Tensor polarisation as a probe for hadrons and hadronic matter

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Quarkonia as Tools 2020, January 14

Outline

- Vector meson angular distributions and frame dependence; geometric model
- Hadronic tensor in inclusive and exclusive cases as density matrix (of virtual photon) and its invariants
- J/Ψ at PHENIX – comparing frames
- Elliptic flow/Wigner function and tensor polarization

Spin-1 density matrix

3 component of wf ->8 parameters

$\varrho_{ij}=\varepsilon_{ijk}S_k+2S_{ij}$

Circular-> 3 components of vector (P-odd)

Linear -> 5 components of symmetric traceless tensor (P-even)

Deuterons - shear forces

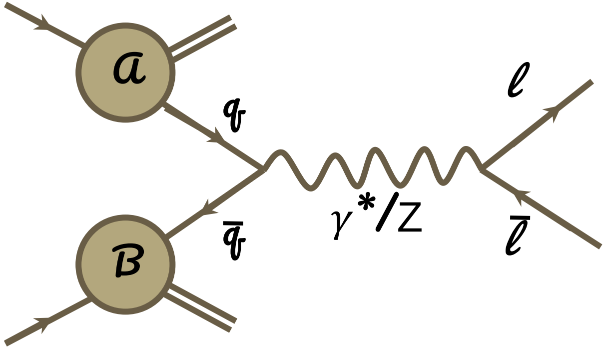
Exotic mesons – a sort of viscosity

Partons collision –> tensor polarized virtual photons -> angular distributions of final particles

Annihilation of (massless) quarks to leptons: $d\sigma \sim 1+\cos^2\theta$

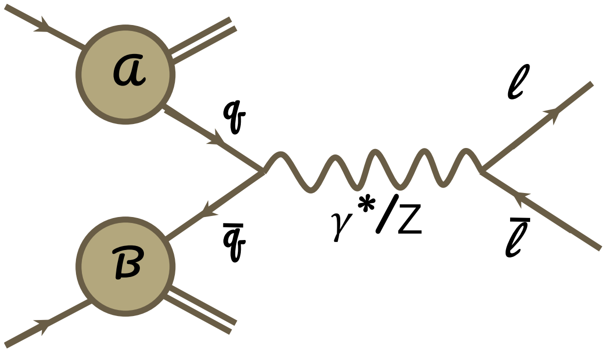
Angular distribution

General form of angular distribution for elastic (2->2) and inclusive production of ANY vector particle. For z axis in the plane formed by colliding hadrons 2 last terms are absent



$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{3+\lambda_\theta}\left(1+\lambda_\theta\cos^2\theta+\lambda_{\theta\phi}\sin 2\theta\cos\phi\right.$$
$$+\ \lambda_\phi\sin^2\theta\cos 2\phi+\lambda_{\perp\phi}\sin^2\theta\sin 2\phi$$
$$+\ \lambda_{\perp\theta\phi}\sin 2\theta\sin\phi+2A_\theta\cos\theta$$
$$+\ \left.2A_\phi\sin\theta\cos\phi+2A_{\perp\phi}\sin\theta\sin\phi\right)$$

Angular distribution



general form of angular distribution :

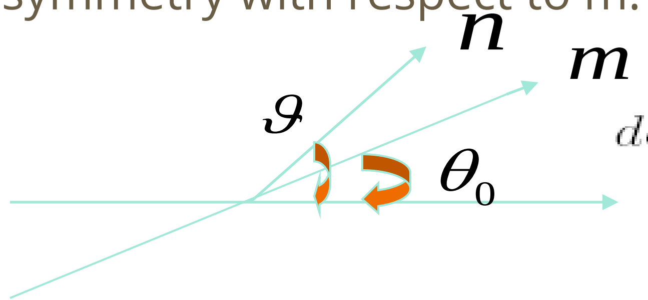
$$
\begin{aligned}
\frac{1}{\sigma}\frac{d\sigma}{d\Omega} &= \frac{3}{4\pi}\frac{1}{3+\lambda_\theta}\left(1+\lambda_\theta\cos^2\theta+\lambda_{\theta\phi}\sin 2\theta\cos\phi\right. \\
&+ \lambda_\phi \sin^2\theta\cos 2\phi + \lambda_{\perp\phi}\sin^2\theta\sin 2\phi \\
&+ \lambda_{\perp\theta\phi}\sin 2\theta\sin\phi + 2A_\theta\cos\theta \\
&+ \left. 2A_\phi\sin\theta\cos\phi + 2A_{\perp\phi}\sin\theta\sin\phi\right)
\end{aligned}
$$

