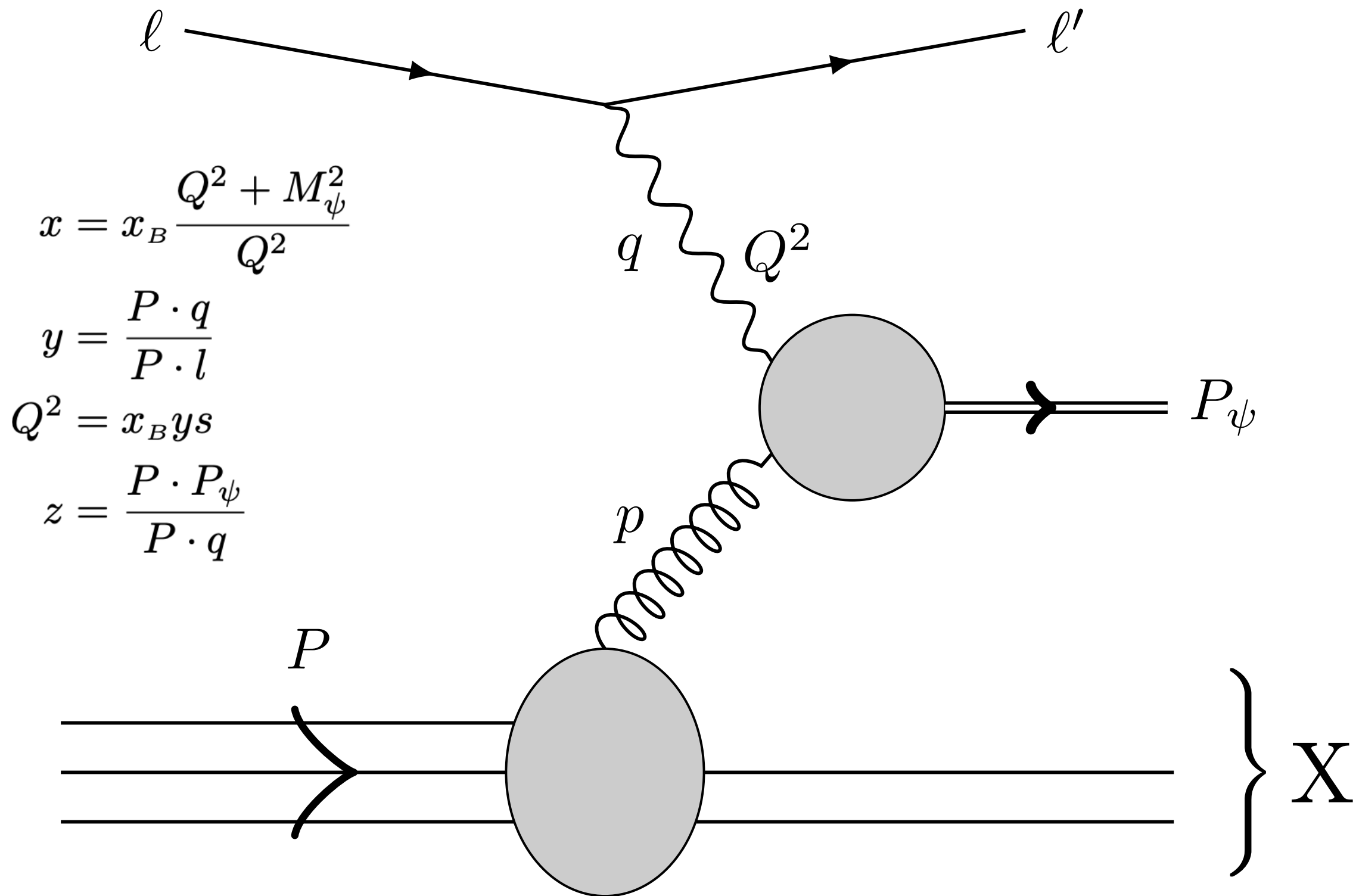




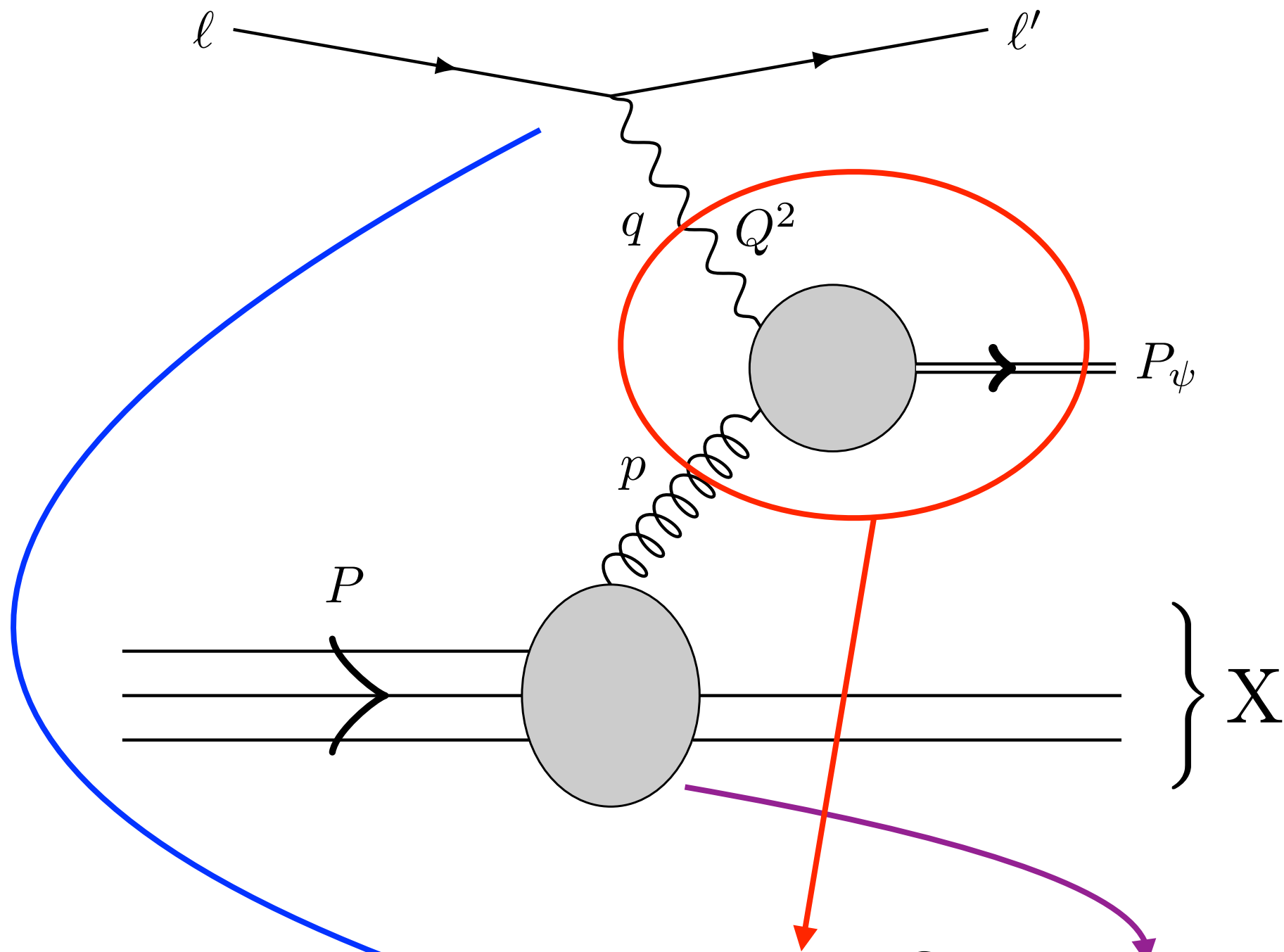


$\ell + p \rightarrow \ell + J/\psi + X$   
as a probe of gluon TMDs

$$\ell + p \rightarrow \ell + J/\psi + X$$



$$\ell + p \rightarrow \ell + J/\psi + X$$



$$d\sigma = \Gamma \otimes |M|^2 \otimes \Phi$$

Lepton tensor

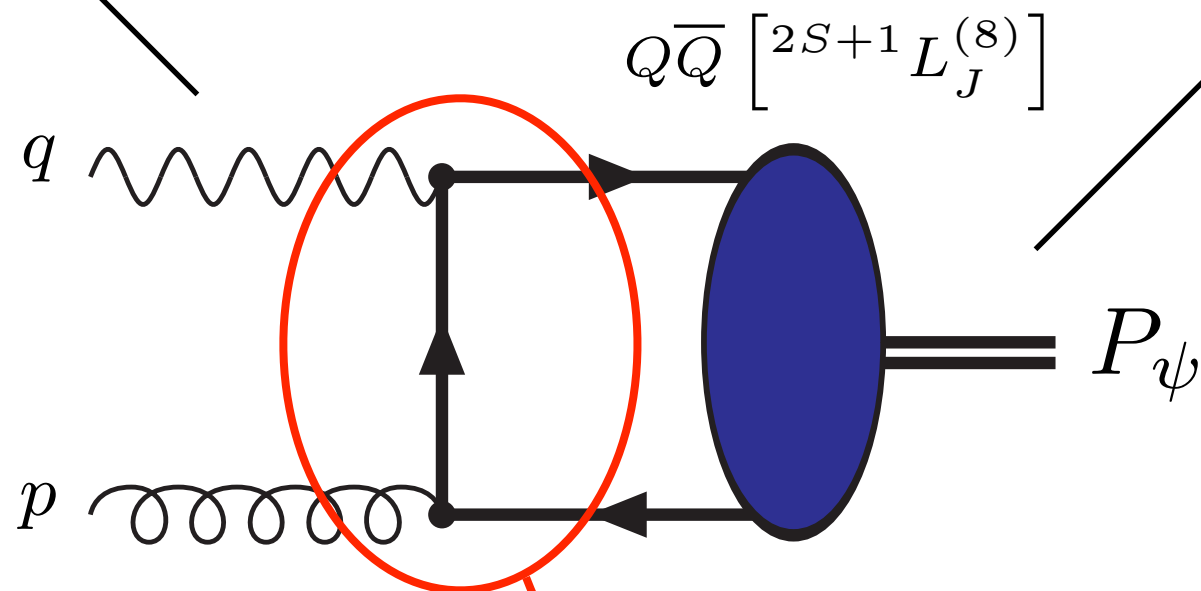
'Hard' part

Hadron tensor

# 'Hard' part

couples to lepton tensor  $\Gamma$

non-relativistic QCD  
(NRQCD)



couples to hadron tensor  $\Phi$

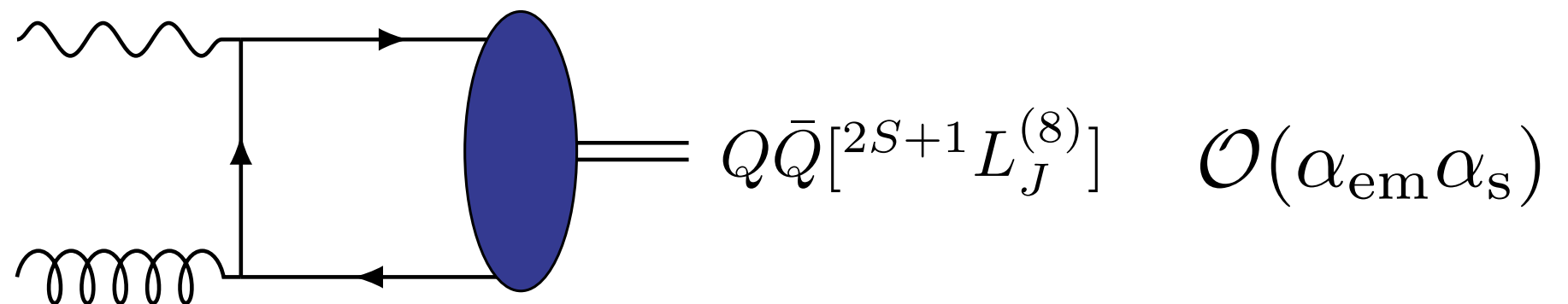
calculable in  
perturbation theory

# Non-relativistic QCD (NRQCD)

Color octet (CO) mechanism: heavy-quark pair is produced with *all allowed quantum numbers* and then hadronizes as encoded in the non-perturbative long-distance matrix elements (LDMEs)

$$\langle |\mathcal{O}_8^{J/\psi}({}^3P_J)|0\rangle$$

$$\langle |\mathcal{O}_8^{J/\psi}({}^1S_0)|0\rangle$$



NRQCD also encompasses color singlet (CS) mechanism where  $J/\psi$  is directly produced with correct quantum numbers

