# Status of DPS Factorisation

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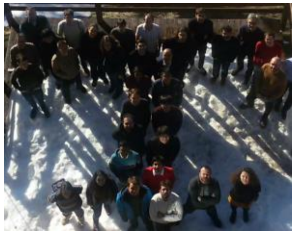
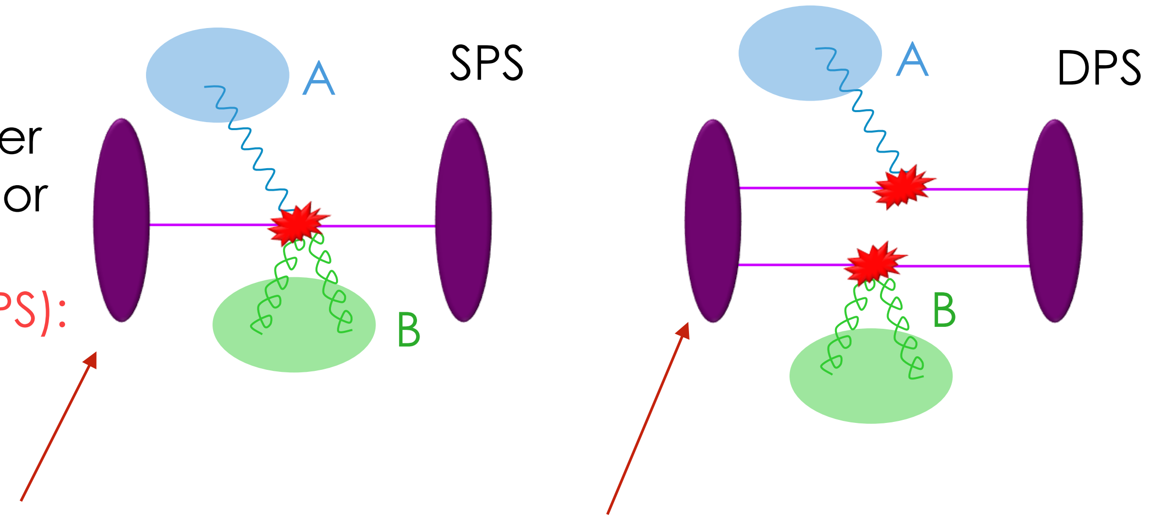
COFUND. A project supported by the European Union



Quarkonia as Tools, Aussois, 18/01/20

# DOUBLE PARTON SCATTERING: BASICS

Certain sets of scattering products can be formed either from one hard collision (SPS), or two separate hard collisions (double parton scattering, DPS):



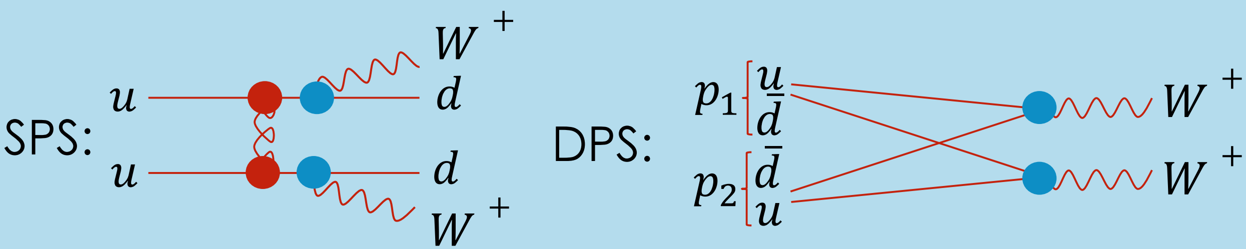

Parton density functions (PDFs)

Double parton densities (DPDs)

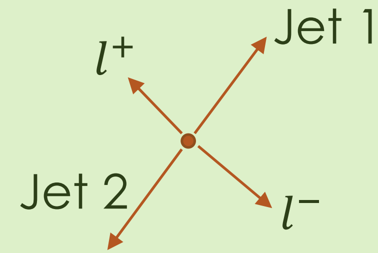
In terms of the total cross section for the production of AB, the DPS mechanism is power suppressed: $\sigma_{DPS}/\sigma_{SPS} \sim \Lambda_{QCD}^2/Q^2$

# WHY STUDY DPS?

DPS can be a significant background to processes suppressed by small/multiple coupling constants…

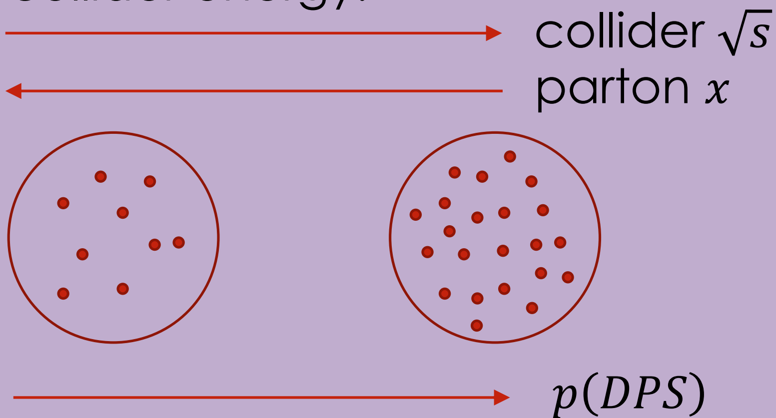



…or in certain phase space regions

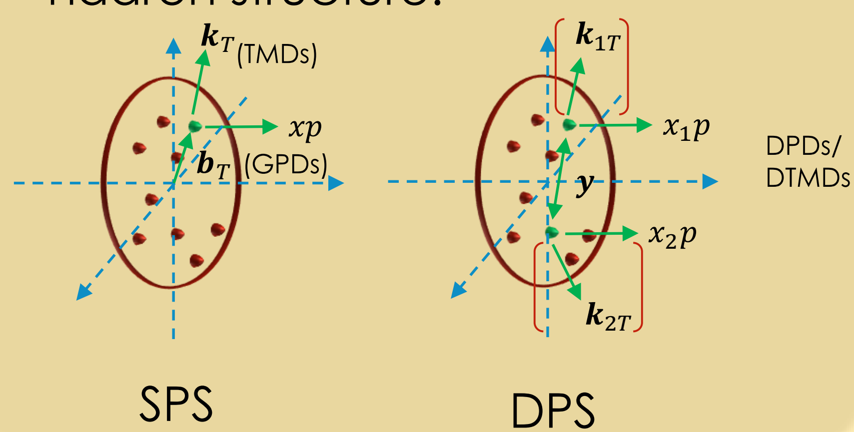



$p_T(l^+l^-), p_T(jj) \ll Q$

DPS importance increases with collider energy:




DPS tells us new information on hadron structure:



# DOUBLE TMD FACTORISATION

Focus on case where we measure transverse momenta of two hard systems: ‘double TMD’ case. Also look only at production of colourless systems (double Drell-Yan).

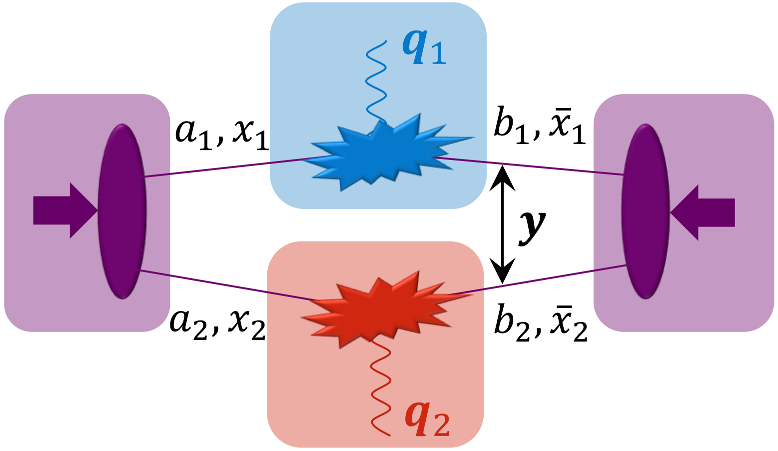
DTMD factorisation formula from parton model analysis:

Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089

$$\frac{d\sigma_{DPS}}{d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} \sim \int d^2\boldsymbol{z}_1 d^2\boldsymbol{z}_1 d^2\boldsymbol{y}\, e^{-i\boldsymbol{q}_1\cdot\boldsymbol{z}_1 - i\boldsymbol{q}_2\cdot\boldsymbol{z}_2}\, \hat{\sigma}_{a_1 b_1} \hat{\sigma}_{a_2 b_2} F_{b_1 b_2}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}) F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y})$$