Invariants

- ➔ Facilitate comparison b/w experiments, theory and experiment
- ➔ Reveal systematic biases

Kinematic azimuthal asymmetry from polar one (OT'05)

Only polar asymmetry with respect to m!



$$d\sigma \propto 1 + \lambda_0 (\vec{n}\vec{m})^2 = 1 + \lambda_0 \cos^2 \theta_{nm}^2$$

$$\cos\theta_{nm} = \cos\theta \cos\theta_0 + \sin\theta \sin\theta_0 \cos\phi$$

$$\lambda = \lambda_0 \frac{2 - 3\sin^2\theta_0}{2 + \lambda_0 \sin^2\theta_0}$$

$$\nu = \lambda_0 \frac{2 sin^2\theta_0}{2 + \lambda_0 \sin^2\theta_0}$$

- azimuthal angle appears with new
- λ_0 - invariant!

Generalized Lam-Tung relation

- Relation between coefficients (high school math sufficient!)
- Invariant!

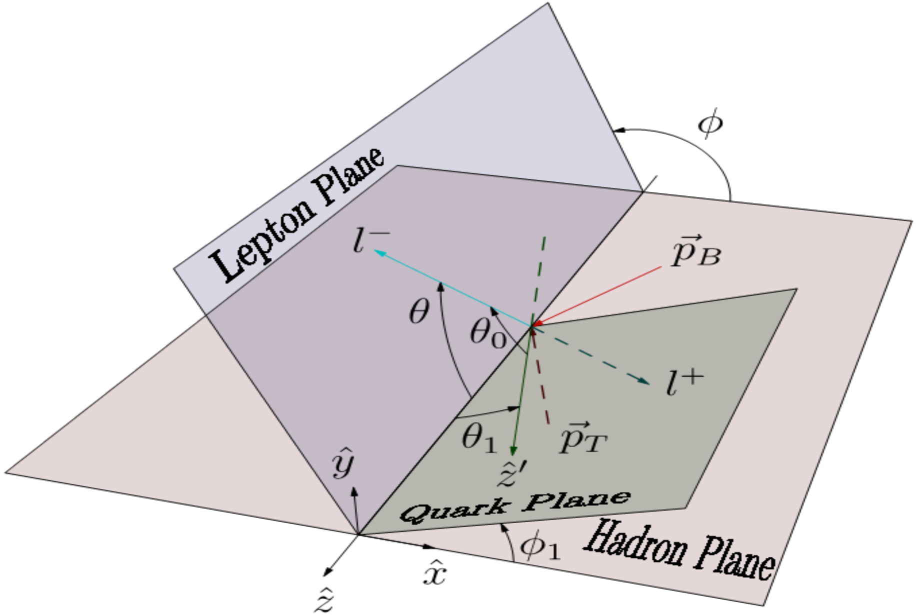
$$\lambda_0 = \frac{\lambda + \frac{3}{2}\nu}{1 - \frac{1}{2}\nu}$$

- Reduced to standard LT relation for transverse polarization (λ_0 =1)
- LT - contains two very different inputs: kinematical asymmetry+transverse polarization
- GLT is applicable for quarkonia for varying λ_0
- Rotational invariance: χ_c (1^+) from off-shell gluons, CO
- Non-coplanarity – violation of (G) LT

Non-coplanarity: Z@LHC

Interpretation of Angular Distributions of Z-boson Production at Colliders; Jen-Chieh Peng,Wen-Chen Chang, Randall Evan McClellan, and Oleg Teryaev' 15-19