At least for lepton production, CO mechanism is the dominant one in the low- $P_{\psi\perp}$  regime Fleming and Mehen (1998)



# Hadron tensor

$$\Gamma_U^{\mu\nu}(x, \mathbf{p}_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu} f_1^g(x, \mathbf{p}_T^2) + \left( \frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\mathbf{p}_T^2}{2M_p^2} \right) h_1^{\perp g}(x, \mathbf{p}_T^2) \right\}$$

↓
↓

unpolarized gluons
linearly polarized gluons

$$\Gamma_T^{\mu\nu}(x, \mathbf{p}_T^2) = \frac{x}{2} \left\{ g_T^{\mu\nu} \frac{\epsilon_T^{\rho\sigma} p_{\perp\rho} S_{\perp\sigma}}{M_p} f_{1T}^{\perp g}(x, \mathbf{p}_T^2) + i\epsilon_T^{\mu\nu} \frac{p_{\perp} \cdot S_{\perp}}{M_p} g_{1T}^g(x, \mathbf{p}_T^2) \right.$$

$$\left. + \frac{p_{\perp\rho} \epsilon_T^{\rho\{\mu} p_{\perp}^{\nu\}}}{2M_p^2} \frac{p_{\perp} \cdot S_{\perp}}{M_p} h_{1T}^{\perp g}(x, \mathbf{p}_T^2) - \frac{p_{\perp\rho} \epsilon_T^{\rho\{\mu} S_{\perp}^{\nu\}} + S_{\perp\rho} \epsilon_T^{\rho\{\mu} p_{\perp}^{\nu\}}}{4M_p} h_{1T}^g(x, \mathbf{p}_T^2) \right\}$$

↑
↑

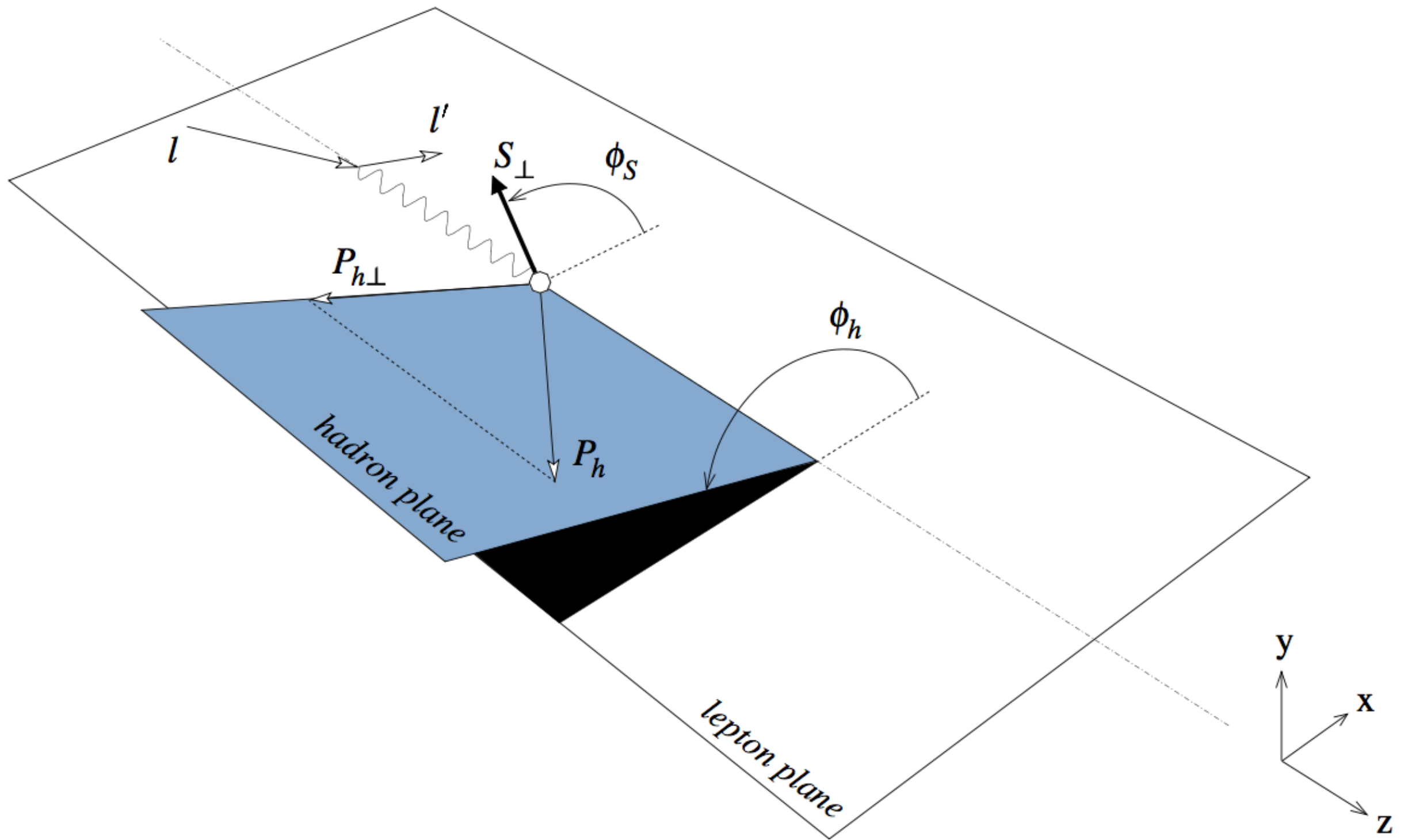
Sivers function
circularly polarized gluons

↓
↓

linearly polarized gluons

Mulders & Rodrigues (2001); Meissner, Metz & Goeke (2007)

# Reference frame



Bacchetta, D'Alesio, Diehl & Miller (2004)



# Cross section

$$\frac{d\sigma}{dy dx_B d^2\mathbf{q}_T} \equiv d\sigma^U(\phi_T) + d\sigma^T(\phi_T, \phi_S)$$

$$d\sigma^U = \mathcal{N} \left[ \underset{\substack{\downarrow \\ \text{unpolarized}}}{A^U f_1^g(x, \mathbf{q}_T^2)} + \frac{\mathbf{q}_T^2}{M_p^2} B^U \underset{\substack{\downarrow \\ \text{linearly polarized}}}{h_1^{\perp g}(x, \mathbf{q}_T^2)} \cos 2\phi_T \right]$$

T even

Sivers function

$$d\sigma^T = \mathcal{N} |\mathbf{S}_T| \frac{|\mathbf{q}_T|}{M_p} \left\{ \underset{\substack{\uparrow \\ \text{Sivers function}}}{A^T f_{1T}^{\perp g}(x, \mathbf{q}_T^2)} \sin(\phi_S - \phi_T) \right. \\ \left. + B^T \left[ h_1^g(x, \mathbf{q}_T^2) \sin(\phi_S + \phi_T) - \frac{\mathbf{q}_T^2}{2M_p^2} h_{1T}^{\perp g}(x, \mathbf{q}_T^2) \sin(\phi_S - 3\phi_T) \right] \right\}$$

**T odd -> only CO!**

linearly polarized

# Azimuthal asymmetries

probe *ratios* of gluon TMDs

$$A^{W(\phi_S, \phi_T)} \equiv 2 \frac{\int d\phi_S d\phi_T W(\phi_S, \phi_T) d\sigma(\phi_S, \phi_T)}{\int d\phi_S d\phi_T d\sigma(\phi_S, \phi_T)}$$

...we have:

$$\int d\phi_S d\phi_T d\sigma(\phi_S, \phi_T) = (2\pi)^2 \mathcal{N} A^U f_1^g(x, \mathbf{q}_T^2)$$

$$A^{\cos 2\phi_T} = H(y, M_\psi, Q) \frac{\mathbf{q}_T^2}{M_p^2} \frac{h_1^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

$$A^{\sin(\phi_S - \phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{f_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

$$A^{\sin(\phi_S + \phi_T)} = H(y, M_\psi, Q) \frac{|\mathbf{q}_T|}{M_p} \frac{h_1^g(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

$$A^{\sin(\phi_S - 3\phi_T)} = -H(y, M_\psi, Q) \frac{|\mathbf{q}_T|^3}{2M_p^3} \frac{h_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

