Fitting (Quarkonium) (Collinear) Fragmentation Functions

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Quarkonia as tools

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3DSPIN
MAPPING THE PROTON IN 3D

Light-hadron FFs

- The **hadronisation** process has an intrinsically **non-perturbative** component:
  - **pQCD** is thus only *partially* applicable.
- In the case of **light-hadron** production (pions, kaons, etc.), **leading-power** (LP) factorisation is typically **sufficient**:

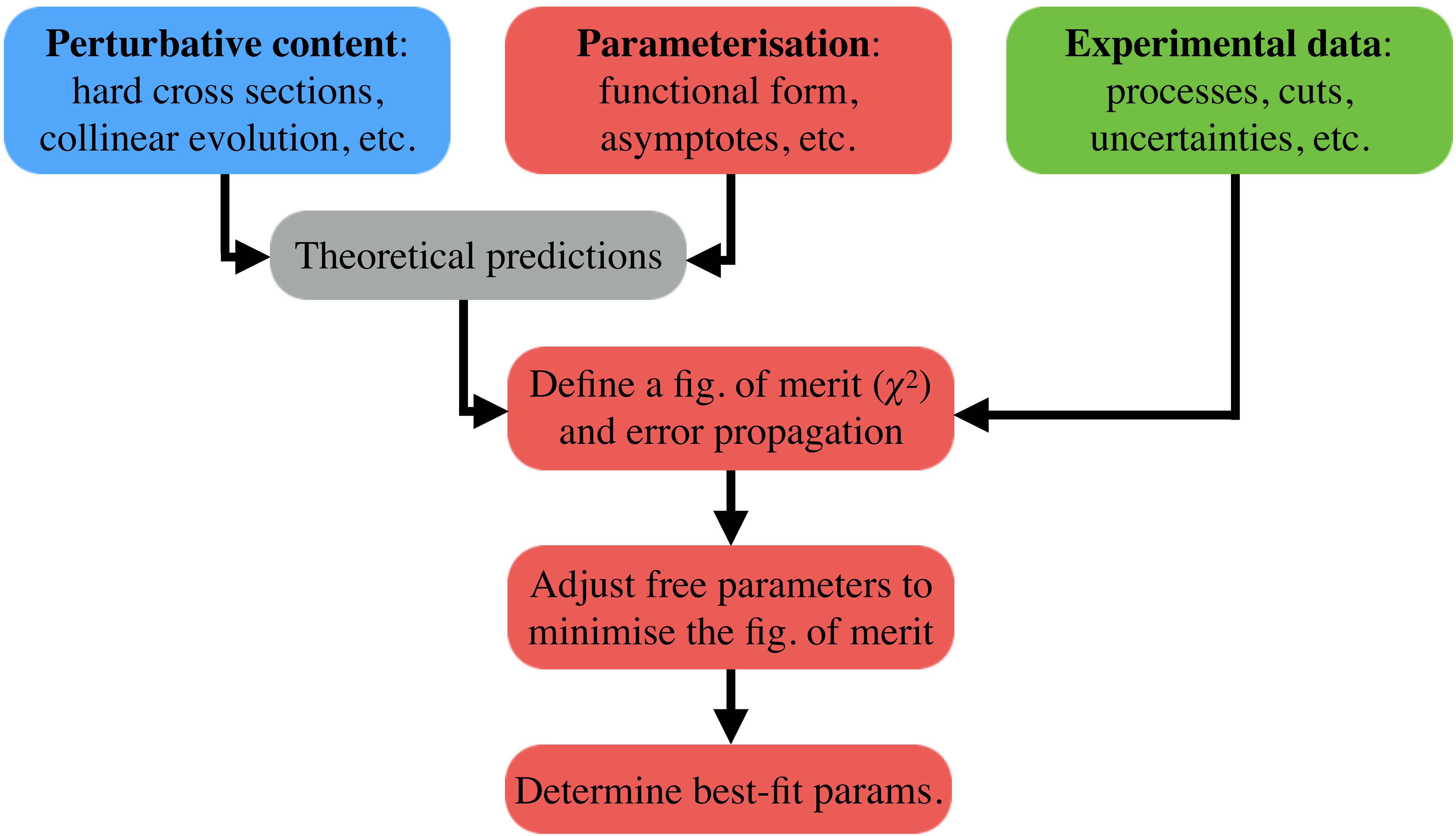
$$\frac{d\sigma_{AB\to L}}{dp_T} = \sum_f \frac{d\hat{\sigma}_{AB\to f}}{dp_T} \otimes D_{f\to L} + \mathcal{O}\left(p_T^{-2}\right)$$

- along with DGLAP equations for the collinear **evolution**:

$$\frac{dD_{i\to L}}{d\ln\mu^2} = \sum_j P_{ij} \otimes D_{j\to L}$$

- P_{ij} and $\sigma_{AB\to f}$ are **calculable in pQCD**.
- $D_{i\to L}$ **universal** but **non-perturbative**:
  - typically determined from **fits to data**.

The general fitting strategy



Fit methodologies

Parameterisation: the "standard" approach

- Distributions are parametrised by means of the function form:

$$f_i(x) = A_i\, x^{\alpha_i} (1-x)^{\beta_i} P_i(x)$$

with:

