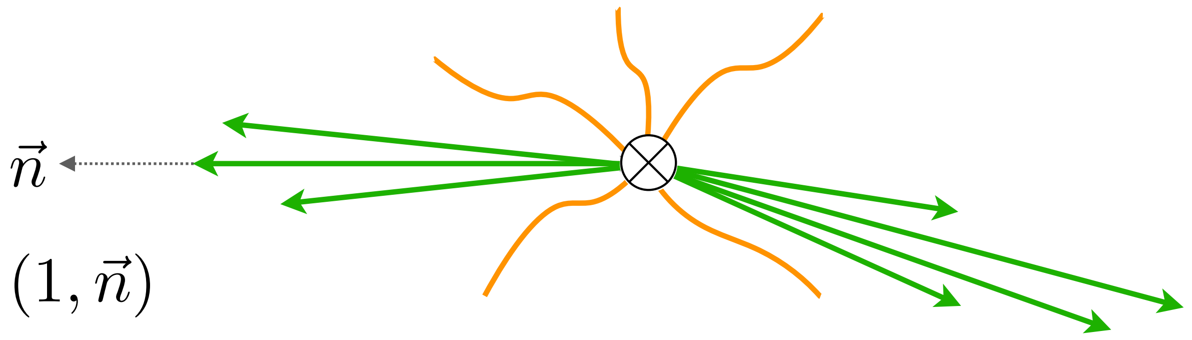
Quarkonium Production **in/with/outside** Jets

Yiannis Makris

SCET scales



$\vec{n}$

$n = (1, \vec{n})$

$$p = (p^+, p^-, p^\perp)$$

$$p^+ = n \cdot p = p^0 - p^3$$

$$p^- = \bar{n} \cdot p = p^0 + p^3$$

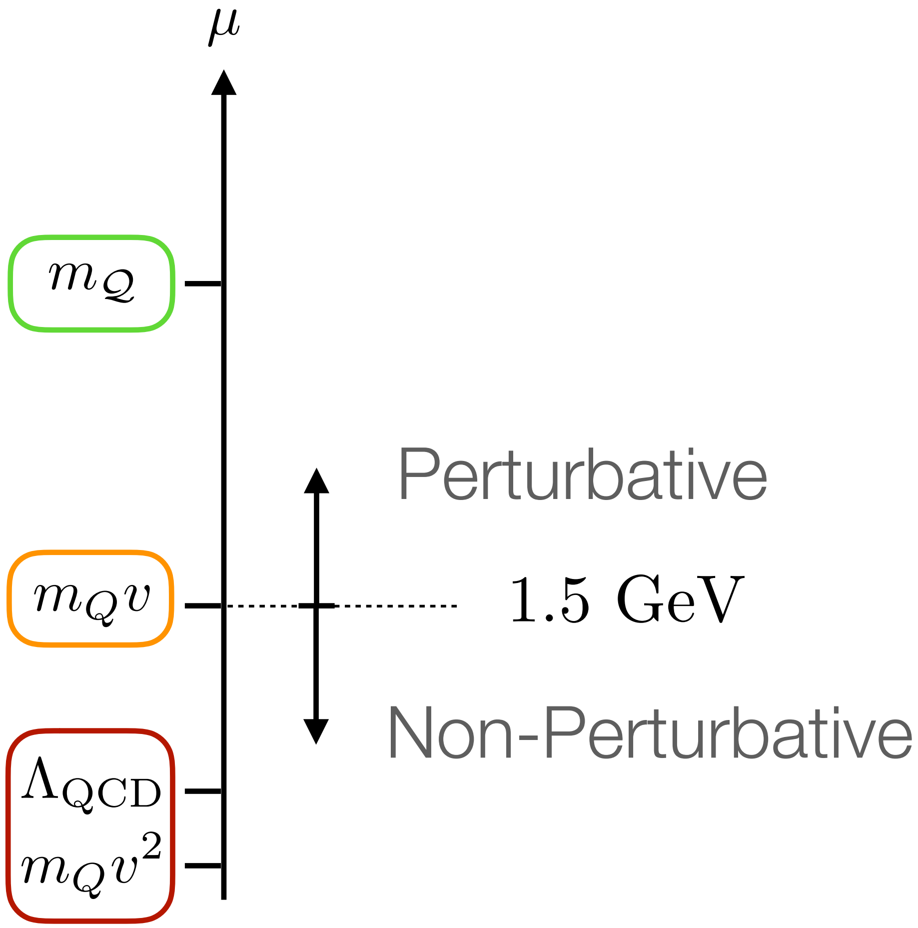
$$n = (1, 0, 0, 1), \quad \bar{n} = (1, 0, 0, -1)$$

	thrust measurement		TMD* measurement	
λ-soft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$\mu_\lambda = Q\tau$	$Q(\lambda, \lambda, \lambda)$	$\mu_\lambda = q_T$
collinear	$Q(\lambda^2, 1, \lambda)$	$\mu_c = Q\sqrt{\tau}$	$Q(\lambda^2, 1, \lambda)$	$\mu_c = q_T$
	$\lambda = \sqrt{\tau} \ll 1$		$\lambda = q_T / Q \ll 1$	

* TMD = transverse momentum distribution

NRQCD scales

NRQCD = Non-Relativistic QCD



μ

m_Q

Perturbative

$m_Q v$

1.5 GeV

Non-Perturbative

Λ_{QCD}

$m_Q v^2$

$b\bar{b}: \quad v^2 \sim 0.1$ bottomonium

$c\bar{c}: \quad v^2 \sim 0.3$ charmonium

Relative velocity of the heavy quark and antiquark in the quarkonium

typical momentum of heavy quark: $|\mathbf{p}_Q| \sim m_Q v$ (soft)

typical kinetic energy of heavy quark: $K_Q \sim m_Q v^2$ (ultra-soft)

In this talk

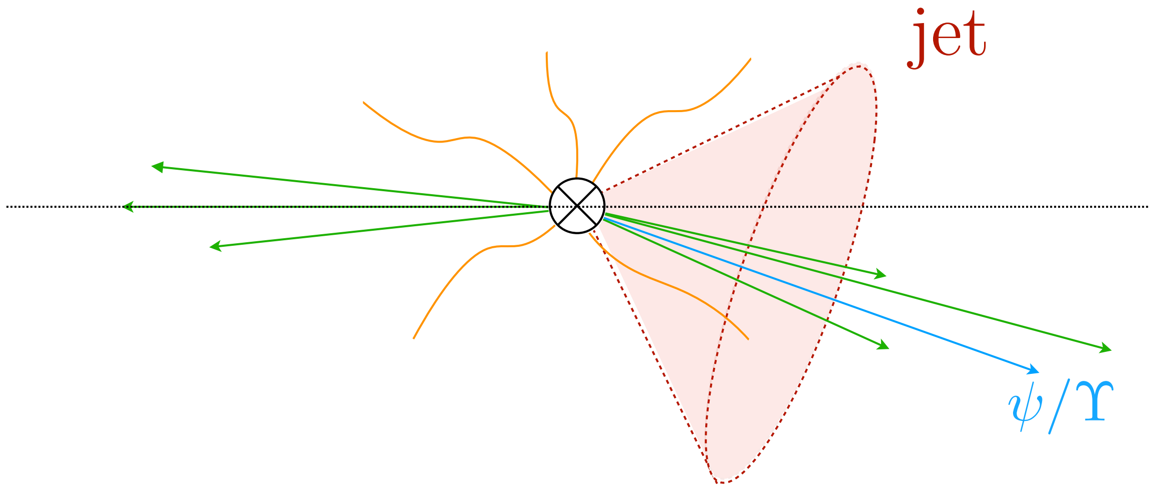
- **In**-jet production
- **Outside**-jet production (w/ jets)
- **Outside**-jet production (w/o jets)
- The game of scales and **the need for a new EFT**

In-jet production

What is a jet?

Intuitively: a jet is a collimated spray of particles initiated by the hadronization of an energetic parton originating from the hard process.