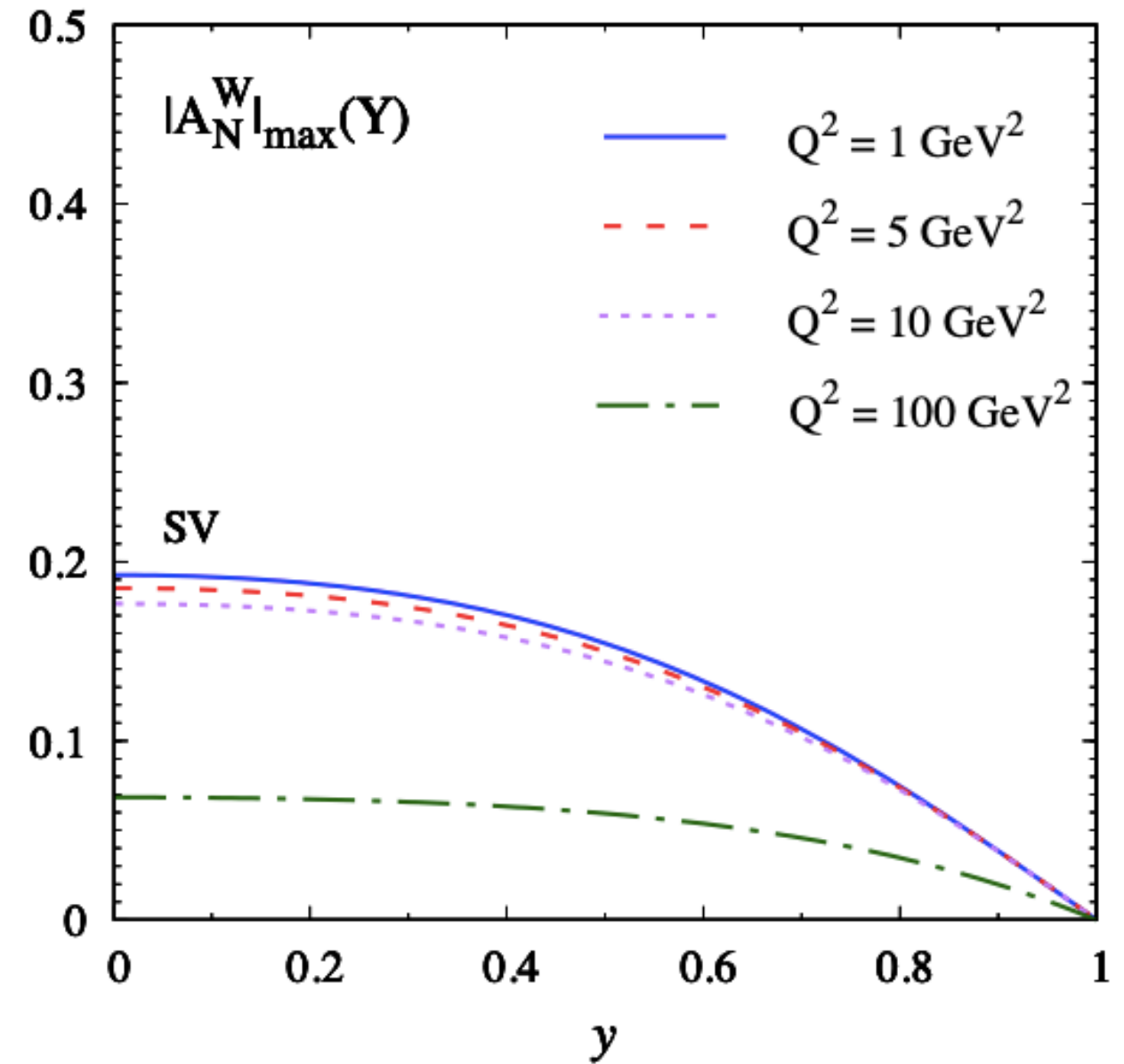
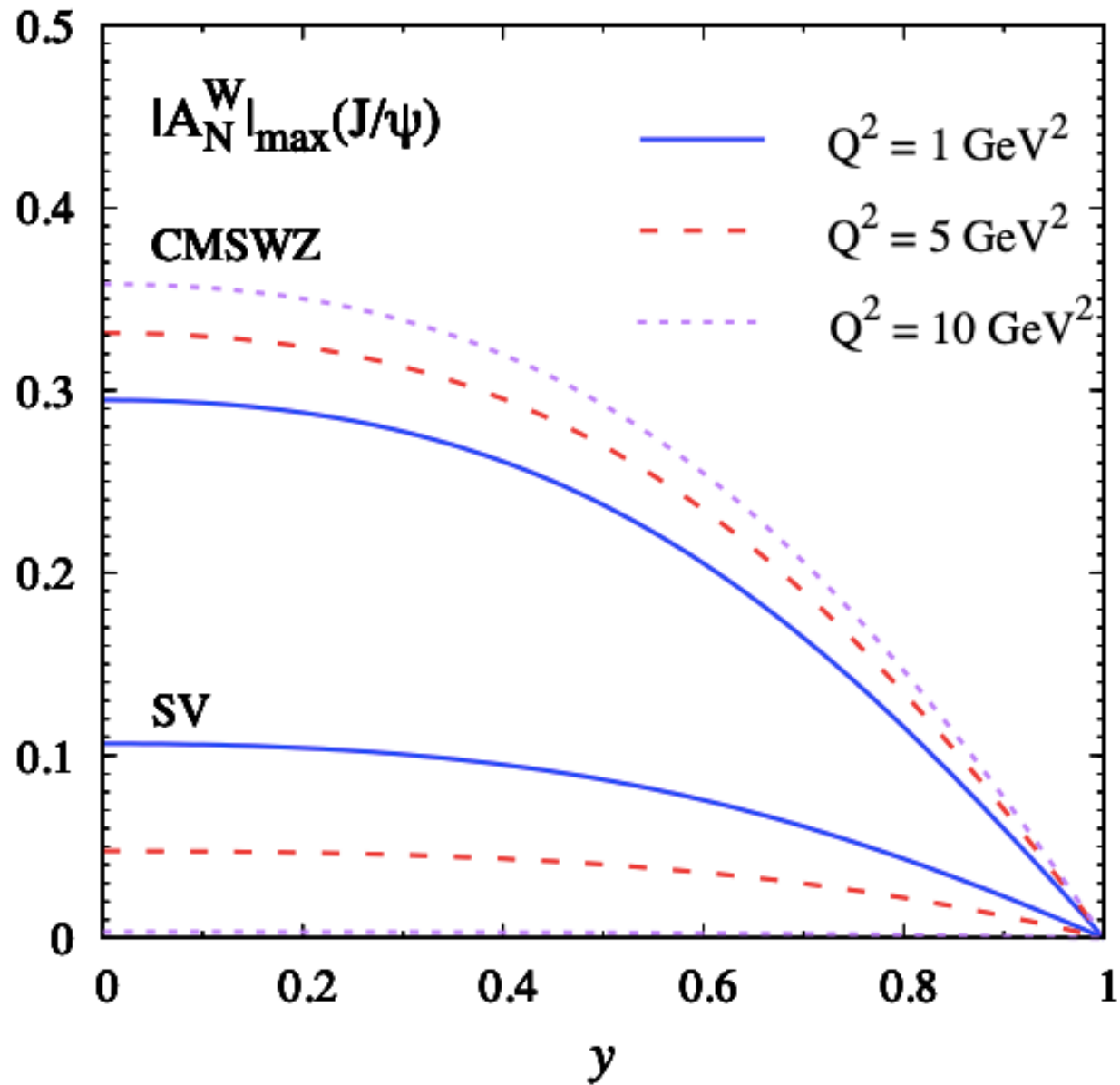
# Upper bounds

Polarized gluon TMDs satisfy the following positivity bounds:

$$\begin{aligned} \frac{|\mathbf{p}_T|}{M_p} |f_{1T}^{\perp g}(x, \mathbf{p}_T^2)| &\leq f_1^g(x, \mathbf{p}_T^2) & \frac{|\mathbf{p}_T|}{M_p} |h_1^g(x, \mathbf{p}_T^2)| &\leq f_1^g(x, \mathbf{p}_T^2) \\ \frac{\mathbf{p}_T^2}{2M_p^2} |h_{1T}^{\perp g}(x, \mathbf{p}_T^2)| &\leq f_1^g(x, \mathbf{p}_T^2) & \frac{|\mathbf{p}_T|^3}{2M_p^3} |h_{1T}^{\perp g}(x, \mathbf{p}_T^2)| &\leq f_1^g(x, \mathbf{p}_T^2) \end{aligned}$$

We can use the maximum allowed values of the gluon TMDs to illustrate the sensitivity of our inclusive charmonium electroproduction to the gluon content of the proton:

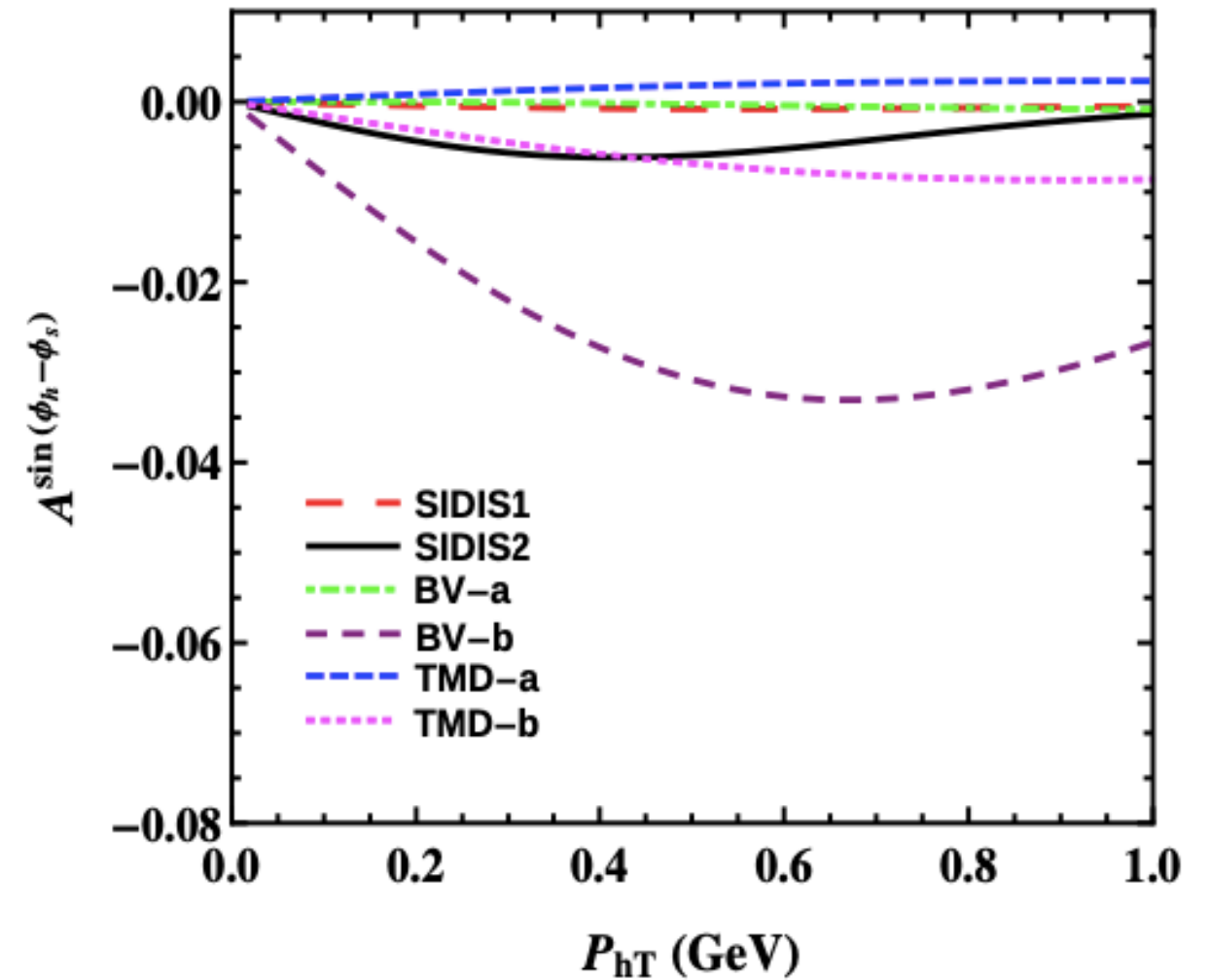
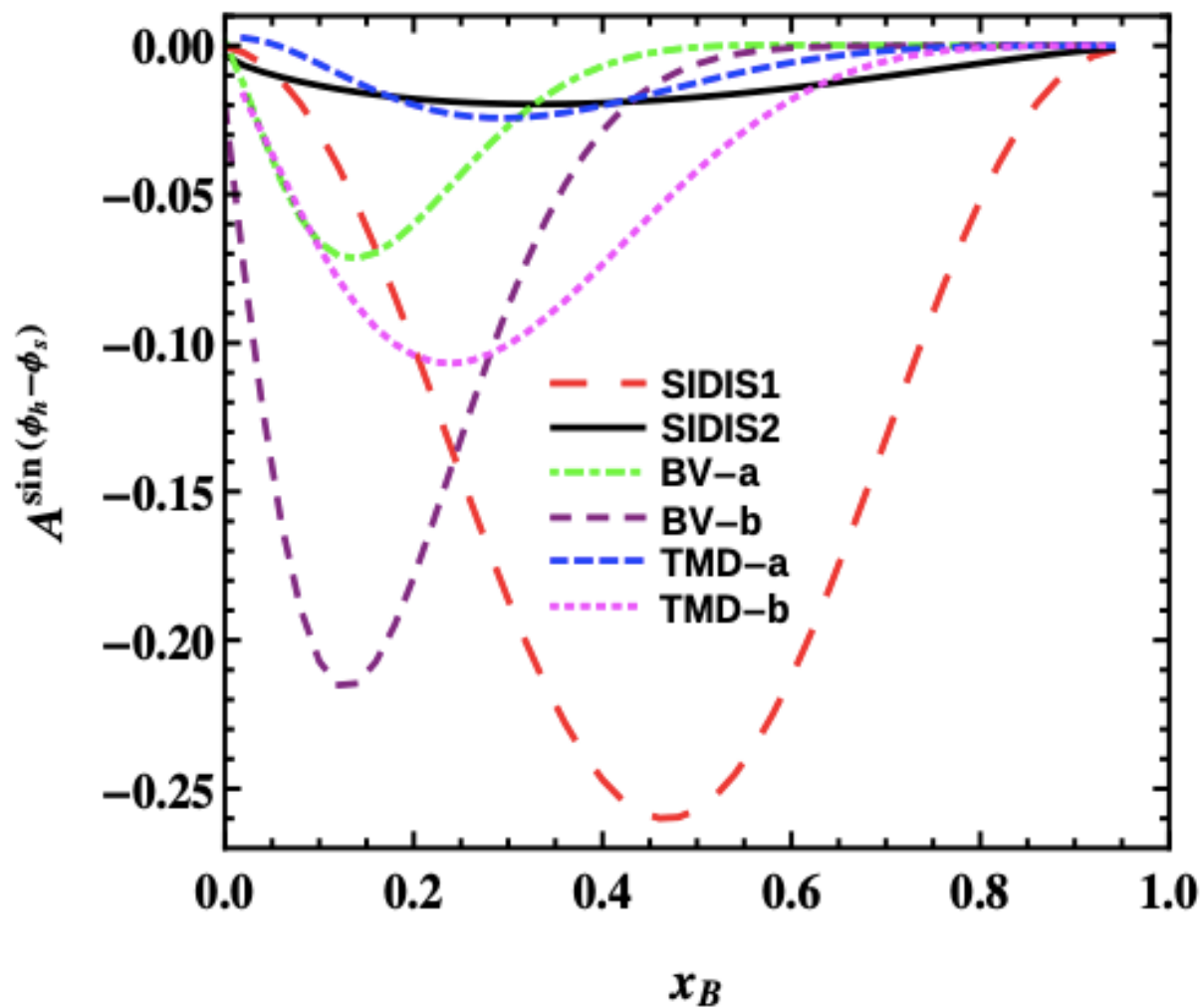
# Upper bounds