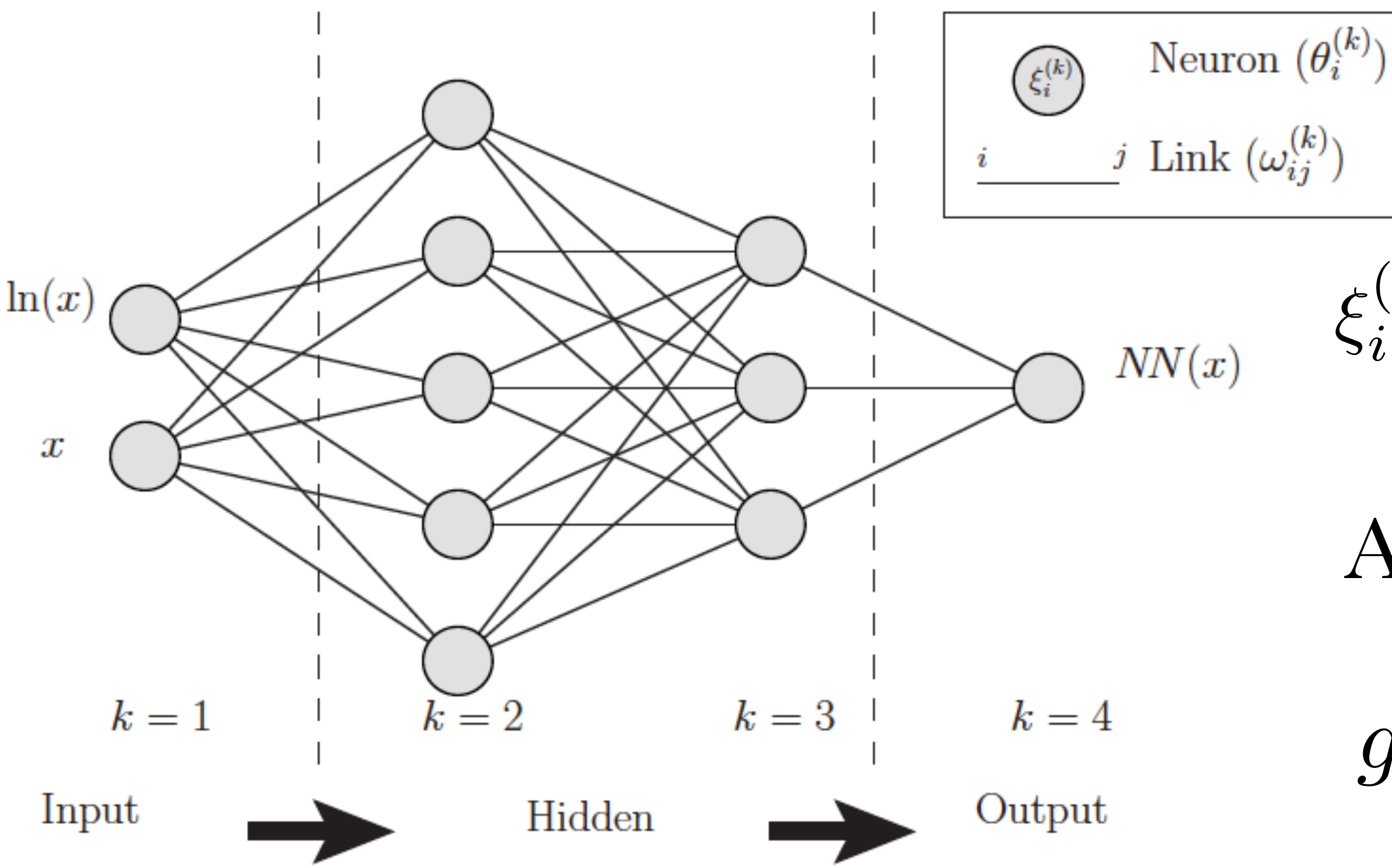
$$P_i(x) = \begin{cases} 1 \\ 1 + \gamma_i x \\ 1 + \gamma_i x + \delta_i \sqrt{x} \\ \ldots \end{cases}$$

- **O(3-5) free parameters** for each distribution.
- **Asymptotic behaviour** defined by the exponents $\boldsymbol{\alpha_i}$ and $\boldsymbol{\beta_i}$.
- Easy to transform analytically in **Mellin space**.
- **Easy to handle** in a fit thanks to its simplicity.
- Potential **source of bias**.

Fit methodologies

Parameterisation: neural networks

- Distributions are parametrised in terms of NNs with arch. (2-5-3-1):



$$\xi_i^{(j)} = g\left(\sum_k^{\text{(j-1)th layer}} \xi_k^{(j-1)} \omega_{ki}^{(j)} - \theta_i^{(j)} \right)$$

Activation function:

$$g(x) = \text{sign}(x)\ln(|x| + 1)$$

- Each NN has **37 free** parameters each.
- Distributions are expressed as $f_i(x) = \text{NN}_i(x) - \text{NN}_i(1)$
  - The $\text{NN}_i(1)$ term ensures that $f_i(x) \underset{x \to 1}{\longrightarrow} 0$
- NNs are **flexible** and thus limit biases but are **harder to handle**.

Fit methodologies

Error propagation

- A faithful determination implies an estimate of the **uncertainty** on FFs/PDFs propagating from the **experimental** dataset.

1. **Hessian** method: the χ^2 is **expanded** around its minimum $\mathbf{a}_0$:

$$\chi^2(\{\mathbf{a}\}) \simeq \chi^2(\{\mathbf{a}_0\}) + \underbrace{\frac{1}{2}\left.\frac{\partial\chi^2}{\partial a_i \partial a_j}\right|_{\mathbf{a}_0}}_{H_{ij}} (a_i - a_{0i})(a_j - a_{0j})$$

The Hessian matrix H_{ij} is **diagonalised** and an uncertainty along each eigenvector is defined as $\Delta\chi^2 = 1$ (sometimes a **tolerance** is introduced).

