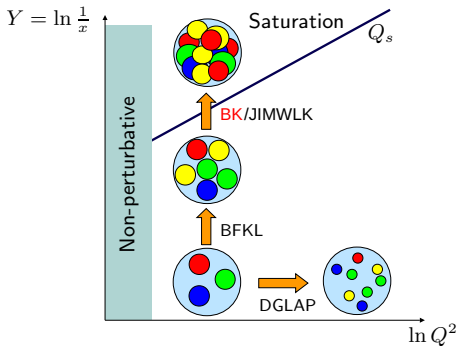


Inclusive particle production in the Color Glass Condensate: recent developments and open points

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Quarkonia as Tools
Aussois, 16/01/2020

A longstanding question in QCD is to find evidence for small x effects, and in particular gluon saturation



The **linear** small x evolution (**BFKL**) is governed by soft gluon emissions. Violates unitarity eventually

At large densities, recombination effects become important:
Non-linear evolution (**BK, JIMWLK**)

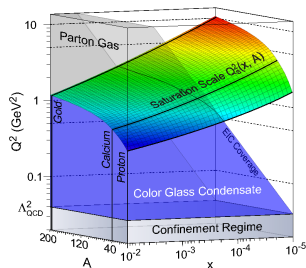


The importance of non-linear effects is quantified by the size of the **saturation scale** Q_s , which is related to the typical transverse momentum of the gluons in the target \Rightarrow need Q_s as large as possible

Roughly we have $Q_s^2 \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$

Probing this regime requires:

- Small x values
 - Large nucleus
 - A hard scale
- } to enhance saturation effects
- } to be in the perturbative regime



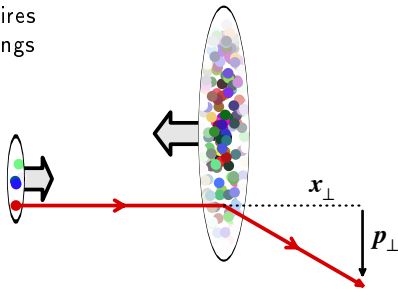
Forward particle production in proton-nucleus collisions (e.g. at the LHC) can be used as a probe of saturation

A simple example: light hadron production $pA \rightarrow h + X$

A quark initially collinear with the proton acquires a **transverse momentum** p_{\perp} via multiple scatterings off the saturated gluons

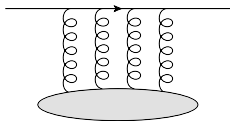
in the **dilute** proton: $x_p = \frac{p_{\perp}}{\sqrt{s}} e^y \sim 1$

in the **dense** nucleus: $x_g = \frac{p_{\perp}}{\sqrt{s}} e^{-y} \ll 1$



At very high energy the gluonic content of a hadron can be described using classical color fields

The eikonal interaction of a dilute probe (e.g. a large x parton coming from a proton) with the dense target (nucleus) is described by a Wilson line V which resums multiple scatterings

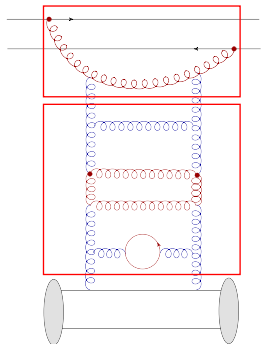


The x evolution of the dipole correlator $S(\mathbf{r} = \mathbf{x} - \mathbf{y}) = \left\langle \frac{1}{N_c} \text{Tr} V(\mathbf{x}) V^\dagger(\mathbf{y}) \right\rangle$ is governed by the **Balitsky-Kovchegov** (BK) equation:

$$\frac{\partial S(\mathbf{r}, x)}{\partial \ln x} = 2\alpha_s N_c \int \frac{d^2 \mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{x^2 (\mathbf{r} - \mathbf{x})^2} [S(\mathbf{r}, x) - S(\mathbf{x}, x) S(\mathbf{r} - \mathbf{x}, x)]$$

The initial condition $S(\mathbf{r}, x_0)$ cannot be computed perturbatively. It can be extracted e.g. by a fit to HERA DIS data (typically $x_0 \sim 0.01$)

Besides the non-linear **evolution** of gluon densities, calculations in this formalism require to know the process-dependent hard part (**impact factor**)



Impact factor: fixed order in α_s

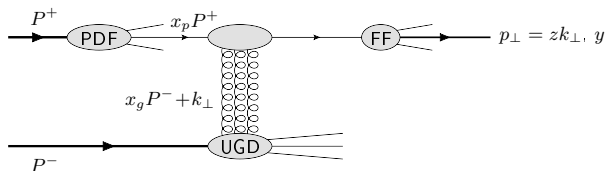
LO: $\mathcal{O}(1)$, NLO: $\mathcal{O}(\alpha_s)$, ...

