k_T-factorization in quarkonium production: single, associated production, and going to one-loop accuracy

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Outline

1. Theory introduction: understanding the “zoo” of UPDFs
2. Single J/ψ and Υ production
3. Associated production: $\Upsilon + D$ and double J/ψ
4. Progress towards NLO

This talk is NOT about:

- **Saturation** and higher-twist effects
- **Exclusive production**
- Small-x resummation in PDF evolution

From collinear to k_T-factorization

Process:

$$p(P_1) + p(P_2) \to J/\psi(p^+, p^-, \mathbf{p}_T) + X,$$

where: $P_1^\mu = P_+ n_-^\mu/2$, $P_2^\mu = P_- n_+^\mu/2$, $p_\pm = p^0 \pm p^3 = n_\pm p$. Rapidity: $y = \ln(p^+/p^-)/2$. Introduce $\boxed{x_\pm = p_\pm/P_\pm}$, then:

$$d\sigma = \int\limits_{x_+}^{1} \frac{dz_+}{z_+} \tilde{f}_i\left(\frac{x_+}{z_+}, \mu_F^2\right) \int\limits_{x_-}^{1} \frac{dz_-}{z_-} \tilde{f}_j\left(\frac{x_-}{z_-}, \mu_F^2\right) \mathcal{H}_{i,j}(z_+, z_-, \alpha_s(\mu_R^2), \ldots),$$

where $\tilde{f}_i(x, \mu^2) = x f_i(x, \mu^2)$, up to power-suppressed corrections in $\mu^2 \simeq \mu_F^2 \simeq \mu_R^2 \sim p_T^2$.

- **Regge limit** for $\mathcal{H}$: $\boxed{z_\pm \ll 1}$.
- **Leading twist $\mathbf{k}_T$-factorization** (for this talk): Understanding higher-order corrections to $\mathcal{H}$ using properties of QCD amplitudes in Regge limit.
- There are important small-x effects in PDF evolution, due to $\ln(1/z)$-corrections in DGLAP splitting functions [Catani, Hautmann, 94';...; R.Ball *et.al.* 2017; ...], but we do not consider them.

From collinear to k_T-factorization

From known all-order results in Regge limit (see below), the following factorization structure can be deduced:

$$\mathcal{H}_{i,j} = \sum_{m,n} \int \frac{dx_1}{x_1} \frac{d^2\mathbf{q}_{T1}}{\pi} \, C_{ik}^{(m)}(\alpha_s^{n_1}(z_+/x_1)^{n_2} \ln^{n_3}(x_1/z_+), \mathbf{q}_{T1}) \times$$

$$\int \frac{dx_2}{x_2} \frac{d^2\mathbf{q}_{T2}}{\pi} C_{lj}^{(n)}(\alpha_s^{n_1'}(z_-/x_2)^{n_2'} \ln^{n_3'}(x_2/z_-), \mathbf{q}_{T2}) \times$$

$$H_{kl}^{(m,n)}(\alpha_s^{m_1} x_1^{m_2} x_2^{m_3}, \mathbf{q}_{T1}, \mathbf{q}_{T2}, \ldots).$$

- Logarithms $\ln(1/z_\pm)$ are factorized to all orders; characteristic q_T-dependence comes with them
- Convolution in transverse momenta $\mathbf{q}_{T1,2} \Rightarrow$ *k_T-factorization* or *High-Energy factorization*
- **Different** from *TMD-factorization*, because **NOT** limited to the region $p_T \ll \mu$.
- There could be several types of *process-dependent* coefficient functions $H^{(m,n)}$, functions $C^{(m)}$ are **universal**.
- Perturbative expansion for H is assumed to be well-behaved

Leading twist k_T-factorization

Substituting $\mathcal{H}_{i,j}$ to factorization formula we get:

$$d\sigma = \sum_{m,n} \int \frac{dx_1}{x_1} \frac{d^2\mathbf{q}_{T1}}{\pi} \Phi_i^{(m)}(x_1, \mathbf{q}_{T1}^2, \mu^2) \times$$

$$\int \frac{dx_2}{x_2} \frac{d^2\mathbf{q}_{T2}}{\pi} \Phi_j^{(n)}(x_2, \mathbf{q}_{T2}^2, \mu^2) \times H_{ij}^{(m,n)} \left(\frac{x_+}{x_1}, \frac{x_-}{x_2}, \mathbf{q}_{T1,2}, \dots \right),$$

where the *unintegrated PDF* (UPDF):

$$\Phi_i^{(n)}(x, \mathbf{q}_T^2, \mu^2) = \int_x^1 \frac{dz}{z} \tilde{f}_j \left(\frac{x}{z}, \mu^2 \right) \times C_{ij}^{(n)}(z, \mathbf{q}_T).$$

What is known about k_T-factorization?

Expansion for $C_{ik}(z)$ at $z \ll 1$, schematically:

LL, [BFKL, 76'; 78']

NLL, [Fadin, Lipatov 98'; Fadin et.al. 2015]

NNLL [Fadin, 2019;...]; $\mathcal{N}=4$ SYM [Caron-Huot, 2018;...]

$$z^0 \times \sum_{n_1} \left\{ (\alpha_s \ln(1/z))^{n_1} + \alpha_s \times (\alpha_s \ln(1/z))^{n_1} + \alpha_s^2 \times (\alpha_s \ln(1/z))^{n_1} + \dots \right\} +$$

NLP-LL, [Bartels, Ermolaev, Ryskin 95'; ...]

NLP-NLL, [Fadin,Sherman 77'; Lipatov, Vyazovsky 2001; ...]

$$+ \quad z^1 \times \sum_{n_2} \left\{ (\alpha_s \ln^2(1/z))^{n_2} + (\alpha_s \ln(1/z))^{n_2} + \dots \right\} +$$

$$+ \quad z^2 \times \{\dots\} +$$

$$+ \quad \dots$$

Convergence: Assuming $f(x) \sim x^{-\lambda}$:

$$\int\limits_x^1 \frac{dz}{z} f\left(\frac{x}{z}\right) \times z^n \ln^m(1/z) \underset{x \ll 1}{\simeq} \frac{m!\ x^{-\lambda}}{(n+\lambda)^{m+1}}$$

Power corrections ($z \to 1$) are important! At fixed order in α_s:

$$\frac{1}{(1-z)_+} \simeq \sum_{k=0}^{\infty} z^k,\ \ln(1-z) = \sum_{k=1}^{\infty} \frac{-z^k}{k},$$