2. **Monte Carlo** sampling: artificial **replicas** of the dataset generated as:

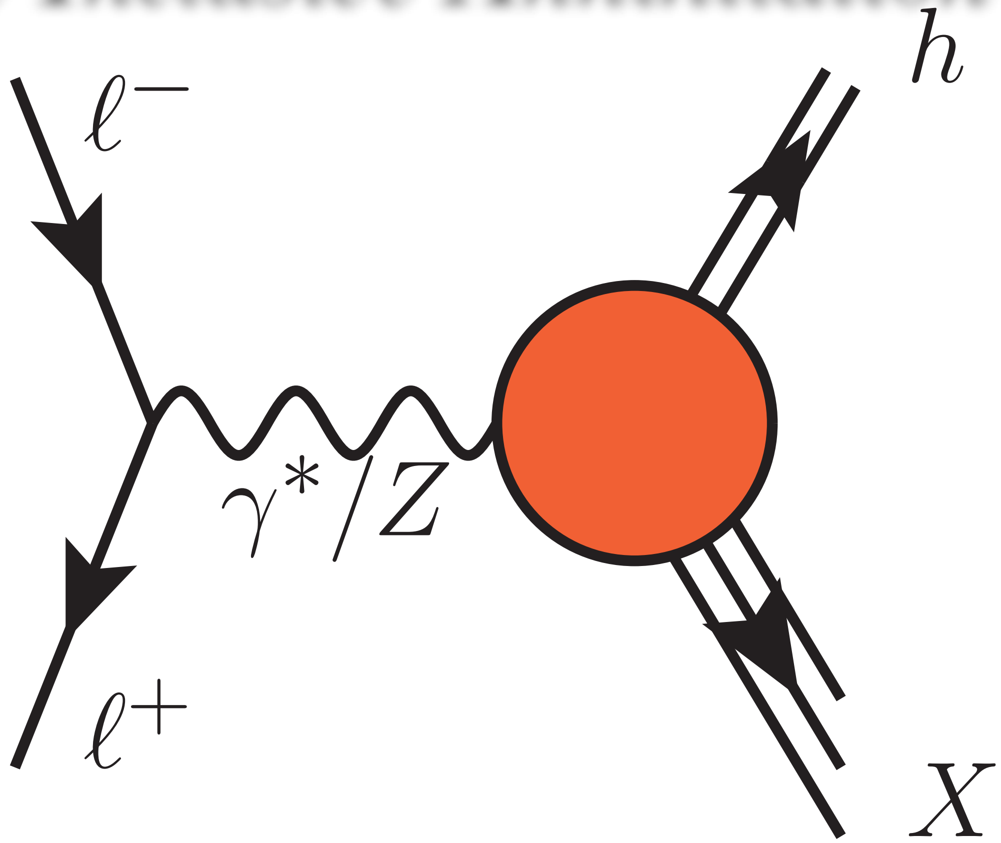
$$D_i^{(k)} = D_i + r_i^{(k)} \sigma_i \,, \qquad \begin{array}{l} k = 1, \dots, N_{\rm rep} \\ i = 1, \dots, N_{\rm dat} \end{array}$$

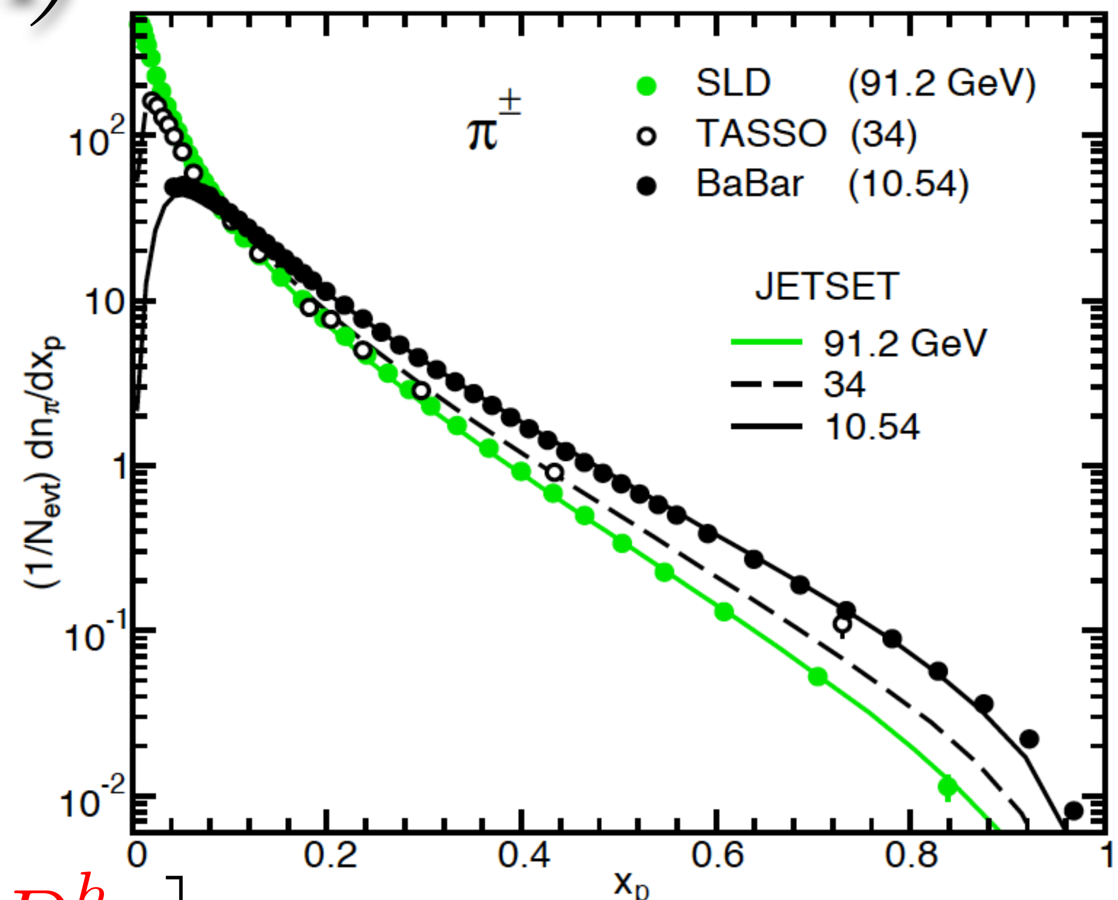
$r_i^{(k)}$ is a *normally distributed* and *univariate* random number. A fit is performed to each replica to produce $N_{\rm rep}$ sets of distributions $\{f_k\}$, such that:

$$\langle \mathcal{O} \rangle = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \mathcal{O}[f_k] \quad \text{and} \quad \sigma_{\mathcal{O}} = \sqrt{\langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2}$$

Experimental data

Single Inclusive Annihilation (SIA)





$$\frac{d\sigma^h}{dz} = \hat{\sigma}_0^h \left[C_q \otimes D_\Sigma^h + C_g \otimes D_g^h + C_{\mathrm{NS}} \otimes D_{\mathrm{NS}}^h\right]$$

- ✅ **Clean** channel: only FFs involved,
- ✅ **higher-order** corrections to NNLO,
- ✅ **precise data** available (BELLE/BABAR).
- ❌ **No flavour separation**,
  - ✅ tagged data for heavy-quark FFs.
- ❌ gluon distribution **suppressed** by α_s.