BK evolution resumming gluon emissions

LO: $(\alpha_s \ln 1/x)^n$, NLO: $\alpha_s (\alpha_s \ln 1/x)^n$, ...

Most phenomenological studies in this formalism: LO impact factor + LO BK evolution with running coupling corrections

Example: forward hadron production at LO in the $q \rightarrow q$ channel:



Dilute projectile: $x_p = \frac{k_\perp}{\sqrt{s}} e^y \sim 1$, described by a collinear PDF

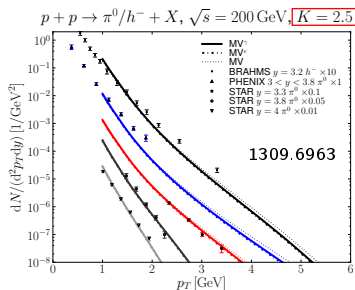
Dense target: $x_g = \frac{k_\perp}{\sqrt{s}} e^{-y} \ll 1$, described by unintegrated gluon distribution \mathcal{S}

$$\frac{dN}{d^2\mathbf{p}_T dy} \propto \text{PDF} \otimes \mathcal{S} \otimes \text{FF} \quad \text{with} \quad \mathcal{S}_{x_g}(k_\perp) = \int d^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r}, x_g)$$

LO calculations cannot be expected to be very accurate (α_s not that small)

Example: forward hadron production (Lappi, Mäntysaari, 2013): large 'K' factor

This factor is needed to reproduce the normalization of the data. Not related to NLO/LO



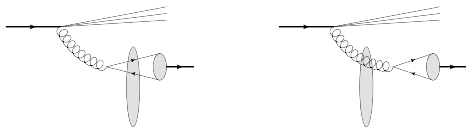
More reliable observables: ratios, e.g. nuclear modification factor $R_{pA} = \frac{1}{A} \frac{\sigma^{pA}}{\sigma^{pp}}$

Expectation in the CGC: because the nucleus has a larger saturation scale than the proton, its gluon density should increase slower at small x : R_{pA} smaller than 1 and decreasing at large rapidity

One interesting observable to study saturation: nuclear modification of forward J/ψ production in pA collision at the LHC

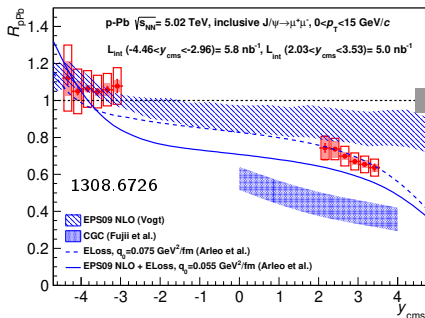
- Gives access to very small x ($\sim 10^{-5}$) in the target
- The charm quark mass should be large enough to provide a hard scale (perturbative treatment) but small enough to be sensitive to saturation
- Relatively easy to reconstruct via dilepton decays \rightarrow lots of data
- Nuclear modification in pA: important reference for AA collisions

Hard part for quark pair production: [Blaizot, Gelis, Venugopalan \(2004\)](#)



x values probed in the projectile and the target: $x_{1,2} = \frac{\sqrt{P_{\perp}^2 + M^2}}{\sqrt{s}} e^{\pm Y}$

The R_{pA} for forward J/ψ production has been measured at the LHC by ALICE and LHCb. Discrepancy with first predictions from [Fujii, Watanabe \(2013\)](#):



However the 'actual' (perturbative) prediction is the **rapidity dependence** coming from the **BK** equation, which governs the evolution of the target from $x_0 \sim 0.01$ down to smaller x . The absolute value depends on the **initial condition** at x_0 which is non-perturbative

The initial condition for a **proton** can be obtained by fitting the HERA DIS data

No similar data at small x ($\lesssim 0.01$) for **nuclei** (yet)

In their original calculation **Fujii & Watanabe** used the same initial condition as for a proton but with an initial saturation scale $Q_{s0,A}^2 \sim A^{1/3} Q_{s0,p}^2$

Some possible ways to constrain the initial condition of a nucleus:

1. **Fit NMC data** on the A dependence of F_2 at $x = 0.0125$

The best fit value for $Q_{s0,A}^2$ depends on the exact form of the initial condition and is $\sim (1.5 - 3) Q_{s0,p}^2$ for a lead nucleus

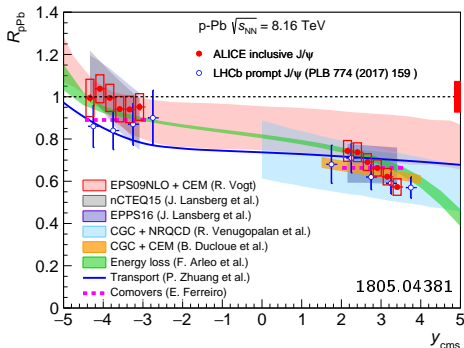
Dusling, Gelis, Lappi, Venugopalan (2009)

2. **Optical Glauber model**: the nuclear density in the transverse plane at the initial condition is given by the standard Woods-Saxon distribution $T_A(\mathbf{b})$

Lappi, Mäntysaari (2013)

The best solution to this problem: future **EIC** data?