Geometrical picture



Non-coplanarity – disbalance of quark and hadron planes – LT violation

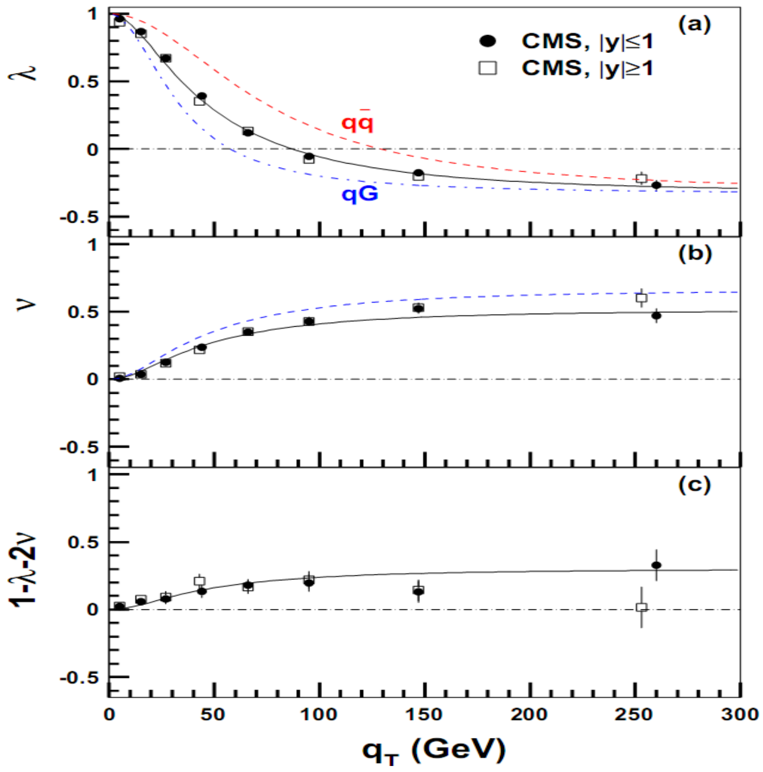
CMS (8 TeV) data

Necessity to account for

qq - 41.5(1.6)%

qG - 58.5(1.6)%

< cos 2φ_1>=0.77



General approach: the number of independent invariants

- ➔ 8 parameters describe distribution
- ➔ 3 Euler angles parameterize rotation

→ **8 - 3 = 5 independent invariants**

Hadronic tensor in terms of observables

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto W^{\mu\nu} L_{\mu\nu}$$

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$$q_\mu W^{\mu\nu} = 0 \quad \longrightarrow \quad W^{ij} = \begin{pmatrix} d_1 & a_1 + ia_2 & b_1 + ib_2 \\ a_1 - ia_2 & d_2 & c_1 + ic_2 \\ b_1 - ib_2 & c_1 - ic_2 & d_3 \end{pmatrix}$$

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$$L_{ij} \propto \delta_{ij} - n_i n_j + ig\epsilon_{ijk} n^k$$

Hadronic tensor in terms of observables

$$
\begin{aligned}
\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \quad \propto & \quad (d_1 + \frac{1}{2}d_2 + \frac{1}{2}d_3) + (-d_1 + \frac{1}{2}d_2 + \frac{1}{2}d_3)\cos^2\theta \\
- & \quad a_1 \sin 2\theta \cos\phi + \frac{1}{2}(d_3 - d_2)\sin^2\theta\cos 2\phi \\
- & \quad b_1 \sin 2\theta \sin\phi - c_1 \sin^2\theta \sin 2\phi \\
- & \quad 2a_2 \sin\theta\sin\phi + 2b_2\sin\theta\cos\phi - 2c_2\cos\theta
\end{aligned}
$$

compare with the general expression

$$
\begin{aligned}
\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \quad = & \quad \frac{3}{4\pi}\frac{1}{3+\lambda_\theta}\left(1 + \lambda_\theta\cos^2\theta + \lambda_{\theta\phi}\sin 2\theta\cos\phi\right. \\
+ & \quad \lambda_\phi \sin^2\theta\cos 2\phi + \lambda_{\perp\phi}\sin^2\theta\sin 2\phi \\
+ & \quad \lambda_{\perp\theta\phi}\sin 2\theta\sin\phi + 2A_\theta\cos\theta \\
+ & \quad \left. 2A_\phi\sin\theta\cos\phi + 2A_{\perp\phi}\sin\theta\sin\phi\right)
\end{aligned}
$$