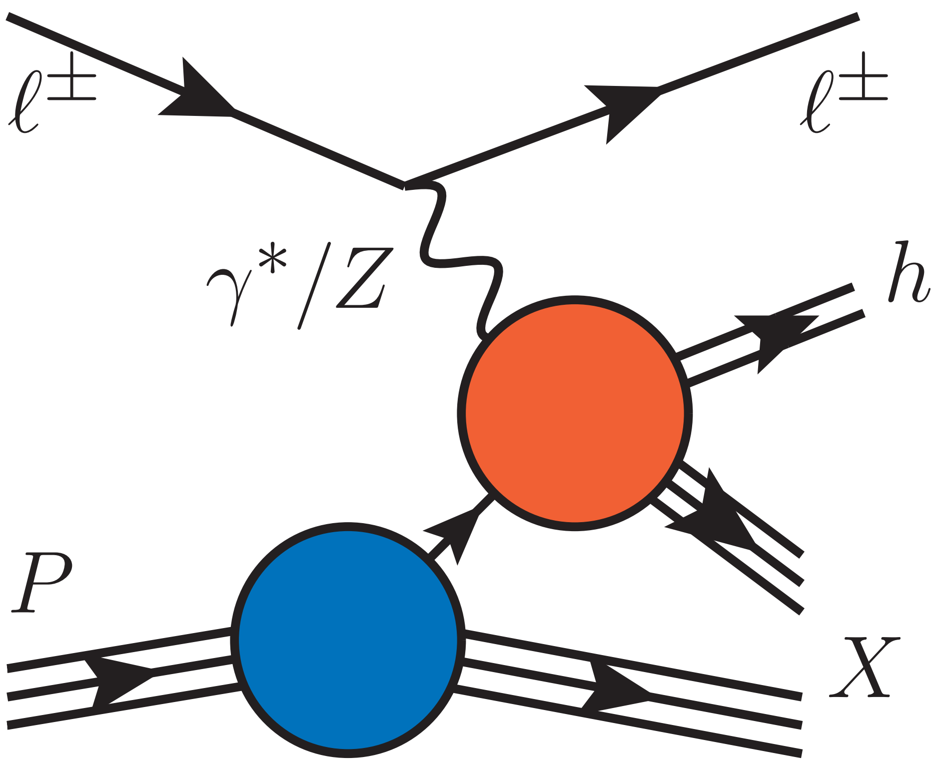
$$D_\Sigma^h = \sum_q \left(D_q^h + D_{\bar{q}}^h\right) = \sum_q D_{q^+}^h$$

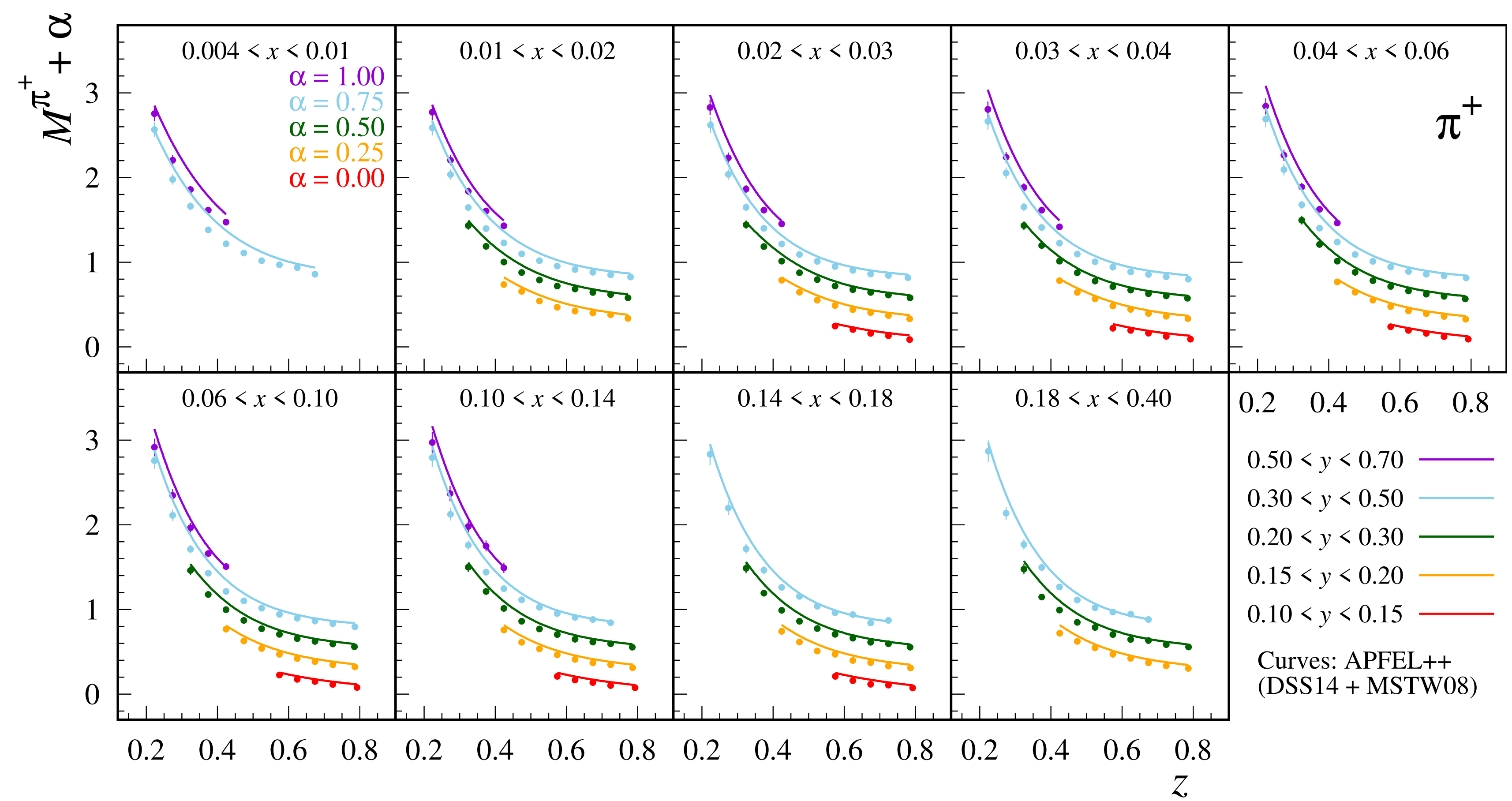
$$D_{\mathrm{NS}}^h = \sum_q \left(\frac{\hat{e}_q^2}{\langle \hat{e}_q^2 \rangle} - 1\right) D_{q^+}^h$$