How to include $z \to 1$ effects? $\Rightarrow$ "zoo" of UPDFs

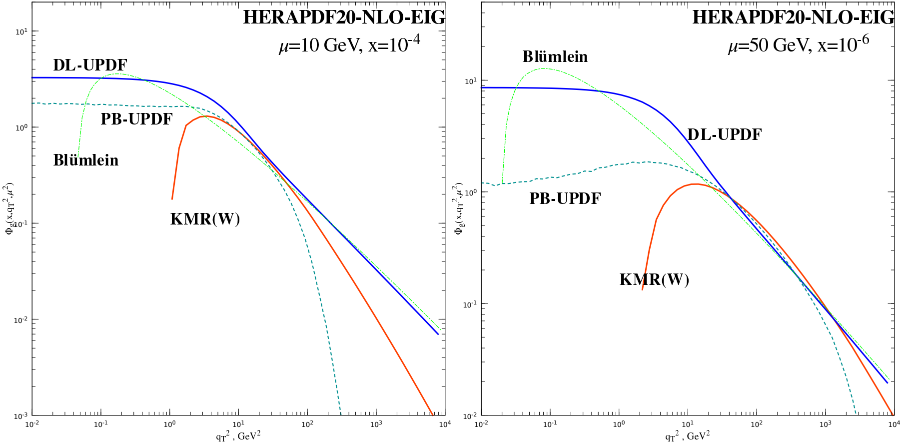
"Zoo" of phenomenological UPDFs

Welcome to the TMDlib/TMDplotter "Zoo": http://tmdplotter.desy.de/ ! All approaches so-far are at LO in α_s for $H \Rightarrow$ LL or NLL for UPDFs. W.r.t. power corrections they can be classified as:

- **BFKL-based – doubly-logarithmic:** ignore power corrections, resum $\alpha_s(\mu^2)\ln(1/z)\ln(\mathbf{q}_T^2/\mu^2)$ using LL BFKL equation: *(Collins-Ellis-Catani-Hautmann-)Blümlein approach* [Collins, Ellis, 91'; Blümlein, 94'; Catani-Hautmann 94'; M.N. 2019].
- **CSS-compatible:** include power corrections, resum $\alpha_s \ln^2(\mathbf{q}_T^2/\mu^2)$ to LL and NLL consistently with Collins-Soper-Sterman small-p_T resummation formalism (**for color-singlet!**):*Kimber-Martin-Ryskin(-Watt) approach* [Kimber, Martin, Ryskin 2001; Watt, Martin, Ryskin 2003; 2009]
- **Intermediate:** partially include power corrections and $\ln(1/z)$ resummation: *CCFM equation*, *Parton-Branching method* [Jung, Hautmann et.al. 2018;...],...

Important constraint: doubly-asymptotic limit $\boxed{x \to 0,\ \mu^2 \to \infty,\ q_T \ll \mu}$ is **universal** in the BFKL, KMRW/CSS and PB fromalisms (at least at LL).

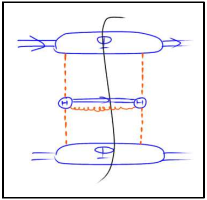
Gluon UPDF

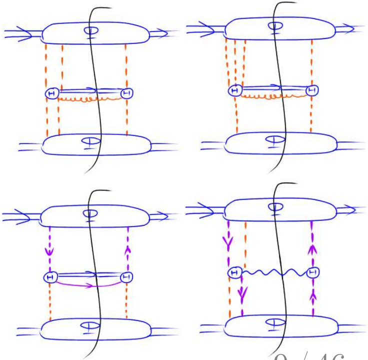


- Region $q_T < 1$ GeV is non-perturbative. No dedicated fits of q_T-dependence for gluon have been done.
- Non-perturbative effects propagate up to $q_T \sim 2-3$ GeV due to evolution.
- Region $2-3$ GeV $< q_T \ll \mu$ – different approaches agree at $\mu^2 \to \infty$, $x \to 0$ due to DL constraint. KMRW is very close to PB.
- Region $q_T > \mu$ – PB UPDF has an (unnaturally) soft tail $\Rightarrow$ strong scale-dependence for multiscale observables

How to compute coefficient function? Parton Reggeization Approach (PRA)

- Lipatov's EFT [Lipatov, 95'] allows to compute **LP** Regge limits for QCD amplitudes directly $\Rightarrow \Phi$ and H are connected by off-shell t-channel "partons" (Reggeons!). **This factorization in gauge-invariant!** (See also [A. van Hameren, P. Kotko, K. Kutak et.al] and `KaTie` code!)
- **LP**: LL, NLL – only single **Reggeized gluon** (R) exchanges contribute to amplitude.
- **LP**: N^{2}LL – EFT diagrams with several Reggeized gluons: can be recast into same factorized form but possibly with different $H^{(n,m)}$ and $\Phi^{(n)}$.
- **NLP**: LL – NLP operator can be understood as effective t-channel exchange (e.g. [Penin, 2019])
- **NLP**: NLL – **Reggeized quarks** (Q) contribute [Fadin, Sherman 77'; Lipatov, Vyazovsky 2001]





Single inclusive quarkonium production in k_T-factorization.

Inclusive heavy quarkonium production

Framework: k_T-factorization (Lipatov's EFT) + NRQCD factorization at LO in α_s.

Reggeized-gluon-initiated subprocesses [Kniehl, Vasin, Saleev 2006] (for ${}^3S_1$ and ${}^3P_J$-quarkonia):

$$\begin{aligned} R(q_1)+R(q_2) &\to c\bar{c}\left[{}^3S_1^{(1)}\right]+g(k), \\ R(q_1)+R(q_2) &\to c\bar{c}\left[{}^1S_0^{(8)},{}^3P_J^{(1,8)},{}^3S_1^{(8)}\right], \end{aligned}$$

where $q_{1,2}^{\mu}=x_{1,2}P_{1,2}^{\mu}+q_{T1,2}^{\mu}$. Off-shell amplitude $R+R\to c+\bar{c}$ in Lipatov's EFT coincides with amplitude obtained using "non-sense" polarization prescription $\varepsilon^{\mu}(q_T)\to q_T^{\mu}/|\mathbf{q}_T|$, **but it is the only "safe" one**.