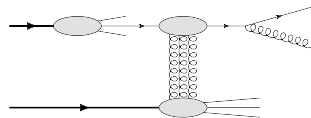
These two approaches lead to a much better agreement with data:



Several other calculations (nPDFs, energy loss, ...) are compatible with data
 The physical mechanism behind the observed suppression is still unclear

Another observable to probe saturation: **azimuthal correlations** between particles produced at forward rapidity

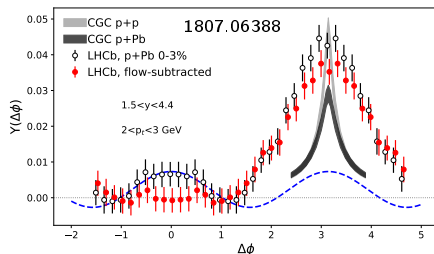
As long as the produced particles have a **small momentum imbalance**, a small q_{\perp} is probed in the target \rightarrow sensitivity to saturation effects



In particular expect a strong suppression of the away-side peak in pA collisions compared to pp because of the larger saturation scale of the nucleus

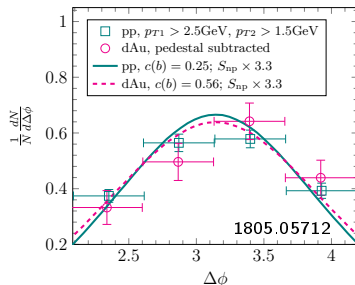
The numerical implementation of the analytical expressions is very challenging
Dramatic simplifications when $\Delta\phi \rightarrow 0$ but limits the range of validity

Example of comparison with LHC data:
(Giacalone, Marquet, 2018)



The back-to-back region is precisely where Sudakov resummation effects can be important due to strong phase-space constraints. Calculations become even more involved

Good description of RHIC data:
(Stasto, Wei, Xiao, Yuan, 2018)



Difficult to draw definitive conclusions at this stage:

- Sudakov is the dominant effect at RHIC (x not so small)
- Need large non-perturbative Sudakov factor to describe RHIC data (negligible at LHC?)
- No BK small x evolution (Golec-Biernat – Wüsthoff parametrization)

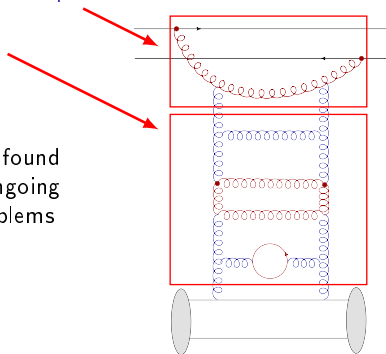
Besides di-hadron correlations, many other possible processes: γ +hadron, double D -meson, dijets at EIC, ...

To obtain more reliable predictions one needs to go **beyond the leading order** approximation

Two sources of NLO corrections in this formalism:

- Corrections to the process-dependent **impact factor**
- Corrections to the BK **evolution**

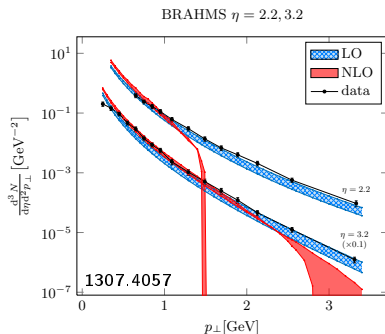
At first both sources of corrections were found to lead to **unphysical** results. Lots of ongoing work to understand and solve these problems



Forward hadron production in pA collisions: first process for which the NLO corrections to the impact factor were evaluated in this formalism

Calculation of the NLO impact factor: [Chirilli, Xiao, Yuan \(2012\)](#)

Numerical implementation ([Stasto, Xiao, Zaslavsky, 2013](#)):



Negative cross sections for $p_\perp \gtrsim Q_s$

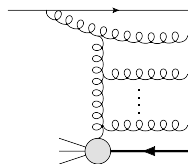
Many works devoted to understanding and solving this issue:

- Kang, Vitev, Xing (2014)
- Stasto, Xiao, Yuan, Zaslavsky (2014)
- Altinoluk, Armesto, Beuf, Kovner, Lublinsky (2014)
- B.D., Lappi, Zhu (2016)
- Iancu, Mueller, Triantafyllopoulos (2016)
- Xiao, Yuan (2018)
- Liu, Ma, Chao (2019)

Most compelling explanation: the origin of the problem is an oversubtraction of the high-energy leading logs from the NLO impact factor (Iancu, Mueller, Triantafyllopoulos, 2016)

Why is there a subtraction?