$$C_q, C_{\mathrm{NS}} \propto \mathcal{O}(1) \quad \text{while} \quad C_g \propto \mathcal{O}(\alpha_s)$$

Experimental data

Semi Inclusive Deep Inelastic Scattering (SIDIS)



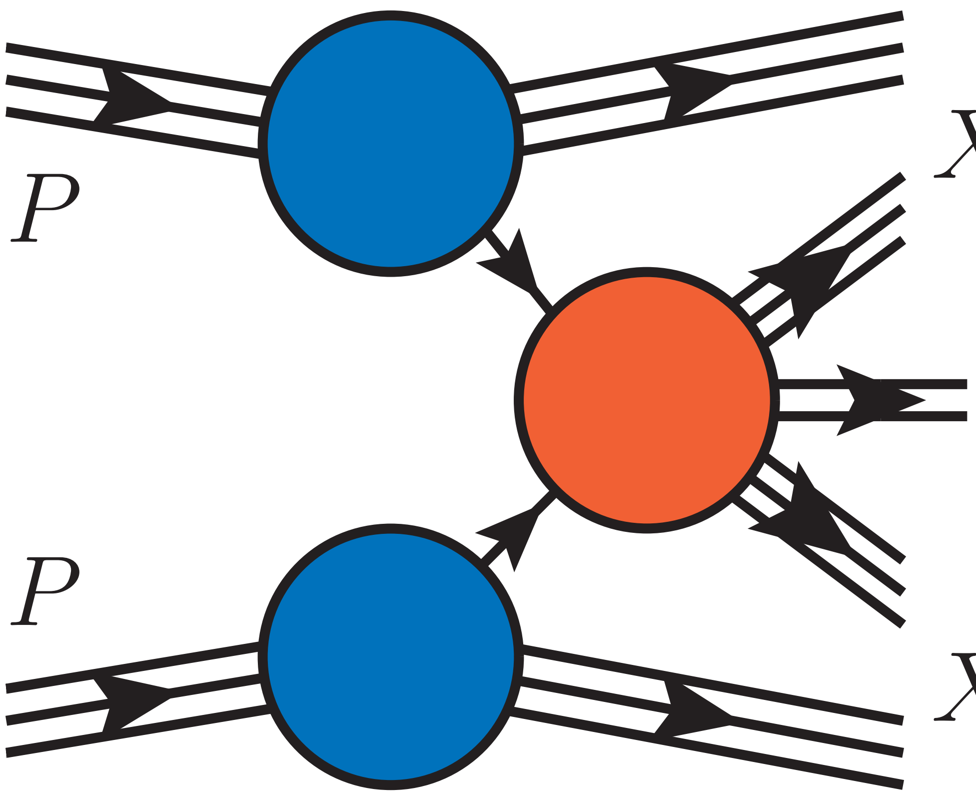


$$\frac{d\sigma^h}{dxdydz} = \hat{\sigma}_0^h \sum_{q,\bar{q}} e_q^2 \left[f_q \otimes C_{qq} \otimes D_q^h + f_g \otimes C_{gq} \otimes D_q^h + f_q \otimes C_{qg} \otimes D_g^h \right]$$

