Status of DPD evolution

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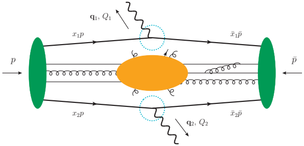
Introduction



DPS factorization: [see Jo's talk]

$$d\sigma_{\text{DPS}} \propto d\sigma^{(1)}_{a_1 b_1}(\mu_1) \otimes d\sigma^{(2)}_{a_2 b_2}(\mu_2)$$

$$\otimes \int d^2y\, F_{a_1 a_2}(y, \mu_1, \mu_2)\, F_{b_1 b_2}(y, \mu_1, \mu_2)$$

theory progress by multiple groups: Politzer; Paver, Treleani; Mekhfi; Gaunt, Stirling; Manohar, Waalewijn; Diehl, Schäfer, Kasemets, Plößl, RN; Blok, Dokshitzer, Frankfurt; Ryskin; Snigirev; ...

- $d\sigma^{(i)}$ are usual partonic cross sections
- F_{ab} are double parton distributions (DPDs)
- y [GeV^{-1}] is inter-parton transverse separation

Focus on DPDs, necessary for pheno applications of full DPS formula

Related issues

- most pheno currently with "**pocket formula**" $d\sigma_{\text{DPS}} \approx d\sigma^{(1)}_{\text{SPS}} d\sigma^{(2)}_{\text{SPS}} / \sigma_{\text{eff}}$

  need to go beyond to account for full physics
- F_{ab} **hard to extract** from experiments

  SPS usually submerges DPS contributions
- complex objects to treat **numerically**: DPDs depend on many variables

  solutions by [Gaunt, Stirling] [Elias, Golec-Biernat, Staśto] [Diehl, RN, Tackmann]

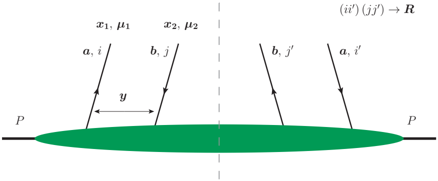
Double parton distributions

A **collinear double parton distribution function** (DPD) in position space is

$$^{R}F_{ab}(x_1, x_2, y, \mu_1, \mu_2) \qquad\qquad x_i \in [0,\, 1];\; x_1 + x_2 < 1;\; |y| > 0$$

and represents the probability of finding two partons a, b in a proton, forming a color representation R, with momentum fractions x_1 and x_2, transverse separation y, and at renormalization scales μ_1 and μ_2.

here assume always R = singlet!



- DPDs **non-perturbative** objects
- renormalization-group equations (**double DGLAP**) can identify DPDs at any scales μ_i, given a set of initial conditions at starting scales μ_i^0
- DPDs obey **sum rules** (like PDFs) $\hookrightarrow$ strong constraint on initial conditions, few studies

[Gaunt, Stirling; Diehl, Plößl, Schäfer; Golec-Biernat, Staśto et al.; Ruiz Arriola et al.; . . .]

DPD evolution: position space

In position space: homogeneous double DGLAP evolution

Position-space DPDs are renormalized in analogy to PDFs.
Generalized DGLAP equations are separate evolution eqn's w.r.t. μ_1 or μ_2

[Diehl, Ostermeier, Schäfer '11]

$$\frac{d}{d \log \mu_1} F_{a_1 a_2}(x_i, y, \mu_i) = 2 \left(P^{(1)}_{a_1 c} \underset{1}{\otimes} F_{c a_2} \right) (x_i, y, \mu_i)$$
$$\frac{d}{d \log \mu_2} F_{a_1 a_2}(x_i, y, \mu_i) = 2 \left(P^{(2)}_{c a_2} \underset{2}{\otimes} F_{a_1 c} \right) (x_i, y, \mu_i)$$

- simpler (disentangled) set of equations $\rightarrow$ independent evol. in μ_1 and μ_2
- unequal-scale evolution $\mu_1 \neq \mu_2$
- hold probabilistic interpretation and enter **DPS cross section**