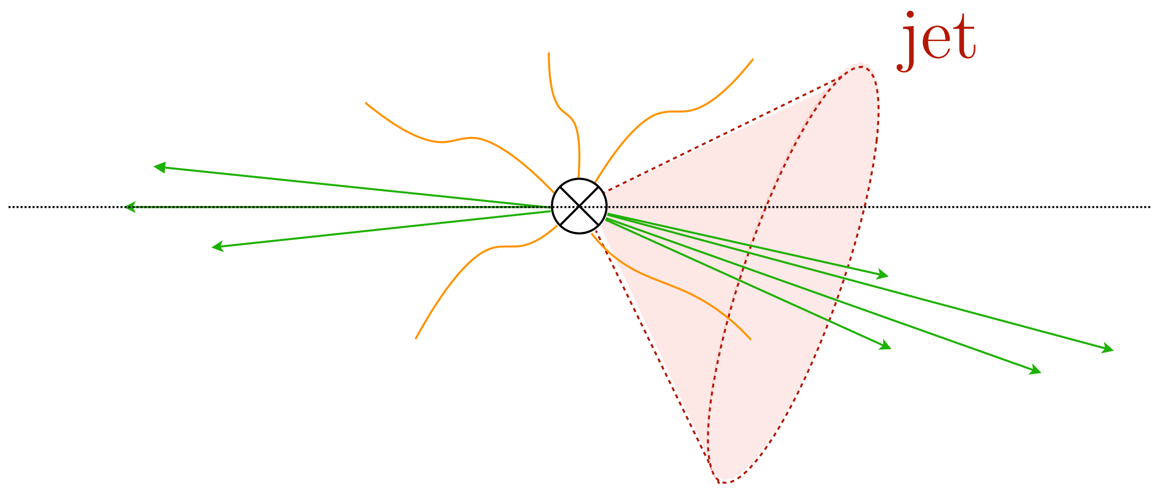
In practice: a jet is defined by a precise and IR-safe algorithm (anti-kT, C/A, Cone). The algorithms collect a subset of the particles in the event into the jet.



In-Jet production = The quarkonium belongs to the jet

Looking outside the jet

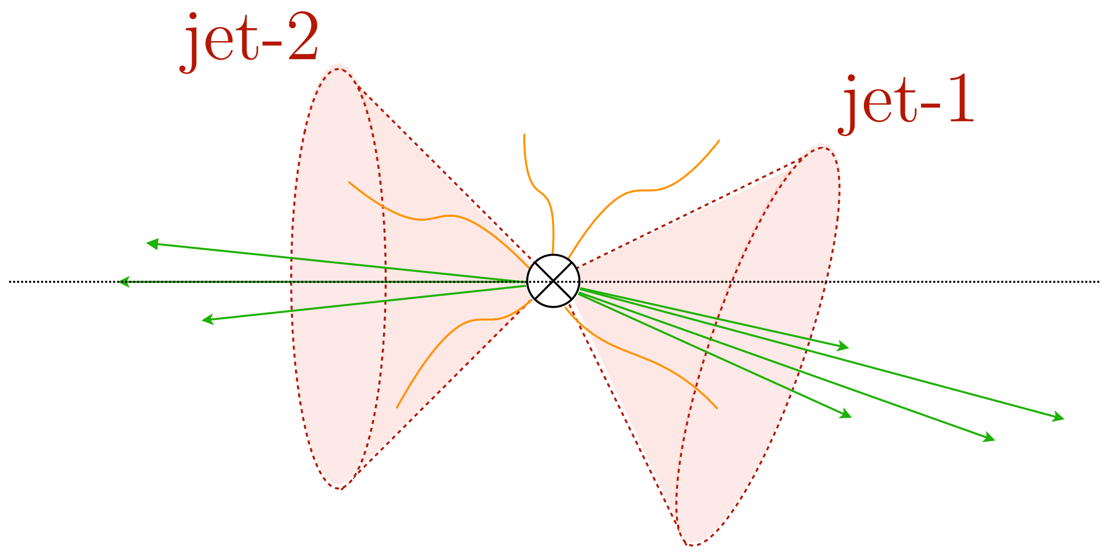
(Semi-)Inclusive:



Collinear radiation
(energetic close to the jet axis)
can escape the jet.
Appropriate for small-R jets.

$$A + B \rightarrow \text{jet} + X$$

Exclusive:



The energetic radiation must
be contained in the jets.
Works well with both small and
large-R jets.

$$A + B \rightarrow \text{di-jet}$$

Looking inside the jet

Measurements of the quarkonium state w.r.t. the jet

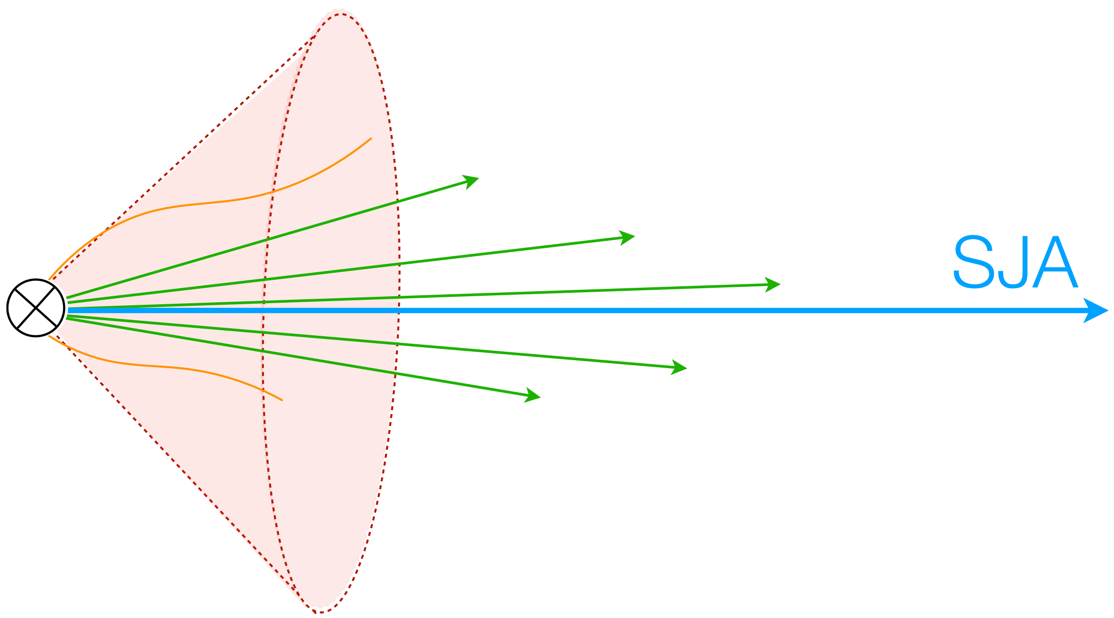
- Transverse momentum w.r.t. (some) jet axis.
- Energy fractions (e.g. $x = E_\psi / E_{\rm jet}$)
- Polarization

Measurements of the jet-substructure

- Angularities, jet-thrust, and other jet-substructure observables
- Jet energy

Standard, modified, and groomed jets

Standard Jets



The Standard Jet Axis is defined using all particles (radiation) within the jet.

Modified/Groomed Jets:

Winner-Take-All (WTA)

Jet-Broadening Axis (JBA)

Groomed Jet Axis (GJA)

…and others

The jet axis is defined using only some of jet constituents. The goal is to focus on the energetic core of the jet while reducing the influence of soft, wide-angle radiation

Formalism of fragmenting-jet-functions

FJFs for the semi-inclusive and exclusive cases are **not** the same. However the formalism and the EFT Feynman rules for calculating those are very similar.