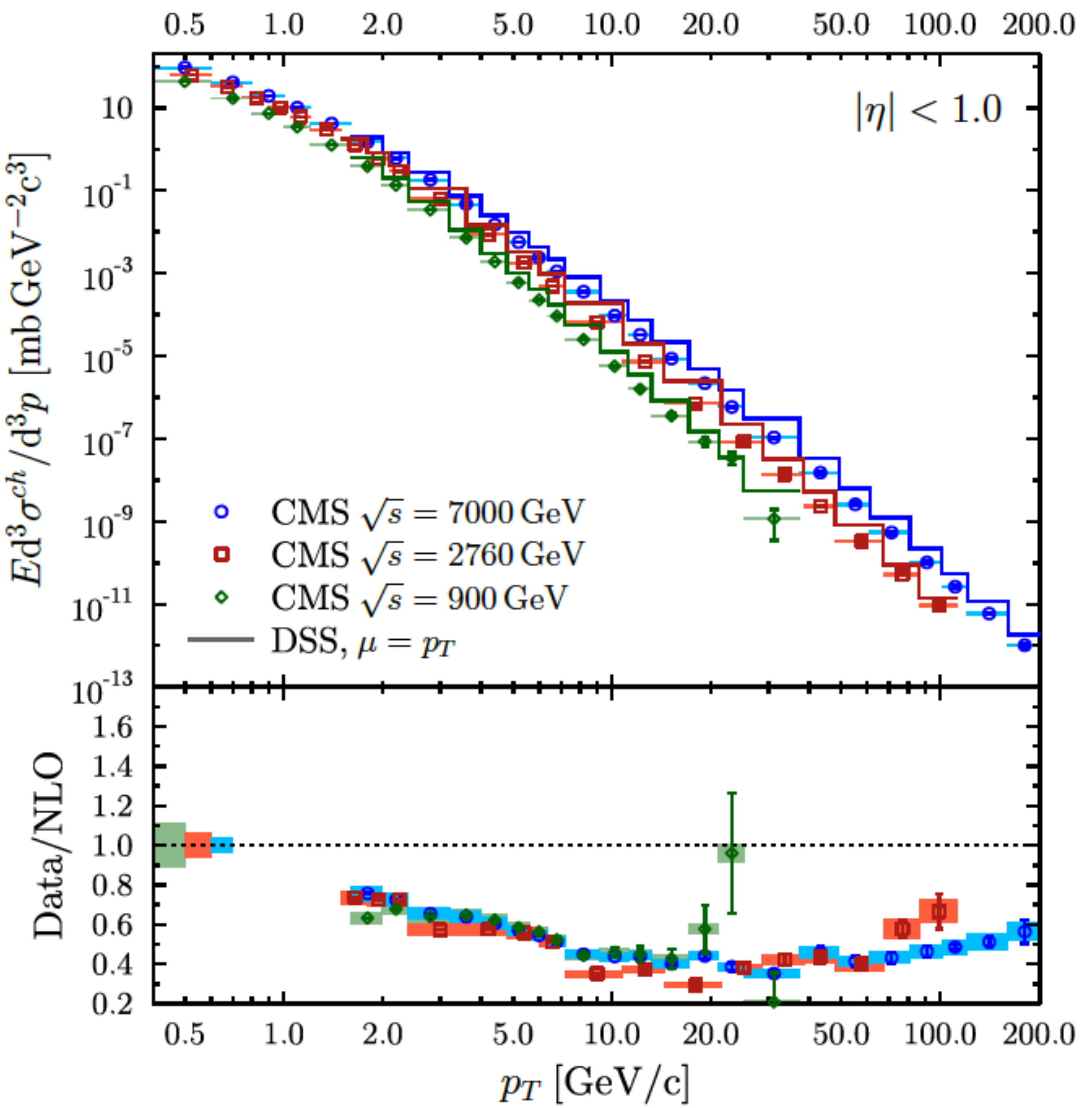
- ✅ handle on **flavour separation**,
- ✅ **precise data** available (HERMES/COMPASS).
- ❌ Involves both **FFs** and **PDFs**,
- ❌ Fully known so far up to $O(\alpha_s)$, *i.e.* NLO.

Experimental data

Hadroproduction in proton-proton collisions (pp)



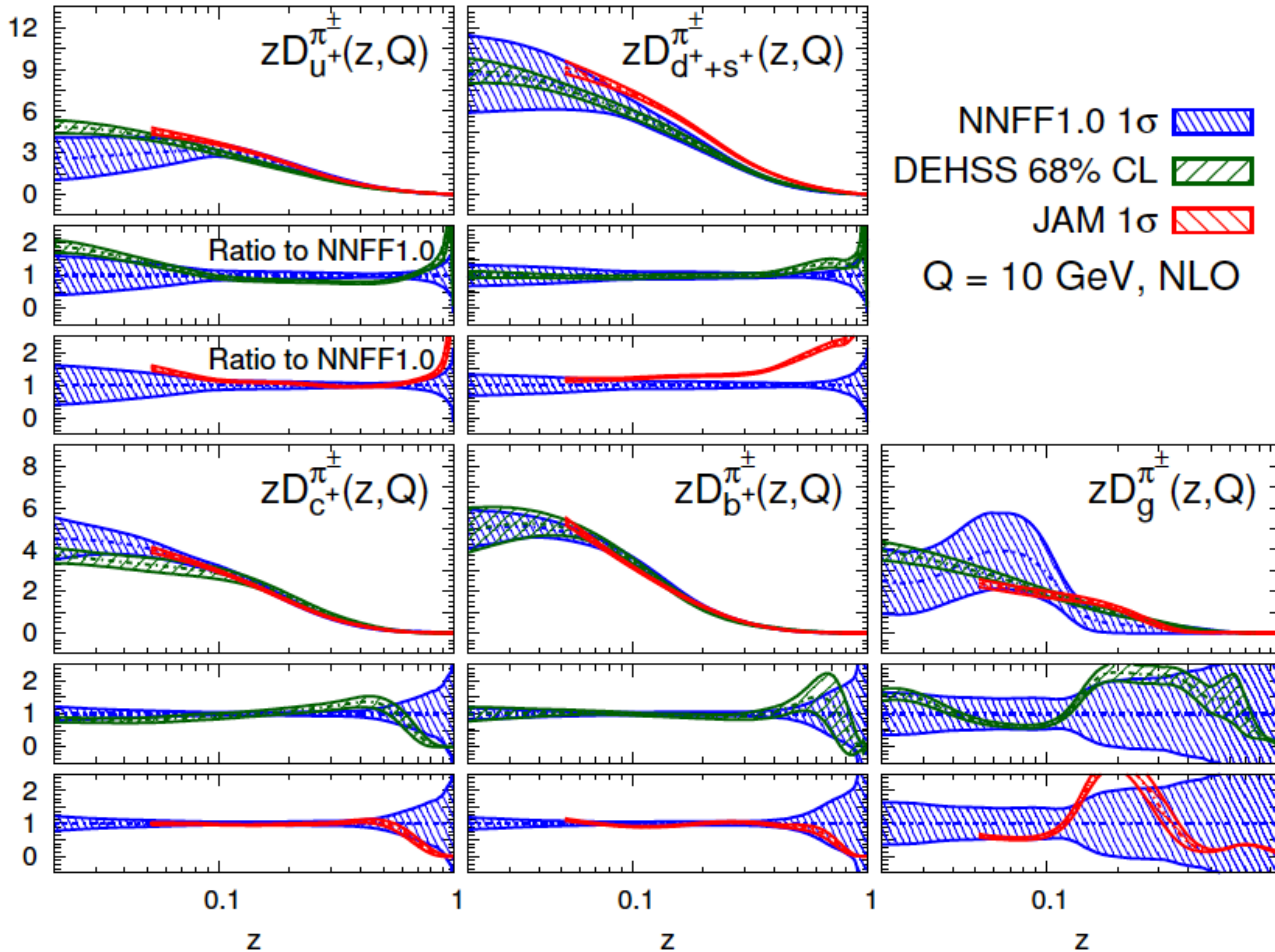
$$\frac{d\sigma^h}{dp_T d\eta} = \sum_{ijk} f_i^{(1)} \otimes f_j^{(2)} \otimes \frac{d\hat{\sigma}_{ijk}}{dp_T d\eta} \otimes D_k^h$$