# Single-spin asymmetries

$$\ell + p \rightarrow \ell + J/\psi + X$$

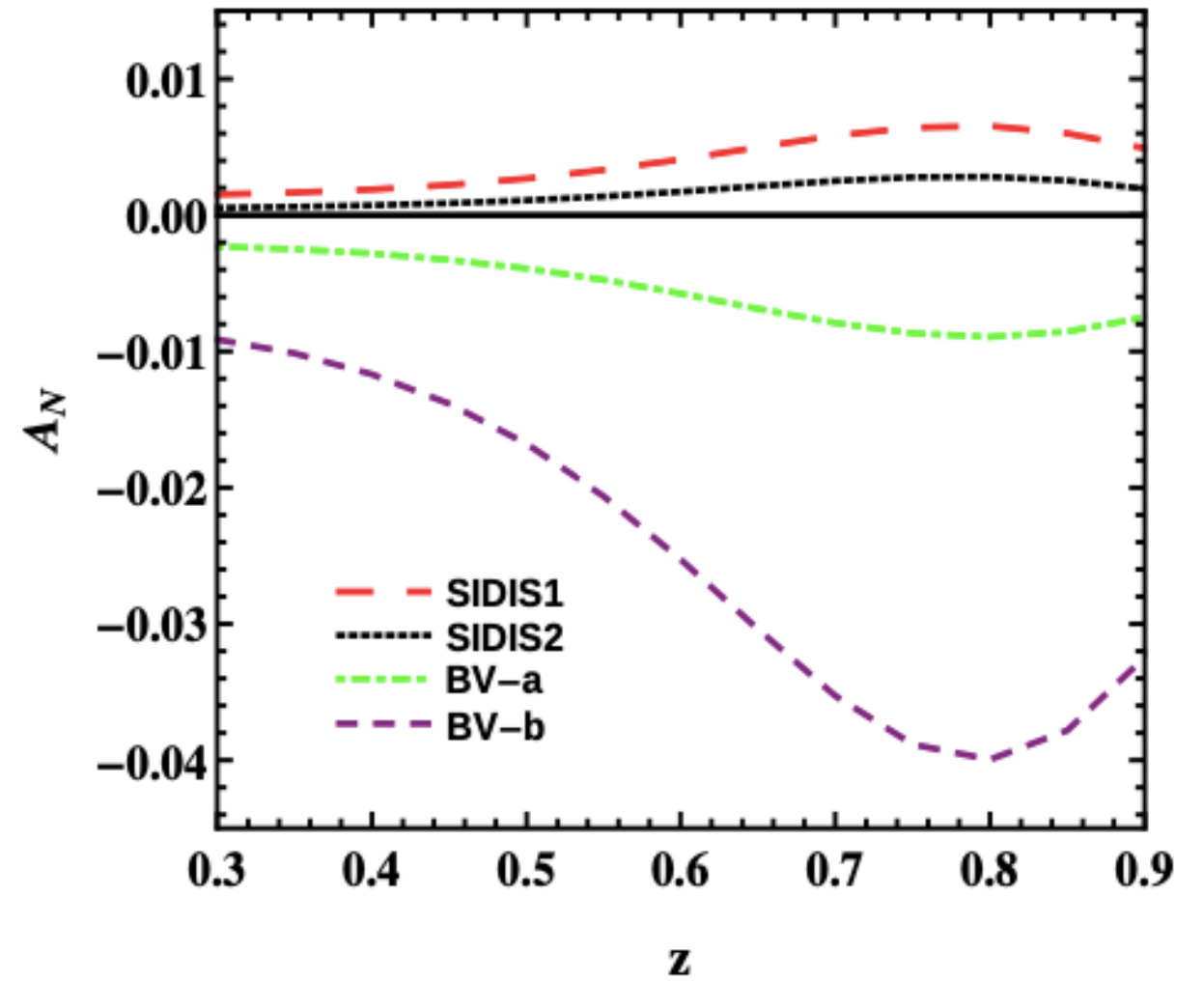
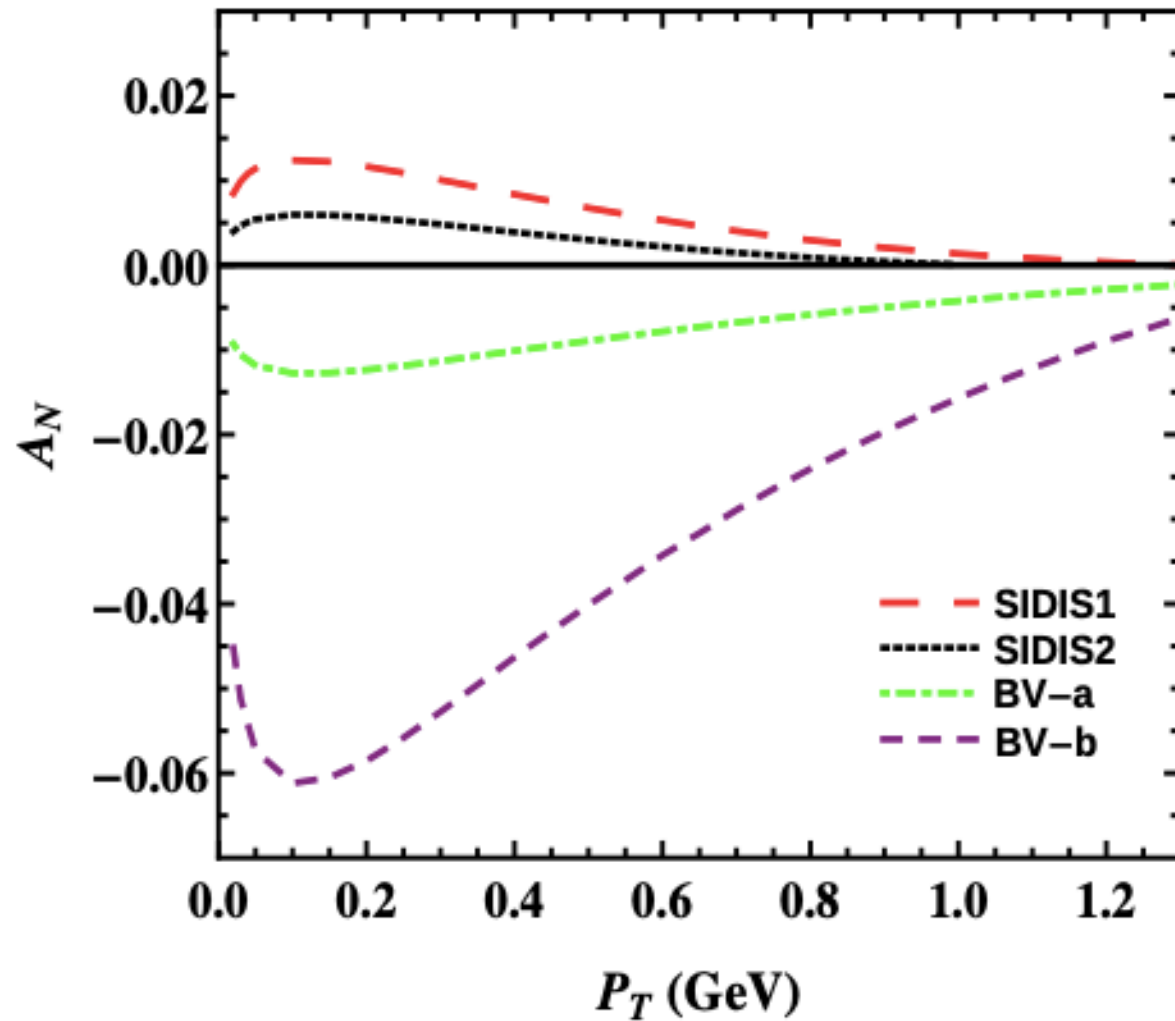


for  $\sqrt{s} = 45$  GeV

Rajesh & Mukherjee (2018)