(Semi-)Inclusive:

$$\frac{d\sigma}{d\mathcal{M}dx}(ab \to \text{jet}(\psi)) = \sum_c \frac{d\sigma}{d\mathcal{M}dx}(ab \to c) \otimes \mathcal{G}_{c/\psi}$$

Exclusive:

$$\frac{d\sigma}{d\mathcal{M}dx}(ab \to \text{dijet}(\psi)) = \sum_{i,j} H_{a,b\to ij} S_G(\mathcal{M}) \otimes J_i(\mathcal{M}) \otimes \mathcal{G}_{c/\psi}(x, \mathcal{M})$$

Fragmenting jet functions (FJFs)

FJFs for quarkonium production

Depending on the measurements inside the jet a “λ-soft” scale is present. The FJFs currently are only studied in the massless limit: $\mu_\lambda \gg m_Q$

(Semi-)Inclusive:

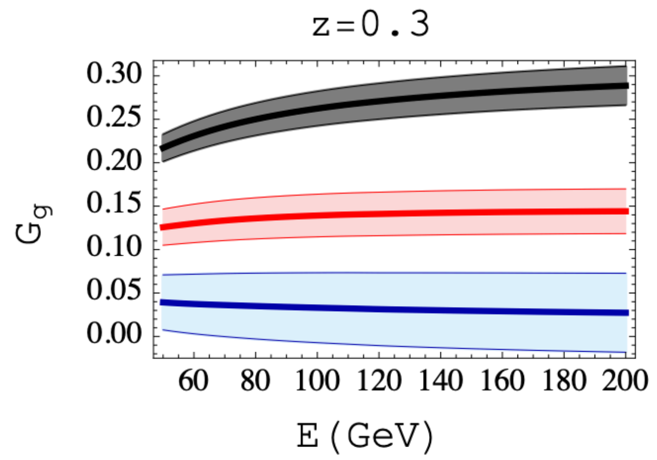
$$\mathcal{G}_c/\psi = H_c \times C_s \otimes \mathcal{J}_c \otimes \sum_n \overbrace{d^{[n]}\langle \mathcal{O}^{[n]} \rangle}^{\text{Quarkonium Fragmentation Functions}}$$

Exclusive:

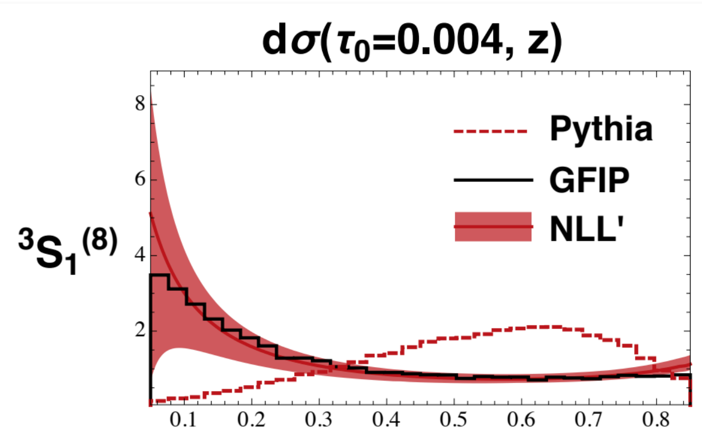
$$\mathcal{G}_c/\psi = C_s \otimes \mathcal{J}_c \otimes \sum_n d^{[n]}\langle \mathcal{O}^{[n]} \rangle$$

Long distance matrix elements

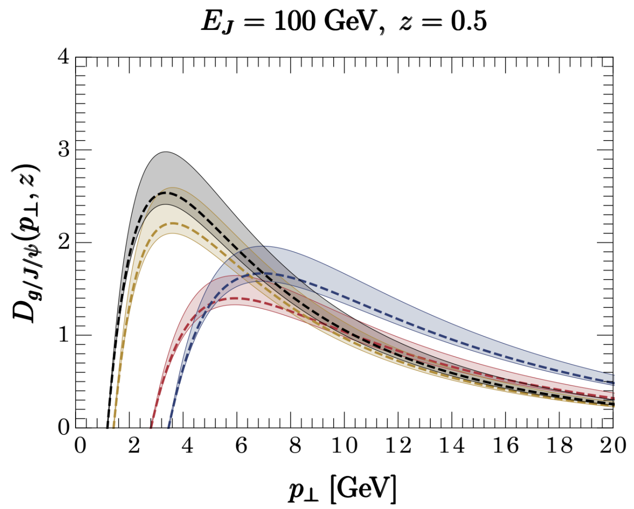
Theoretical works to date



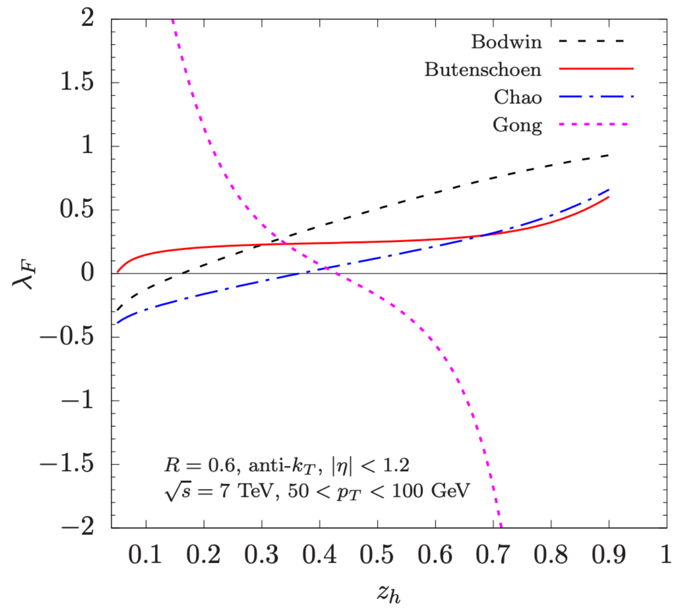
Jet energy, energy fraction
arXiv:1406.2295



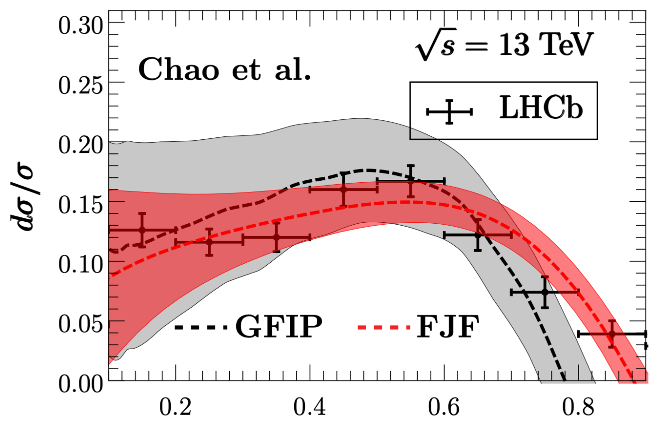
Angularities, energy fraction
arXiv:1603.06981