Partonic cross sections

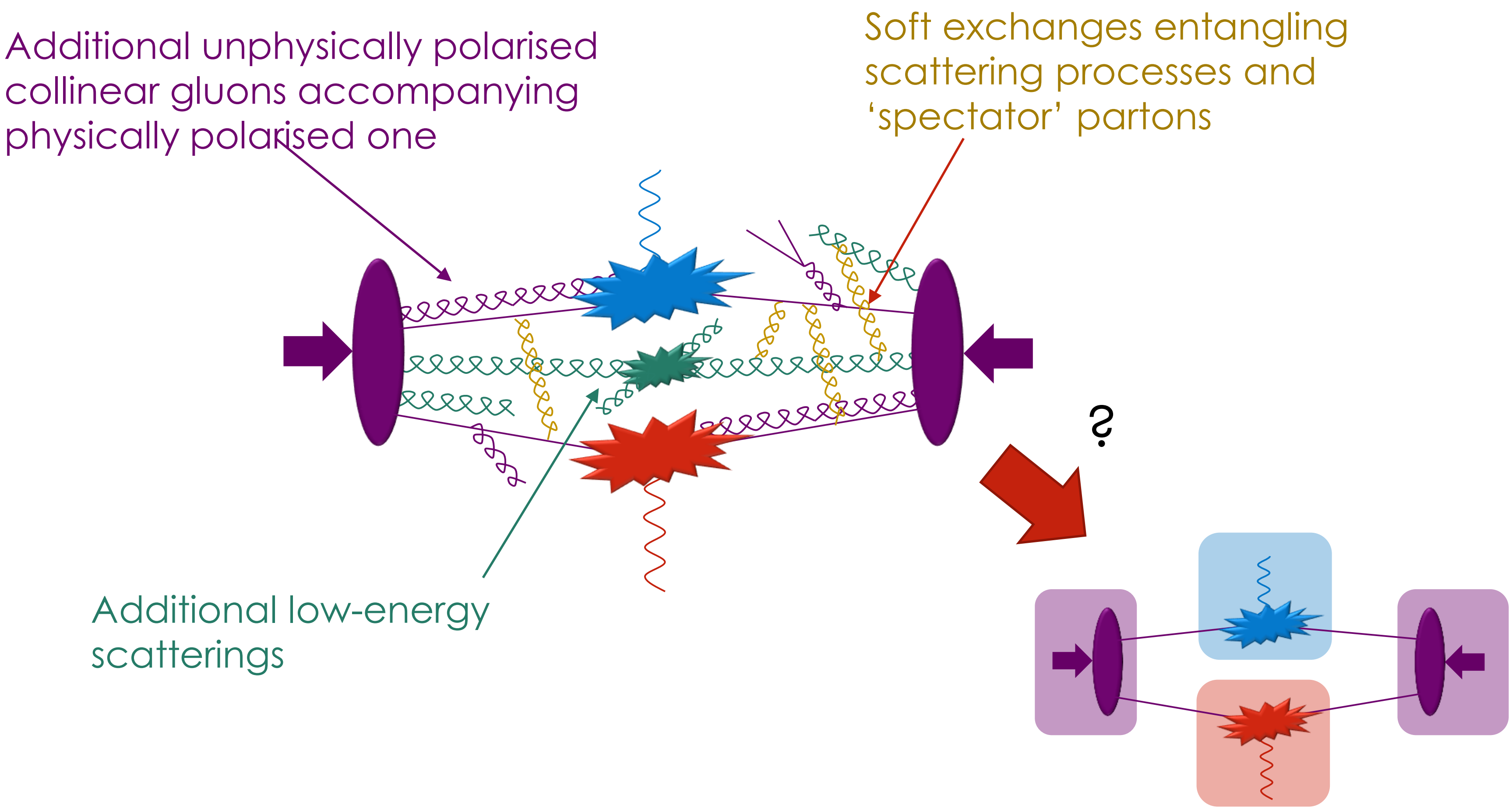
‘Double transverse momentum dependent parton distributions’/DTMDs




Does this hold in full QCD?

# THE FULL PICTURE

Full picture of events containing a double scattering:



# LEADING REGIONS

Step 1: Consider arbitrary Feynman graph contributing to DPS production of state of interest.

What regions of loop momenta can give leading contributions to DTMD cross section?

+ component - component T component

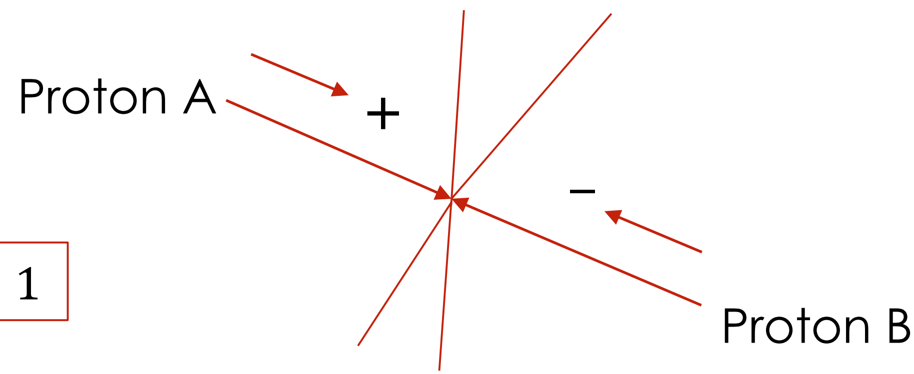
Hard: $k \sim Q(1,1,1)$

Collinear: $k \sim Q(1, \lambda^2, \lambda)$

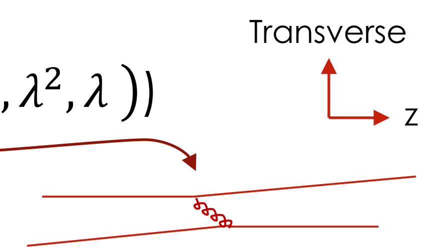
$\boxed{\lambda \ll 1}$

Soft: $k \sim Q(\lambda^n, \lambda^n, \lambda^n)$

Glauber: $|k^+ k^-| \ll \boldsymbol{k}^2 \ll Q^2$

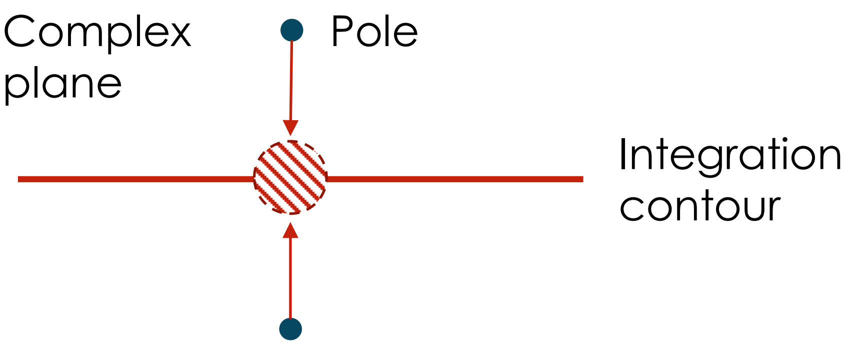



(e.g. $k \sim Q(\lambda^2, \lambda^2, \lambda)$)

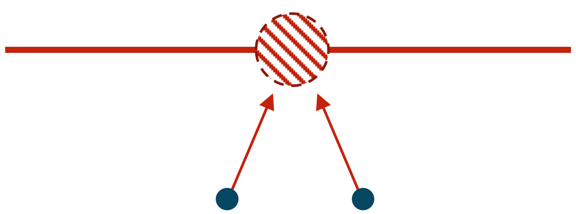


# PINCH SINGULARITIES

More specifically, looking for leading regions around pinch singularities:



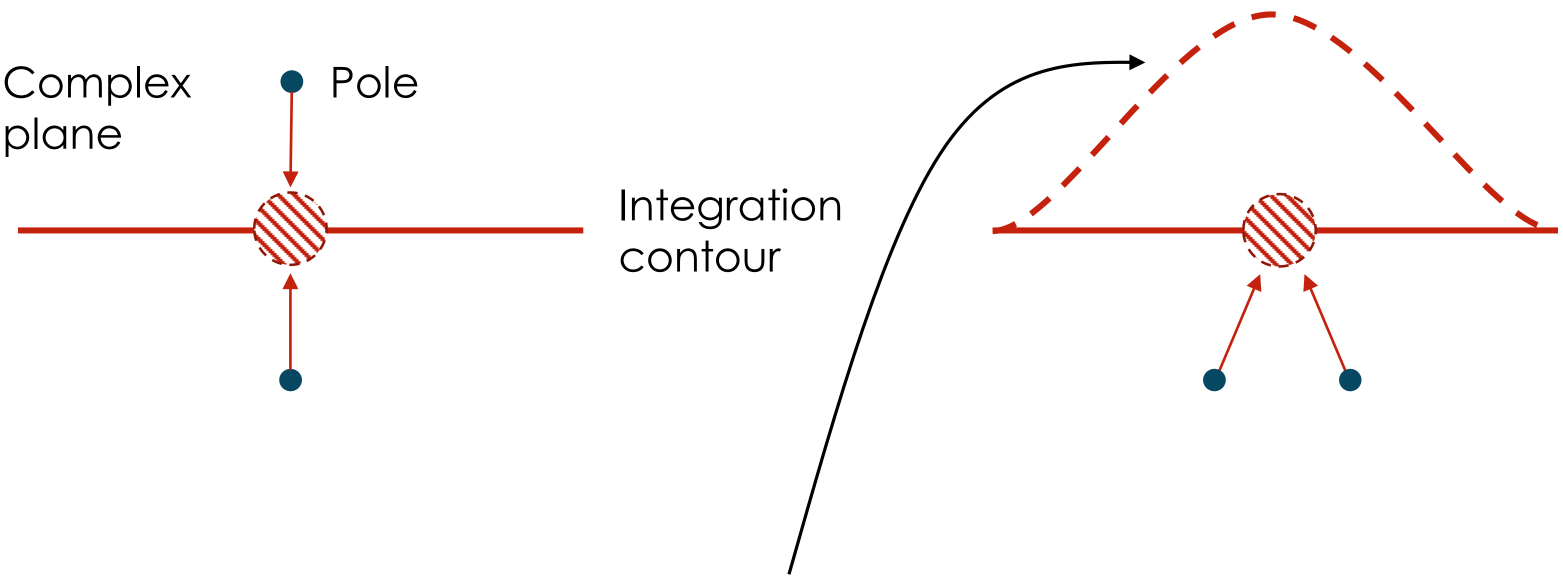

Pinch singularity



Non-pinched singularity

# PINCH SINGULARITIES

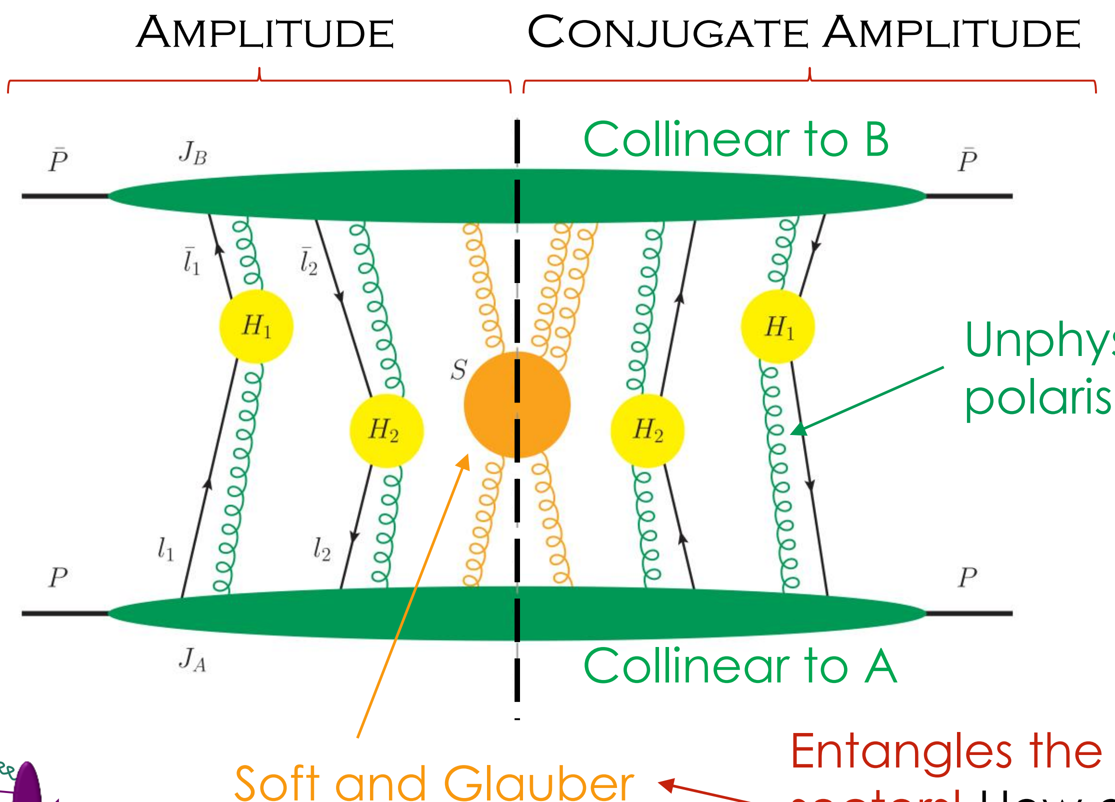
More specifically, looking for leading regions around pinch singularities:

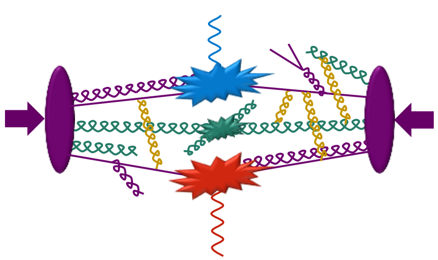



If singularity is not pinched, can deform contour in complex plane away from poles into another momentum region
→ don't have to consider unpinched region explicitly.

# LEADING REGIONS FOR DDY

Leading region for DTMD process producing two colourless systems (double DY):



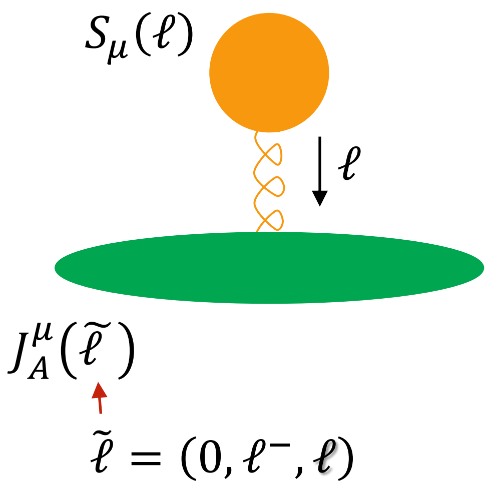



Entangles the two collinear sectors! How do we deal with these exchanges?

# SOFT ATTACHMENTS: APPROXIMATIONS

Consider soft/Glauber attachment into A:

$v_R$ is 'almost' lightcone +




$$\approx S^-(\ell) J_A^+(\tilde{\ell}) = S^-(\ell) \frac{v_R^+ \tilde{\ell}^-}{v_R^+ \tilde{\ell}^- + i\epsilon} J_A^+(\tilde{\ell})$$

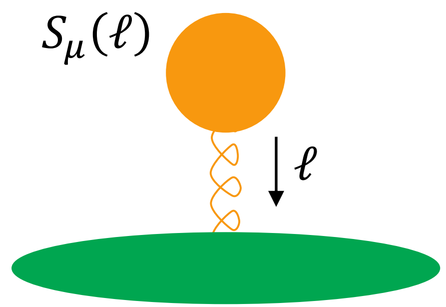
If $\ell$ is SOFT:

$$\approx S_\mu(\ell) \frac{v_R^\mu \tilde{\ell}_\nu}{v_R \cdot \tilde{\ell} + i\epsilon} J_A^\nu(\tilde{\ell}) \qquad \tilde{\ell}_\nu J_A^\nu(\tilde{\ell}) = \underbrace{\tilde{\ell}^- J_A^+(\tilde{\ell})}_{\lambda^n} + \underbrace{\boldsymbol{\ell} \cdot \boldsymbol{J}_A(\tilde{\ell})}_{\lambda^{n+1}}$$

# SOFT ATTACHMENTS: APPROXIMATIONS

Consider soft/Glauber attachment into A:

$v_R$ is 'almost' lightcone +




$\tilde{\ell} = (0, \ell^-, \ell)$

$$\approx S^-(\ell) J_A^+(\tilde{\ell}) = S^-(\ell) \frac{v_R^+ \tilde{\ell}^-}{v_R^+ \tilde{\ell}^- + i\epsilon} J_A^+(\tilde{\ell})$$

If $\ell$ is SOFT:

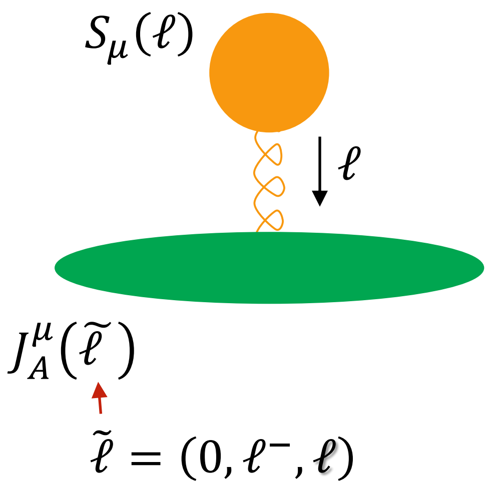
$$\approx S_\mu(\ell) \frac{v_R^\mu \tilde{\ell}_\nu}{v_R \cdot \tilde{\ell} + i\epsilon} J_A^\nu(\tilde{\ell}) \qquad \tilde{\ell}_\nu J_A^\nu(\tilde{\ell}) \approx \tilde{\ell}^- J_A^+(\tilde{\ell})$$

'Grammar-Yennie' approx.: Suitable for application of WARD IDENTITIES.

After summing over all attachments to A: Soft attachment to A → soft attachment to Wilson line(s). See later!

# SOFT ATTACHMENTS: APPROXIMATIONS

Consider soft/Glauber attachment into A:




$$\approx S^-(\ell) J_A^+(\tilde{\ell}) = S^-(\ell) \frac{v_R^+ \tilde{\ell}^-}{v_R^+ \tilde{\ell}^- + i\epsilon} J_A^+(\tilde{\ell})$$

If $\ell$ is GLAUBER $\ell \sim Q(\lambda^n, \lambda^2, \lambda)$, $n \geq 1$:

$$\cancel{\approx S_\mu(\ell) \frac{v_R^\mu \tilde{\ell}_\nu}{v_R \cdot \tilde{\ell} + i\epsilon} J_A^\nu(\tilde{\ell})} \qquad \tilde{\ell}_\nu J_A^\nu(\tilde{\ell}) = \underset{\lambda^2}{\tilde{\ell}^- J_A^+(\tilde{\ell})} + \underset{\lambda^2}{\boldsymbol{\ell} \cdot \boldsymbol{J}_A(\tilde{\ell})}$$

Can't use Ward identities here. How can one handle Glauber attachments?