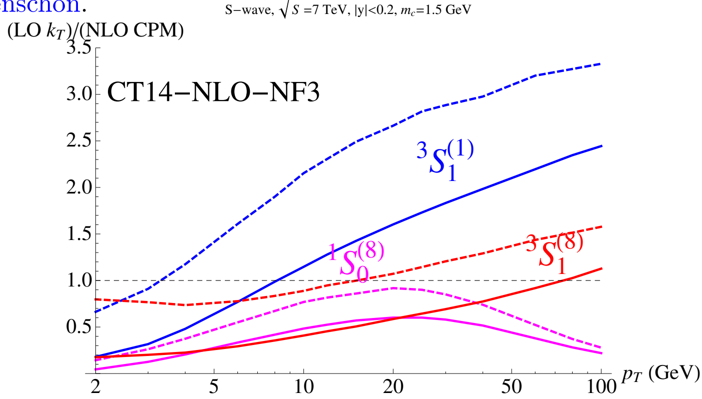
Reggeized-quark-initiated subprocesses(NLP, numerically small but important at low energies):

$$\begin{aligned} Q(q_1)+\overline{Q}(q_2) &\to c\bar{c}\left[{}^3S_1^{(8)}\right], \\ C(q_1)+\overline{C}(q_2) &\to c\bar{c}\left[{}^3S_1^{(1)},{}^3S_1^{(8)},{}^1S_0^{(8)},{}^3P_J^{(1,8)}\right], \end{aligned}$$

where Q – light Reggeized quark and C – heavy Reggeized quark [Lipatov, Vyazovsky, 2001].

Comparison to NLO CPM

Numerical results at NLO of CPM provided by B. Kniehl and M. Butenschön.



- Solid lines – KMRW UPDF, dashed lines – Blümlein UPDF
- Spectra for P-wave states from NLO CPM turn negative at $p_T \simeq 7$ GeV. In LO PRA cross-section is always positive
- Large corrections for ${}^3S_1^{(1)}$ are predicted at high p_T. Other S-states $\sim$ compatible with NLO of CPM

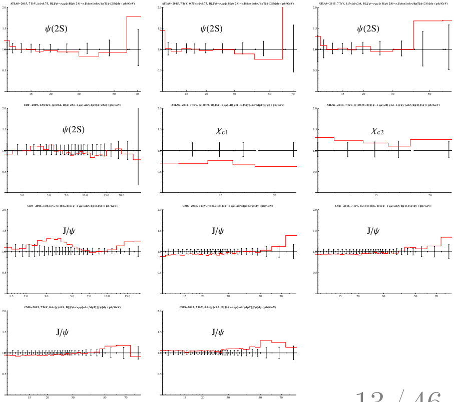
Example LDME fit (LDMEs used in double- J/ψ calculation)

Framework: LO KMR(W) UPDF [MSTW-08 LO PDF], included p_T-shift in cascade decays, included LL fragmentation for $^3S_1^{(8)}$ at high p_T, $m_c = 1.5$ GeV.

LDME, GeV^3
$\left\langle \mathcal{O}^{J/\psi}[^3S_1^{(1)}] \right\rangle = 1.16$
$M_0^{J/\psi} = 3.61 \times 10^{-2}$
$\left\langle \mathcal{O}^{J/\psi}[^3S_1^{(8)}] \right\rangle = 1.25 \times 10^{-3}$
$\left\langle \mathcal{O}^{\psi'}[^3S_1^{(1)}] \right\rangle = 0.76$
$M_0^{\psi'} = 2.19 \times 10^{-2}$
$\left\langle \mathcal{O}^{\psi'}[^3S_1^{(8)}] \right\rangle = 3.41 \times 10^{-4}$
$\left\langle \mathcal{O}^{\chi_{c0}}[^3P_0^{(1)}] \right\rangle / m_c^2 = 4.77 \times 10^{-2}$
$\left\langle \mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}] \right\rangle = 5.29 \times 10^{-4}$

where $M_0^{\mathcal{H}} = \left\langle \mathcal{O}^{\mathcal{H}}[^3S_0^{(8)}] \right\rangle + \frac{R}{m_c^2} \left\langle \mathcal{O}^{\mathcal{H}}[^3P_0^{(8)}] \right\rangle$.

Only LHC data with $p_T > 10$ GeV where fitted. χ^2/d.o.f.$\simeq 1$.



Review of some recent results on single production

NRQCD factorization:

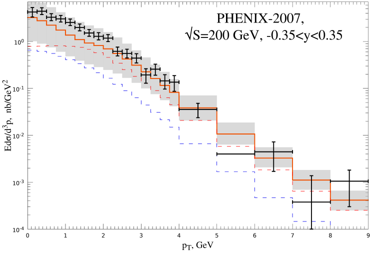
- [M.N., Saleev, Shipilova 2012; 2013] Global fits for prompt J/ψ, $\psi(2S)$, χ_{cJ}, $\Upsilon(nS)$ and χ_{bJ} hadroproduction in KMR(W) framework
- [M.N., Kniehl, Saleev 2016] $\psi(2S)$ and $\Upsilon(3S)$ re-considered. Polarization predictions – *transverse polarization for $\psi(2S)$ at high p_T.*
- [Baranov, Lipatov, Malyshev 2016;...;2019] Similar framework, but taking into account soft-gluon and decay-photon recoils in amplitude-based approach. Similar hierarchy of CO LDMEs found. *Conclusion that (Electric-dipole?!) soft-gluon effects cure polarization problem is questionable!* (Our reply coming soon...)

k_T-factorization + other factorization approaches for quarkonium:

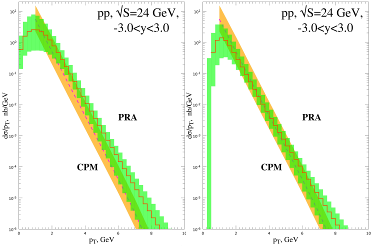
- [Cheung, Vogt 2018] *Improved Color Evaporation Model.* De-polarization at high-p_T is found! Mechanism – requires investigation.
- [Babiarz, Pasechnik, Schäfer, Szczurek 2019] *Light-front wavefunction approach* for η_c-production. Results reasonably stable w.r.t. choice of LFWF.

Going to low energies

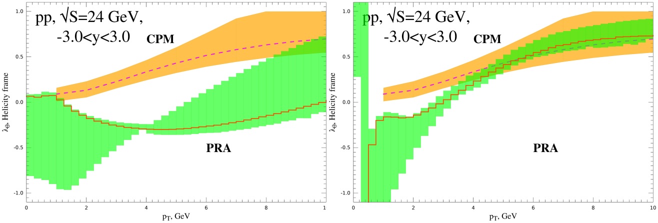
RHIC ($\sqrt{S} = 200$ GeV):



Planned JINR NICA ($\sqrt{S} = 24$ GeV; PRA vs. CPM LDMEs; vs. NLO CPM [Kniehl, Butenschön]):



Polarization (PRA vs. CPM LDMEs; vs. NLO CPM):



$\Upsilon(1S) + D$ associated production.