Hadronic tensor in terms of observables

$$W^{ij} = \frac{2}{3+\lambda_\theta} \begin{pmatrix} \frac{1-\lambda_\theta}{2} & -\lambda_{\theta\phi} - iA_{\perp\phi} & -\lambda_{\perp\theta\phi} + iA_\phi \\ -\lambda_{\theta\phi} + iA_{\perp\phi} & \frac{1+\lambda_\theta-2\lambda_\phi}{2} & -\lambda_{\perp\phi} - iA_\theta \\ -\lambda_{\perp\theta\phi} - iA_\phi & -\lambda_{\perp\phi} + iA_\theta & \frac{1+\lambda_\theta+2\lambda_\phi}{2} \end{pmatrix}$$

(normalization condition Tr W = 1)

Hadron tensor in terms of observables

$$W^{ij} = \frac{2}{3+\lambda_\theta}\begin{pmatrix} \frac{1-\lambda_\theta}{2} & -\lambda_{\theta\phi} - iA_{\perp\phi} & -\lambda_{\perp\theta\phi} + iA_\phi \\ -\lambda_{\theta\phi} + iA_{\perp\phi} & \frac{1+\lambda_\theta-2\lambda_\phi}{2} & -\lambda_{\perp\phi} - iA_\theta \\ -\lambda_{\perp\theta\phi} - iA_\phi & -\lambda_{\perp\phi} + iA_\theta & \frac{1+\lambda_\theta+2\lambda_\phi}{2} \end{pmatrix}$$

(normalization condition Tr W = 1)

$$W = \frac{1}{3}\cdot\mathbf{1} + W_s + iW_a$$

Spin 1: symmetric and antisymmetric part do not mix (cf linear and circular photon polarisations): rotational invariants can be found as eigenvalues of matrices:

$$W, W_s, W_a, W_sW_a, W_sW_aW_s, \ldots$$

Invariants (Gavrilova,OT'19)

$$U_1 = \frac{A_\theta^2 + A_\phi^2 + A_{\perp\theta\phi}^2}{(3+\lambda_\theta)^2}, \qquad U_2 = \frac{\lambda_\theta^2 + 3\left(\lambda_\phi^2 + \lambda_{\theta\phi}^2 + \lambda_{\perp\phi}^2 + \lambda_{\perp\theta\phi}^2\right)}{(3+\lambda_\theta)^2}$$

$$T = \frac{\left(\lambda_\theta + 3\lambda_\phi\right)\left(2\lambda_\theta^2 - 6\lambda_\theta\lambda_\phi + 9\lambda_{\theta\phi}^2\right) + 9\left(\lambda_\theta\lambda_{\perp\theta\phi}^2 - 2\lambda_\theta\lambda_{\perp\phi}^2 + 6\lambda_{\theta\phi}\lambda_{\perp\theta\phi}\lambda_{\perp\phi} - 3\lambda_\phi\lambda_{\perp\theta\phi}^2\right)}{\left(3+\lambda_\theta\right)^3}$$

$$R = \frac{1}{\left(\lambda_\theta + 3\right)^3}\left(54\left(A_\theta A_\phi \lambda_{\theta\phi} + A_\theta A_{\perp\phi}\lambda_{\perp\theta\phi} + A_{\perp\phi}A_\phi\lambda_{\perp\phi}\right) + 9\lambda_\theta\left(2A_\theta^2 - A_{\perp\phi}^2 - A_\phi^2\right)\right.$$
$$\left. +27\lambda_\phi\left(A_\phi^2 - A_{\perp\phi}^2\right)\right)$$