# Single-spin asymmetries

$$\ell + p \rightarrow \ell + J/\psi + X$$



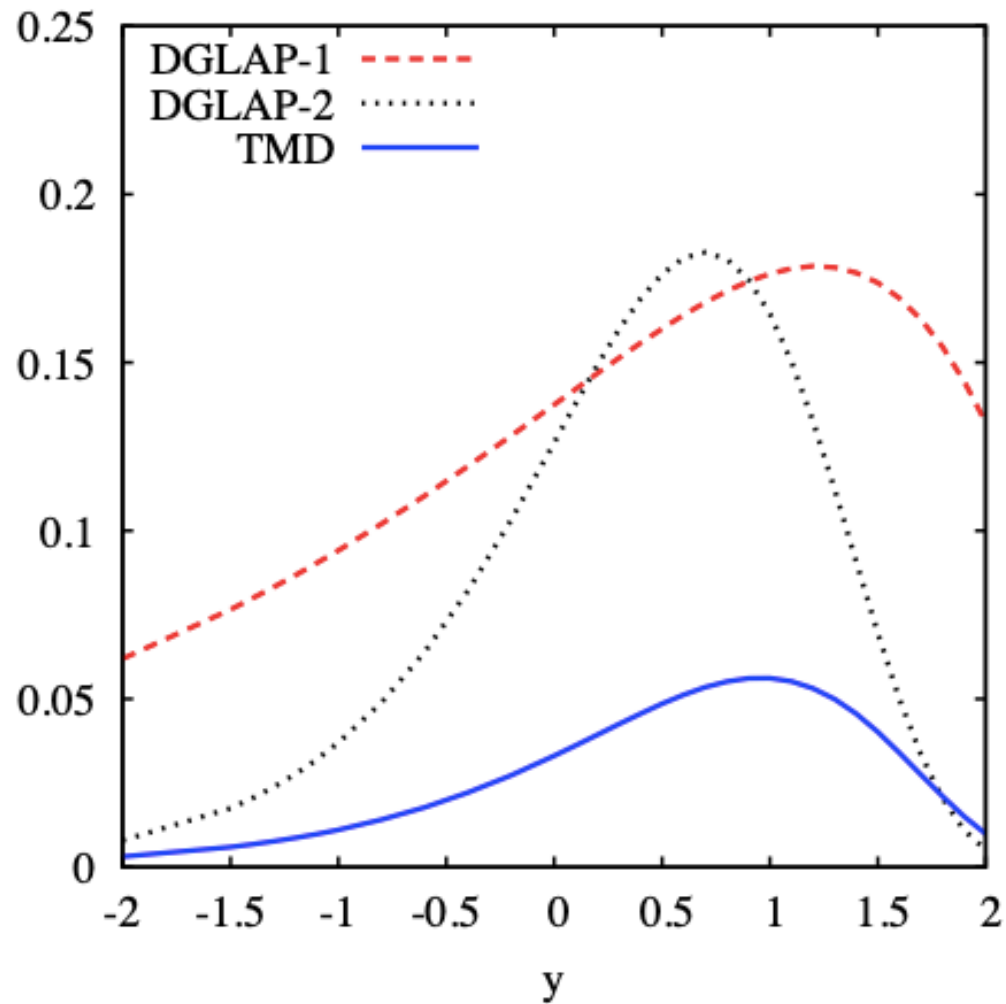
for  $\sqrt{s} = 45$  GeV, calculated at NLO\*

Rajesh, Kishore & Mukherjee (2018)

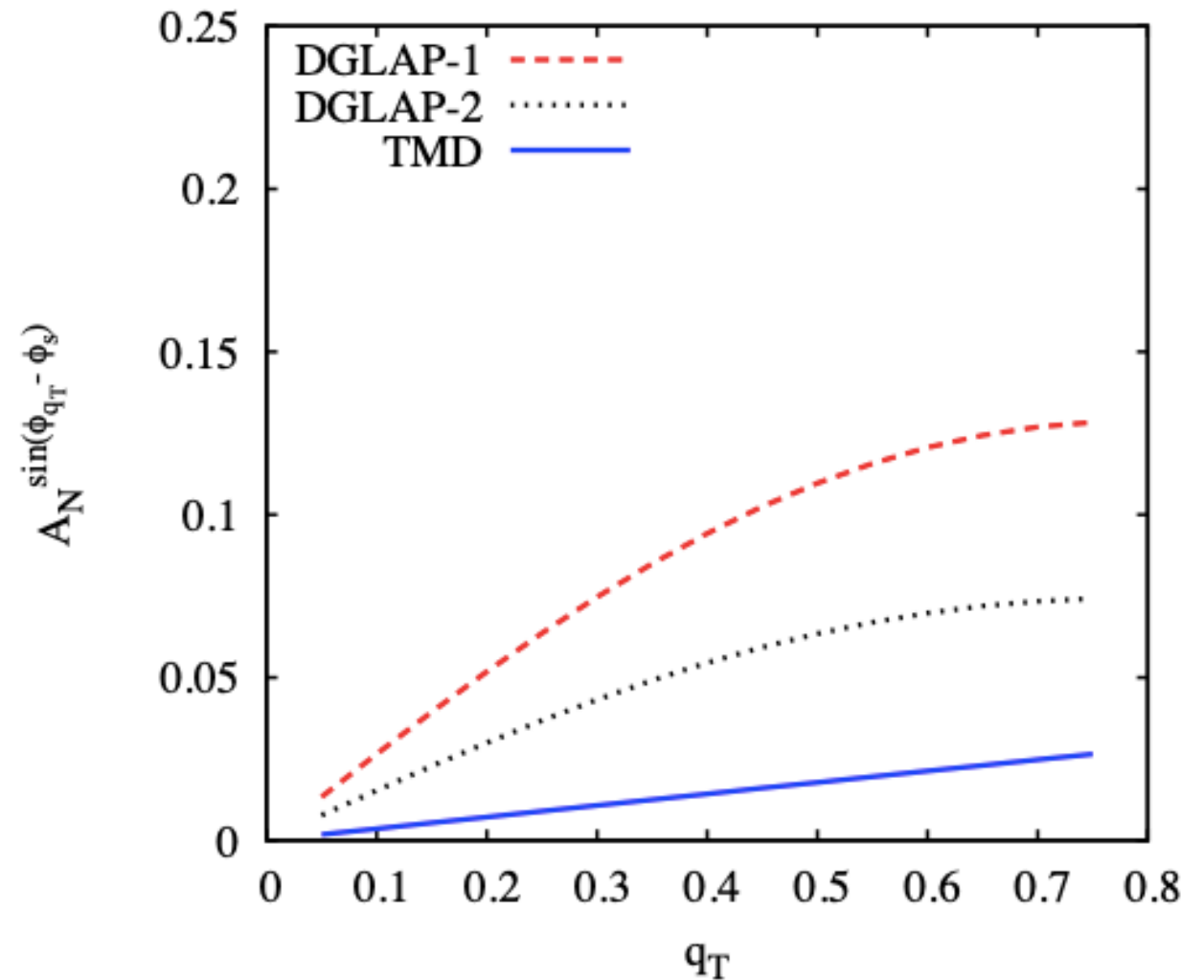
# Single-spin asymmetries

$$\gamma + p \rightarrow \gamma + J/\psi + X$$

eRHIC-1,  $\sqrt{s} = 31.6$  GeV



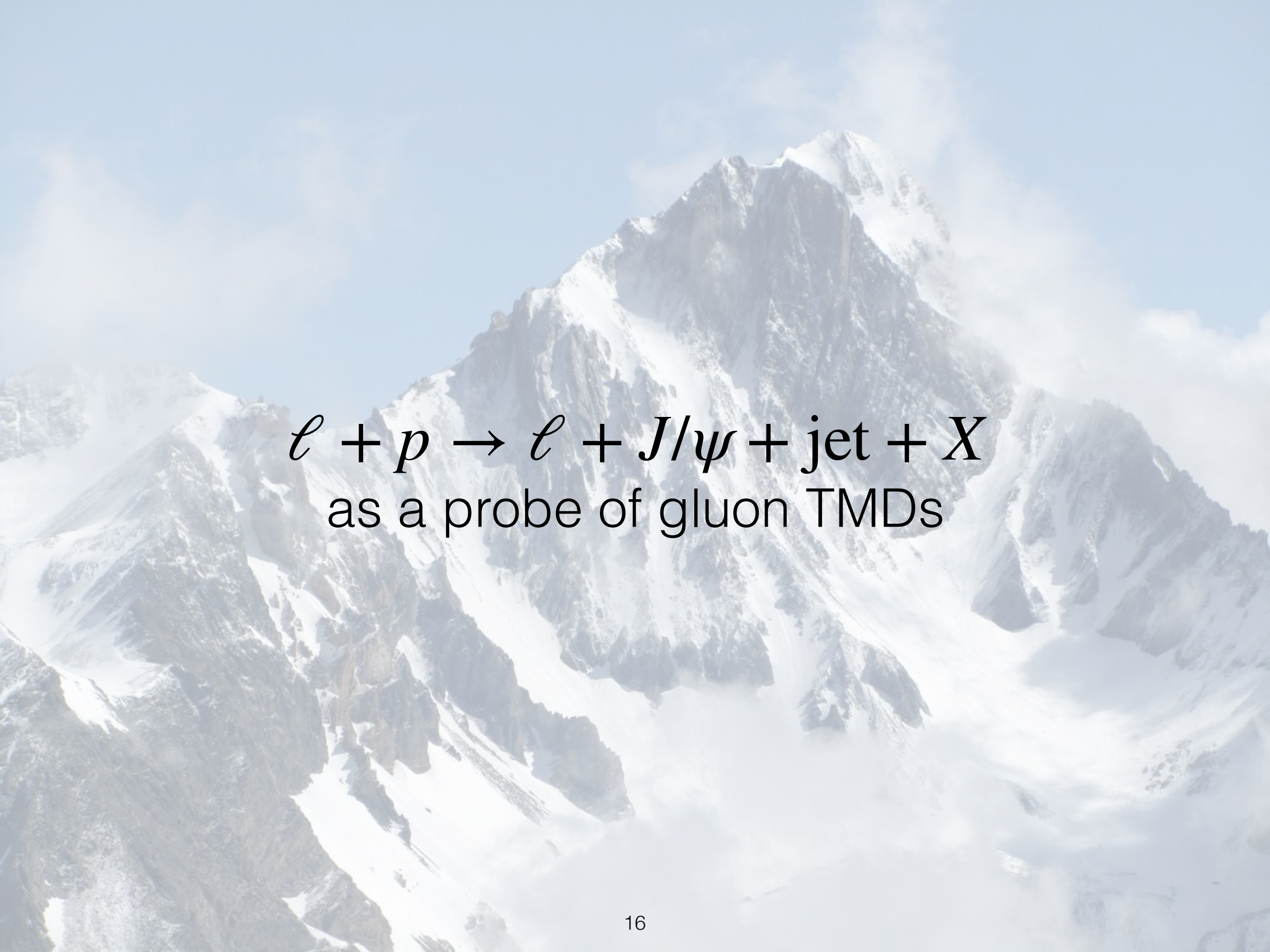
eRHIC-1,  $\sqrt{s} = 31.6$  GeV



In Color Evaporation Model

Godbole, Misra, Mukherjee & Rawoot (2018)