Based on [Phys. Rev. D 99, 096021 (2019)]

Associated production: $\Upsilon(1S) + D$, experiment

See [Karpishkov, M.N., Saleev 2019]. In the experimental paper [LHCb 2016] this process was considered as sensitive to DPS.

Arguments:

- Significant cross-section ~ 100 pb was measured at $\sqrt{S} = 7$ and 8 TeV, $R_{\text{exp.}} = \sigma_{\text{exp.}}(\Upsilon + D)/\sigma_{\text{exp.}}(\Upsilon) \sim 8\%$, while SPS LO (color-singlet) CPM calculation [Berezhnoy, Likhoded 2015] predicted $R_{\text{LO CPM}} \sim (0.2 - 0.4)\%$. $\Rightarrow$ *SPS production is strongly suppressed by α_s?*
- Various measured kinematic distributions where found to be consistent with phase-space distributions for un-correlated production of Υ and $D \Rightarrow$ *no sign of kinematic correlations from common SPS production subprocess?*

$\Rightarrow$ **DPS?**

Associated production: $\Upsilon(1S) + D$, LO PRA

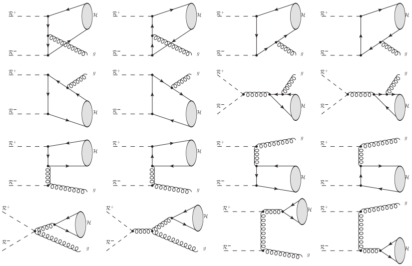
- LO Subprocesses ($\mathcal{O}(\alpha_s^3)$):

$$R + R \to \Upsilon(3S, 2S, 1S)(\to \Upsilon(1S)) + g(\to D)$$
$$R + R \to \chi_b(2P, 1P)(\to \Upsilon(1S)) + g(\to D)$$

$b\bar{b}\left[{}^3S_1^{(1)}, {}^3S_1^{(8)}, {}^3P_J^{(1)}\right]$-states are taken into account.

- **LO scale-dependent FF** [Kniehl, Kramer, Schienbein, Spiesberger 2006] fitted on LEP data is used to describe $g \to D$ transition.
- Υ LDMEs from our fit of 2013 has been used.

Diagrams of Lipatov's EFT:



Ratio ($g \to D$ vs. $c \to D$ fragmentation):

$$\frac{\sigma^{D^0}_{\mathrm{dir.}}[R + R \to b\bar{b}[{}^3S_1^{(1+8)}] + g]}{\sigma^{D^0}_{\mathrm{dir.}}[R + R \to b\bar{b}[{}^3S_1^{(1+8)}] + c + \bar{c}]} \simeq 2.6 \div 2.5,$$

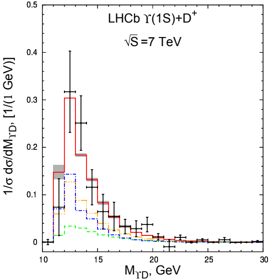
$\Rightarrow$ gluon fragmentation is the dominating mechanism

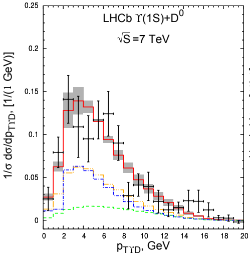
Associated production: $\Upsilon(1S) + D$, results

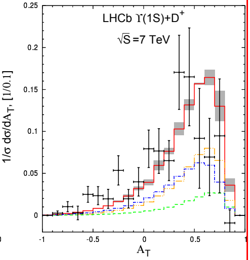
Total cross-section:

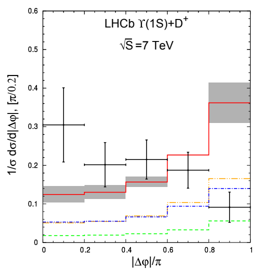
	$\Upsilon(1S)D^0$, pb	$\Upsilon(1S)D^+$, pb
Direct:	51	20
$\Upsilon[^3S_1^{(1)}]$	37	15
$\Upsilon[^3S_1^{(8)}]$	14	5
Feed-down	40	16
Total CS, LO PRA	91^{+48}_{-41}	36^{+19}_{-16}
Total CS, exp.	155 ± 28	82 ± 24

Radiative corrections lead to significant SPS CS $\sim 1/2$ exp. CS.









Almost all distributions are in reasonable agreement with SPS hypothesis, except $\Delta\phi$.

Prompt J/ψ pair production.

Based on [Phys. Rev. Lett. 123, 162002 (2019)]

J/ψ pair production: (Selected) theory results

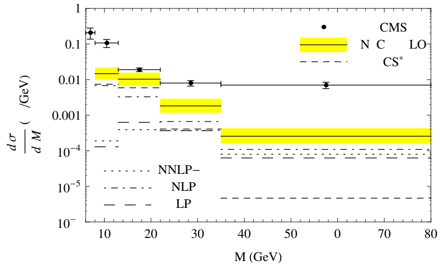
- The total cross-section is dominated by double-${}^3S_1^{(1)}$ contribution [Kartvelishvili, Esakiya 83'; Humpert, Mery 83'; Qiao 2002]
- The CO-states contribute, [Barger et.al. 96'] proposed double-J/ψ production as a test of CO mechanism. Relativistic corrections to $2^3S_1^{(1)}$ and $2^3S_1^{(8)}$-channels where also considered [Li, et.al. 2013].
- The full calculation in the LO of CPM, including all CO states and feed-down, was done by [He, Kniehl 2015]. The double-CO contributions are very important at large-$M_{\psi\psi}$ and $\Delta Y_{\psi\psi}$.
- The Double Parton Scattering (DPS) contributes to the same kinematic region [Lansberg, et.al. 2015] ! But DPS contribution is flat or decreasing with $\Delta Y_{\psi\psi}$.
- The full NLO corrections in CPM for double-${}^3S_1^{(1)}$ channel has been calculated by [Sun, et.al. 2016].
- The CS-model computation in non-gauge-invariant k_T-factorization with CCFM-based UPDFs [Baranov, et.al. 2015] fails to describe data.
- And more...