# TREATING GLAUBERS: SINGLE DY

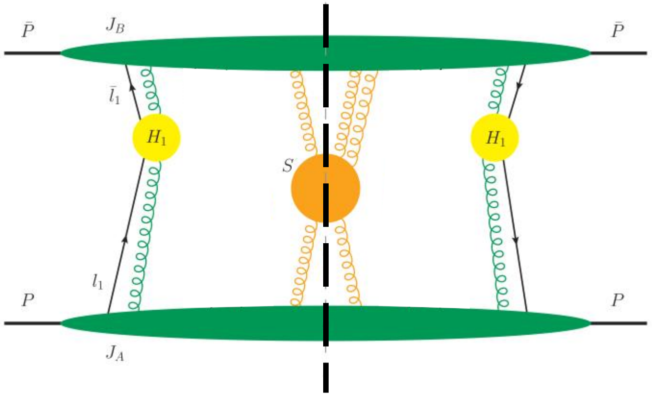
Only known strategy – try to show that pinched Glauber exchanges ‘cancel’.

Shown at all orders for SPS production of colour singlet $V$ by Collins, Soper, Sterman (both TMD + collinear cross section).

Collins, Soper, Sterman, Nucl. Phys. B308 (1988) 833, Collins, pQCD book

Need to sum over all possible cuts of a contributing graph.

If you only measure properties of $V$, Glauber pinches disappear due to unitarity.



# TREATING GLAUBERS: SINGLE DY

Very rough picture:

Glauber pinches are due to possibility of Glauber exchange processes happening 'before' and 'after' hard process in spacetime.

If you measure just properties of $V$, insensitive to what happens in 'final state' after production event.

Σ(all possibilities for final state activity) = 1 (unitarity)

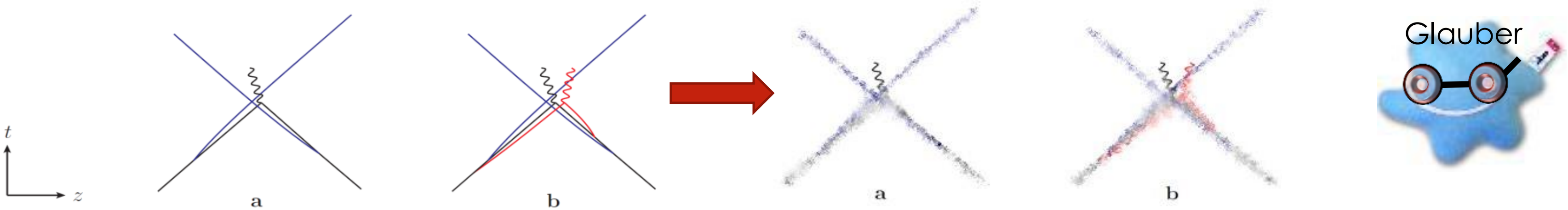
'Final-state' poles blocking deformation out of Glauber region disappear.

N.B. Cancellation of poles does not mean that there are no contributions from Glauber momentum region!
Some contributions cancel to zero…but others can be absorbed into pieces that appear in factorisation formula.

# TREATING GLAUBERS: DOUBLE DY

All order proof of Glauber cancellation for DPS production of two colour singlet systems (both DTMD and collinear) obtained in Diehl, JG, Ostermeier, Plößl, Schäfer, JHEP 1601 (2016) 076.

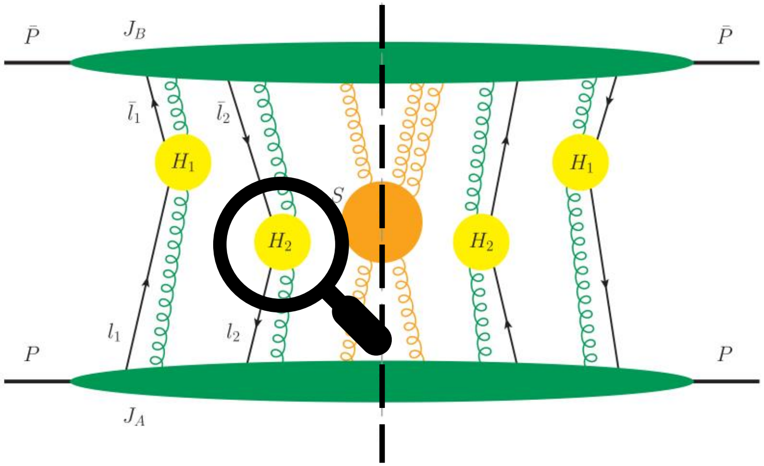
Full proof is very technical, but can get some insight as to why it works by looking at spacetime pictures of single and double scattering:




Argument can be extended to *n*-parton scattering.

# WARD IDENTITIES: COLLINEAR

Next step: use Ward identities to decouple unphysical gluons coupling to hard blobs, physically polarised partons detached using a projector:

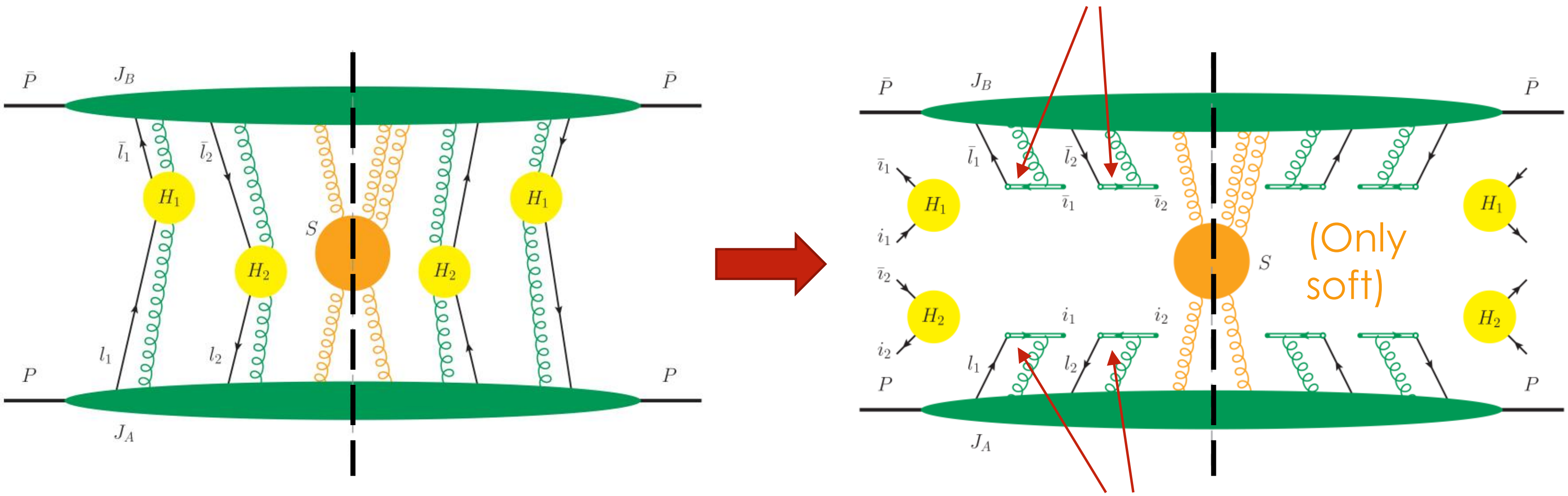



Argument can be copied <u>exactly</u> from the single scattering (SPS) case!

# WARD IDENTITIES: COLLINEAR

Next step: use Ward identities to decouple unphysical gluons coupling to hard blobs, physically polarised partons detached using a projector:

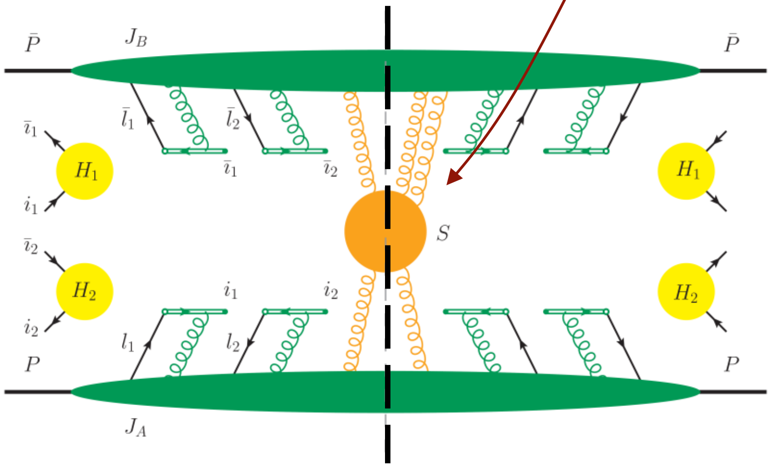
Initial-state Wilson lines, almost light-like along direction of A, $v_B$




Initial-state Wilson lines, almost light-like along direction of B , $v_A$

# TREATING SOFT ATTACHMENTS

Next task is to decouple soft attachments to A and B collinear blobs.




Achieved in Diehl, Nagar, JHEP 1904 (2019) 124.

Not trivial copy of SPS argument. Why?

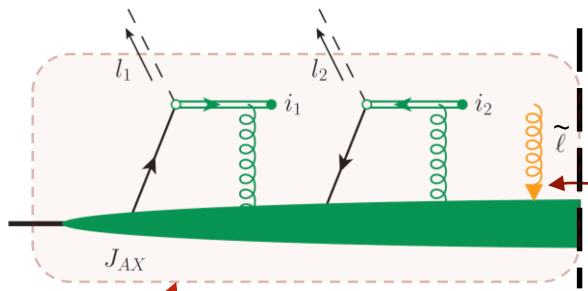
(1) Original SPS argument could ignore transverse component of soft $\ell$ in A/B. For DPS you cannot do this, even for collinear case.