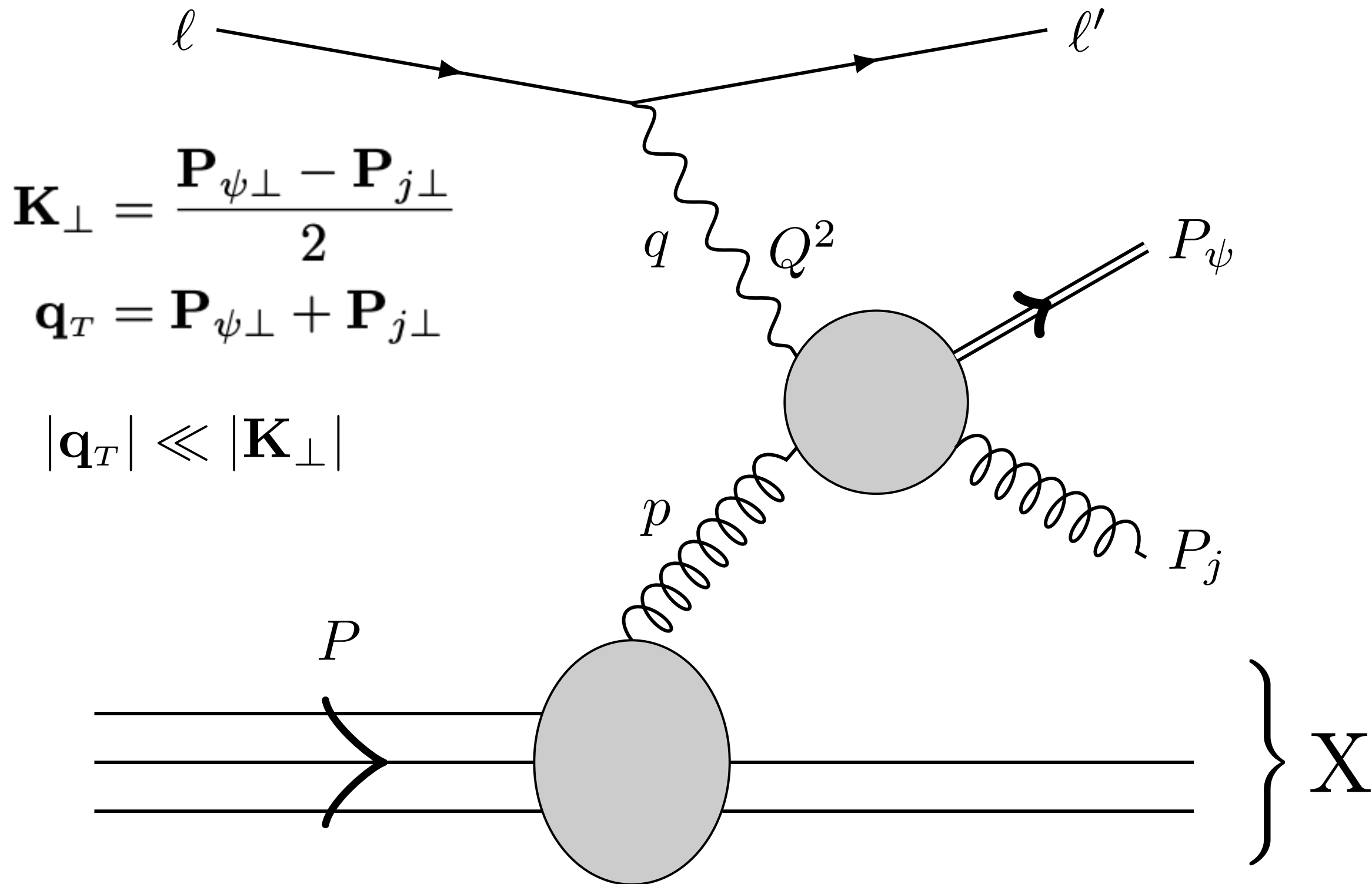




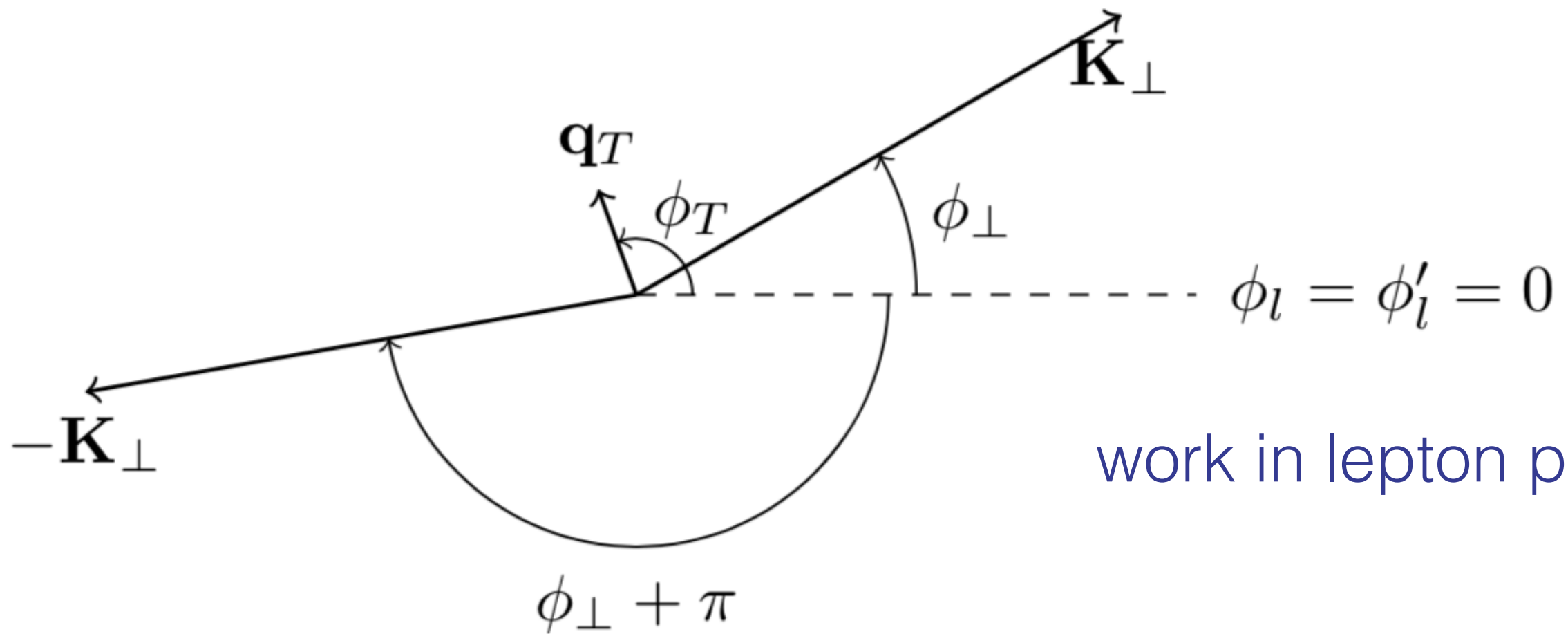
$\ell + p \rightarrow \ell + J/\psi + \text{jet} + X$   
as a probe of gluon TMDs



$$\ell + p \rightarrow \ell + J/\psi + g + X$$



# Definition of the angles



work in lepton plane

$$\mathbf{K}_\perp = \frac{\mathbf{K}_{\psi_\perp} - \mathbf{K}_{j_\perp}}{2}$$

$$\mathbf{q}_T = \mathbf{K}_{\psi_\perp} + \mathbf{K}_{j_\perp}$$

$$|\mathbf{q}_T| \ll |\mathbf{K}_\perp|$$

# Cross section

$$\frac{d\sigma}{dz dy dx_B d^2\mathbf{q}_T d^2\mathbf{K}_\perp} \equiv d\sigma(\phi_S, \phi_T, \phi_\perp) = d\sigma^U(\phi_T, \phi_\perp) + d\sigma^T(\phi_S, \phi_T, \phi_\perp)$$

$$d\sigma^U = \mathcal{N} \left[ \begin{aligned} & (\mathcal{A}_0^{eg} + \mathcal{A}_1^{eg} \cos \phi_\perp + \mathcal{A}_2^{eg} \cos 2\phi_\perp) f_1^g(x, \mathbf{q}_T^2) \\ & + (\mathcal{B}_0^{eg} \cos 2\phi_T + \mathcal{B}_1^{eg} \cos(2\phi_T - \phi_\perp) + \mathcal{B}_2^{eg} \cos 2(\phi_T - \phi_\perp) \\ & + \mathcal{B}_3^{eg} \cos(2\phi_T - 3\phi_\perp) + \mathcal{B}_4^{eg} \cos(2\phi_T - 4\phi_\perp)) \frac{\mathbf{q}_T^2}{M_p^2} h_1^{\perp g}(x, \mathbf{q}_T^2) \end{aligned} \right]$$

Similar structure as in the case of heavy-quark pair production

Pisano, Boer, Brodsky, Buffing & Mulders (2013);  
Boer, Mulders, Pisano, Zhou (2016)