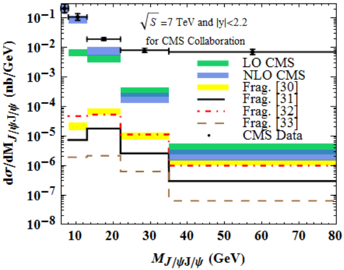
J/ψ pair production: Experimental data

- First measurements of $2J/\psi$ by LHCb (pp @ 7 TeV) [LHCb 2012; 2017]. The p_T^{ψ}-spectrum at $2 < y^{\psi} < 4.5$ agrees reasonably with LO CPM+NRQCD [He, Kniehl 2015] with LDMEs fitted for inclusive single-J/ψ hadroproduction.
- Total cross-section measurements by D0 ($p\bar{p}$ @ 1.96 TeV) [D0 2014] are also reproduced in LO CPM + NRQCD.
- We will concentrate on CMS (pp @ 7 TeV) [CMS 2014] and ATLAS (pp @ 8 TeV) [ATLAS 2017] measurements which provide a rich set of spectra vs.: $M_{\psi\psi}$, $\Delta Y_{\psi\psi}$, $p_T^{\psi\psi}$ and $p^{\psi}_{T,\text{ lead.}}$.

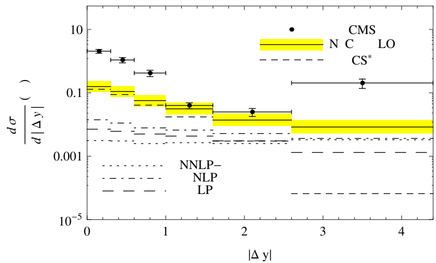
Description in Collinear Parton Model

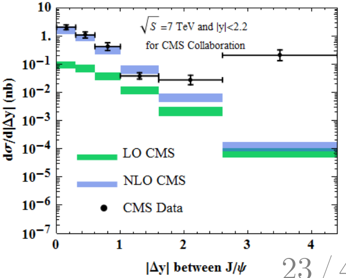
- The $M_{\psi\psi}$-spectrum (CMS-data, Full LO vs. $2^3S_1^{(1)}$ NLO CPM):





- The $\Delta Y_{\psi\psi}$-spectrum (CMS-data, Full LO vs. $2^3S_1^{(1)}$ NLO CPM):





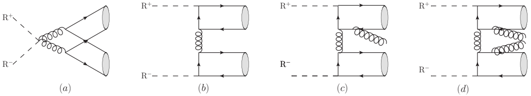
Fixed-order contributions in PRA

We have calculated contributions of **all** diagrams at $\mathcal{O}(\alpha_s^4)$ (LO) to all direct and feed-down partonic channels in PRA:

$$R_+(q_1) + R_-(q_2) \to c\bar{c}[m] + c\bar{c}[n],$$

with $m, n = {}^{2S+1}L_J^{(c)}$.

The dominant *asymptotics* at large $M_{\psi\psi}$ ($\Delta Y_{\psi\psi}$) is provided by diagrams with t-channel (Reggeized) gluon exchange **between** $c\bar{c}$-states. Partonic channels can be classified according to the order in α_s in which the t-channel gluon exchange first occur:

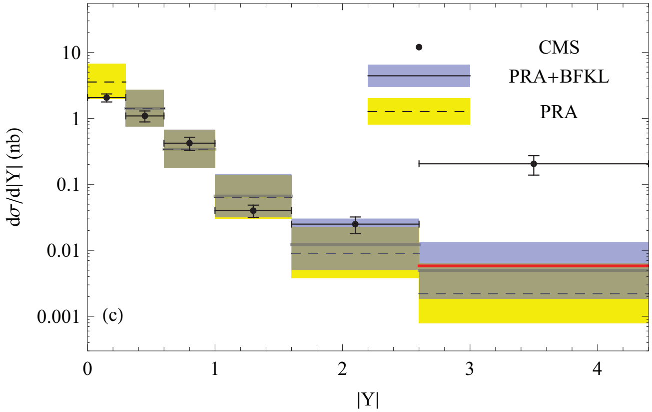


(b) **LT** : $m, n = {}^1S_0^{(8)}, {}^3S_1^{(8)}, {}^3P_J^{(1,8)}$,

(c) **NLT**: $m = {}^3S_1^{(1)}$ and $n = {}^1S_0^{(8)}, {}^3S_1^{(8)}, {}^3P_J^{(1,8)}$,

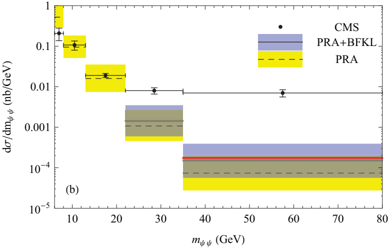
(d) **NNLT** : $m, n = {}^3S_1^{(1)}$.

$\Delta Y_{\psi\psi}$ spectrum, CMS

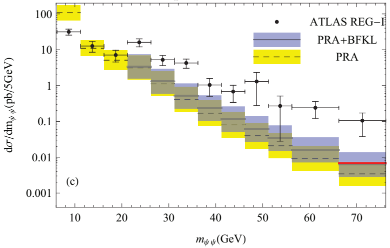


Yellow band and black dashed line – LO PRA.
Unfortunately, ATLAS provides only *fiducial* $\Delta Y_{\psi\psi}$-spectrum which is hard to compare with our predictions.

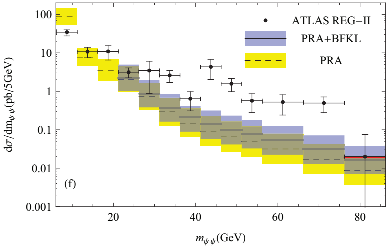
$M_{\psi\psi}$ spectra, CMS and ATLAS



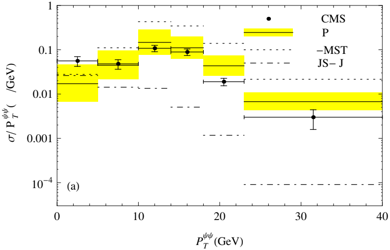
$|y^{\psi}_{\text{subl. } p_T}| < 1.05$



$1.05 < |y^{\psi}_{\text{subl. } p_T}| < 2.1$

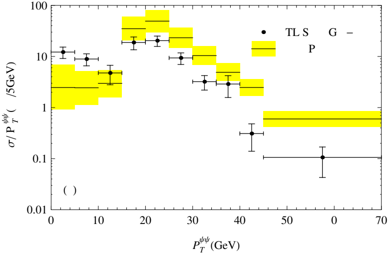


The $p_T^{\psi\psi}$- spectra, CMS and ATLAS

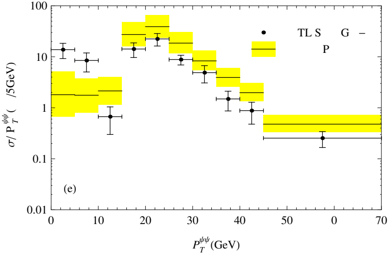


- Solid line – KMR(W) UPDF,
- Dashed line – Blümlein UPDF,
- Dash-dotted line – CCFM-based Jung-Hautmann UPDF (the result from PB UPDF will be similar).