transverse momentum,
energy fraction
arXiv:1610.06508

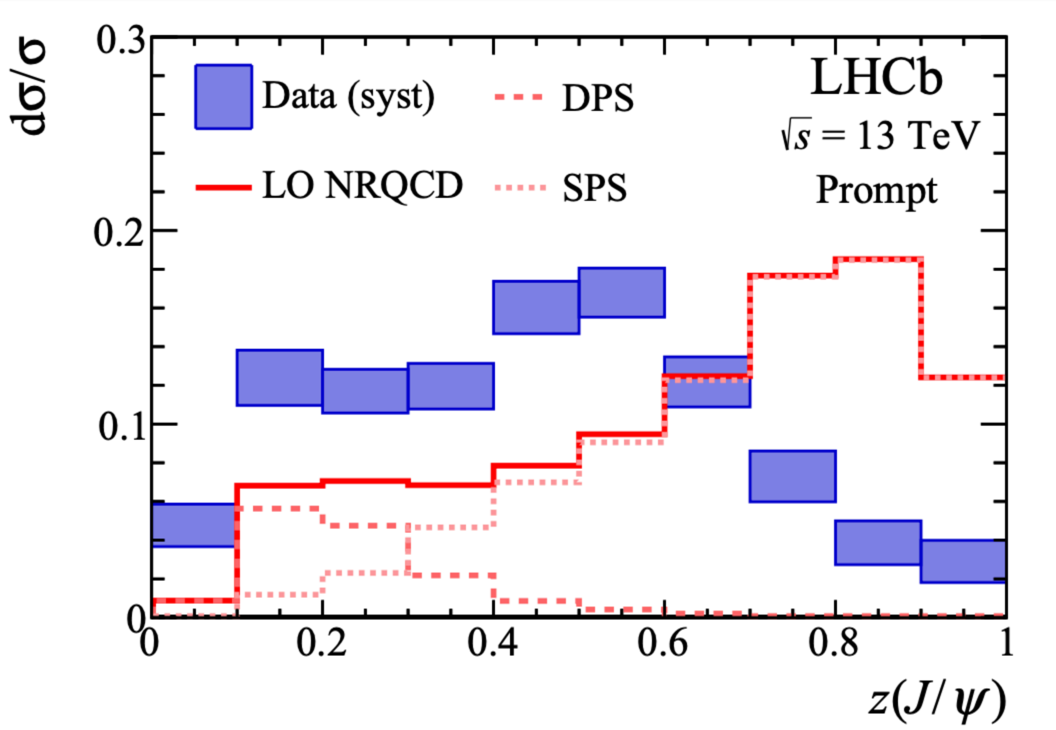


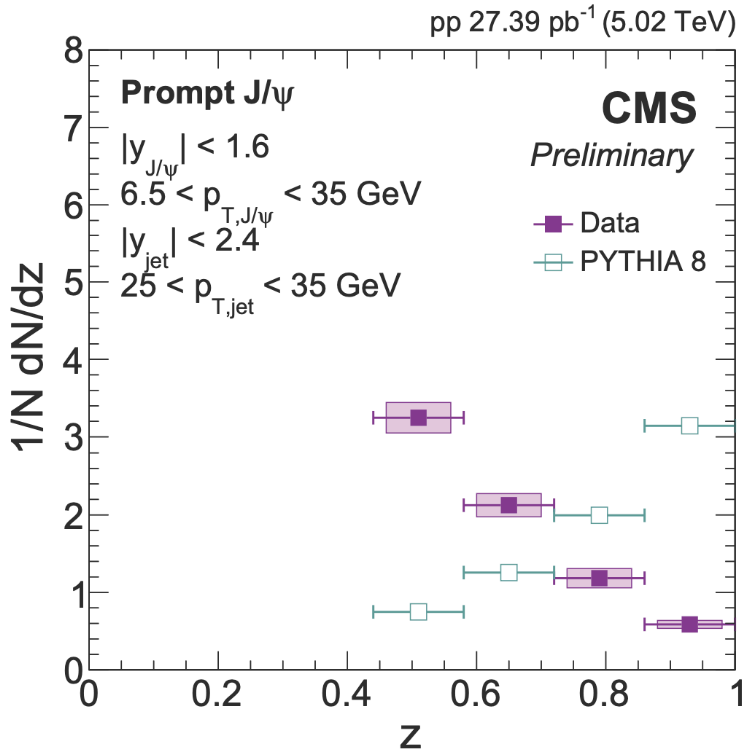
Polarization, energy fraction
arXiv:1702.03287