(2) Structure of A/B is more complicated than SPS case – two active lines in amplitude/conjugate with Wilson lines attached.

# WARD IDENTITY FOR ONE SOFT GLUON

Consider one soft attachment into A.

Enough to consider just amplitude, $J_{AX}$:




Arrow means insertion of:

$$\frac{v_R^\mu \tilde{\ell}_\nu}{v_R^+ \tilde{\ell}^- + i\epsilon}$$

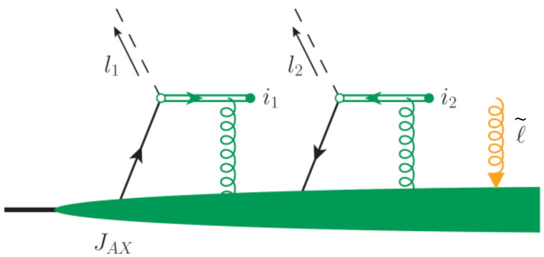
Ward identity: if we were to sum over all possible attachments of soft line *everywhere* in this Green's function, we would get zero.

What do we have? All attachments of soft line into internal lines in $J_{AX}$ blob, and not Wilson lines.

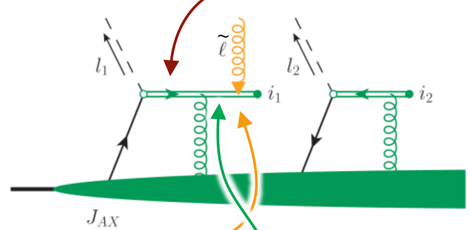
Diehl, Nagar, JHEP 1904 (2019) 124

# WARD IDENTITY FOR ONE SOFT GLUON

Soft attachments 'inside' collinear attachments to WL suppressed.




$= -$

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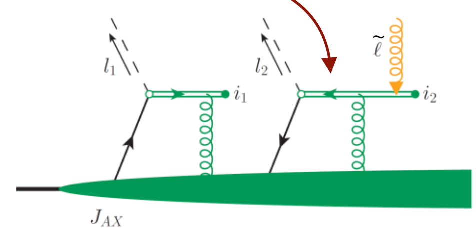
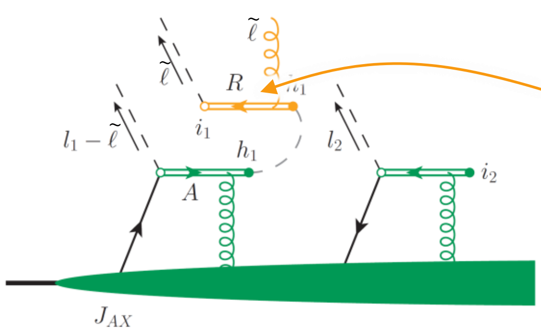
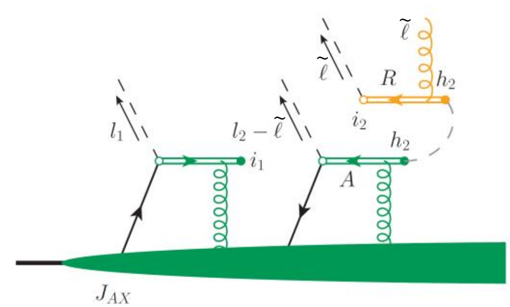



$$\frac{v_R^\mu \tilde{\ell}_\nu}{v_R \cdot \tilde{\ell} + i\epsilon} \frac{v_A^\nu}{v_A \cdot \tilde{\ell} + i\epsilon}$$




$$= \frac{v_R^\mu}{v_R \cdot \tilde{\ell} + i\epsilon}$$




Soft attachments converted to soft attachments to WLs along $v_R$!

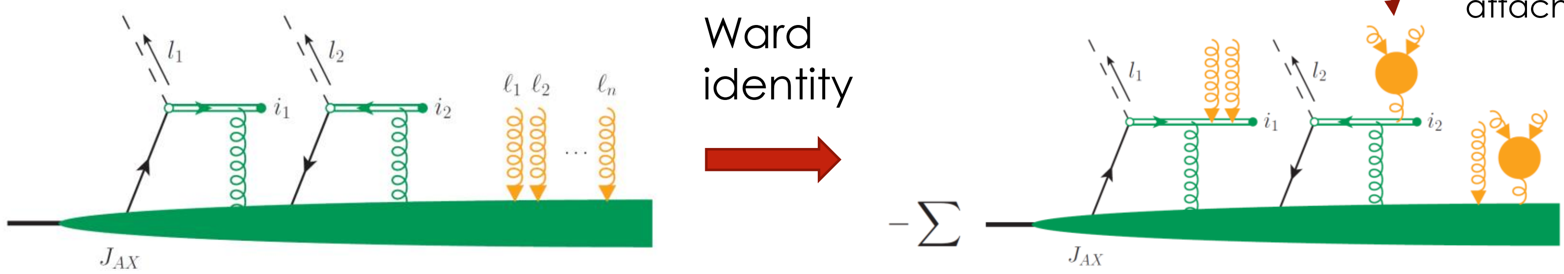
Diehl, Nagar, JHEP 1904 (2019) 124

# WARD IDENTITY FOR MANY SOFT GLUONS

General case:

Ward identity

In initial set, also missing diagrams where soft gluons attach together




Diehl, Nagar, JHEP 1904 (2019) 124

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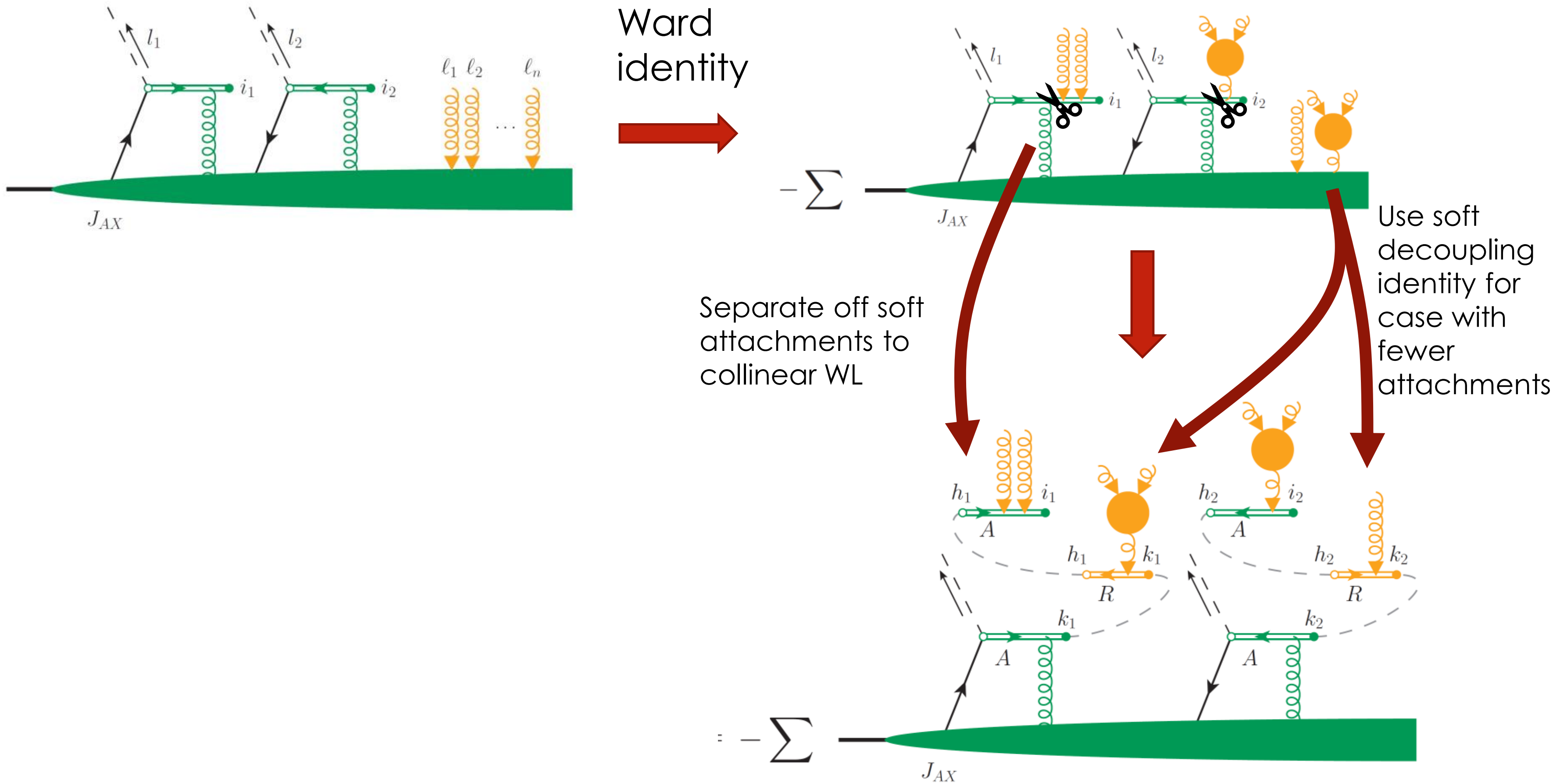