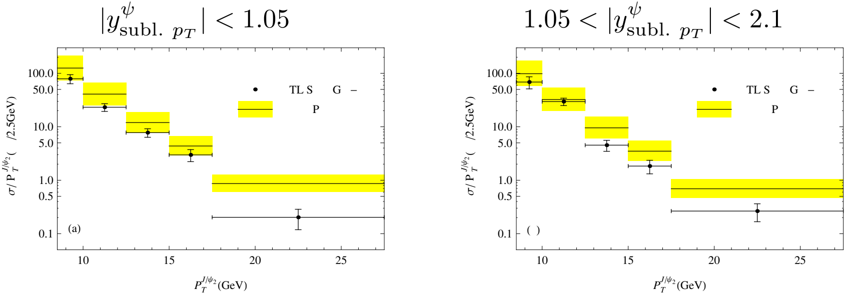
$|y^{\psi}_{\text{subl. } p_T}| < 1.05$



$1.05 < |y^{\psi}_{\text{subl. } p_T}| < 2.1$

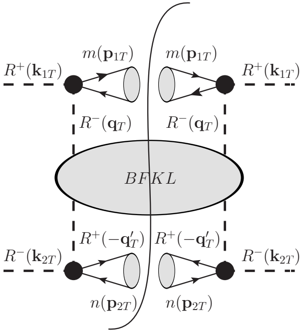


The $p^{\psi}_{T,\text{ lead.}}$ spectra from ATLAS



BFKL-resummation contribution

Overall agreement of LO PRA calculation with data is quite reasonable, except large $\mathcal{O}(10 - 100)$ deficit at large $M_{\psi\psi}$ and $\Delta Y_{\psi\psi}$. But radiative corrections to **LT** and **NLT** contributions could be significant!



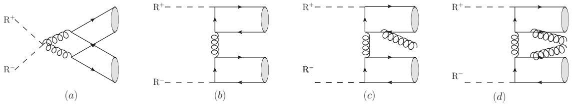
- We resum higher-order corrections $\sim (\alpha_s \Delta Y_{\psi\psi})^n$ to LT-channels using LLA BFKL Green's function with suitable BLM-type renormalization-scale setting [Brodsky, et.al., 99'] to take into account large running-coupling effects.
- Resummation is performed for $\Delta Y_{\psi\psi}$ and $M_{\psi\psi}$-spectra. For other spectra effect is negligible.
- The LO $R_+ R_- \to c\bar{c}[n]$ impact-factors are well-known [Kniehl, Vasin, Saleev 2006].

NLT contribution

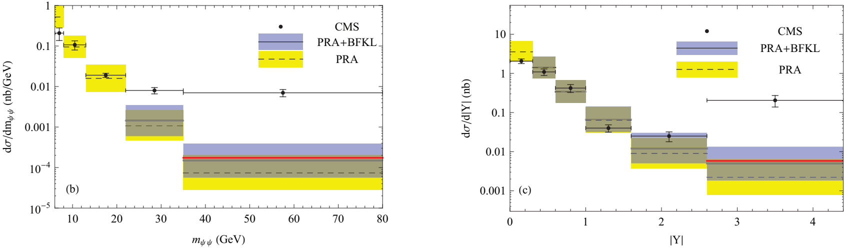
- Since $\left\langle \mathcal{O}^{J/\psi}[^3S_1^{(1)}] \right\rangle \sim (10^2 - 10^3) \times \left\langle \mathcal{O}^{J/\psi}[^3L_J^{(8)}] \right\rangle$, the NLT contribution could be numerically significant.
- The

$$R_+ + R_- \to c\bar{c}\left[{}^3S_1^{(1)}\right] + g \qquad (1)$$

amplitude **does not** have any singularities for $E_g \to 0$ or $k_{Tg} \to 0$ since $\mathcal{M}\left(R_+ + R_- \to c\bar{c}\left[{}^3S_1^{(1)}\right]\right) = 0$.
- There is **no** *rapidity divergence* for integration over rapidity of gluon in (1), so no double-counting with BFKL resummation or UPDF.
- So we can construct *gauge-invariant and IR-finite* large-$\Delta Y_{\psi\psi}$ asymptotics for $\mathcal{O}(\alpha_s^5)$ NLT squared amplitudes by replacing ordinary t-channel gluon with Reggeized one in the diagram (c).



Combined effect on $M_{\psi\psi}$ and $\Delta Y_{\psi\psi}$ spectra



- Effect of BFKL-resummation is significant, up to a factor of two.
- Large $\mathcal{O}(100)$ K-factors are found in some NLT channels in the last $\Delta Y_{\psi\psi}$-bin. The effect on direct production is +45%, but after addition of feeddown the overall effect of NLT-contributions reduces to +16% (the thick red line).
- Apparent growth of CMS cross-section with $\Delta Y_{\psi\psi}$ is a complete mystery. One needs enormous Pomeron intercept ($> \alpha_P^{\text{LL BFKL}}$???) to fit this.
- Effects in ATLAS $M_{\psi\psi}$ spectra are roughly the same (see corresponding plots on slide 15).

New idea: Pomeron contribution

In [Shao, Zhang 2016; Lansberg et.al. 2019] the loop-induced contribution with two-gluon exchange in t-channel was discussed:

$$g + g \to c\bar{c}\left[{}^3S_1^{(1)}\right] + c\bar{c}\left[{}^3S_1^{(1)}\right].$$

Which is **IR-finite**, of LP at large ΔY and enhanced by two ${}^3S_1^{(1)}$ LDMES!

- It is a LO approximation to the Pomeron exchange amplitude.
- We can write-down a simple asymptotic expression for this amplitude with two Reggeized gluons in t-channel.
- Additional gluon emissions can be added to this exchange, by solving Bartels-Kwiecinski-Praszalowicz equation, leading to a growing cross-section

Loop corrections in Lipatov's EFT.