Some theory

Position-space DPDs generalize the Collins-Soper definition for PDFs:

$$F^{\text{bare}}_{a_1 a_2}(x_i, y) \propto \int \mathrm{d}y^- \mathrm{d}z_i^- \, e^{-i\, x_i z_i^- \, p^+} \, \langle p | \, \mathcal{O}_{a_1}(y, z_1) \, \mathcal{O}_{a_2}(0, z_2) \, | p \rangle \, \big|_{z_i^+ = y_i^+ = 0} \,,$$

where $\mathcal{O}_a(y, z)$ is Wilson-line operator, same as for PDFs [see Collins' book]

- two $\mathcal{O}$'s at finite space-like distance $\Rightarrow$ renormalize each separately

DPD evolution: momentum space

In momentum space: inhomogeneous DGLAP evolution

Reminder: y is transverse spatial separation between two partons

$$F_{ab}^{\text{bare}}(x_i, y, \mu_i) \xrightarrow{\mathcal{F}_y} F_{ab}^{\text{bare}}(x_i, \Delta, \mu_i)$$

Fourier-transform gives **momentum-space** $F_{ab}(x_i, \Delta, \mu_i)$ up to additional renormalization

$\Rightarrow$ **inhomogeneous DGLAP eqn's** (take equal scales $\mu_1 = \mu_2 = \mu$)

$$\frac{d}{d\log\mu} F_{a_1 a_2}(x_i, \Delta, \mu, \mu) = 2 \Bigg(P^{(1)}_{a_1 c} \underset{1}{\otimes} F_{c a_2} + P^{(2)}_{c a_2} \underset{2}{\otimes} F_{a_1 c}$$

$$+ P^{s}_{a_0 \to a_1 a_2} \underset{12}{\otimes} f_{a_0} \Bigg)(x_i, \Delta, \mu, \mu) \qquad \text{LO } P^s = \text{regular part of splt fct's}$$

- enter the **DPD sum rules**, but **not** the X-sec [Gaunt, Stirling '09; Diehl, Plößl, Schäfer '18]
- also depend on PDF set
- also valid for **integrated** DPDs $F_{ab}(x_i, \mu)$ [Kirschner '79; Shelest, Snigirev, Zinovjev '82]
- **note:** different divergent behaviour from position-space DPDs
  $\hookrightarrow$ non-trivial matching to PDFs to go back to position-space

Issues with numerical evolution

Challenges in DPD evolution

- $F_{a_1 a_2}(x_1, x_2, y, \mu_1, \mu_2)$ are $5D$ (compared to $2D$ PDFs)
- one set contains $\mathcal{O}(4\, N_f^2)$ DPDs
- **many variables** & **multiplicity** $\Rightarrow$ extension of current approaches for PDFs has *large* memory impact optimistic estimate ≈ 1 TB!
- can reduce # of variables, i.e. limit to $\mu_1 = \mu_2$ or integrate over y
  $\hookrightarrow$ but we would miss important information!

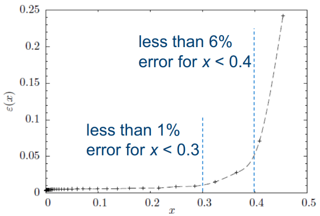
What is desirable for DPD phenomenology

- fast and accurate **evolution** and **interpolation** of DPDs
- **different factorization scales** (μ_1, μ_2) (needed e.g. for $W + J/\Psi$)
- higher-order and higher-N_f DGLAP, possibly include polarization (needed e.g. for like-sign WW)
- full dependence on transverse separation $y \Rightarrow$ see DPS cross-section formula
- **flexibility** on choice of DPD model for starting conditions

Evolution codes

Gaunt-Stirling DPD code (private) [Gaunt, Stirling '09]

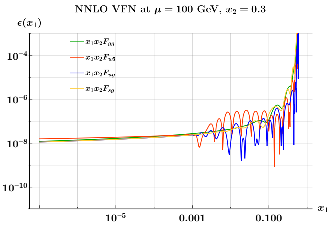
- same scale $(\mu_1 = \mu_2)$ can also do $\mu_1 < \mu_2$: evolve to μ_1, then 2nd parton $\mu_1 \to \mu_2$
- LO DGLAP, fixed $N_f = 3$ (push to $N_f = 4$?)
- up to $\mathcal{O}(10^{-3} - 10^{-2})$ relative accuracy



[J. Gaunt's talk @ MPI10]

