Quarkonium dynamics with a 1D quantum master equation

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Quarkonia As Tools 2020

Collaborators:

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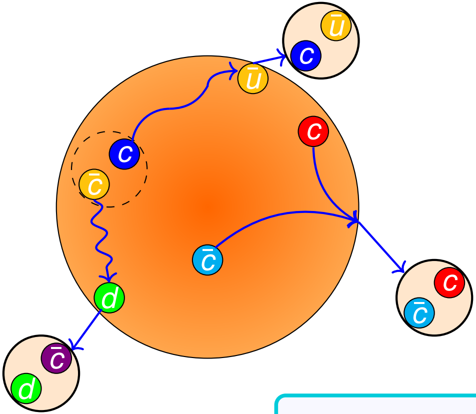
Subatech

Quarkonium in heavy-ion collisions



- Multiple effects at play
- Very dynamic problem

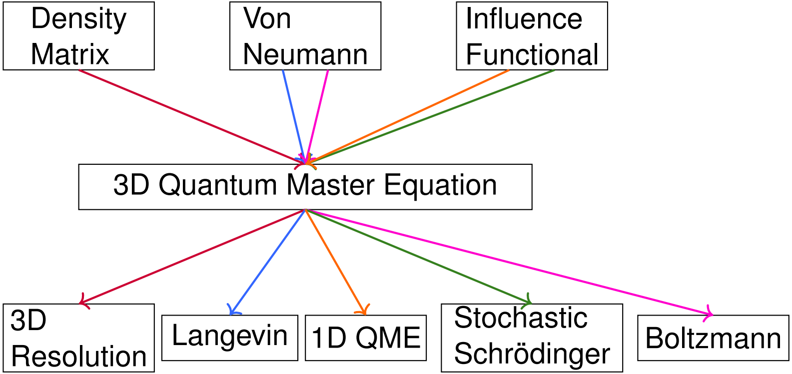
Dynamical and continuous description of dissociation, recombinaison and transitions between states

Theoretical models

- Statistical Recombinaison:
  - Quasi-stationnary medium
  - $Q\bar{Q}$ dissociated
  - Recombinaison at freeze-out
- Transport:
  - Based on dilute medium approximation
  - Transitions rates guided by cross-sections
  - Cross-sections can't treat everything
- Quantum (OQS):
  - Allows to treat those effects
  - Dynamic medium
  - Recombinaison of $c\bar{c}$ still problematic $\Rightarrow$ Semi-classical approximations

Use OQS to understand the impact of semi-classical approximations on the dynamics

Quantum Master Equations Approaches



Brambilla et al. (2017)
arXiv:1711.04515

Blaizot & Escobedo (2017)
arXiv:1711.10812

De Boni (2017)
arXiv:1705.03567

Akamatsu (2014)
arXiv:1403.5783

Miura et al. (2019)
arXiv:1908.06293

Yao & Mehen (2018)
arXiv:1811.07027

Several ways to treat the problem from open quantum systems

3D model

$$i\frac{d\mathcal{D}}{dt} = [\mathcal{H}, \mathcal{D}] \xrightarrow[\text{representation}]{\text{Interaction}} i\frac{d\mathcal{D}^I(t)}{dt} = [\mathcal{H}_1(t), \mathcal{D}^I(t)]$$

$$\mathcal{D}^I(t) = \mathcal{U}_I(t, t_0)\mathcal{D}(t_0)\mathcal{U}_I^\dagger(t, t_0)$$

$$\mathcal{H} = \mathcal{H}_Q + \mathcal{H}_1 + \mathcal{H}_{pl}$$

- $\mathcal{H}_Q$: Free Quark Hamiltonian
- $\mathcal{H}_1$: Quark-Plasma Interactions
- $\mathcal{H}_{pl}$: Plasma Hamiltonian

Average over plasma d.o.f

↓

Equation on $\mathcal{D}_Q$

- HQ-Plasma interactions weak $\Rightarrow \mathcal{H}_1$ treated as a perturbation
- One gluon exchanged at most
- No chromomagnetic fields (Coulomb gauge)

3D Quantum Master Equation

singlet density matrix

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t)\\ \mathcal{D}_o(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t)\end{pmatrix}$$

octet density matrix

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

singlet-octet transitions

Coupled partial differential equations on $\mathcal{D}_{s,o}(\mathbf{r}, \mathbf{y}, t)$

Linked to $Q\bar{Q}$ positions, more convenient in their approach

$$\mathbf{r} = \frac{1}{2}(\mathbf{r}_{rel} + \mathbf{r}'_{rel}) \qquad \mathbf{y} = \mathbf{r}_{rel} - \mathbf{r}'_{rel}$$