- ✅ Direct sensitivity to the **gluon FF**,
- ✅ **Scale scan** ($\mu_F \propto p_T$),
- ✅ Precise data from LHC/Tevatron
- ❌ Involves both **FFs** and **PDFs**,
- ❌ Known so far up to NLO,
  - ❌ large scale variations at low p_T,
- ❌ cumbersome to compute.

Light-hadron FFs

- **Many** determinations of light-hadron FFs on the market:



V. Bertone *et al.*, *Eur. Phys. J.* C78 (2018), no. 8 651

Quarkonium FFs

- In the case of **quarkonium** production, **leading-power** (LP) factorisation is often **insufficient**:
  - **next-to-leading-power** (NLP) corrections may be significant.

$$\frac{d\sigma_{AB\to H}}{dp_T} = \sum_f \frac{d\hat{\sigma}_{AB\to f}}{dp_T} \otimes D_{f\to H} + \sum_{[Q\overline{Q}]} \frac{d\hat{\sigma}_{AB\to [Q\overline{Q}]}}{dp_T} \otimes^{\mathbf{3}} D_{[Q\overline{Q}]\to H} + \mathcal{O}\left(p_T^{-4}\right)$$

- A **new set** of (NLP) distributions enters the computation:
  - in principle, they need to be parameterised and **fitted** to data,
  - **NRQCD** helps constrain these distributions

Quarkonium FFs

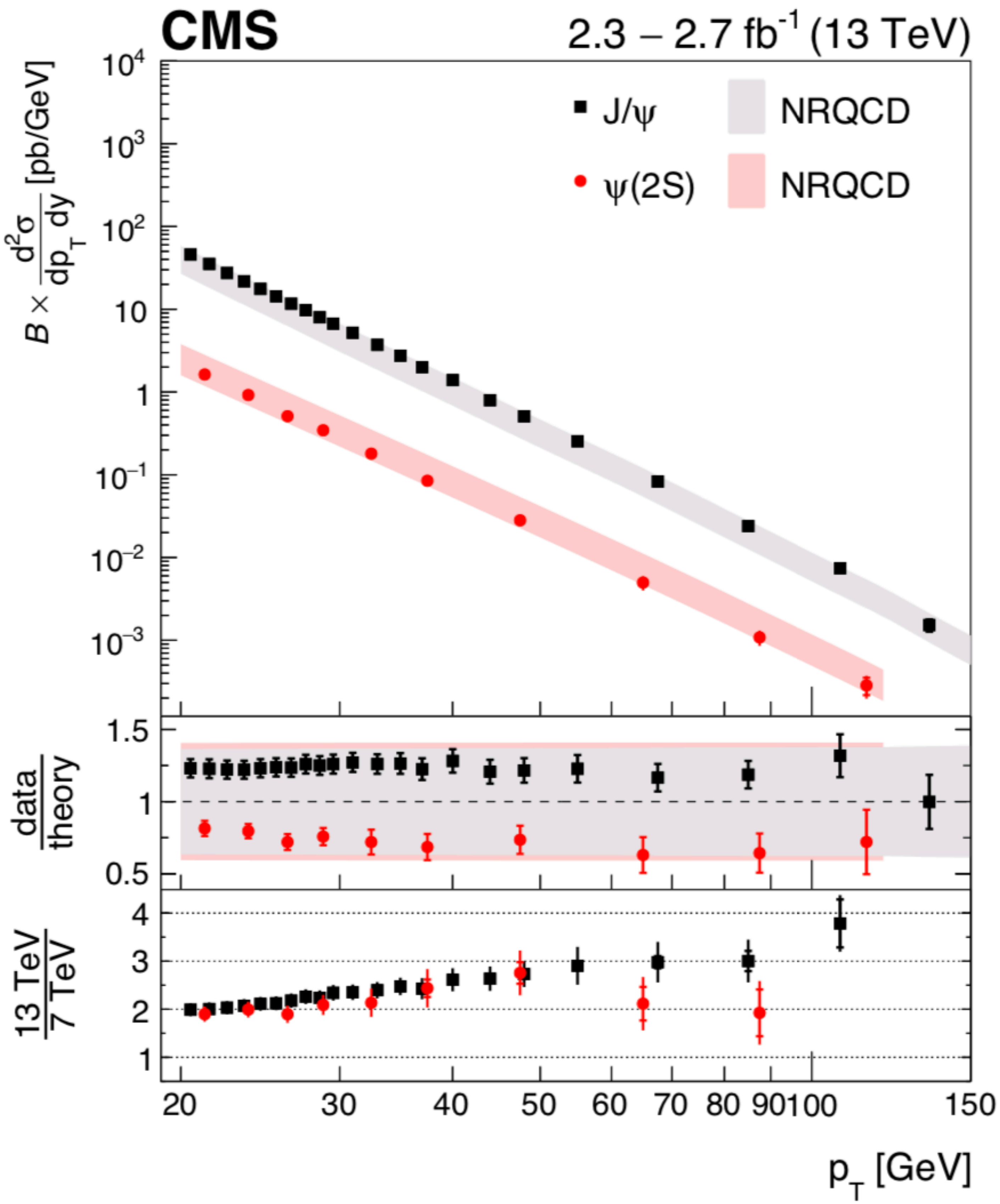
- The DGLAP evolution, including NLP corrections, are **more complicated** and **couple** LP and NLP FFs

