High-Energy Take on Transverse-Momentum Resummation

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Quarkonia as tools

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Resummation formalisms

- Different formulations of the q_T spectrum:

$$\left(\frac{d\sigma}{dq_T}\right)_{\mathbf{res.}} \propto \begin{cases} e^{2S}\left[f_1 \otimes \mathcal{H} \otimes f_2\right] & : \textbf{Resum.} \\ H \times F_1 \times F_2 & : \textbf{TMD} \\ H \times B_1 \times B_2 \times S & : \textbf{SCET} \end{cases} + \mathcal{O}\left[\left(\frac{q_T}{Q}\right)^m\right]$$

- Dictionary:

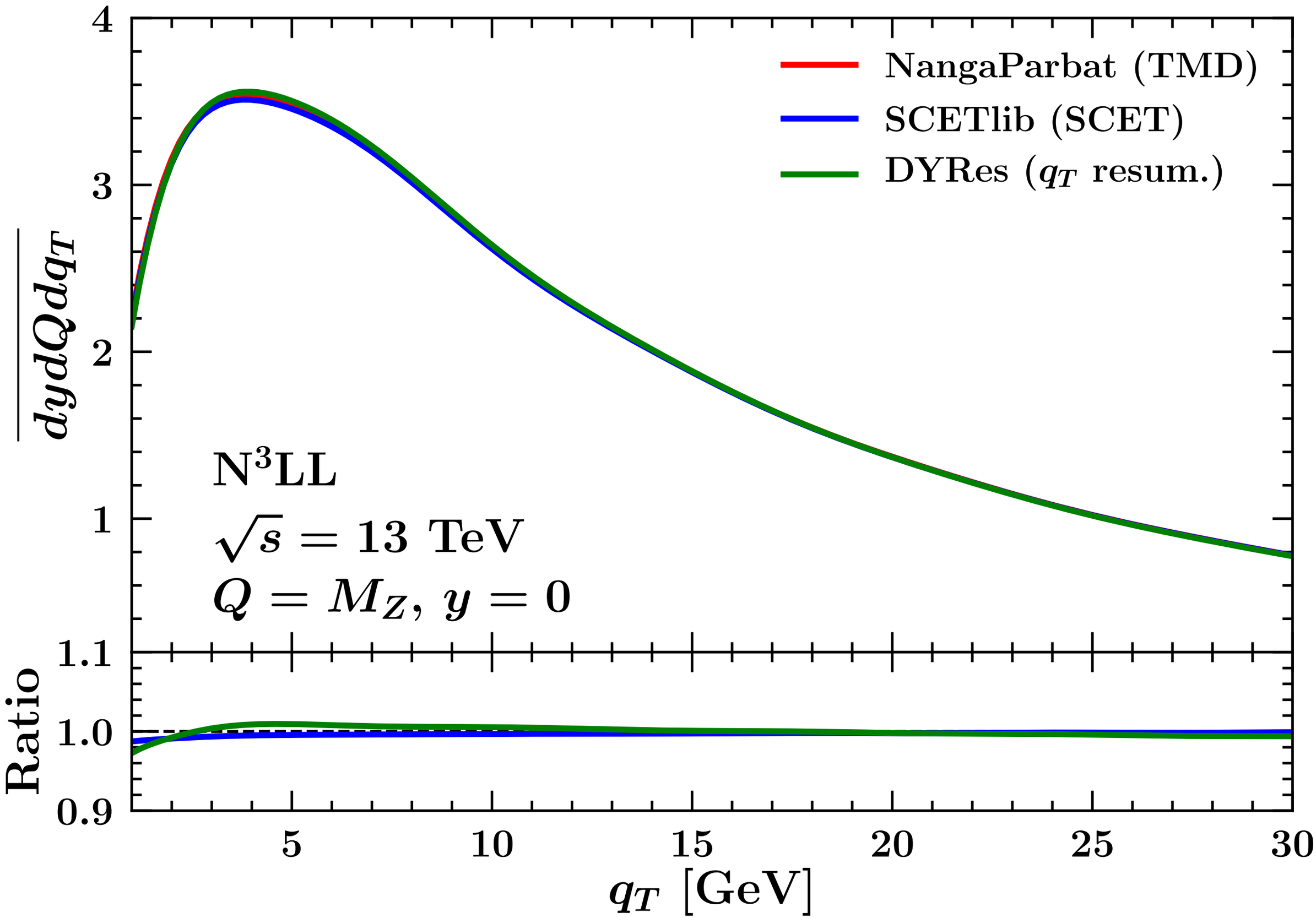
$$\mathcal{H} = HC_1C_2$$

$$F_i = e^S C_i \otimes f_i$$

$$F_i = \sqrt{S} \times B_i$$

- All **equivalent** for *exponentiating* processes.