Diehl, Nagar, JHEP 1904 (2019) 124

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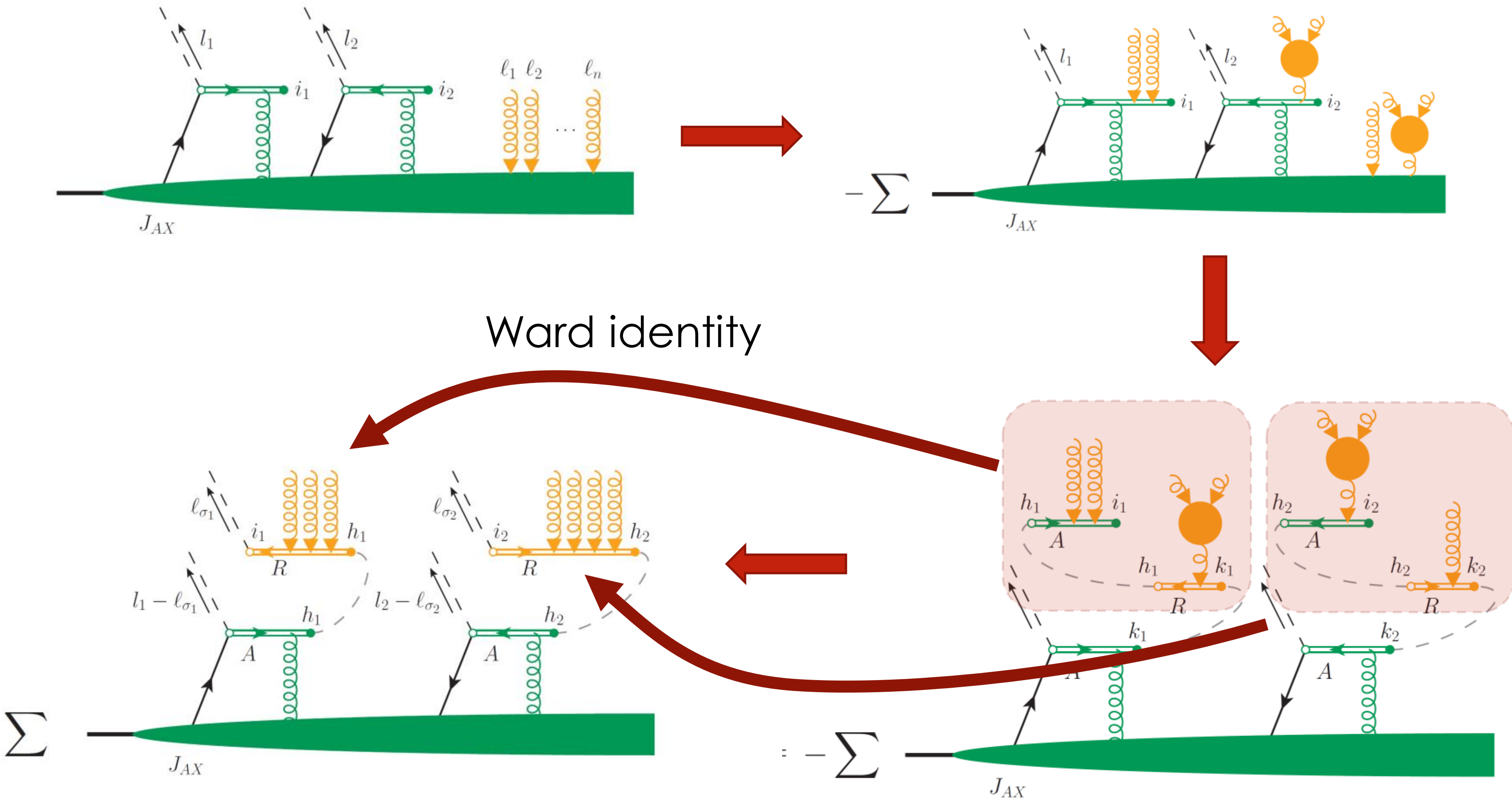
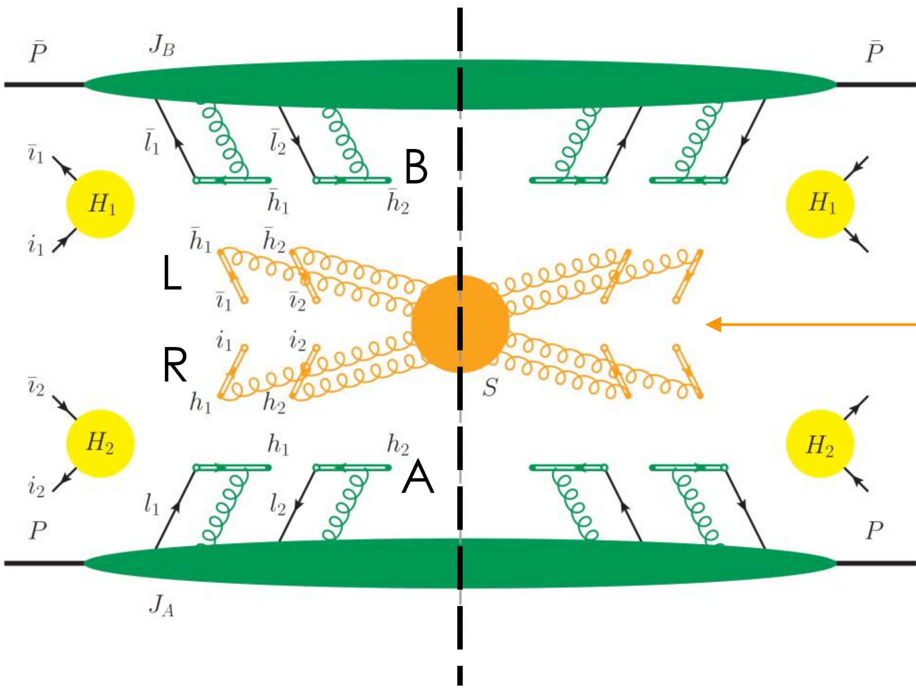



Diehl, Nagar, JHEP 1904 (2019) 124

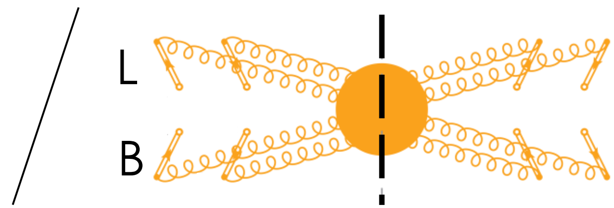
Extends also to *n*-parton scattering!

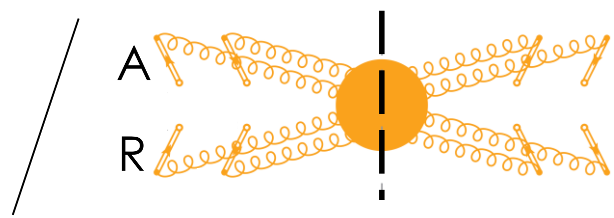
# STATUS AFTER SOFT GLUON DECOUPLING

Each collinear factor contains subtraction terms, which can be factorised into:




Soft factor



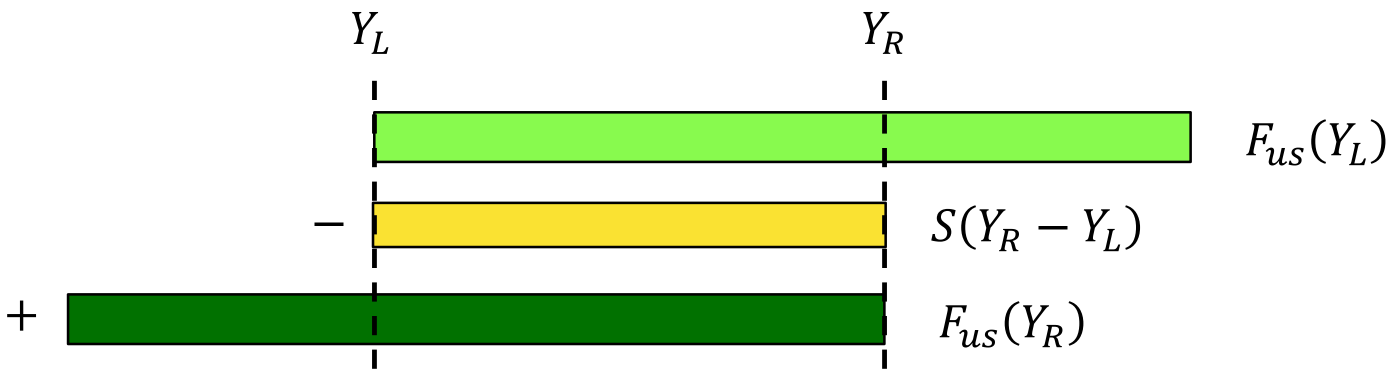




We’ve still not reached the DTMD factorisation formula, which has only DTMDs and partonic cross sections!

# DIVIDING UP THE SOFT FACTOR

For simplicity set $v_A = v_L$, $v_B = v_R$.

Rapidities of WLs act as rapidity cutoffs, define rapidity coverage of different factors:

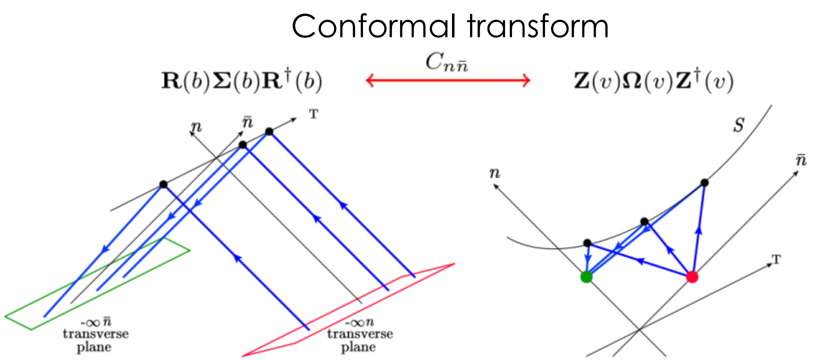



To obtain factorisation formula with just DTMDs, want to ‘chop’ soft function in two and partition between collinear factors. Can one do this?