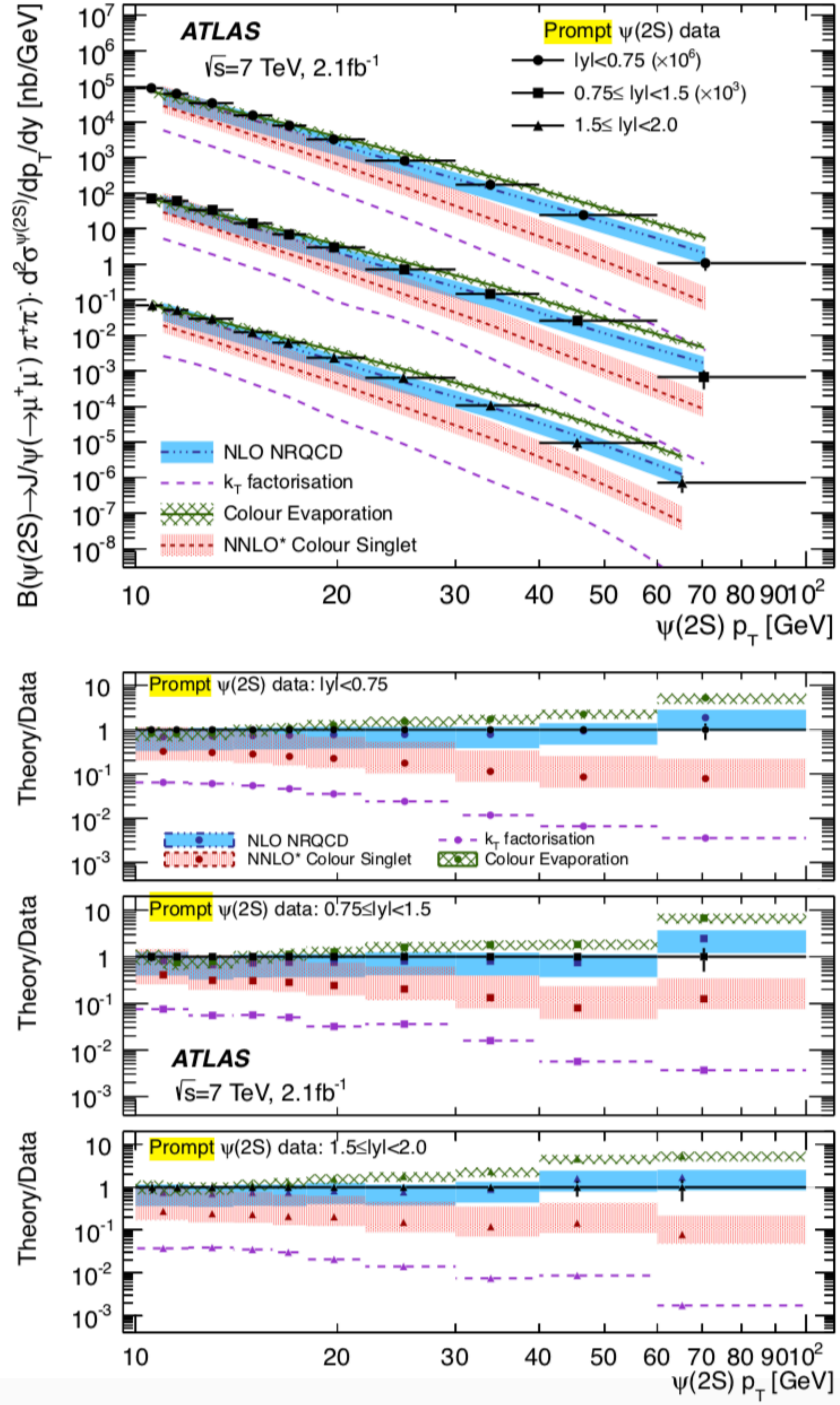
$$\frac{dD_{i\to H}}{d\ln\mu^2} = \sum_j P_{ij}\otimes D_{j\to H} + \frac{1}{\mu^2}\sum_{[Q\overline{Q}]} P_{i\to[Q\overline{Q}]}\otimes^{\mathbf{3}} D_{[Q\overline{Q}]\to H}$$

$$\frac{dD_{[Q\overline{Q}]\to H}}{d\ln\mu^2} = \sum_{[Q\overline{Q}]'} P_{[Q\overline{Q}]\to[Q\overline{Q}]'}\otimes^{\mathbf{3}} D_{[Q\overline{Q}]'\to H}$$

- The **perturbative** components have been computed.
- However, an efficient solution of this set of equations is a **numerically challenging** task:
  - due to the **triple convolutions**.
- Significant **technological** effort required to achieve a fit of quarkonium FFs

Quarkonium FFs





Summary

- The technology to extract **light-hadron FFs** is **highly developed**:
  - mostly inherited from **PDFs**.
  - Perturbative quantities known to **high accuracy**.
  - refined treatment of the **uncertainties**.
  - A **wealth of data** currently available.
  - Based on **collinear leading-power** factorisation framework.
- The determination of **quarkonium FFs** may require including **next-to-leading power** corrections:
  - NLP corrections **complicate** the framework significantly.
  - **Not many perturbative corrections** currently known.
  - **NRQCD** helps determine part of the non-perturbative elements.