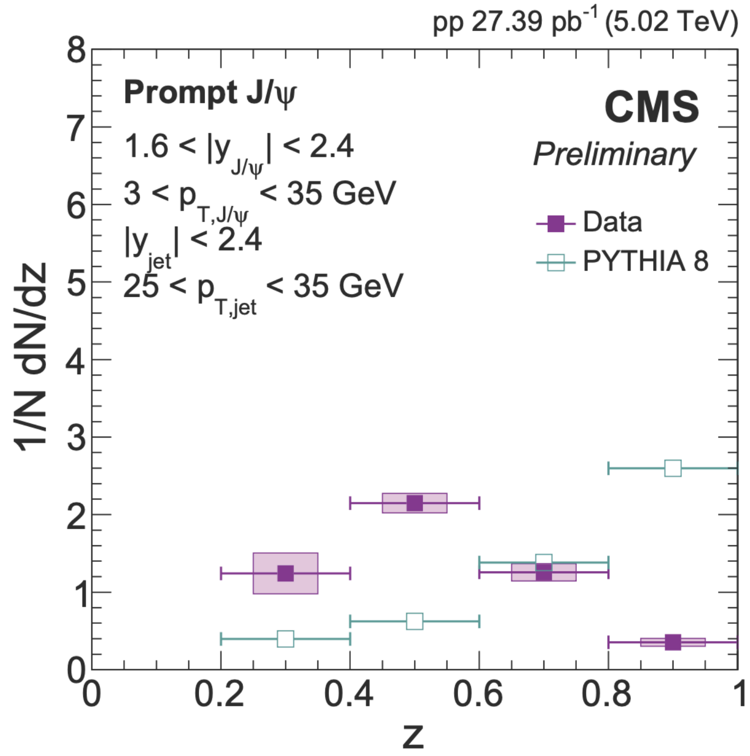


Energy fraction
arXiv:1702.05525

Measurements performed until now

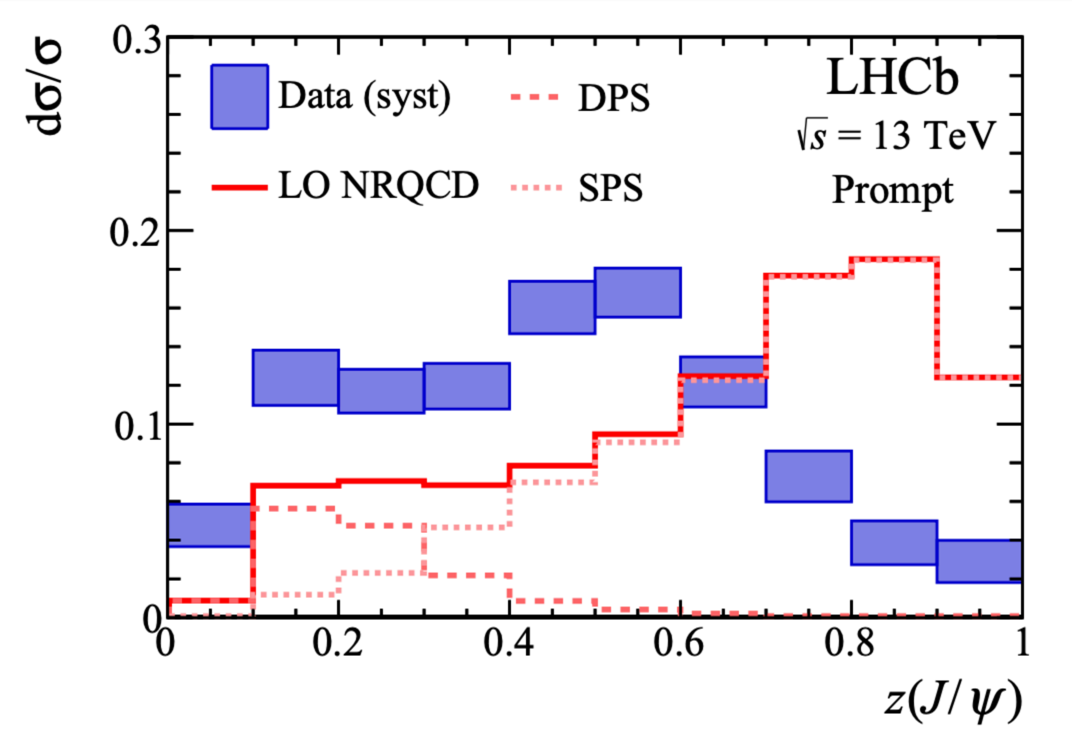




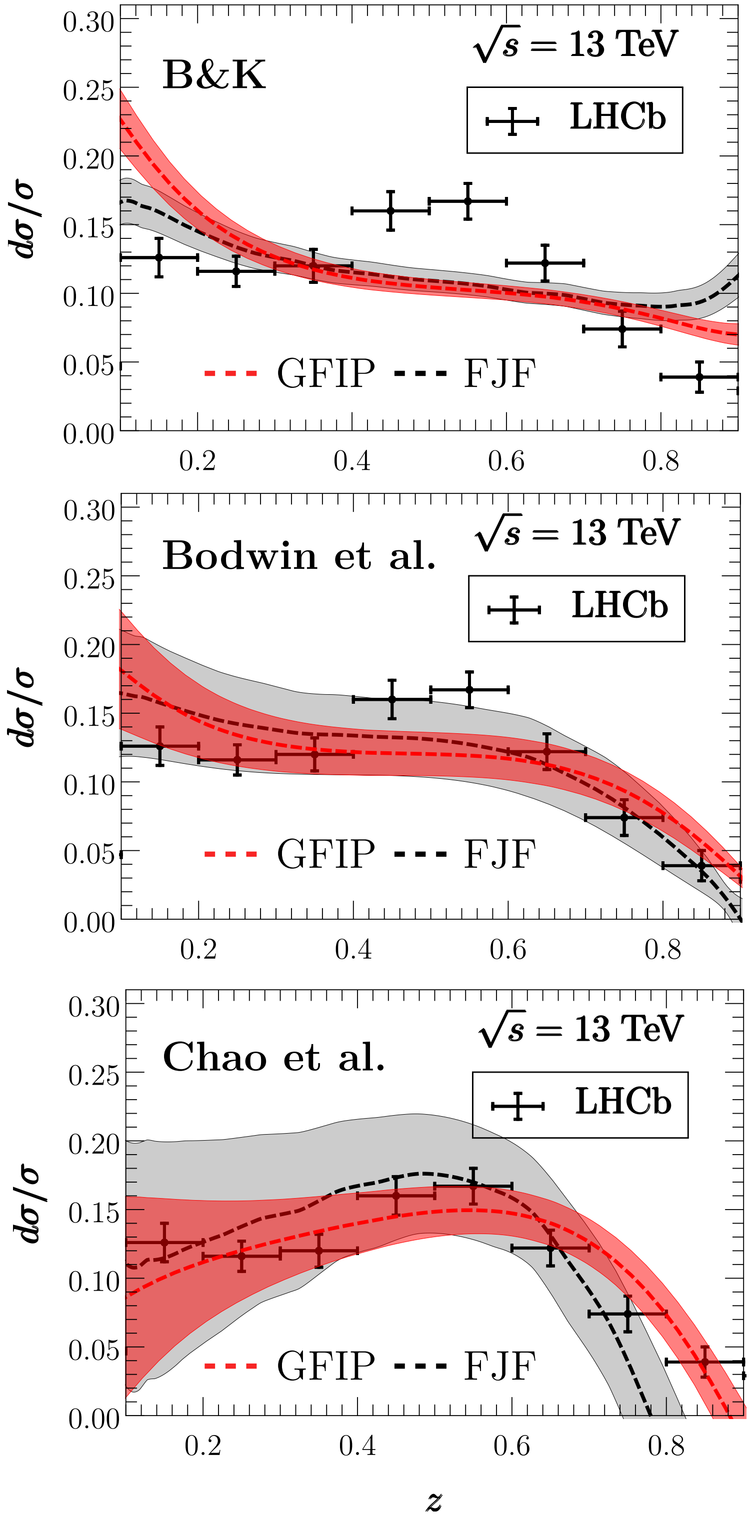


Only for the simplest measurement there are available data:
arXiv:1701.05116 (LHCb)
Nucl.Phys. A982 (2019) 186-188 (CMS)

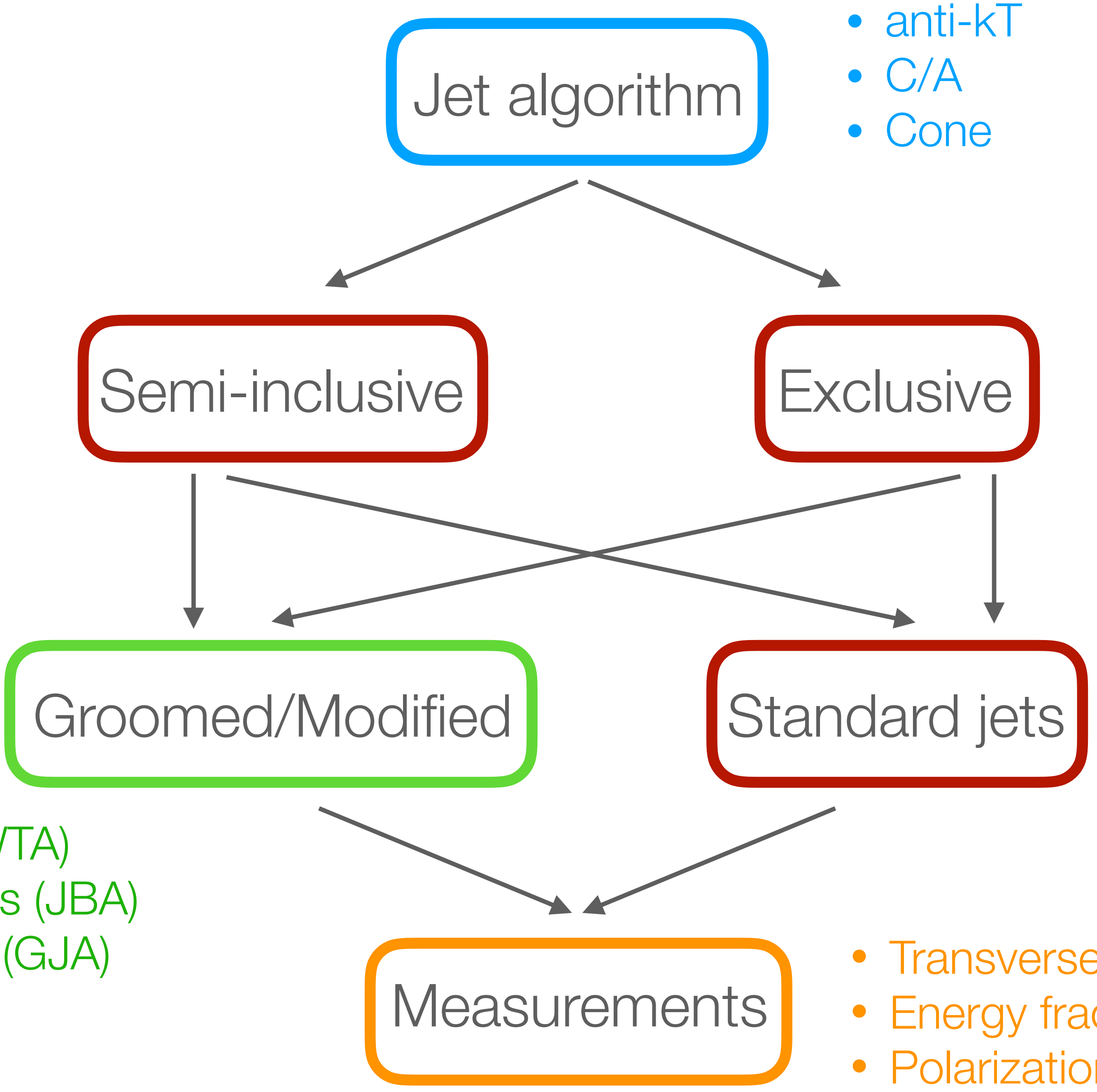
Measurements performed until now



Only for the simplest measurement there are available data:
arXiv:1701.05116 (LHCb)
arXiv:1702.05525 (Theory)