$$M = \frac{1}{(3+\lambda_\theta)^4}\left\{A_\theta^2\left(\lambda_\theta^2 - 9\lambda_\phi^2 - 9\lambda_{\perp\phi}^2\right) - A_\phi^2\left(2\lambda_\theta\left(\lambda_\theta + 3\lambda_\phi\right) + 9\lambda_{\perp\theta\phi}^2\right)\right.$$
$$+A_{\perp\phi}^2\left(6\lambda_\theta\lambda_\phi - 2\lambda_\theta^2 - 9\lambda_{\theta\phi}^2\right) + 6A_\theta A_{\perp\phi}\left(\lambda_{\perp\theta\phi}\left(\lambda_\theta - 3\lambda_\phi\right) + 3\lambda_{\theta\phi}\lambda_{\perp\phi}\right)$$
$$\left. +6A_\phi\left[A_\theta\left(\lambda_{\theta\phi}\left(\lambda_\theta + 3\lambda_\phi\right) + 3\lambda_{\perp\theta\phi}\lambda_{\perp\phi}\right) + A_{\perp\phi}\left(3\lambda_{\theta\phi}\lambda_{\perp\theta\phi} - 2\lambda_\theta\lambda_{\perp\phi}\right)\right]\right\}$$

Restrictions on Invariants

Positivity condition together with normalization lead to the following inequalities:

$$0 \leq w_{1,2,3} \leq 1$$
$$0 \leq w_1 w_2 + w_1 w_3 + w_2 w_3 \leq \frac{1}{3}$$
$$0 \leq w_1 w_2 w_3 \leq \frac{1}{27}$$

Constraints for Invariants

Positivity condition together with normalization lead to the following inequalities:

$$0 \leq w_{1,2,3} \leq 1$$
$$0 \leq w_1 w_2 + w_1 w_3 + w_2 w_3 \leq \tfrac{1}{3}$$
$$0 \leq w_1 w_2 w_3 \leq \tfrac{1}{27}$$

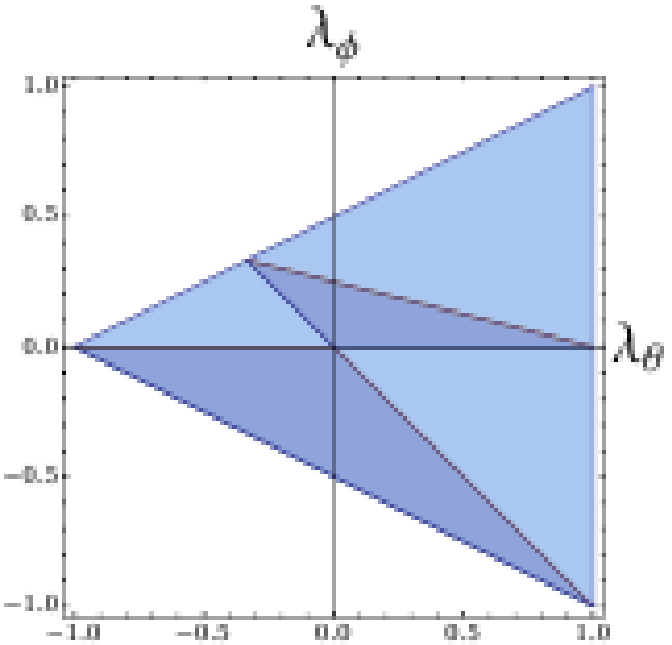
$$\boxed{U_1 + \frac{1}{3} U_2 \leq \frac{1}{12} \quad \rightarrow \quad U_1 \leq \frac{1}{12}, \; U_2 \leq \frac{1}{4}}$$