Use the complex potential V + iW

Laine et al. (2006) arXiv:0611300

Beraudo et al. (2007) arXiv:0712.4394

Semi-classical approximation

- **y** measures the deviation from the classical limit:
  - mesures how $\mathcal{D}$ deviates from a diagonal matrix
- Small-**y** expansion viewed as a semi-classical approximation:

$$W(\mathbf{y}) = W(0) + \frac{1}{2}\mathbf{y} \cdot \mathcal{H}(0) \cdot \mathbf{y} + \cdots$$

Treat semi-classicaly the relative motion while taking in account the color transitions

Numerical resolution of the semi-classical equation

- Regime where color exchanges are fast enough to equilibrate color
- Equation for the maximal entropy state reduces to QED case:
  - Dynamics studied near this state
  - Color transitions treated in perturbative approach
- Derivation of a Langevin equation
  - Possible to generalize to the case of multiple $Q\bar{Q}$ pairs

- Random color force $\propto \frac{1}{\Gamma(\mathbf{r})}$
- $\Gamma(\mathbf{r}) = W(\mathbf{r}) - W(\mathbf{0})$
- Amplified force when the size of the pair becomes too small $\Rightarrow$ Unphysical behaviour

Only valid as long as singlet-octet transitions are fast compared to the rest of the dynamics!

Attempt to resolve the full equations

Main objectives

- Comparison with the semi-classical approach
- Gain insight on the quarkonium dynamics inside the QGP
    - Evolution of states probabilities over time
    - Evolution of the trace of the density matrix
    - Singlet-octet transitions
    - Color relaxation time
- Resolution to be done in the 1D case
    - Reduced computationnal cost
    - Sufficient to gain insight

1D Quantum Master Equation

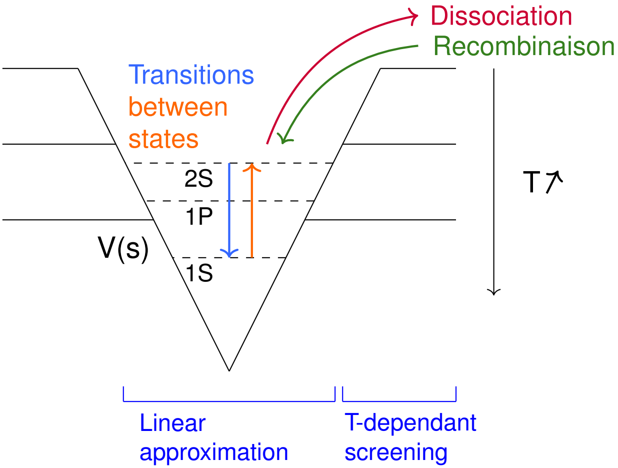
Kinematics   Color screening

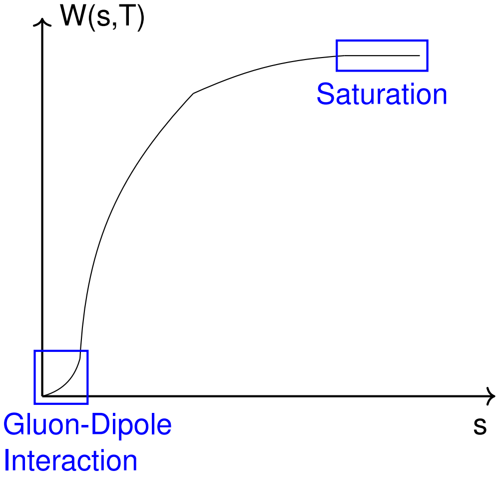
$$\hbar\frac{d\mathcal{D}_Q}{dt} = \frac{i(\hbar c)^2}{M}\left[\partial_s^2 - \partial_{s'}^2\right]\mathcal{D}_Q - iC_F\left[V(s) - V(s')\right]\mathcal{D}_Q$$
$$+ C_F\left[2W(0) - W(s) - W(s')\right]\mathcal{D}_Q$$
$$+ \frac{C_F(\hbar c)^2}{4MT}\left[2W''(0) - W''(s) - W''(s')\right]\mathcal{D}_Q$$
$$- \frac{C_F(\hbar c)^2}{2MT}\left[W'(s)\partial_s + W'(s')\partial_{s'}\right]\mathcal{D}_Q$$

Collisions, dissipation...