Only publicly available set: `GS09` [gsdpdf.hepforge.org]

- starting set based on products of `MSTW2008`
- inhomogeneous DGLAP equations



Recent progress: **ChiliPDF** project [Diehl, RN, Tackmann]

- unequal-scale evolution $(\mu_1 \neq \mu_2)$
- NNLO DGLAP, with NNLO matching at flavor-transition scales
- flexible input (numerical, analytical, ...) of starting conditions
- fast computation (1 to 100 GeV in $\mathcal{O}(10$ s$)$)

To the best of my knowledge, private DPD evolution codes have been developed by other groups (e.g. in [Elias, Golec-Biernat, Staśto '18])

DGLAP evolution in a code (for PDFs)

Discretized DGLAP equations

Set of integro-differential equations expressing the RGE of the PDFs

$$\boxed{\frac{d}{d\log\mu} f_a(x,\mu) = 2\,(P_{ab}\otimes f_b)(x,\mu),} \quad \text{with } P_{ab} = \frac{\alpha_s}{4\pi}\sum P_{ab}^{(k)}\left(\frac{\alpha_s}{4\pi}\right)^k$$

becomes linear **system of differential equations** when discretized as

$$\frac{d}{d\log\mu}\tilde{f}_m = 2\,\tilde{P}_{mn}\,\tilde{f}_n \quad \Rightarrow \text{ solve numerically with **Runge-Kutta** method}$$

Flavor transition

Introducing the mass m of an heavy quark, transition between theories with different N_f at arbitrary scale $\tilde{\mu}$ is done by imposing **matching conditions**

$$f_a^{(N_f+1)}(x,\tilde{\mu}) = \left(A_{ab}(\tilde{\mu})\otimes f_b^{(N_f)}(\tilde{\mu})\right)(x)$$

[Buza et al. '96, '98]

- A_{ab} can be expanded in orders of α_s and $\log(\tilde{\mu}/m)$
- lowest-order matching given by $A_{ab}^{(0)} = \delta_{ab}$.

Extension to position-space DPDs

- **double DGLAP equations** in position space:

$$\frac{d}{d\log\mu_1} F_{a_1a_2}(x_i, y, \mu_i) = 2\left(P^{(1)}_{a_1c} \underset{1}{\otimes} F_{ca_2}\right)(x_i, y, \mu_i)$$
$$\frac{d}{d\log\mu_2} F_{a_1a_2}(x_i, y, \mu_i) = 2\left(P^{(2)}_{ca_2} \underset{2}{\otimes} F_{a_1c}\right)(x_i, y, \mu_i)$$

- use mom. conservation $F_{a_1a_2}(x_1, x_2, y) = 0$ for $x_1 + x_2 > 1$
  ↪ obtain μ_1-evolution independent of x_2 and viceversa
- 2D interpol. $F(x_1, x_2) = \sum_{ij} F_{ij}\, w_i(x_1)\, w_j(x_2)$
- DGLAP kernels same as for PDFs → matrices $\tilde{P}_{mn}$ are the same!

The y-dependence

- y-dependence also interpolated on grids $y \in [y_{\text{min}}, \infty[$
- evolution **does not depend on y** ⇒ often recycle evolution operators

For momentum-space (or integrated) DPDs

- two-step procedure cannot be applied
- inhomogeneous term: evolve DPDs and PDFs at the same time

State of the art of ChiliPDF [Diehl, RN, Tackmann]

ChiliPDF (**Ch**ebyshev-based **I**nterpolation **Li**brary for **PDF**s) is a C++14 library currently under development, intended for public release

- based on Chebyshev interpolation
  $\longrightarrow$ reduce memory impact, enhance numerical accuracy
- up to **NNLO DGLAP evolution** [Moch et al. '04]
  and $\mathcal{O}(\alpha_s^2)$ **flavour matching** [Ablinger et al. '14] for PDFs and DPDs!
- control DGLAP **parameters** (start., match. scales, ...) and **flexible input** (PDFs & DPDs)
- **results for PDFs**: evolution from 1 to 100 GeV (timings on my laptop)
  - variable-flavor NNLO, **70 pts**, RK step $\mathbf{0.02}$ $\longrightarrow t \approx$ **35 ms** (+ 3 s init.)
  - $\epsilon_{\text{rel}} \leq \mathcal{O}(10^{-8})$ for $x < 0.8$
- **results for DPDs**: full DPD set evolution from 1 to 100 GeV
  - variable-flavor NNLO, **70 pts**, RK step $\mathbf{0.05}$ $\longrightarrow t \approx$ **35 s** (+ 3 s init.)
  - $\epsilon_{\text{rel}} \leq \mathcal{O}(10^{-4})$ for $x_1 + x_2 < 0.8$

Currently also...