$$\boxed{-\frac{1}{8} \leq R + T \leq \frac{3}{8}}$$

Positivity domain and geometric model for PHENIX J/Ψ data

$$\lambda_0 = \frac{\lambda_\theta + 3\lambda_\phi}{1 - \lambda_\phi}$$

$$\sin^2 \xi = \frac{2\lambda_\phi}{\lambda_\theta + 3\lambda_\phi}$$



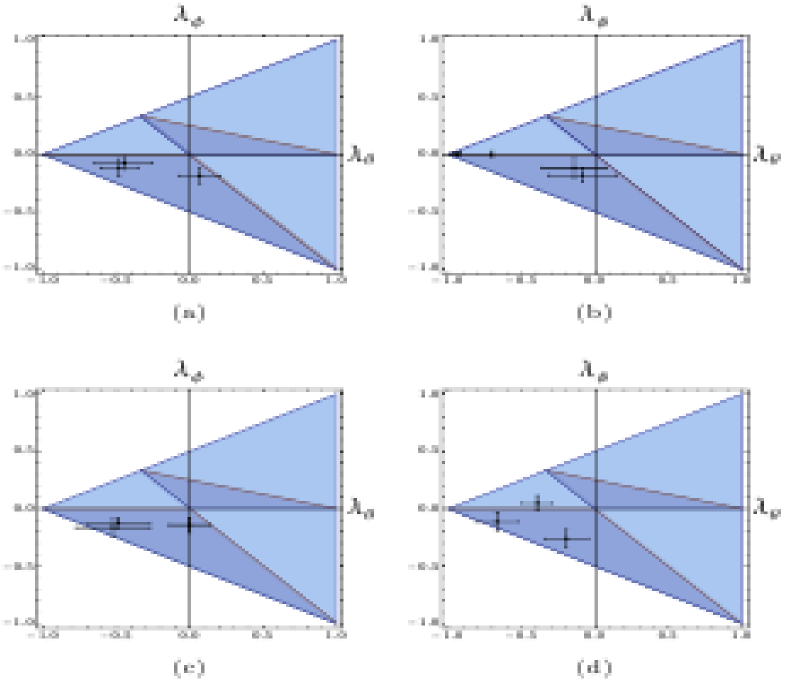


FIG. 2. Angular coefficients λ_θ and λ_ϕ measured b PHENIX [4]: (a) – HX frame, (b) – CS frame, (c) – GJ frame, (d) – GJF frame. Different points correspond to di ferent values of transverse momentum. Only statistical erro are shown.

How invariant are they?

TABLE V. The values of invariants U_2 and T calculated for angular coefficients measured by PHENIX collaboration [4] in $J/\psi \to \mu^+\mu^-$ decays for $1.2 < y < 2.2$ in four reference frames at different values of p_T: (2 − 3 GeV), (3 − 4 GeV) and (4 − 10 GeV). Only statistical errors are taken into account.

	U_2			T		
p_T [GeV/c]	2 − 3	3 − 4	4 − 10	2 − 3	3 − 4	4 − 10
HX	3.0 ± 2.5	2.8 ± 1.7	1.2 ± 0.8	4.8 ± 6.5	1.7 ± 3.7	1.3 ± 1.4
CS	$> (6.2 \pm 0.4)$	1.0 ± 0.8	0.5 ± 0.8	$> (15.2 \pm 1.5)$	-0.1 ± 1.6	-0.3 ± 0.8
GJB	5.0 ± 3.6	3.8 ± 3.6	1.0 ± 0.6	10.5 ± 12.1	0.7 ± 6.9	0.9 ± 0.8
GJF	8.7 ± 4.4	3.0 ± 1.7	4.3 ± 2.2	24.3 ± 20.1	3.3 ± 3.5	7.4 ± 7.1

Invariants discussed in literature

➔ Faccioli et al. (PRL 105 (2010) 061601)

$$\mathcal{F} = \frac{1 + \lambda_\theta + \lambda_\phi}{3 + \lambda_\theta}$$

➔ Faccioli et al. (PRD 81 (2010) 111502), Palestini (PRD 83 (2011) 031503)

(λ_0 (OT'05) =)$\tilde{\lambda} = \frac{\lambda_\theta + (3/2)\lambda_\phi}{1 - (1/2)\lambda\phi} = \frac{3\mathcal{F} - 1}{1 - \mathcal{F}}$

rotations along the y-axis in the dilepton rest frame

Faccioli et al. (PRD 83 (2011) 056008)

$$G = \frac{1 + \lambda_\theta - \lambda_\phi}{3 + \lambda_\theta}$$

along the x-axis

➔ Faccioli et al. (PRD 83 (2011) 056008)

$$\lambda_\theta$$

along the z-axis

➔ Ma,Qiu,Zhang (arXiv: 1703.04752 (2017))