In-jet production-summary



- anti-kT
- C/A
- Cone

Jet algorithm

Semi-inclusive

Exclusive

Groomed/Modified

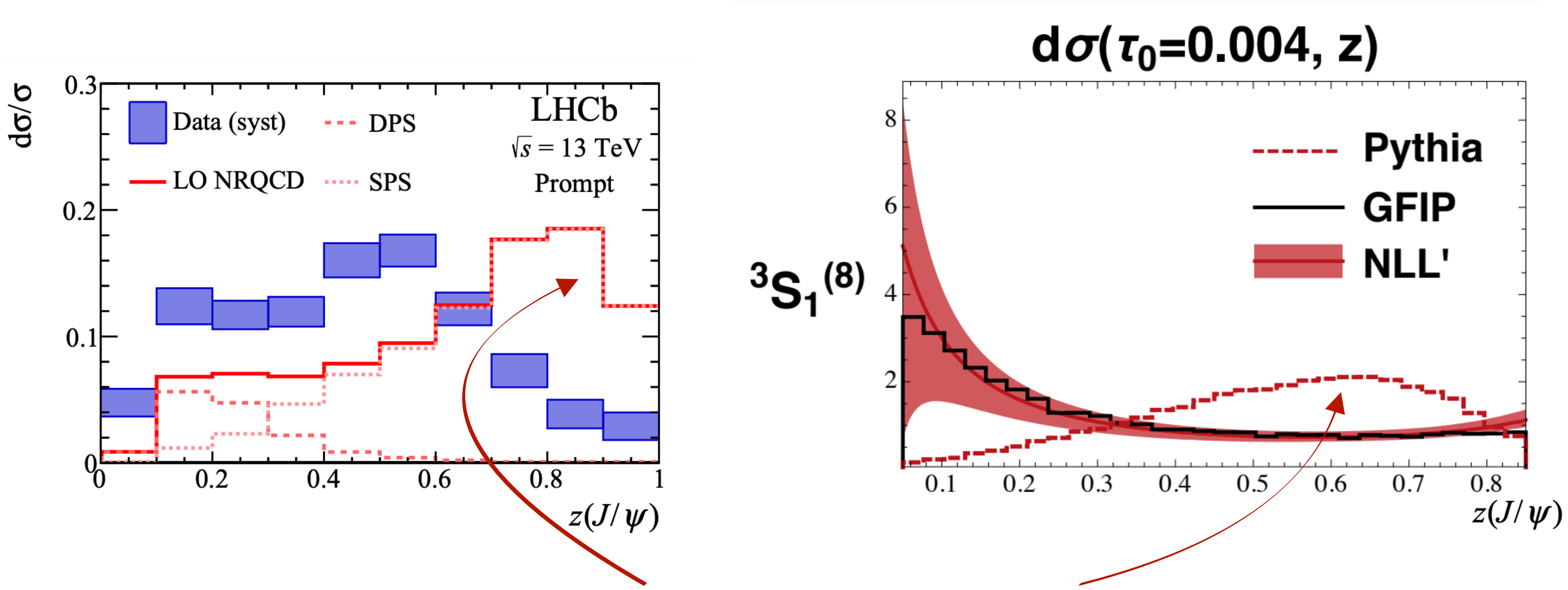
Standard jets

Measurements

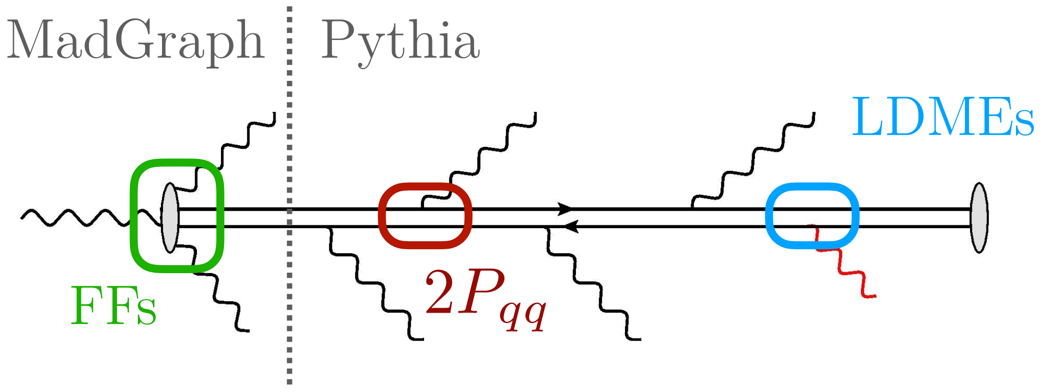
- Winner-Take-All (WTA)
- Jet-Broadening Axis (JBA)
- Groomed Jet Axis (GJA)

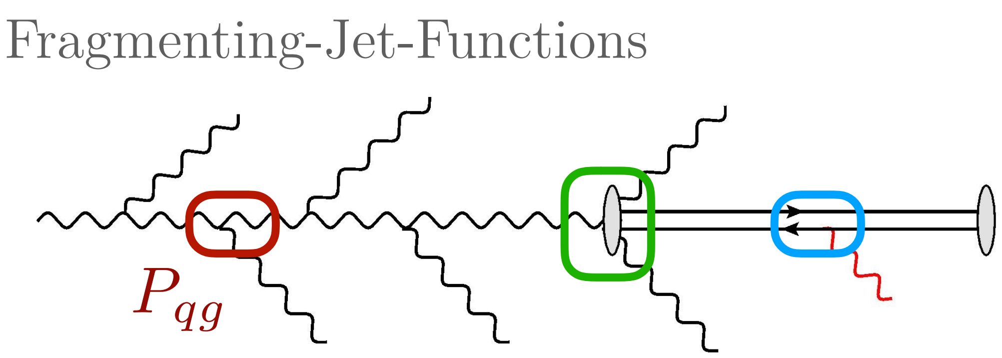
- Transverse momentum
- Energy fractions
- Polarization
- Jet substructure
- Jet energy

Comparison with event generators



- Pythia generates consistently harder spectrum than theory and experiment





- In-jet quarkonium provides a reference process for improving quarkonium fragmentation in EGs

Outside-jet production (w/ jets)

Quarkonium recoils against a jet or jet-like configuration.

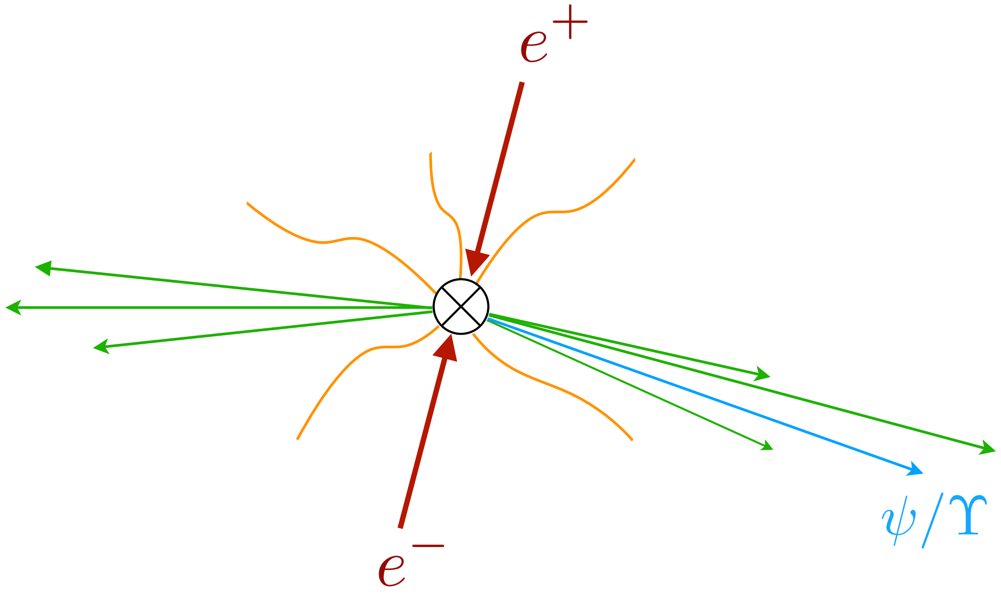
Away from the kinematic endpoint and no additional measurement

NRQCD Fragmentation approach

Soft-collinear sensitive measurement

New SCET-like approach is needed similar to in-jet factorization

Example: electron-positron annihilation



- Reconstructed recoiling jet
- Event shape measurement: $\tau \ll 1$
- TMD measurement: $q_T \ll Q$

Formalism (factorization) for 2-“jet” limit

“2-jet” limit : $\tau \ll 1$

$(q_T/Q) \ll 1$

This factorization is only true for :

$$E_\psi \ll \mu_\lambda \ll m_\psi$$

$$\frac{d\sigma}{d\mathcal{M}} = \sum_{i,j,n} H_{ij}\; S_G(\mathcal{M}) \otimes J_i(\mathcal{M}) \otimes \mathcal{G}_{j/\psi}(\mathcal{M})$$

$$\downarrow$$

$$\mathcal{G}_{j/\psi} = \mathcal{J}_j \otimes \sum_n d^{[n]} \langle \mathcal{O}^{[n]} \rangle$$

The formalisms of fragmenting-jets-functions can also be applied to event shapes in the soft and collinear limit even though no jet-algorithm has been introduced.

Outside-jet production (w/o jets)

Quarkonium recoils against soft and collinear init. state.