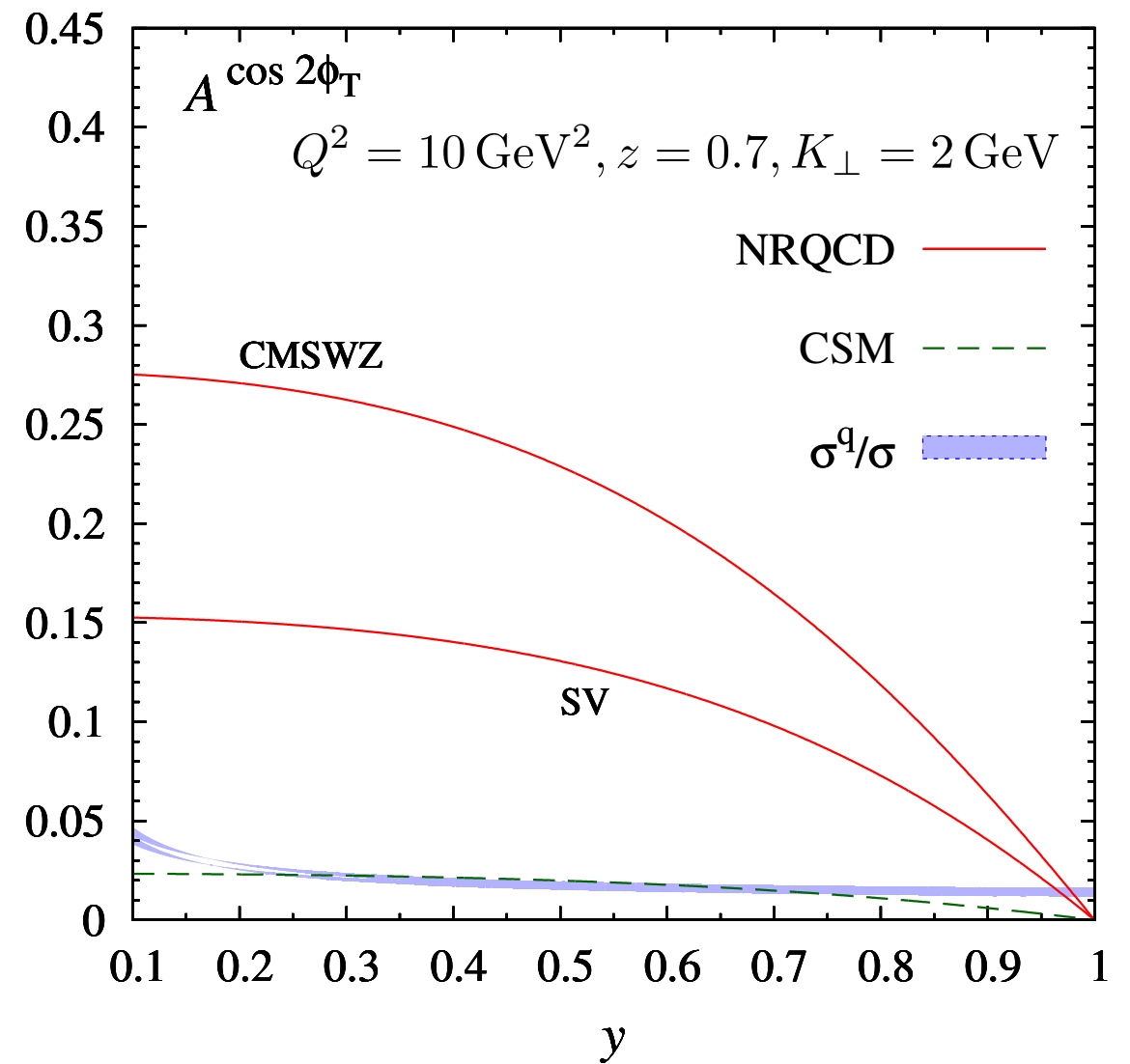
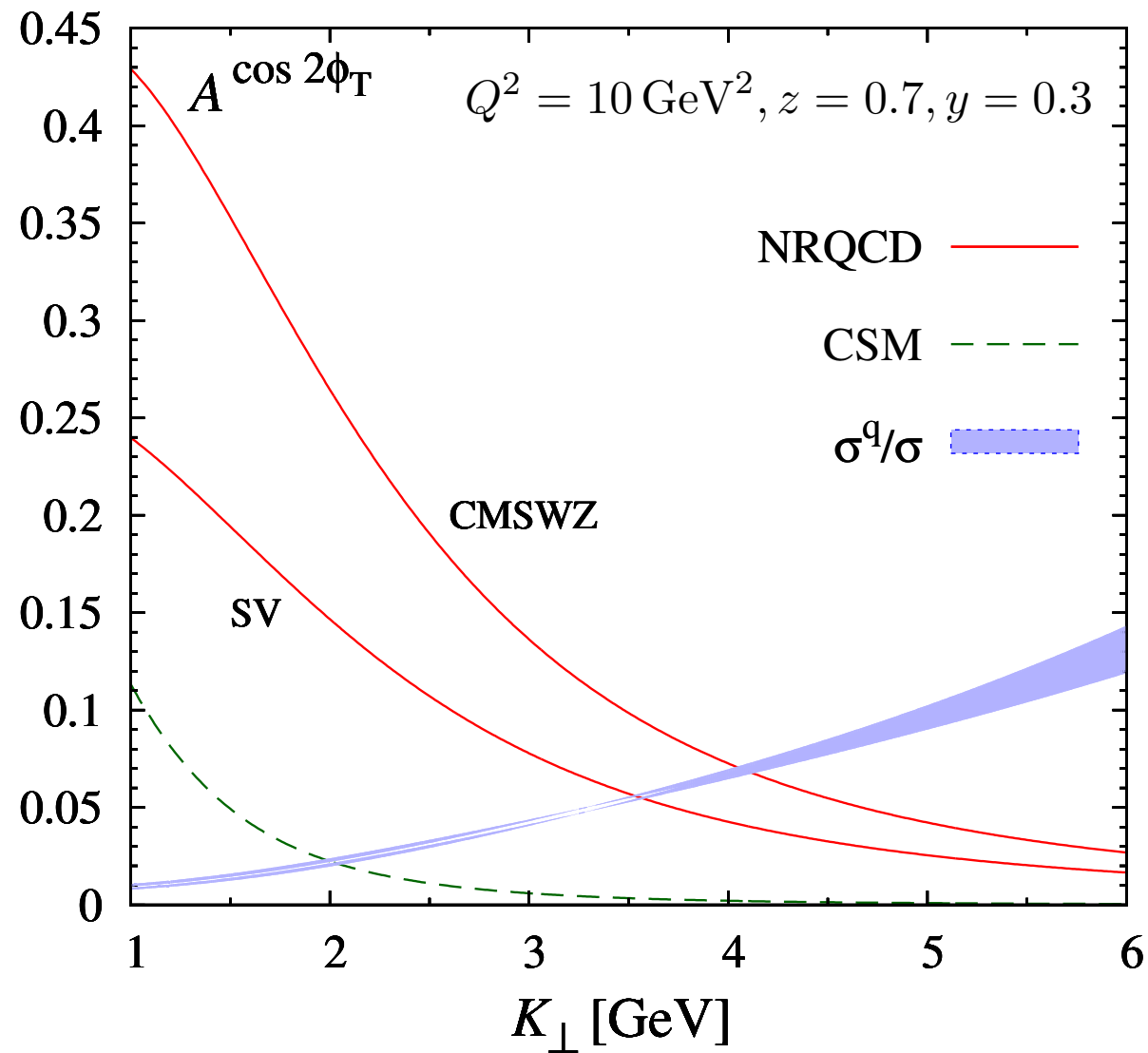
# Cross section

$$\frac{d\sigma}{dz dy dx_B d^2\mathbf{q}_T d^2\mathbf{K}_\perp} \equiv d\sigma(\phi_S, \phi_T, \phi_\perp) = d\sigma^U(\phi_T, \phi_\perp) + d\sigma^T(\phi_S, \phi_T, \phi_\perp)$$

$$\begin{aligned} d\sigma^T = \mathcal{N} |\mathbf{S}_T| & \left[ \sin(\phi_S - \phi_T) (\mathcal{A}_0^{eg} + \mathcal{A}_1^{eg} \cos \phi_\perp + \mathcal{A}_2^{eg} \cos 2\phi_\perp) \frac{|\mathbf{q}_T|}{M_p} f_{1T}^{\perp g}(x, \mathbf{q}_T^2) \right. \\ & + \cos(\phi_S - \phi_T) (\mathcal{B}_0^{eg} \sin 2\phi_T + \mathcal{B}_1^{eg} \sin(2\phi_T - \phi_\perp) + \mathcal{B}_2^{eg} \sin 2(\phi_T - \phi_\perp) \\ & + \mathcal{B}_3^{eg} \sin(2\phi_T - 3\phi_\perp) + \mathcal{B}_4^{eg} \sin(2\phi_T - 4\phi_\perp)) \frac{|\mathbf{q}_T|^3}{M_p^3} h_{1T}^{\perp g}(x, \mathbf{q}_T^2) \\ & + (\mathcal{B}_0^{eg} \sin(\phi_S + \phi_T) + \mathcal{B}_1^{eg} \sin(\phi_S + \phi_T - \phi_\perp) + \mathcal{B}_2^{eg} \sin(\phi_S + \phi_T - 2\phi_\perp) \\ & \left. + \mathcal{B}_3^{eg} \sin(\phi_S + \phi_T - 3\phi_\perp) + \mathcal{B}_4^{eg} \sin(\phi_S + \phi_T - 4\phi_\perp)) \frac{|\mathbf{q}_T|}{M_p} h_{1T}^g(x, \mathbf{q}_T^2) \right] \end{aligned}$$

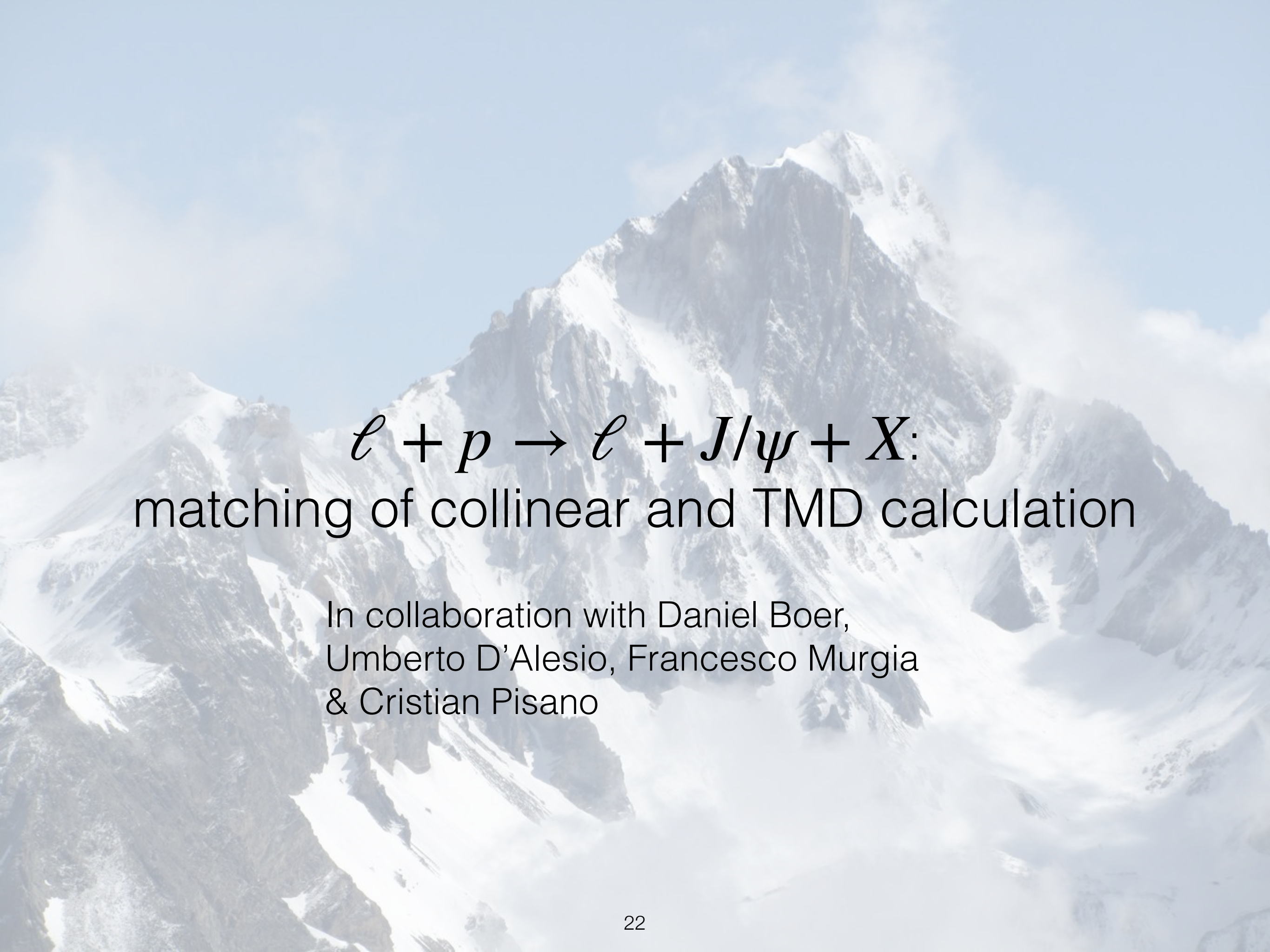


$$A_{\max}^{\cos 2\phi_T} \text{ in } e + p \rightarrow J/\psi + \text{jet} + X$$



D'Alesio, Murgia, Pisano, PT (2019)

See the talk by Rajesh for SSA's in  $\gamma + p \rightarrow \gamma + J/\psi + \text{jet} + X$



$\ell + p \rightarrow \ell + J/\psi + X$ :  
matching of collinear and TMD calculation