general method

Connection to invariants from arXiv: 1703.04752

$$\widetilde{U}_1 = \tfrac{3}{\pi} U_1$$

$$\widetilde{U}_2 = \tfrac{1}{5\pi} U_2$$

$$\widetilde{W}_3 = \frac{1}{70\pi^2}\left(T + 7R\right)$$

$$\widetilde{W}_4 = \tfrac{9}{20\pi}\widetilde{U}_1^{\,2} + \tfrac{15}{28\pi}\widetilde{U}_2^{\,2} + \tfrac{27}{14\pi}\widetilde{U}_1\widetilde{U}_2 - \tfrac{9}{35\pi^3}\tfrac{1}{144}\left(45U_1 + 10R + 36M\right)$$

$$\widetilde{W}_5 = \tfrac{5}{2\pi}\left(\tfrac{3}{7}\widetilde{U}_1 + \tfrac{5}{11}\widetilde{U}_2\right)\widetilde{W}_3 + \frac{3}{539\pi^4}\left(U_1\left(-\tfrac{143}{4} + 429U_2 - 297T\right) + \tfrac{7}{3}U_2 R\right)$$

Invariants in terms of eigenvalues

$$w_1^{(a)} w_2^{(a)} + w_1^{(a)} w_3^{(a)} + w_2^{(a)} w_3^{(a)} = w_2^{(a)} w_3^{(a)} = 4U_1$$

$$w_1^{(s)} w_2^{(s)} + w_1^{(s)} w_3^{(s)} + w_2^{(s)} w_3^{(s)} = -\frac{4}{3} U_2$$
$$w_1^{(s)} w_2^{(s)} w_3^{(s)} = \frac{8}{27} T$$

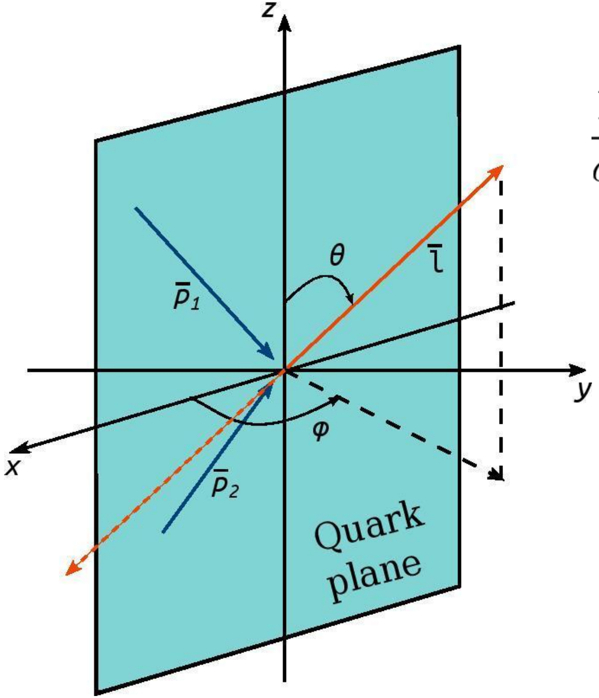
$$w_1 w_2 + w_1 w_3 + w_2 w_3 = -4U_1 - \frac{4}{3} U_2$$
$$w_1 w_2 w_3 = \frac{8}{27} (T + R)$$

$$w_1^{(as)} w_2^{(as)} + w_1^{(as)} w_3^{(as)} + w_2^{(as)} w_3^{(as)} = w_2^{(as)} w_3^{(as)} = 4M$$

Special case

- ➔ Gottfried-Jackson frame - z || quark momenta
- ➔ Collins-Soper frame - z || bisection

related by rotation along the y-axis



$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{1}{3+\lambda_\theta}\left(1+\lambda_\theta\cos^2\theta+\lambda_{\theta\phi}\sin 2\theta\cos\phi+\lambda_\phi\sin^2\theta\cos 2\phi\right)$$

↓

$$W^{ij} = \frac{1}{3+\lambda_\theta}\begin{pmatrix} 1-\lambda_\theta & -2\lambda_{\theta\phi} & 0 \\ -2\lambda_{\theta\phi} & 1+\lambda_\theta-2\lambda_\phi & 0 \\ 0 & 0 & 1+\lambda_\theta+2\lambda_\phi \end{pmatrix}$$