Complications relative to SPS case: multiple sets of WLs in amplitude/ conjugate, $S$ is a matrix in colour space.

# DIVIDING UP THE SOFT FACTOR

It has been shown that this can be done: Vladimirov, JHEP 1804 (2018) 045 , Buffing, Diehl, Kasemets, JHEP 1801 (2018) 044

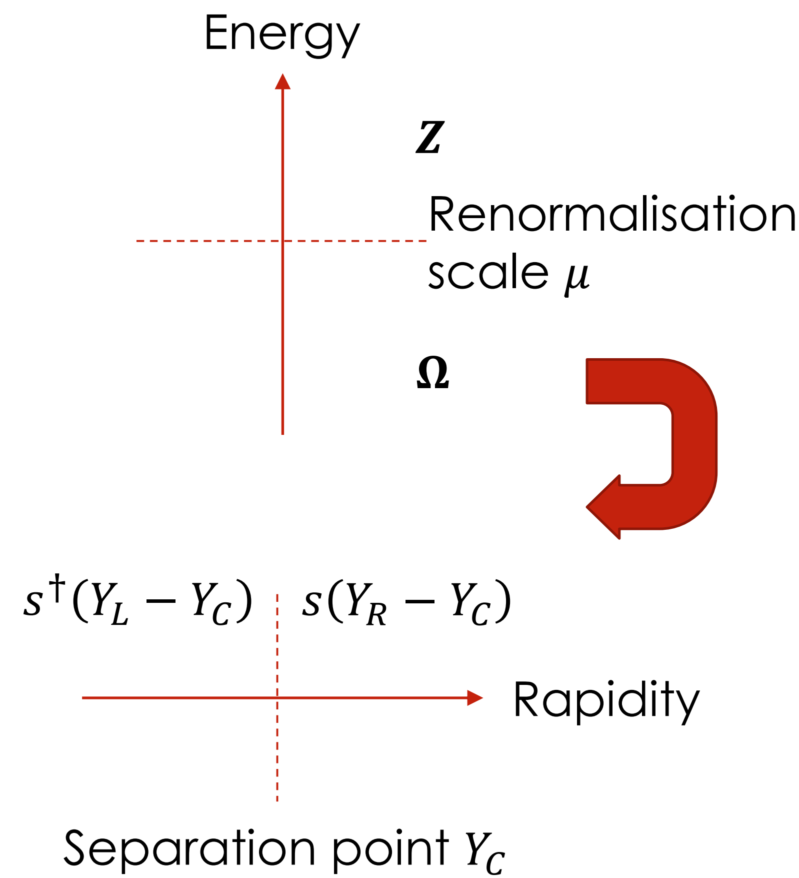



Vladimirov, JHEP 1804 (2018) 045

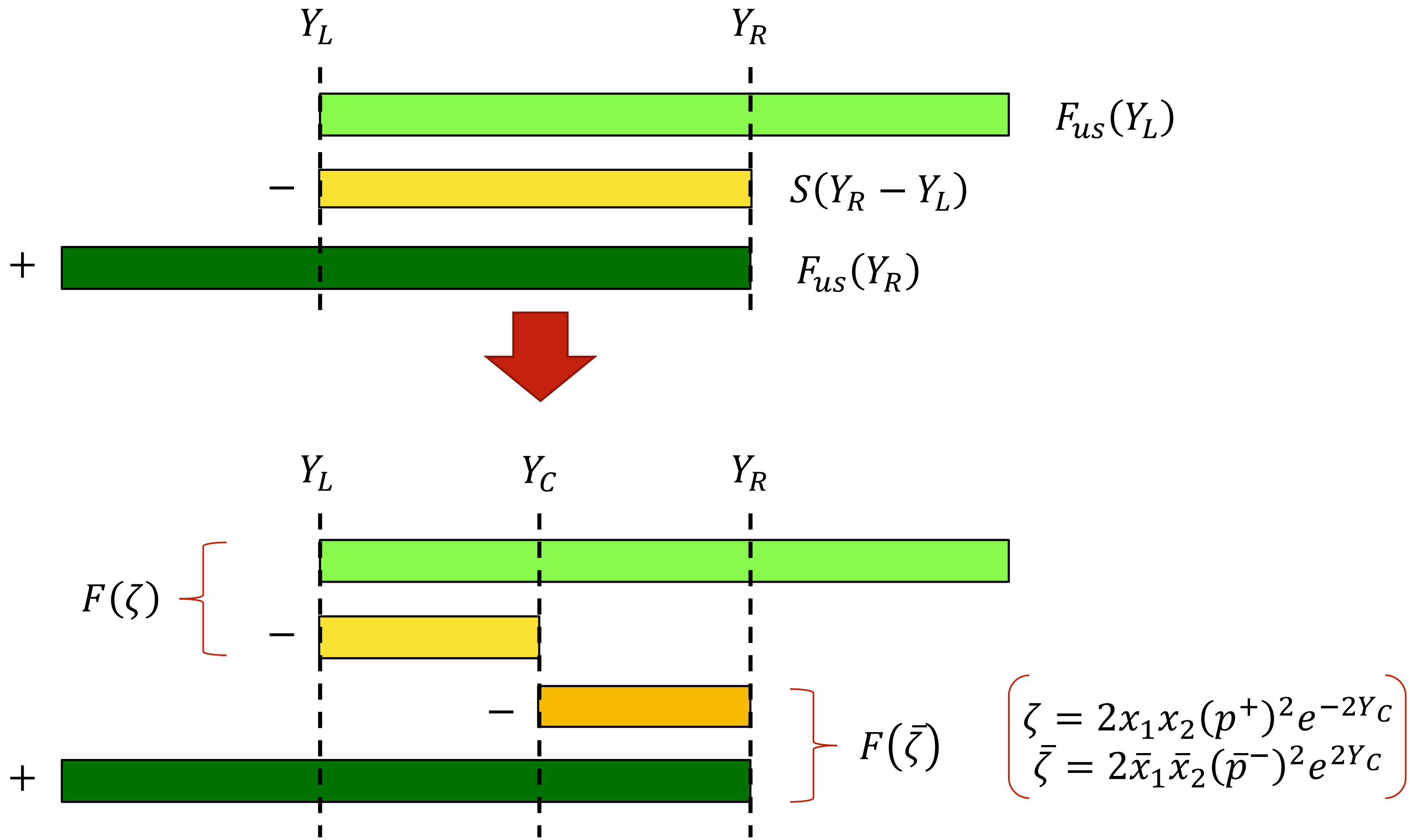
In conformal theory: connection between rapidity divergences and UV divergences.

Fixes structure of ‘rapidity renormalisation factor’ $\boldsymbol{R}$.

Can show that this structure also holds in QCD using inductive argument.



# DIVIDING UP THE SOFT FACTOR




JG, Kasemets, Adv.High Energy Phys. 2019 (2019) 3797394

# SUMMARY

Now we finally achieved TMD factorisation formula for the DPS production of two colourless systems ('double Drell-Yan'):

$$\frac{d\sigma_{DPS}}{d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} \sim \int d^2\boldsymbol{z}_1 d^2\boldsymbol{z}_1 d^2\boldsymbol{y}\, e^{-i\boldsymbol{q}_1\cdot\boldsymbol{z}_1 - i\boldsymbol{q}_2\cdot\boldsymbol{z}_2}\, \hat{\sigma}_{a_1 b_1}\hat{\sigma}_{a_2 b_2}\, F_{b_1 b_2}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \bar{\zeta}) F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta)$$

Most steps of DPS proof also work for general multiple scattering.

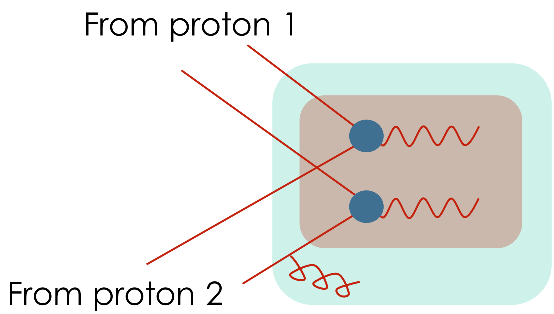
Steps also work in collinear case.

Full factorisation proof obtained for double Drell-Yan (up to some small remaining technical issues that aren't expected to cause problems). Status equivalent to single Drell-Yan proof.

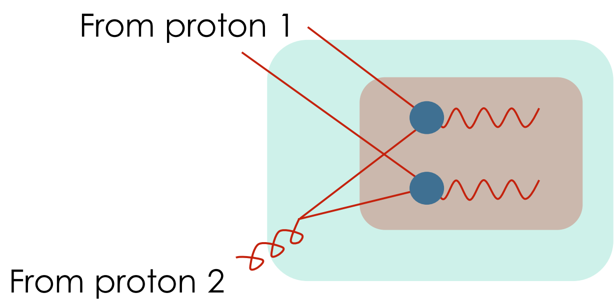
For coloured particle production, factorisation status less clear – will be problems for DTMD factorisation with final-state colour.