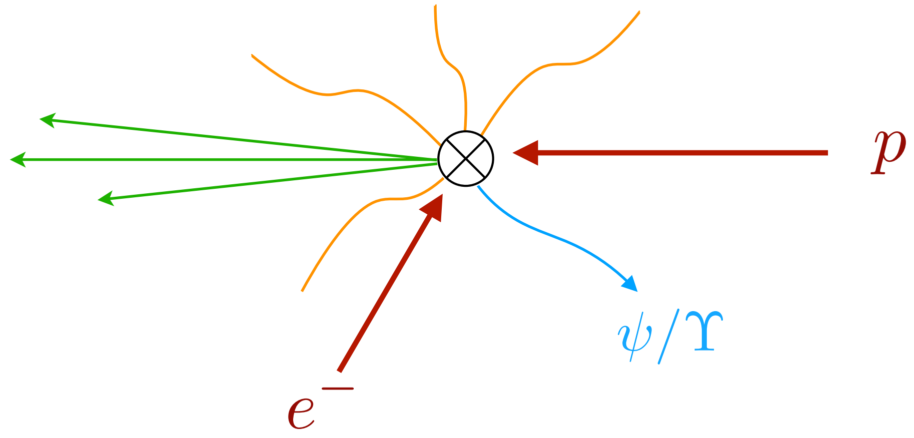
Away from the kinematic endpoint and no additional measurement

Fixed order (traditional) NRQCD approach

Soft-collinear sensitive measurement

New SCET-like approach is needed similar to in-jet factorization

Example: Semi-Inclusive DIS (photo/lepto-production)



Particularly interesting processes for accessing the momentum of gluons in the IS.

Factorization for soft-collinear limit

Factorized cross section: $$\frac{d\sigma}{d\mathcal{M}} = \sum_n H_{\gamma^* g \to c\bar{c}[n]}\, S_\psi^{[n]}(\mathcal{M}) \otimes B_{g/P}(\mathcal{M})$$

No LDMEs

This factorization is only true for: $\mu_\lambda \sim \underbrace{p_\psi^\perp \ll m_\psi}$

Important hierarchy to access the unintegrated gluon distribution functions

The game of scales

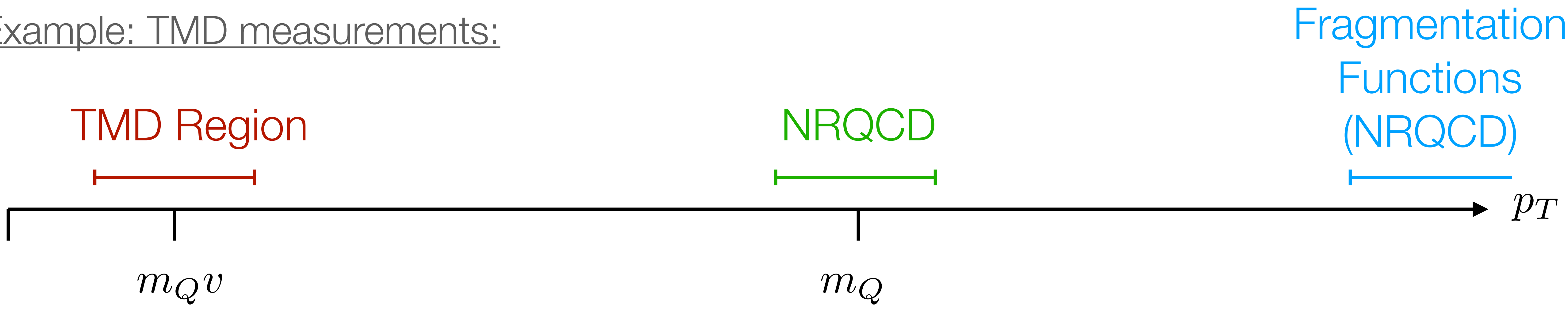
Often we are interested in the case where λ-soft is particularly small: $\mu_\lambda \ll m_\psi$
e.g. hadronization and non-perturbative effects, parton distribution in nucleons

see also talk by Thomas Mehen this morning

Warning!

λ-soft and NRQCD-soft interplay. Overlap of modes requires new EFT and new factorization approach.

Example: TMD measurements:



Expansion in v and in λ

$$\lambda = q_\perp / M$$

NRQCD in brief (Lagrangian)

arXiv:hep-ph/9910209 (M. E. Luke, A. V. Manohar, I. Z. Rothstein)

soft: $p^\mu_s \sim m_Q v(1,1,1,1)$

ultra-soft: $p^\mu_{us} \sim m_Q v^2(1,1,1,1)$

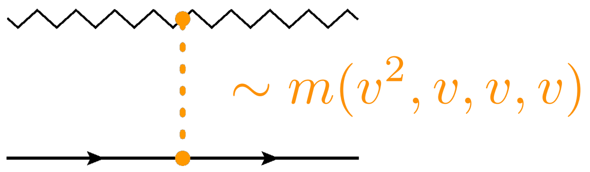
ultra-soft

subleading

$$\mathcal{L} = \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left\{ iD^0 - \frac{(\mathbf{p} - \boxed{i\mathbf{D}})^2}{2m} \right\} \psi_{\mathbf{p}}$$

$$-4\pi\alpha_s \sum_{q,q'\mathbf{p},\mathbf{p}'} \left\{ \frac{1}{q^0}\psi^\dagger_{\mathbf{p}'}\left[A^0_{q'}, A^0_q\right]\psi_{\mathbf{p}} + \frac{g^{\nu 0}(q'-p+p')^\mu - g^{\mu 0}(q-p+p')^\nu + g^{\mu\nu}(q-q')^0}{(\mathbf{p}'-\mathbf{p})^2}\psi^\dagger_{\mathbf{p}'}\left[A^\nu_{q'}, A^\mu_q\right]\psi_{\mathbf{p}} \right\}$$

soft



$$+\sum_{\mathbf{p},\mathbf{q}} \frac{4\pi\alpha_s}{(\mathbf{p}-\mathbf{q})^2}\psi^\dagger_{\mathbf{q}}T^A\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{q}}\bar{T}^A\chi_{-\mathbf{p}} + \ldots$$

NRQCD in brief (factorization)

LDME: Long Distance Matrix Elements

$$d\sigma(a+b\to \mathcal{Q}+X) = \sum_n \boxed{d\sigma(a+b\to Q\overline{Q}(n)+X)}\boxed{\langle \mathcal{O}_n^{\mathcal{Q}}\rangle}$$

Perturbative expansion in the strong coupling.

NRQCD Scaling Rules

$$\boxed{d\sigma_0(1+\alpha_s C_1+\alpha_s^2 C_2+\ldots)}\quad\boxed{\langle \mathcal{O}(^{2S+1}L_J^{[1,8]})\rangle \sim v^{3+2L+2E+4M}}$$

$$Q\bar{Q}(n) \xrightarrow{\langle \mathcal{O}_n^{\mathcal{Q}}\rangle} \mathcal{Q}$$

$$\mathcal{O}_n^{\mathcal{Q}} = \mathcal{O}_2^{n\dagger}\left(\sum_X |X+\mathcal{Q}\rangle\langle X+\mathcal{Q}|\right)\mathcal{O}_2^n$$

$$\mathcal{O}_2^n = \psi^\dagger \mathcal{K}^n \chi$$

$$n = {}^{2S+1}L_J^{[c]}$$

ultra-soft
\+ soft

NRQCD is an expansion of QCD in the relative velocity of the heavy quark pair

Factorization in NR-SCET

Dark matter bound-state decay spectrum (NRDM-SCET):

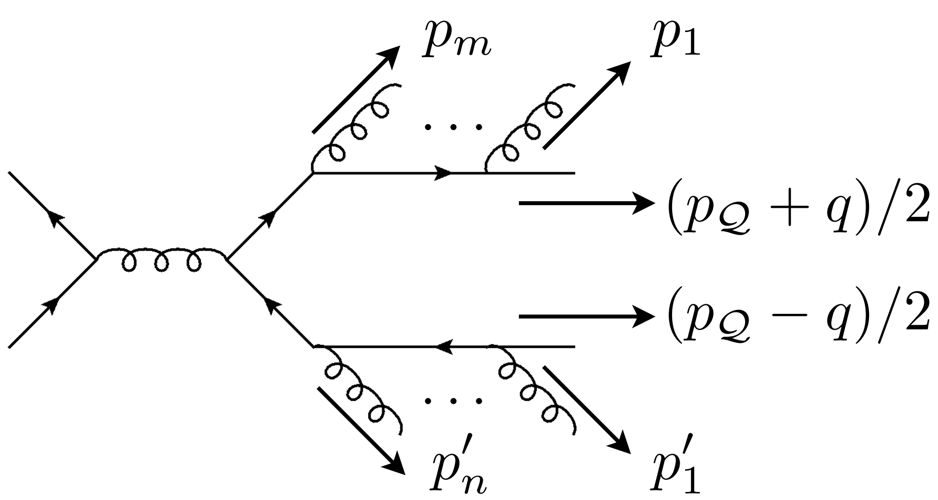
arXiv:1409.4415 (M. Baumgart, I. Z. Rothstein, and V. Vaidya)
arXiv:1409.7392 (M. Bauer, T. Cohen, R. J. Hill, and M. P. Solon)
arXiv:1409.8294 (G. Ovanesyan, T. R. Slatyer, and I. W. Stewart)
arXiv:1412.8698 (M. Baumgart, I. Z. Rothstein, and V. Vaidya)
arXiv:1510.02470 (M. Baumgart and V. Vaidya)
arXiv:1612.04814 (G. Ovanesyan, N. L. Rodd, T. R. Slatyer, and I. W. Stewart)
arXiv:1712.07656 (M. Baumgart, T. Cohen, I. Moult, N. L. Rodd, T. R. Slatyer, P. Solon, I. W. Stewart, and V. Vaidya)
arXiv:1808.04388 (L. Rinchiuso, N. L. Rodd, I. Moult, E. Moulin, M. Baumgart, T. Cohen, T. R. Slatyer, I. W. Stewart, and V. Vaidya)
arXiv:1808.08956 (M. Baumgart, T. Cohen, E. Moulin, I. Moult, L. Rinchiuso, N. L. Rodd, T. R. Slatyer, I. W. Stewart, and V. Vaidya)