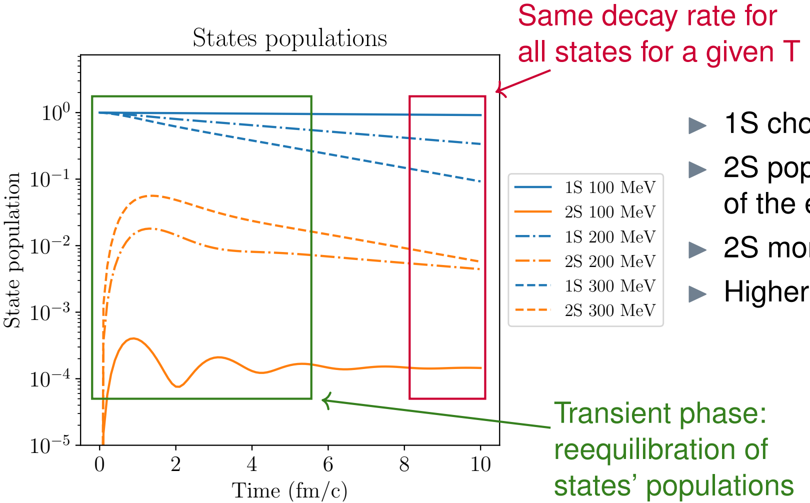
- Equation for $\mathcal{D}_s$
- Direct transformation to 1D + trace over c.o.m d.o.f

Model dynamics





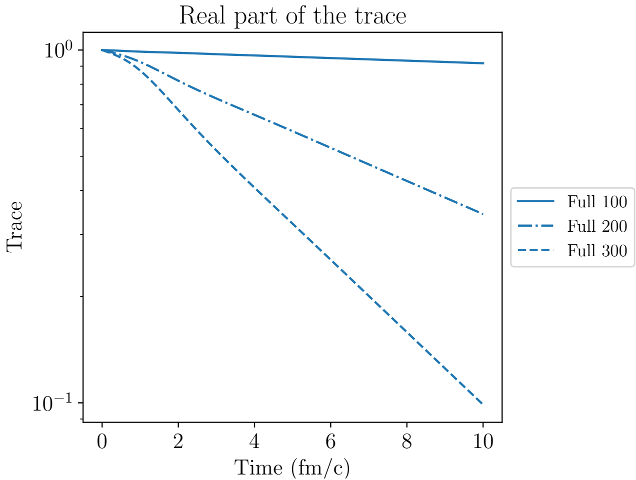
State probabilities



- 1S chosen as initial state
- 2S populated from 1S at the start of the evolution
- 2S more populated at 300 MeV
- Higher decay rate at 300 MeV

Interplay between various processes

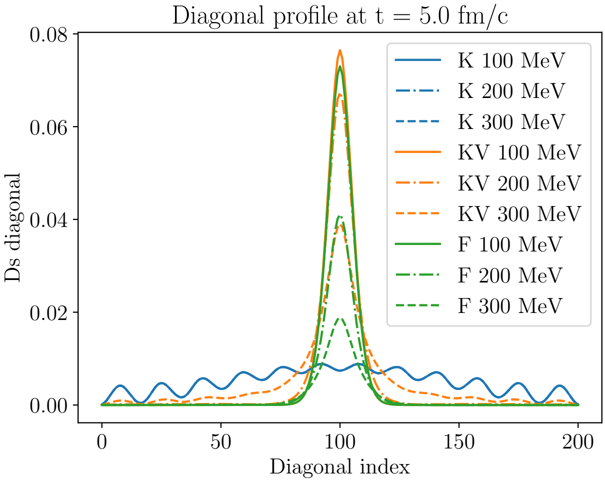
Trace conservation



- Exponential trace loss in all cases
- Higher loss for higher T
  - Expected, linked to the unbound/octet transitions

No loss expected for the complete equations ($\mathcal{D}_s$,$\mathcal{D}_o$)

Diagonal profile



- K, KV: Ballistic regime: $c\bar{c}$ outside of the potential evolving freely $\Rightarrow$ oscillations (bouncing)
- Full: Diffusive regime: $c\bar{c}$ stay close $\Rightarrow$ No bouncing

Main ideas

- Need for a complete description of real-time quarkonium dynamics
- Lot of recent efforts on open quantum systems
- Several ways of treating the problem
- Need to understand the impact of semi-classical approaches on the dynamics (especially to treat many $c\bar{c}$ pairs)

Results & Perspectives

- Excited states populated from ground state
- 2 phases: equilibration of populations and global decay
- Trace not conserved and loss depends on the temperature
  - Should be conserved for the full equations ($\mathcal{D}_s$,$\mathcal{D}_o$)
- Quantum and semi-classical effects

- Implement the full equations (singlet-octet transitions)
- Improve the potentials used
- Add a more realistic background (cooling medium...)
- Go to 3D ? (depends on the results of the 1D case)
- Add more $Q\bar{Q}$ pairs ? (recombinaison, probably too difficult)

Schwinger-Keldysh contour