When the additional gluon at NLO is **soft**, the integration over its phase space becomes logarithmic and generates a contribution $\sim \alpha_s \ln 1/x$ already included in the LO piece which resums $(\alpha_s \ln 1/x)^n$: need to avoid **double counting**

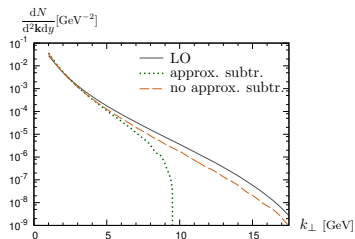


First implementation of the NLO impact factor: **approximations** in the subtraction term. But the cross section is the sum of large contributions with opposite signs \rightarrow not positive definite result

Not doing these approximations: no negativity

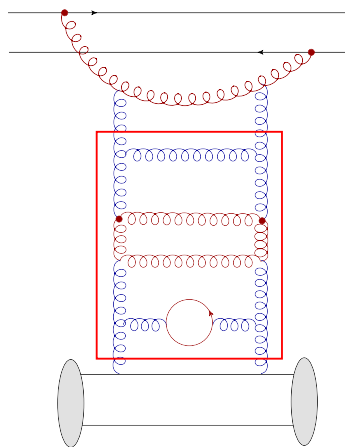
B.D., Lappi, Zhu (2017)

Very similar results in DIS with the NLO impact factor computed by Beuf (2017)

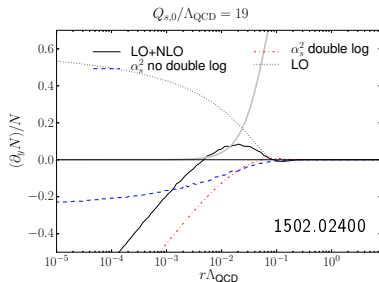


In addition to the NLO corrections to the impact factor, a complete NLO calculation must also include the corrections to the **BK evolution**

NLO BK derived by [Balitsky, Chirilli \(2007\)](#)



The first numerical implementation of the NLO corrections to BK showed that they lead to instabilities in the evolution because of large collinear contributions (Lappi, Mäntysaari, 2015):

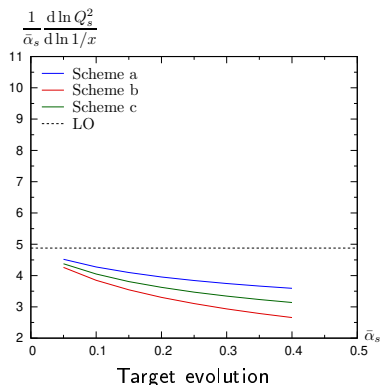
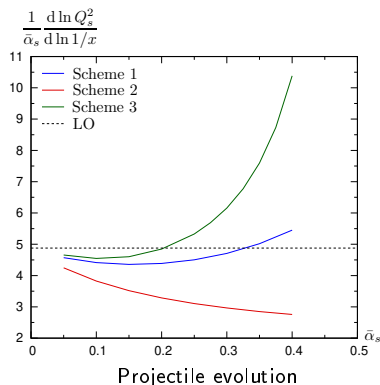


A similar observation was made for NLO BFKL a long time ago and was solved by resumming the collinear logarithms to all orders (Salam et al., 98-2003)

First proposals of similar resummations in the non-linear regime: Beuf (2014); Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2015). Considered projectile evolution. Makes the evolution stable, but:

- A boundary value problem, not an initial condition one
- Large resummation scheme dependence

When considering target evolution instead of projectile one, the resummation scheme dependence is of the expected magnitude (B.D., Iancu, Mueller, Soyez, Triantafyllopoulos, 2019):



Resummed evolution + LO impact factor: good description of HERA DIS data

After adding the remaining $\mathcal{O}(\alpha_s)$ corrections not enhanced by collinear logarithms, the evolution can in principle be convoluted with NLO impact factors to perform calculations with full NLO accuracy

LHC data on forward particle production in proton-nucleus collisions provides unique information to test the CGC formalism

- Recent CGC calculations show less suppression than early ones
This is due to a better treatment of the initial condition/nuclear geometry
- Weakness of current calculations: limited to LO + subset of NLO corrections
Progress towards NLO accuracy but still no full NLO study

Outlook:

- Future NLO calculations should provide much more accurate predictions
- It would be good to have pA data for processes such as inclusive light hadron or photon production: much simpler to calculate at NLO than e.g. J/ψ
- EM probes (photons, Drell-Yan) can also be used to study initial vs. final state effects and would in addition provide new constraints for nPDFs
- Also expect valuable information from EIC