Based on [Nucl.Phys., B946, 114715 (2019)]

NLO calculations: Why to bother?

- To show the self-consistency of the approach. The statement is, that most of corrections which determine the **shape** of various multiscale kinematic distributions are already included, so *NLO corrections must be small.*
- For quarkonium physics another motivation is that at NLO in PRA, the process:

$$R + R \to g \to c\bar{c}\left[{}^3S_1^{(1)}\right] + 2g,$$

  appears, which is by factor p_T^2 “stronger” than LO process $R + R \to c\bar{c}\left[{}^3S_1^{(1)}\right] + g$. In CPM, the corresponding “fragmentation-type” process:

$$g + g \to g + g(\to c\bar{c}\left[{}^3S_1^{(1)}\right] + 2g),$$

  contributes only at NNLO.

Eikonal denominators in the induced vertices

A closer look at $R_\pm g$-interaction [Lipatov 95'; 97'; Bondarenko, Zubkov 2018]:

$$S_{\rm int.} = \int dx \frac{i}{g_s} \,{\rm tr}\left[R_+(x) \partial_\perp^2 \partial_- \left(W_{x_+}\left[A_-\right] - W^\dagger_{x_+}\left[A_-\right] \right) + (+ \leftrightarrow -) \right],$$

where $\partial_\pm = 2\partial/\partial x_\mp$, $x_\pm = x^\pm = (n_\pm x) = x^0 \pm x^3$, fields $R_\pm$ satisfy MRK constraint $\boxed{\partial_\mp R_\pm(x) = 0}$ and

$$\begin{aligned} W_{x_\mp}[x_\pm, {\bf x}_T, A_\pm] &= P\exp\left[\frac{-ig_s}{2} \int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, {\bf x}_T) \right] \\ &= \left(1 + ig_s \partial_\pm^{-1} A_\pm\right)^{-1}, \end{aligned}$$

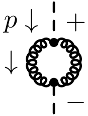
so that $\boxed{\partial_\pm^{-1} \to -i/(k_\pm + i\varepsilon)}$ in the Feynman rules.

$\Rightarrow$ multiple *induced vertices* with light-cone (Eikonal) denominators appear. Pole prescription is fixed by Hermitian form of $R_\pm g$-interaction.

Rapidity divergences and regularization

Due to the presence of the $1/q^{\pm}$-factors in the induced vertices, loop integrals in EFT contain the light-cone (Rapidity) divergences:

$$\Pi^{(1)}_{ab} = q\downarrow$$



$$= g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{\left(\mathbf{p}_T^2 (n_+ n_-)\right)^2}{q^2 (p-q)^2 q^+ q^-}$$

The regularization by explicit cutoff in rapidity was proposed by Lipatov [Lipatov, 1995] ($q^{\pm} = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$, $p^+ = p^- = 0$):

$$\int \frac{dq^+ dq^-}{q^+ q^-} = \int\limits_{y_1}^{y_2} dy \int \frac{dq^2}{q^2 + \mathbf{q}_T^2},$$

then

$$\Pi^{(1)}_{ab} \sim \delta_{ab} \mathbf{p}_T^2 \times \underbrace{C_A g_s^2 \int \frac{\mathbf{p}_T^2 d^{D-2}\mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)-\text{one}-\text{loop Regge trajectory}} \times (y_2 - y_1) + \text{finite terms}$$

Covariant regularization

To regularize RDs covariantly one have to "tilt" Wilson lines from the light-cone [Hentschinski, Sabio Vera, Chachamis et.al. 2012-2013; Collins 2011]:

$$S_{\text{int.}}^{(\text{reg.})} = \int dx \frac{i}{g_s} \,\text{tr}\left[R_+(x) \partial_\perp^2 \tilde{\partial}_- \left(W_{\tilde{x}_+}\left[\tilde{A}_-\right] - W^\dagger_{\tilde{x}_+}\left[\tilde{A}_-\right]\right) + (+ \leftrightarrow -)\right],$$

where $\boxed{\tilde{x}_\pm = x_\pm + r \cdot x_\mp}$ with $0 < r \ll 1$, and modify the kinematic constraint [M.N. 2019]:

$$\boxed{\tilde{\partial}_\mp R_\pm(x) = 0,}$$

$\Leftrightarrow \tilde{p}_\mp = p_\mp + r \cdot p_\pm$ for $R_\pm$ (Necessary to regularize $R_+ R_+ \to R_- R_-$ Green's function at one loop!).

Rapidity divergences at one loop

Only log-divergence $\sim \log r$ (Blue cells in the table) is related with Reggeization of particles in t-channel.
Integrals which do not have log-divergence *before expansion in* ϵ may still contain the power-like dependence on r:

- $r^{-\epsilon} \to 0$ for $r \to 0$ and $\epsilon < 0$.
- $r^{+\epsilon} \to \infty$ for $r \to 0$ and $\epsilon < 0$ – **weak-power divergence** (Pink cells in the table)
- $r^{-1+\epsilon} \to \infty$ – **power divergence.** (Red)

(# LC prop.) \ (# quadr. prop.)	1	2	3	4
1	$A_{[-]}$	$B_{[-]}$	$C_{[-]}$	…
2	$A_{[+-]}$	$B_{[+-]}$	$C_{[+-]}$	…
3	…	…	…	…

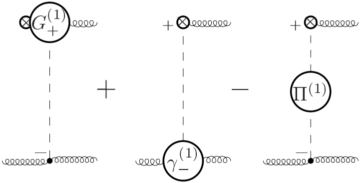
The **weak-power** and **power-divergences** cancel between Feynman diagrams describing one region in rapidity, so only log-divergences are left.