- mixed **QCD×QED** DGLAP and flavor matching — with F. Fabry
- **polarized** distributions (important for DPS [see T. Kasemets talk]) — with P. Plößl
- **WIP**: extend latter to DPDs; finally make the code public

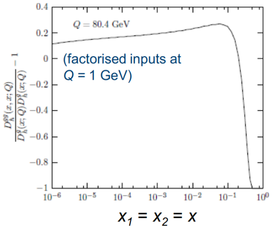
Determining DPDs from theory

Factorized Ansatz

Assumption: $F_{ab} = f_a(x_1, \mu_1)\, f_b(x_2, \mu_2)\, G(y)$
leads to **DPS pocket formula**:

$$\sigma_{\text{DPS}} = \frac{\sigma^{(1)}_{\text{SPS}} \sigma^{(2)}_{\text{SPS}}}{\sigma_{\text{eff}}}$$

- DGLAP evolution deviates from factorized form
- violate mom. conservation for $x_1 + x_2 \geq 1$
- miss pQCD splitting from sPDF



[J. Gaunt talk 2010; Gaunt, Stirling '09]

Lattice QCD

First couple of moments of DPDs computed for the pion [Bali et al. '18], ongoing work on nucleon [Bali et al.; see Zimmermann talk at MPI2019]

More models

- constituent quark models [Rinaldi, Scopetta, Ceccopieri]; "bag" model [Manohar, Waalewijn]; valence quark models [Broniowski, Ruiz Arriola]
- KMR approach [Golec-Biernat, Staśto]

A QCD-motivated Ansatz

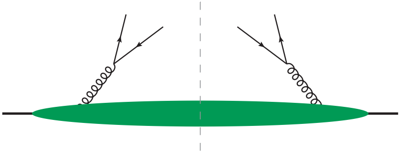
A class of DPD Ansätze at small y

When $y \to 0$ DPDs are sum of "**splitting**" and "**intrinsic**" piece

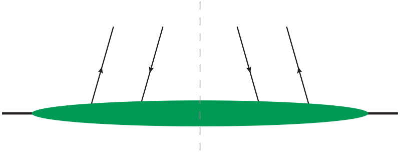
$$F(y)|_{y\to 0} = F_{\text{spl}}(y) + F_{\text{int}}(y)$$

this comes from OPE!

At larger y, DPDs can be modeled so that $F(y) \to 0$ as $y \to \infty$.



splitting (F_{spl} or "$\mathbf{1}$")



intrinsic (F_{int} or "$\mathbf{2}$")

perturbative splitting factorization
$F_{ab}(y;\mu) = V_{a_0,\,ab}(y;\mu) \otimes f_{a_0}(\mu)$

- LO: $F_{ab} \propto P_{a_0 \to ab} \cdot f_{a_0}$ [Diehl et al. '11]
- NLO: see [Diehl et al. '19]
- multiply with y-damping factor for large y

twist-4 distribution at small y, model for now

- take GS product Ansatz $F_{ab} = f_a \cdot f_b \cdot \Phi$ with $\Phi(x_1, x_2, y)$ suppr. factor
- possible to reduce model-dependence by adding twist-4 dist. renormalization

DPD luminosities

DPDs-dependent term in DPS cross section

$$\mathcal{L}_{a_1 a_2 b_1 b_2}(x_i, \bar{x}_i, \mu_i) = \int d^2y \, F_{a_1 a_2}(x_i, y, \mu_i) \, F_{b_1 b_2}(\bar{x}_i, y, \mu_i) \, ,$$