↓

$$w_1 = \frac{1+\lambda_\theta+2\lambda_\phi}{3+\lambda_\theta} \qquad w_{2,3} = \frac{1-\lambda_\phi\pm\sqrt{(\lambda_\theta-\lambda_\phi)^2+4\lambda_{\theta\phi}^2}}{3+\lambda_\theta}$$

Faccioli invariant

Other special rotations

$$W^{ij} = \frac{1}{3+\lambda_\theta}\begin{pmatrix} 1-\lambda_\theta & -2\lambda_{\theta\phi} & -2\lambda_{\perp\theta\phi} \\ -2\lambda_{\theta\phi} & 1+\lambda_\theta-2\lambda_\phi & -2\lambda_{\perp\phi} \\ -2\lambda_{\perp\theta\phi} & -2\lambda_{\perp\phi} & 1+\lambda_\theta+2\lambda_\phi \end{pmatrix} \qquad \bar{e}_x = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \bar{e}_z = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

↓

$$e_x^\mu W_{\mu\nu} e_x^\nu = \frac{1+\lambda_\theta-\lambda_\phi}{3+\lambda_\theta} = G$$

$$e_z^\mu W_{\mu\nu} e_z^\nu = \frac{1-\lambda_\theta}{3+\lambda_\theta} \Rightarrow \lambda_\theta = \text{invariant}$$

FROM INCLUSIVE TO (SEMI)EXCLUSIVE

- From mixed to pure states
- Density matrix: strongly ordered eigenvalues
- Positivity constraints: close to saturation
- May be used to discriminate between inclusive and exclusive production

Elliptic flow in HIC and Elliptic WF

- $1+2v_2 \cos 2\varphi$: Fourier expansion of hadron
- Common features of VERY different final states
- Y. Hatta et al: WF for gluons at low x may be measured to lp and UP pA collisions
- Quarkonia: polarisation vector similar to relative momentum of jets(also $\cos 2\varphi$)?!

Elliptic flow and tensor polisation

- Common features of VERY different final states
- WF for gluons at low x may be measured to lp and UP pA (Hatta et al) collisions
- Quarkonia: polarisation vector similar to relative momentum of jets

New ALICE data on K_0* and φ (longitudinal wrt to normal to reaction plane) polarization : arXiv:1910.14408 **[nucl-ex]**

- VM from quarks: Efremov, OT'82: product of quark polizations (small!) vs correlations

$$\rho_{00} = \frac{1 - c_{zz} + c_{xx} + c_{yy}}{3 + c_{zz} + c_{xx} + c_{yy}} = \frac{1 - Sp_z c + Sp_\perp c}{3 + Sp\, c} \qquad d_{ij} = \langle \vec{P}_{1i} \vec{P}_{2j} \rangle - \langle \vec{P}_{1i} \rangle \langle \vec{P}_{2j} \rangle = c_{ij} - a_i b_j$$

- Can geometrical model help?
- Recall $\lambda = \lambda_0 \frac{2 - 3\sin^2\theta_0}{2 + \lambda_0 \sin^2\theta_0}$ ~ 1- 3 $\cos^2$ Θ o
- $\nu = \lambda_0 \frac{2 sin^2 \theta_0}{2 + \lambda_0 \sin^2\theta_0}$

- Any correlations smears to zero

Are there sources of longitudinal polarization?

- Average with the weight due to elliptic flow – non zero polarization (OT, in preparation)!
- $<\lambda> = - 16 v_2 \lambda_0 /15$
- Positive $v_2 \lambda_0$ - longitudinal polarization
- Another source: strong magnetic field; Lattice calculations – robust conclusion on longitudinal polarization E.V. Luschevskaya , O.V. Teryaev ,D.Yu. Golubkov, O.V. Solovjeva, R.A. Ishkuvatov
  Published in **JHEP 1811 (2018) 186**

CONCLUSIONS/OUTLOOK

- Tensor polarization – important complementary tool
- Frame (in)dependence) important ingredient of its study
- PHENIX J/Ψ data – invariants at workat
- Positivity bounds check of self-consistency and hint for mechnisms

- Exclusive limits: stauration of positivity bounds?
- Elliptic flow: exclusive and HIC
- Polarization of VMs @ ALICE: geometric model vs magnetic field? Tests for Zr/Ru like CME?