# QCD EVOLUTION EFFECTS IN DPS

Consider “zooming out” from the hard processes. What kind of QCD effects can occur?




Emission from single leg. Familiar from single scattering.




‘1→2 splitting’. New effect!

Perturbative calculation at small ***y***

Single PDF

$$F(x_1, x_2, y) \propto \alpha_s \frac{f(x_1 + x_2)}{x_1 + x_2} P\left(\frac{x_1}{x_1+x_2}\right) \frac{1}{y^2}$$

Dimensionful part

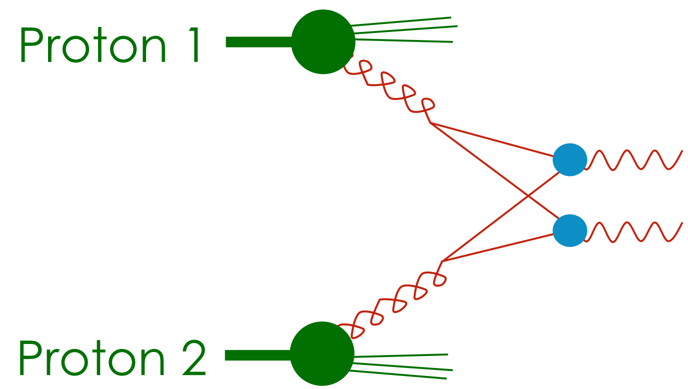
Perturbative splitting kernel

Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

# DOUBLE COUNTING PROBLEMS

Perturbative splitting can occur in both protons (1v1 graph) – gives power divergent contribution to DPS cross section!

$$\int \frac{d^2y}{y^4} = ?$$




This is related to the fact that this graph can also be regarded as an SPS loop correction


$$\frac{\Lambda^2}{Q^4} \longrightarrow \frac{1}{Q^2}$$

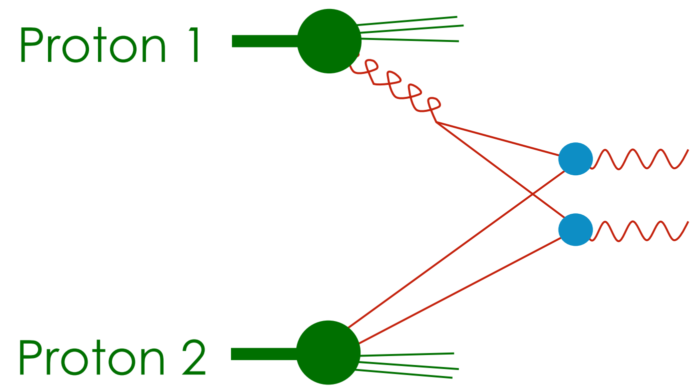
Power divergence!

Diehl, Ostermeier and Schafer (JHEP 1203 (2012)), Manohar, Waalewijn Phys.Lett. 713 (2012) 196, **JG and Stirling, JHEP 1106 048 (2011),** Blok et al. Eur.Phys.J. C72 (2012) 1963 Ryskin, Snigirev, Phys.Rev.D83:114047 ,2011, Cacciari, Salam, Sapeta JHEP 1004 (2010) 065

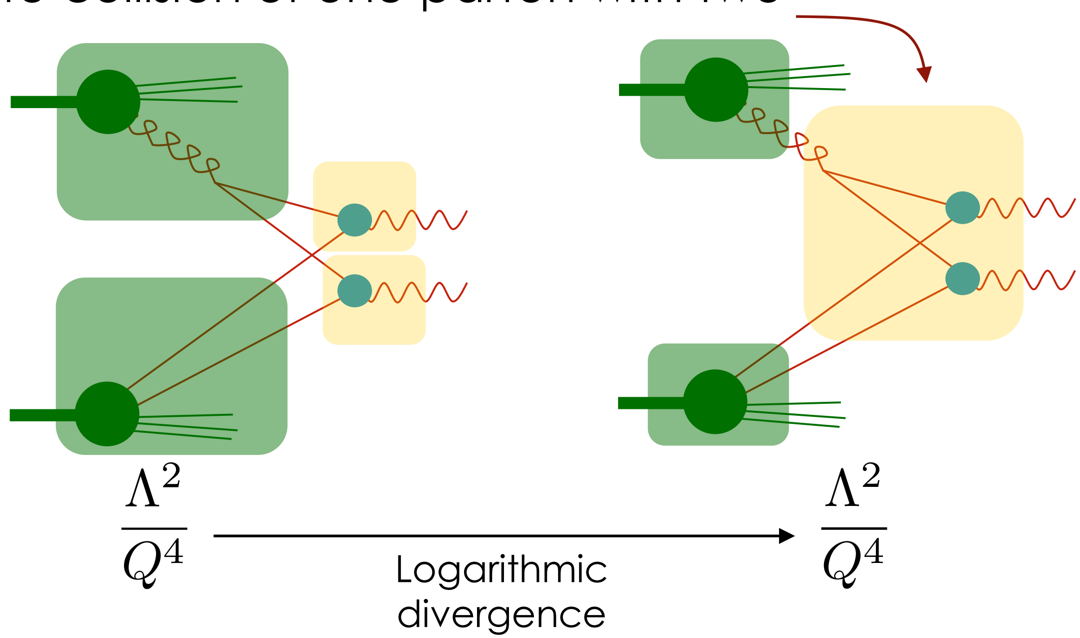
# DOUBLE COUNTING PROBLEMS

Also have graphs with perturbative 1→2 splitting in one proton only (2v1 graph).

This has a log divergence: $\int d^2y/y^2 F_{\text{non-split}}(x_1, x_2; y)$




Related to the fact that this graph can also be thought of as an NLO correction to collision of one parton with two

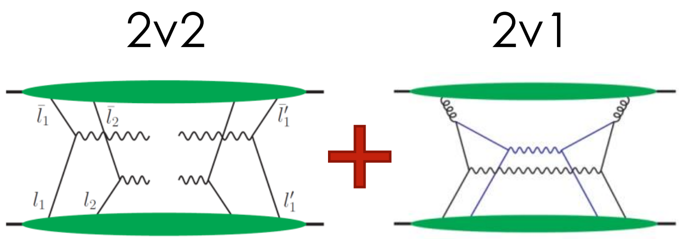



Blok et al., Eur. Phys. J. C72 (2012) 1963
Ryskin, Snigirev, Phys. Rev. D83:114047,2011,
**JG, JHEP 1301 (2013) 042**

# AVOIDING DOUBLE COUNTING: A SOLUTION?

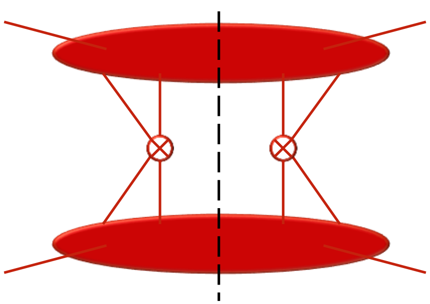
Just remove 1v1 graphs from DPS, include 2v1 graphs?

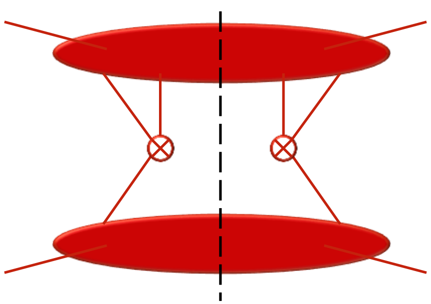
JG, Stirling, JHEP 1106 (2011) 048
Also: Blok et al. Eur.Phys.J. C72 (2012) 1963
Manohar, Waalewijn Phys.Lett. B713 (2012) 196




$$\sigma_{DPS,2v2}^{(A,B)} + \sigma_{DPS,2v1}^{(A,B)} \qquad \sigma_{DPS}^{(A,B)} = \int d^2\boldsymbol{y} \mathrm{d}x_i F_{ik}(\boldsymbol{y}) F_{jl}(\boldsymbol{y}) \hat{\sigma}_{ij\to A} \hat{\sigma}_{kl\to B}$$

$$A^2 + 2AB \neq (A+B)\cdot(A+B)!$$





No well-defined DPD for single hadron.

NLO corrections?

# A NEW SCHEME

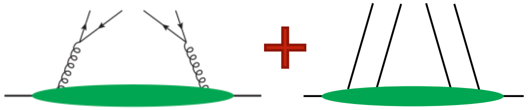
Diehl, Gaunt, Schönwald, JHEP 1706 (2017) 083

Alternative idea: *can* regard some of the 1v1 loop diagrams as DPS if we want.

Split diagrams up into DPS/SPS according to value of partonic separation $y$.

<u>Step1</u>: Insert cutoff into DPS cross section:

$$\sigma_{\mathrm{DPS}} = \int d^2y\, \Phi^2(\nu y)\, F(x_1, x_2; y) F(\bar{x}_1, \bar{x}_2; y)$$



If one just combined this with SPS naively, would have double counting up to scale $\nu$!

# A NEW SCHEME

<u>Step 2:</u> For total cross section for production of *AB*, include a subtraction term to remove double counting.

$$\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$$

Subtraction term constructed by adapting techniques used by Collins in single scattering factorisation treatment.


By construction:

$$\sigma_{tot}(y{\sim}1/Q) \approx \sigma_{SPS} \qquad \sigma_{tot}(y{\sim}1/\Lambda) \approx \sigma_{DPS}$$

Framework extended to DTMD case in Buffing, Diehl, Kasemets, JHEP 1801 (2018) 044