State of the art

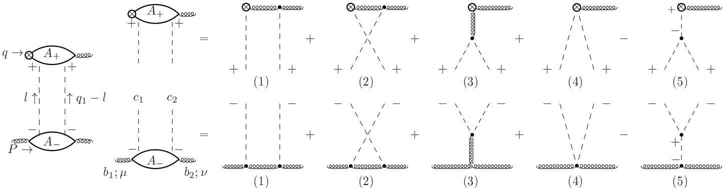
- LO BFKL kernel comes-out as rapidity-divergent part of $R_+R_+ \to R_-R_-$ Green's function [Bartels, Lipatov, Vacca 2012]
- Known QCD results for one-loop impact-factors of gluon and quark with one scale of virtuality are reproduced [Hentschinski, Sabio Vera, Chachamis et.al. 2012-2013]
- Two-loop Regge trajectory of a gluon is reproduced [Hentschinski, Sabio Vera, Chachamis et.al. 2013]
- Consistency of Reggeized quark formalism is verified at one loop on example of the process $\gamma\gamma \to q\bar{q}$ [M.N., Saleev 2017]
- New one-loop impact-factors $\mathcal{O}(q) + R_+(q_1) \to g(q+q_1)$ (with $\mathcal{O}(x) = \mathrm{tr}[G_{\mu\nu}G^{\mu\nu}]$) and $\gamma^\star(q) + Q(q_1) \to q(q+q_1)$ with additional scale $Q^2 = -q^2$ besides $q_1^2 = -\mathbf{q}_{1T}^2$ are computed [M.N. 2019] and consistency of Regge limits of one-loop amplitudes:
$$g(P) + \mathcal{O}(q) \to g(P-q_1) + g(q+q_1),$$
$$\gamma(P) + \gamma^\star(q) \to q(P-q_1) + \bar{q}(q+q_1),$$
between EFT and QCD is checked.
- NLO BFKL is in progress...

Contributions in the EFT, gluon case

One-Reggeon contribution (*negative signature*, Re+Im parts @ 1 loop, $\log r$-divergences cancel):



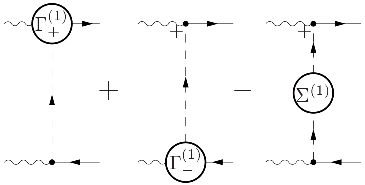
Two-Reggeon contribution (*positive signature*, does not contribute due to color):



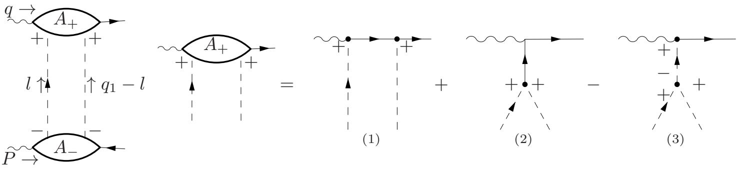
The one-Reggeon contribution reproduces QCD result exactly.

Contributions in the EFT, photon case

One-Reggeon contribution (*positive signature*, Re+Im parts @ 1 loop, $\log r$-divergences cancel):



Two-Reggeon contribution (*negative signature*, Im part @ 1 loop):



Sum of one- and two-Reggeon contributions reproduces QCD result exactly.

Subtraction problem for real NLO corrections.

Example

The (over-)subtraction problem is a common disease of NLO calculations in BFKL-related formalisms (NLO BFKL, forward hadron produciton in CGC, etc.). Let's consider an example: a forward-jet production in a fusion of one collinear (large-x) and one Reggeized (small-x) gluon. At LO

$$g(p_+) + R(q_-, \mathbf{q}_{T1}) \to g(p_+, q_-, \mathbf{k}_{T1} = \mathbf{q}_{T1}),$$

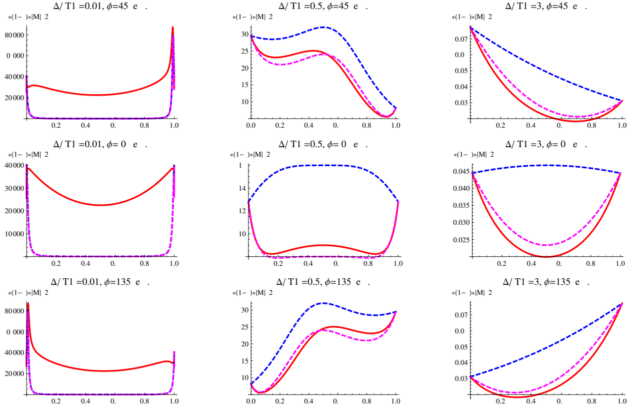
at NLO:

$$g(p_+) + R(q_-, \mathbf{q}_{T1}) \to g(k_1^+, zq_-, \mathbf{k}_{T1}) + g(k_2^+, (1-z)q_-, \mathbf{k}_{T2}),$$

where $z \ll 1$ or $1 - z \ll 1$ (gluons are identical!) corresponds to the Regge limit.
Asymptotic expression for the squared amplitude in Regge limit should be subtracted from NLO amplitude, because emissions with $z \ll 1$ are already taken into account in UPDF. Typically the LP asymptotic expression is subtracted, often leading to **negative cross-section** at NLO.

Example

The reason is, that LP asy. expression (dashed blue line) is a very poor approximation to exact squared amplitude (red line) outside Regge limits :



Here $\mathbf{k}_{T1} = z\mathbf{q}_{T1} + \mathbf{\Delta}$, $\mathbf{k}_{T2} = (1-z)\mathbf{q}_{T1} - \mathbf{\Delta}$, $|\mathbf{\Delta}| \ll |\mathbf{q}_{T1}|$ – final-state collinear limit.

Quasi-Eikonal approximations

This problem is another manifestation of importance of power-suppressed corrections.
It was noticed in several approaches(e.g. in HEJ-approach [J. Andersen, et.al.]) that minor kinematic improvements to the LP BFKL amplitudes lead to dramatic improvement of their agreement with exact amplitudes.
Minimal version of such improvement is just to relax the standard BFKL approximation for the t-channel propagator:

$$\frac{-1}{\mathbf{q}_T^2} \to \frac{1}{t_{QE}},\ t_{QE} = -\mathbf{q}_T^2 - \frac{z\mathbf{k}_T^2}{1-z},$$

Result of such *Quasi-Eikonal* approximation is shown by magenta line in the previous plot.
This approximation should be applied not only to the subtraction term, but also **iterated** to all orders, to produce the UPDF. This turns out to be technically similar to above-mentioned DL/Blümlein approach.

Conclusions

- k_T-factorization is most suitable for doubly-asymptotic limit $x \to 0,\ Q^2 \to \infty$. Outside this limit different approaches diverge.
- Fits for single J/ψ and Υ production look quite reasonable, work over wide range of energies
- As a phenomenological tool – produces lot of interesting results for challenging (multiscale, differential) observables.
- For $\Upsilon + D-$ production, the SPS mechanism is not as much supressed as it was in LO CPM
- Reasonable description of all distributions for double-J/ψ, except of large $M_{\psi\psi}$ and ΔY was found. Significant SPS contributions in latter regions where identified
- Development of NLO formalism is in progress. Single J/ψ (electro-)production is on top of the list of processes to consider, but similar problems with P-wave contributions as in CPM are to be expected

Thank you for your attention!