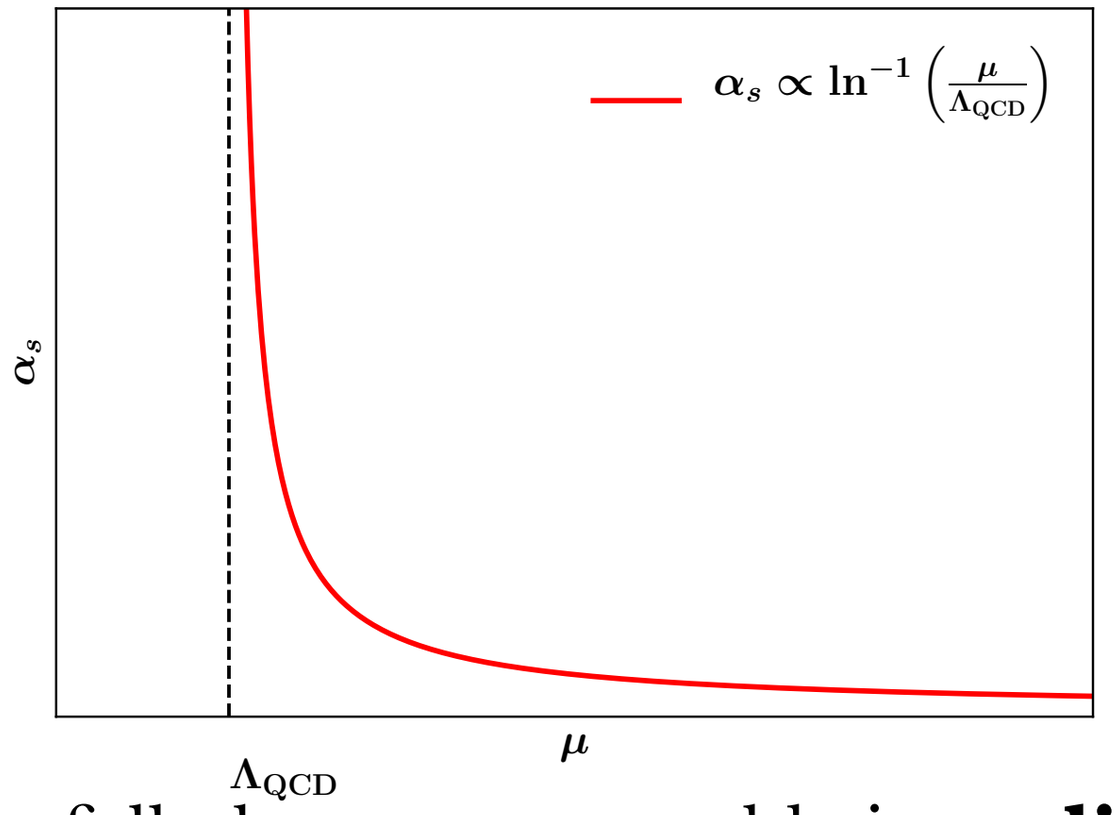
Resummation formalisms



Landau pole regularisation

Origin of TMD non-perturbative corrections

$$\sigma \propto \int_0^{\infty} db_{\mathrm{T}}\, \alpha_s^p \left(\frac{1}{b_{\mathrm{T}}}\right) \cdots \sim \int_0^{Q} dk_{\mathrm{T}}\, \alpha_s^p \left(k_{\mathrm{T}}\right) \ldots$$



- Integrating over the full phase space would give a **divergent** result.
- **Prescriptions** to avoid integrating over the **Landau pole**:
  - introduction of **power corrections** $(\Lambda/Q)^n$.
- TMD non-perturbative effects thus connected with **low-energy** region:
  - no matter how large Q is.

Logarithmic counting

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \overset{\text{TMD}}{=} \sigma_0 H(Q) \int d^2\mathrm{b}_T e^{i\mathrm{b}_T\cdot\mathrm{q}_T} F_1(x_1,\mathrm{b}_T,Q,Q^2) F_2(x_2,\mathrm{b}_T,Q,Q^2)$$

$$F_f(x,\mathrm{b}_T,\mu,\zeta) = \sum_j C_{f/j}(c,b_T;\mu_b,\zeta) \otimes f_j(x,\mu_b)$$

$$\times \exp\left\{ K(b_T,\mu_b) \ln\frac{\sqrt{\zeta}}{\mu_b} + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F - \gamma_K \ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\}$$

Accuracy	γ_K	γ_F	K	$C_{f/j}$	H
LL	α_s	-	-	1	1
NLL	α_s^2	α_s	α_s	1	1
NLL’	α_s^2	α_s	α_s	α_s	α_s
N^{2}LL	α_s^3	α_s^2	α_s^2	α_s	α_s
N^{2}LL’	α_s^3	α_s^2	α_s^2	α_s^2	α_s^2
N^{3}LL	α_s^4	α_s^3	α_s^3	α_s^2	α_s^2
N^{3}LL’	α_s^4	α_s^3	α_s^3	α_s^3	α_s^3

Additive matching and counting

- Accurate predictions for all q_T's by **additive matching**, order by order in perturbation theory, of collinear and TMD calculations:

$$\left(\frac{d\sigma}{dq_T}\right)_{\mathbf{add.match.}} = \left(\frac{d\sigma}{dq_T}\right)_{\mathbf{res.}} + \left(\frac{d\sigma}{dq_T}\right)_{\mathbf{f.o.}} - \left(\frac{d\sigma}{dq_T}\right)_{\mathbf{d.c.}}$$

- In order for the match to actually take place:

$$\left(\frac{d\sigma}{dq_T}\right)_{\mathbf{res.}} \xrightarrow[\mathbf{f.o.}]{} \left(\frac{d\sigma}{dq_T}\right)_{\mathbf{d.c.}} \xleftarrow[q_T \ll Q]{} \left(\frac{d\sigma}{dq_T}\right)_{\mathbf{f.o.}}$$

- Therefore, the "fixed-order" parts have to match in the relevant limits:

Log Accuracy	Minimal f.o. accuracy
NLL'	α_s (LO)
N^{2}LL	α_s (LO)
N^{2}LL'	α_s^2 (NLO)
N^{3}LL	α_s^2 (NLO)
N^{3}LL'	α_s^3 (NNLO)