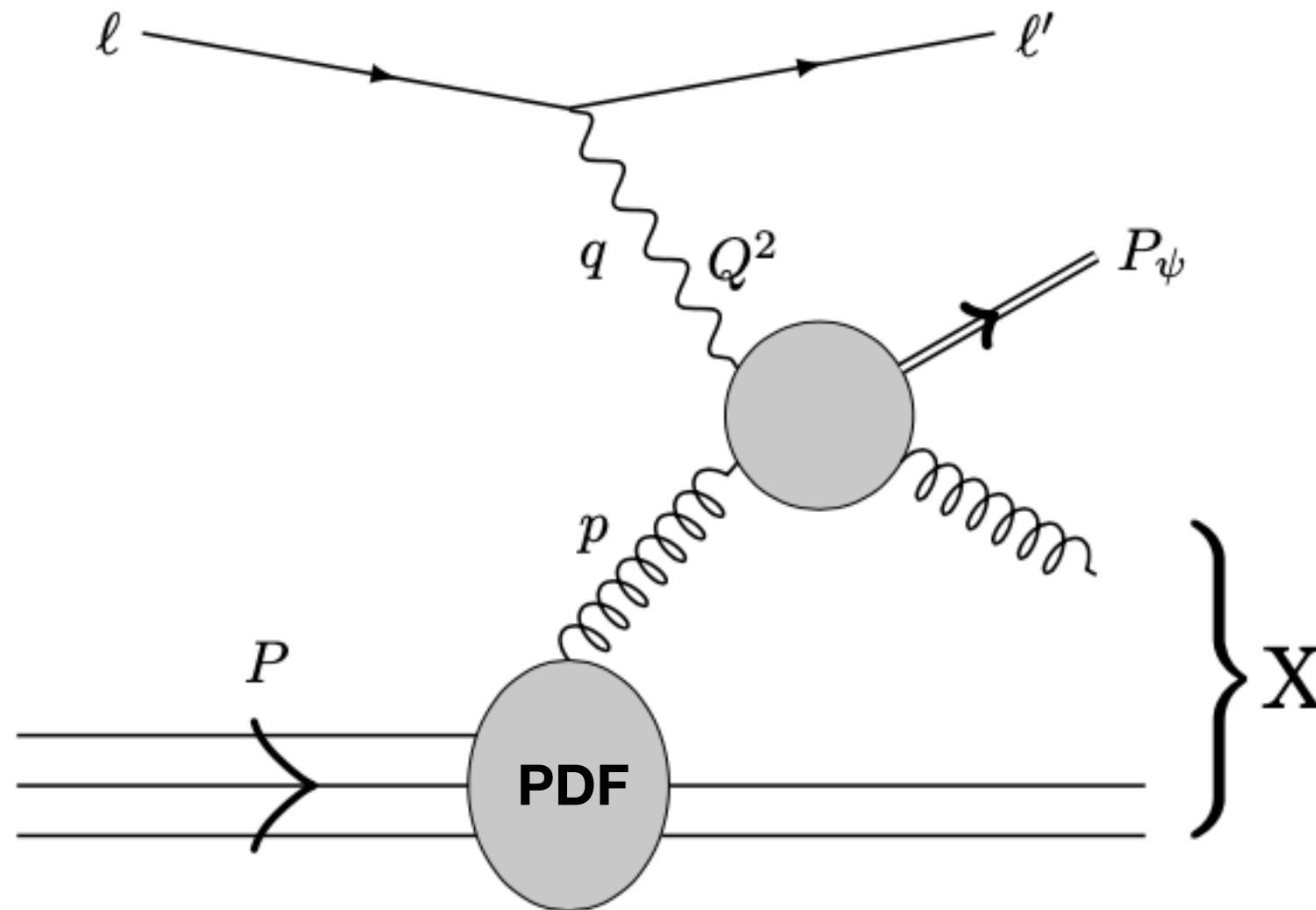
In collaboration with Daniel Boer,  
Umberto D'Alesio, Francesco Murgia  
& Cristian Pisano

$$\ell + p \rightarrow \ell + J/\psi + X:$$

matching of collinear and TMD calculation

In the collinear regime:  $\mu \sim P_{\psi T} \gg M_p$  with  $\mu = \sqrt{Q^2 + M_\psi^2}$

$P_{\psi T}$  is generated by recoil off hard parton

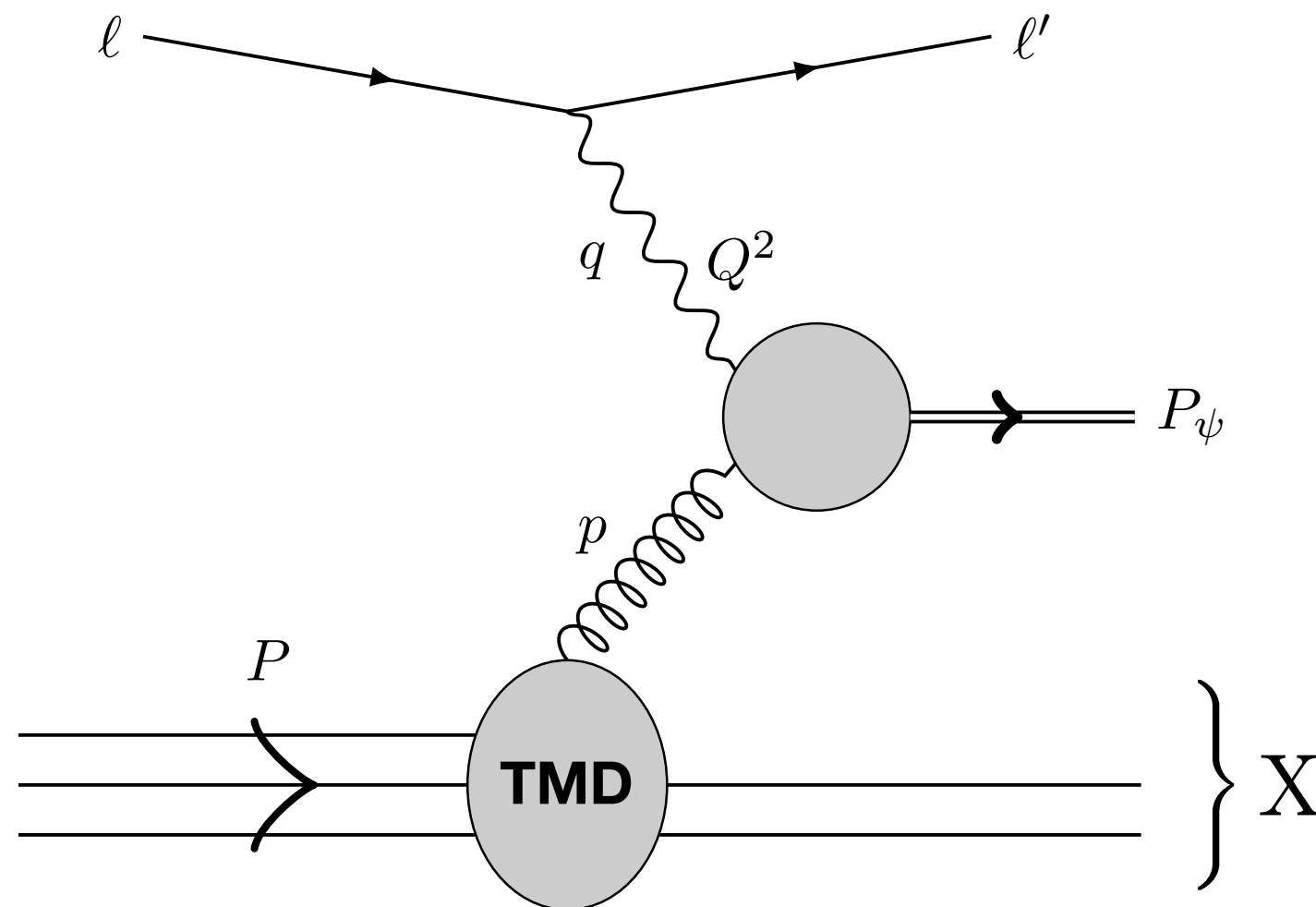


$$\ell + p \rightarrow \ell + J/\psi + X:$$

matching of collinear and TMD calculation

In the TMD regime:  $\mu \gg P_{\psi T} \gtrsim M_p$

$P_{\psi T}$  stems from intrinsic transverse momentum in target,  
or from soft emissions





$$\ell + p \rightarrow \ell + J/\psi + X:$$

matching of collinear and TMD calculation

If TMD factorization holds, both the collinear and the TMD calculation should match in overlapping kinematic region

$$\mu \gg P_{\psi T} \gg M_p$$

From collinear calculation: hard emission is absorbed into DGLAP

From TMD calculation: high- $p_{\perp}$  tail also matches to DGLAP

Bacchetta, Boer, Diehl & Mulders (2008)

Bacchetta, Bozzi, Echevarria, Pisano, Prokudin & Radici (2019)

$$\ell + p \rightarrow \ell + J/\psi + X:$$

matching of collinear and TMD calculation

Collinear calculation at small  $q_T$ :

$$\frac{d\sigma}{dy dx_B dz dq_T^2 d\phi_\psi} = \frac{\alpha^2}{y} \left[ \frac{1+(1-y)^2}{Q^2} F_{UU,T} + 4(1-y) F_{UU,L} + (1-y) \cos 2\phi_\psi F_{UU}^{\cos 2\phi_\psi} \right]$$

**transverse  $\gamma^*$** 
**longitudinal  $\gamma^*$** 
**lin. pol.  $\gamma^*$**

TMD calculation at high  $q_T$ :

$$\frac{d\sigma}{dy dx_B dz d\mathbf{q}_T^2 d\phi_\psi} = \frac{\alpha^2}{y} \left[ \frac{1+(1-y)^2}{Q^2} \mathcal{F}_{UU,T} + 4(1-y) \mathcal{F}_{UU,L} + (1-y) \cos 2\phi_\psi \mathcal{F}_{UU}^{\cos 2\phi_\psi} \right] \delta(1-z)$$

In kinematic region of overlap:

$$\mathcal{F}_{UU,T} = F_{UU,T} - \sigma_{UU,T} C_A \ln \left( \frac{Q^2 + M_\psi^2}{q_T^2} \right)$$

$$\mathcal{F}_{UU,L} = F_{UU,L} - \sigma_{UU,L} C_A \ln \left( \frac{Q^2 + M_\psi^2}{q_T^2} \right)$$

$$\mathcal{F}_{UU}^{\cos 2\phi_\psi} = F_{UU}^{\cos 2\phi_\psi}$$

$$\ell + p \rightarrow \ell + J/\psi + X:$$

matching of collinear and TMD calculation

Therefore, assuming both TMD factorization and enforcing the matching:

$$\mathcal{F}_{UU,T} = \sum_n \mathcal{H}_{UU,T}^{[n]} \mathcal{C}[f_1^g \Delta^{[n]}](x, \mathbf{q}_T^2)$$

$$\mathcal{F}_{UU,L} = \sum_n \mathcal{H}_{UU,L}^{[n]} \mathcal{C}[f_1^g \Delta^{[n]}](x, \mathbf{q}_T^2)$$

$$\mathcal{F}_{UU}^{\cos 2\phi_\psi} = \sum_n \mathcal{H}_{UU, \cos 2\phi_\psi}^{[n]} \mathcal{C}[wh_1^\perp g \Delta^{[n]}](x, \mathbf{q}_T^2)$$

we have to introduce the following ‘smearing’ function:

$$\Delta^{[n]}(\mathbf{k}_T^2, \mu^2) = \frac{\alpha_s}{2\pi^2 \mathbf{k}_T^2} C_A \langle 0 | \mathcal{O}_c(n) | 0 \rangle \ln \frac{\mu^2}{\mathbf{k}_T^2}$$

Are these the perturbative tails of the ‘shape’ functions introduced by Miguel, Tom & Yannis?

Echevarria (2019); Fleming, Makris, Mehen (2019)



# Conclusions & outlook

# Conclusions & outlook

Leptoproduction of  $J/\psi$  (+jet) at the Electron-Ion Collider seems a very promising process to probe gluon TMDs

Largest source of theoretical uncertainty are the nonperturbative LDMEs

For a real accurate extraction, we need a dedicated fit at low  $q_{\perp}$  of the LDMEs + shape function (and an NLO calculation, probably)

Thanks to the organizers,  
thanks for your attention!