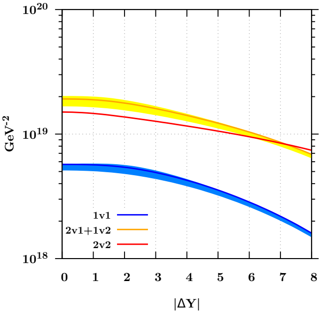
with $F = F_{\text{spl}}^{\text{“1”}} + F_{\text{int}}^{\text{“2”}}$

$$\mathcal{L} = \mathcal{L}_{\text{1v1}} + \mathcal{L}_{\text{1v2}} + \mathcal{L}_{\text{2v1}} + \mathcal{L}_{\text{2v2}}$$

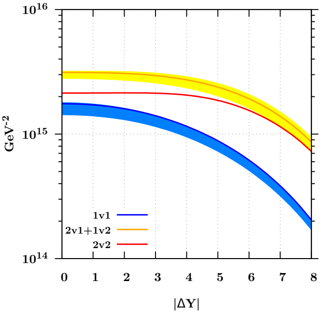
respectively from $F_{\text{spl}} F_{\text{spl}}$ (1v1), $F_{\text{spl}} F_{\text{int}}$ (1v2), $F_{\text{int}} F_{\text{spl}}$ (2v1), $F_{\text{int}} F_{\text{int}}$ (2v2)

bands from variation of cutoff ν on integration $\int d^2y$

ΔY = rapidity separation of two systems



$\mathcal{L}_{gggg}\,(m_{J/\Psi}, m_{J/\Psi})$



$\mathcal{L}_{gggg}\,(m_W, m_{J/\Psi})$

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with $F = F_{\text{spl}}^{\text{"1"}} + F_{\text{int}}^{\text{"2"}}$

$$\mathcal{L} = \mathcal{L}_{1\text{v}1} + \mathcal{L}_{1\text{v}2} + \mathcal{L}_{2\text{v}1} + \mathcal{L}_{2\text{v}2}$$

respectively from $F_{\text{spl}} F_{\text{spl}}$ (1v1), $F_{\text{spl}} F_{\text{int}}$ (1v2), $F_{\text{int}} F_{\text{spl}}$ (2v1), $F_{\text{int}} F_{\text{int}}$ (2v2)

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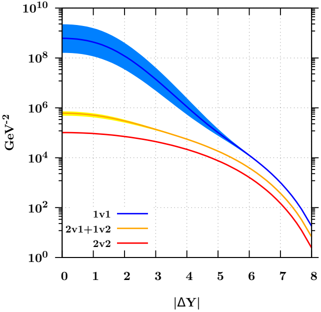
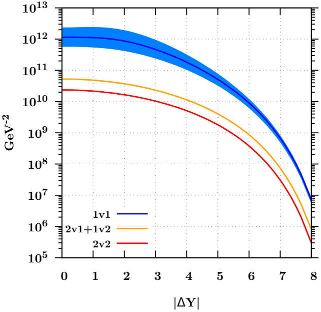
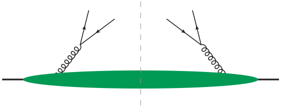


$\mathcal{L}_{b\bar{b}\bar{b}b+\bar{b}bb\bar{b}}\,(m_Z,\, m_H)$



$\mathcal{L}_{gggg}\,(m_H,\, m_H)$

Quark-mass effects on LO splitting



DPD massless splitting formula:

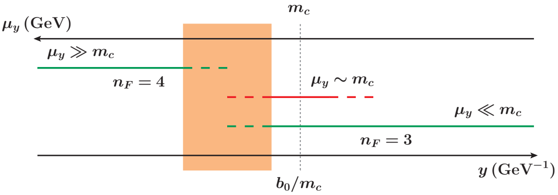
- **at LO**:

$$F_{ab}^{(\mathrm{spl})}(x_i, y; \mu_y, \mu_y) = \frac{\alpha_s}{2\pi^2}\frac{1}{y^2}\, T^{(0)}_{a_0\to ab}\left(\frac{x_1}{x_1+x_2}\right)\frac{f_{a_0}(x_1+x_2;\mu_y)}{x_1+x_2}$$

- $T^{(0)}_{a_0\to ab}(x)$ analog of $P^{(0)}_{aa_0}(x)$; quarks are treated as **massless**
- set $\mu_y \simeq b_0/y$ (with $b_0 = 2e^{-\gamma}$) to cancel large $\log(y\,\mu/b_0)$ from h.o.
- evaluate PDF $f_{a_0}(\mu_y)$ with different N_f at different scales μ_y: smaller y corresponds to higher N_f and viceversa

Arbitrariness in the choice of the transition scales between regions with different starting N_f!

or also: What to do in the region where m_c cannot be neglected?