Quarkonium at the kinematic end-point and TMDs:

arXiv:hep-ph/0211303 (S. Fleming and A. K. Leibovich)
arXiv:hep-ph/010631 (C. W. Bauer, C-W Chiang, S. Fleming, A. K. Leibovich, and I. Low)
arXiv:hep-ph/0306139 (S. Fleming, A. K. Leibovich, and T. Mehen)
arXiv:hep-ph/0607121 (S. Fleming, A. K. Leibovich, and T. Mehen)
arXiv:1907.06494 (M. Echevarria)
arXiv:1910.03586 (S. Fleming, Y. Makris, T. Mehen)

Decay to light hadrons

Decay to light quarks nice example that exhibits operator mixing as well as color-octet modification of the CSS kernel:



S-wave octet: $^3S_1^{(8)}$

$$\mathcal{S}_v^{cd}\left(\psi^\dagger\sigma^i T^d\chi\right)\times\left(\bar\chi_{\bar n}\gamma^i S_{\bar n}^\dagger T^c S_n\chi_n\right)\xrightarrow{\text{LO matching}} C_{q\bar q}({}^3S_1^{[8]}) = \alpha_s\frac{\pi}{m^2}$$

NRQCD

SCET

P-wave singlet: $^3P_J^{(1)}$

Confirmed at LO:

$$\sqrt{\frac{2}{N_c}}\times\mathcal{S}_v^{cd}\left[\frac{g\,B_s^{d,j}}{m\;v\cdot\mathcal{P}}\right]\left[\psi^\dagger\frac{\sigma^i\overleftrightarrow{\mathcal{P}}^j}{2\sqrt{2N_c}}\chi\right]\times\left(\bar\chi_{\bar n}\gamma^i S_{\bar n}^\dagger T^c S_n\chi_n\right)\longrightarrow C_{q\bar q}({}^3P_J^{[1]}) = C_{q\bar q}({}^3P_J^{[8]}) = C_{q\bar q}({}^3S_1^{[8]})$$

Factorization

$$\frac{d\Gamma}{dz_1 dz_2 d\boldsymbol{q}_\perp} = \Gamma_0 \sum_{ij} H^{ij}_{{}^3S_1^8}\ D^\perp_{i/H_1}(b, z_1)\ D^\perp_{j/H_2}(b, z_2)\ S^\perp_{ij}(b)$$

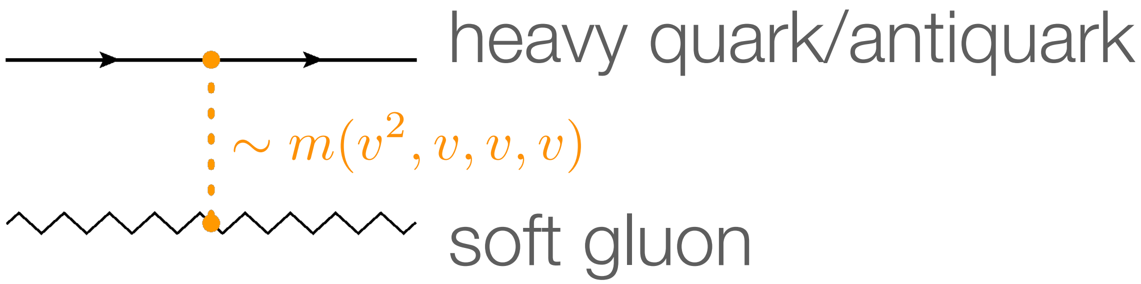
Hard function: The same for S and P wave mechanism for this process. Importance consequence for IR finiteness of the observable

Standard unsubtracted TMD fragmentation functions

Quarkonium shape functions

$$S^\perp_{ij}(b) = \sum_{n \in \{{}^3S_1^8, {}^3P_J^1\}} S^{[n]\perp}_{ij}(b)$$

Quarkonium **TMD-shape functions*** encode both soft and non-perturbative quarkonium related effects. Further factorization is not possible due to Coulomb-like interactions:



*See: arXiv:1907.06494 (M. Echevarria)

TMD-shape functions

$$\frac{d\Gamma}{dz_1 dz_2 d\boldsymbol{q}_\perp} = \Gamma_0 \sum_{ij} H^{ij}_{{}^3S_1^8}\ D^\perp_{i/H_1}(b, z_1)\ D^\perp_{j/H_2}(b, z_2)\ S^\perp_{ij}(b)$$

Quarkonium shape functions

$$S^\perp_{ij}(b) = \sum_{n \in \{{}^3S_1^8, {}^3P_J^1\}} S^{[n]\perp}_{ij}(b)$$

S-wave octet:

$$S^\perp_{\chi_J \to {}^3S_1^{[8]}}(\mathbf{k}_\perp) = \frac{d-2}{(d-1)t_F} \mathrm{tr}\Big\langle \chi_J \Big| \psi^\dagger \sigma^i T^a \chi \mathcal{S}_v^{ba} (S_{\bar{n}}^\dagger T^b S_n) \delta^{(2)}(\mathbf{k}_\perp - \boldsymbol{\mathcal{P}}_\perp) \times (S_n^\dagger T^c S_{\bar{n}}) \mathcal{S}_v^{dc} \chi^\dagger \sigma^i T^d \psi \Big| \chi_J \Big\rangle$$

Ultra-soft gluons (Wilson-lines) are not shown to keep the expressions simple

P-wave singlet:

$$S^\perp_{\chi_J \to {}^3P_J}(\mathbf{k}_\perp) = (2J+1) \frac{g^2}{N_c^2\ t_F}\ \mathcal{A}_J^{ij}\ \mathrm{tr}\Big\langle \chi_J \Big| \psi^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\boldsymbol{\mathcal{P}}}\ \chi \Big[\frac{B_s^{a,i}}{m\ v \cdot \mathcal{P}} \Big] \mathcal{S}_v^{ba} (S_{\bar{n}}^\dagger T^b S_n)$$
$$\times\ \delta^{(2)}(\mathbf{k}_\perp - \boldsymbol{\mathcal{P}}_\perp)(S_n^\dagger T^c S_{\bar{n}}) \mathcal{S}_v^{dc} \Big[\frac{B_s^{d,j}}{m\ v \cdot \mathcal{P}} \Big] \chi^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\boldsymbol{\mathcal{P}}}\ \psi \Big| \chi_J \Big\rangle$$

To be resolved

- In-jet quarkonium polarization. Could that give an insight into quarkonium production mechanisms? What is the possibility of future measurement?

- Reliable event generators in the collinear and soft limits.

- Quarkonium fragmentation in the limit: $\mu_\lambda \sim p_\psi^\perp \ll m_\psi \ll E_\psi$
  e.g. quarkonium TMD FF, useful for TMD studies in EIC.

- Relative velocity and λ-soft power counting. Subleading factorization/resummation for quarkonium photo production.

Thank you

Supported by the INFN (PV), the FELLINI project, and the European Commission