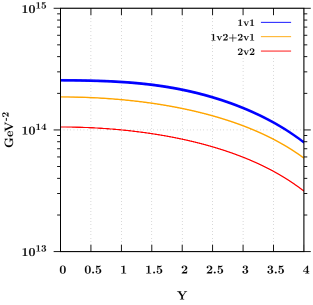


Under scrutiny: effect of transition-scale variations

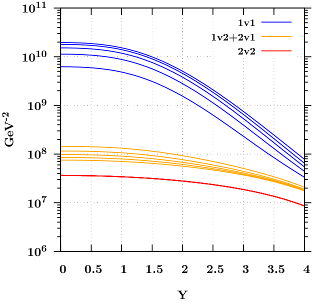
Variations of $y_{c,b}$ can produce sizeable scale uncertainties in $\mathcal{L}$!

- scale arbitrariness introduced by transition scales y_c $(3 \to 4)$ and y_b $(4 \to 5)$
- take $y_i = b_0/(k\, m_i)$ with $k = 1 \ldots 5$
- must have $\mu_y \gg m_i$ to safely assume that quark i is massless

$\mathcal{L}_{gggg}$ (25 GeV, 25 GeV)



$\mathcal{L}_{b\bar{b}\bar{b}b+\bar{b}bb\bar{b}}$ (25 GeV, 25 GeV)



here plot luminosity for 1^{st} system at zero rapidity, 2^{nd} system at rapidity Y

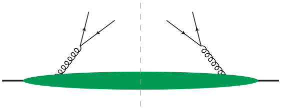
Under scrutiny: mass dependence of splitting DPDs

Work in progress

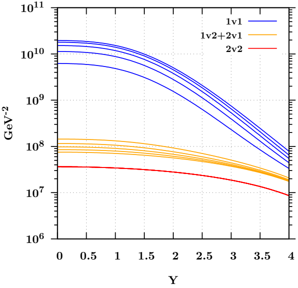
Include the mass dependence in the LO splitting (**multiscale problem**)

$$F_{ab}(y;\mu) = V_{a_0,\,ab}(y, m_Q;\mu) \otimes f_{a_0}(\mu)$$

$\hookrightarrow$ can include $Q\bar{Q}$ production at $\mu_y \sim m_Q$



$\mathcal{L}_{b\bar{b}\bar{b}b+\bar{b}bb\bar{b}}\,(25\text{ GeV}, 25\text{ GeV})$
massless splitting



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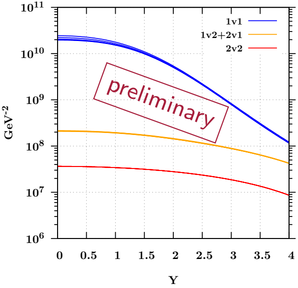
massive splitting



Scale variation bands shrink ⇒ result not much dependent on $y_{c,b}$!

Summary

Double parton distributions:

- complex objects, but **necessary for** full-QCD DPS **phenomenology**
- need **starting conditions** for evolution, must be assumed for now...
- **sum rules** offer strong constraint $\rightarrow$ need further studies

Tools:

- **Gaunt-Stirling** `GS09` only publicly available set of DPDs
  $\hookrightarrow$ LO, $n_F = 3$, based on `MSTW08` PDF set
- some studies done using private tools [Elias, Golec-Biernat, Staśto '18]
- new `ChiliPDF` under development [Diehl, RN, Tackmann *to appear*]
  $\hookrightarrow$ intended for publication
  $\hookrightarrow$ extend to NNLO, include flavor-matching (e.g. $n_F = 3 \ldots 5$)
  $\hookrightarrow$ faster computation & flexible input of boundary conditions

Work in progress:

- ongoing work on inclusion of **polarization** in evolution
- ongoing study on effects of **scale choices** and treatment of **heavy quarks**
- **colour non-singlet** representations: in principle suppressed, how much?

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