universite PARIS-SACLAY

Constraining gluon PDFs with quarkonium production

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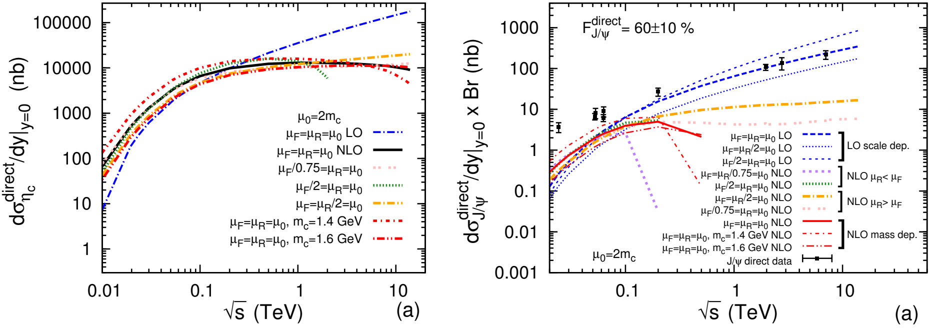
Quarkonia as Tools - Aussois
13-19 January 2020

Quarkonia & PDFs: Phenomenology

problem of negative cross-sections - η_c and J/ψ at NLO



comparison of η_c (left) and J/ψ (right) differential cross-sections at NLO with different scale choices of μ_R and μ_F with CTEQ6M

[Y. Feng, J.-P. Lansberg, J.X. Wang, Eur.Phys.J. C75 (2015) no.7, 313]

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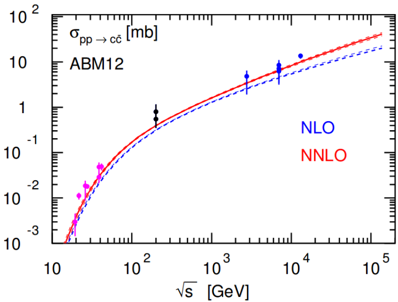
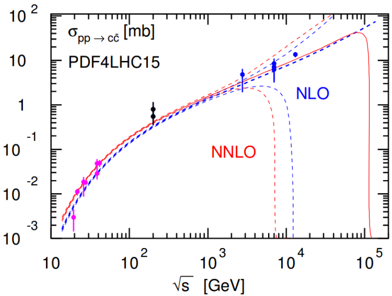
negative cross-sections - open $c\bar{c}$ production at N2LO





open $c\bar{c}$ production at NLO/N2LO, comparison with different PDFs (ABM12, PDF4LHC15) **[Accardi et al., Eur.Phys.J. C76 (2016) no.8, 471]**

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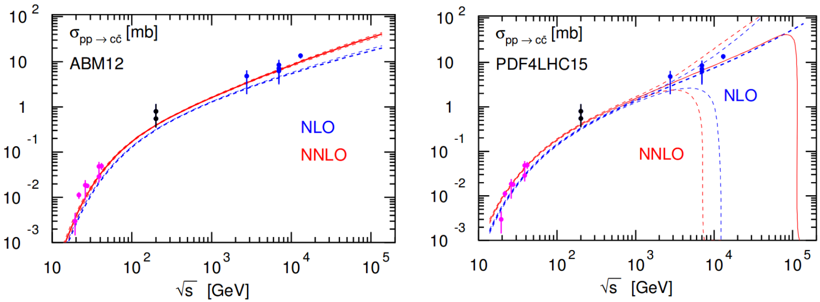

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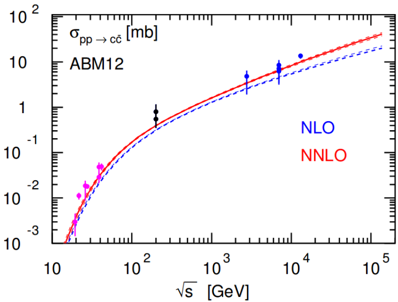

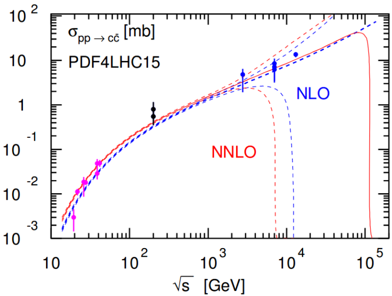

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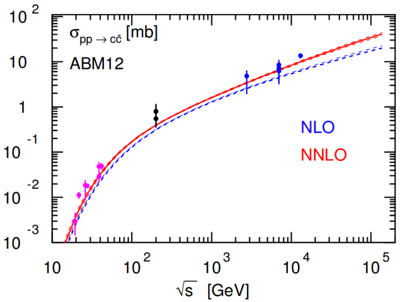

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- full scale analysis not yet performed
  $\to$ therefore one cannot rule out the possibility of negative cross-sections with positive PDFs

η_c at NLO - historical development

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- I confirm that everybody above was correct ;-)

Schuler 1994 - Review

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  - small x-behaviour of gluon and sea-quark distributions
  - behaviour of partonic cross-sections away from threshold

Mangano 1997 - Proceedings

¶Most of the remarks which follow have already been made by G. Schuler in his '94 review [9]. Schuler at the time had available the full NLO corrections to η production, as well as the leading small-x behaviour of the χ cross sections. It is a pity that those remarks have passed almost unnoticed in the community!

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- arrives to similar conclusions that steeper gluon PDF choices will give better results because real corrections become less relevant (see Schuler's table) at high hadronic energies
- confirms that partonic high-energy limit has the general structure,

$$\lim_{z\to 0} \hat{\sigma}_{gg} = 2C_A \frac{\alpha_s}{\pi} \hat{\sigma}_{\mathrm{Born}} \left(\log \frac{M^2}{\mu_F^2} - C_J \right), \quad (1)$$

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where C_J is a process-dependent quantity

high energy behaviour - partonic cross-section

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- some values for C_J:
  - $C_J = 1$ for pseudo-scalar quarkonia $\eta_{c/b}$
  - $C_J = 43/27$ for $\chi_{c/b,0}$
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- but $\log\mu_F^2$-dependence is universal and process-independent $\rightarrow$ with flat PDFs there is strong dependence on factorisation scale μ_F

η_c - factorisation scale μ_F

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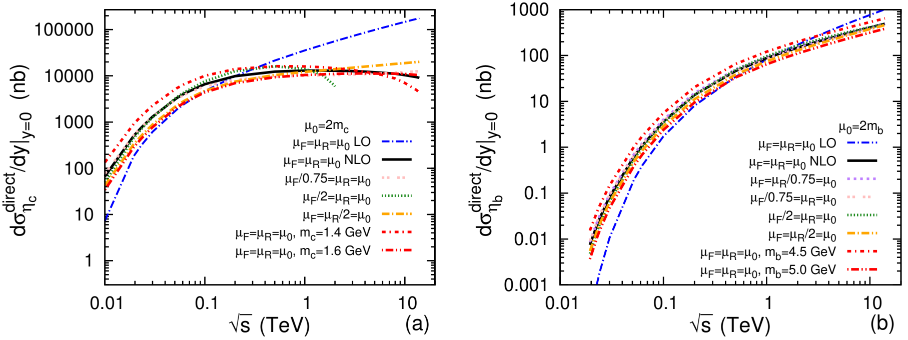
- let's make a comparison with η_b, why do we not encounter negative cross-sections?

η_c versus η_b



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  - note however that the NLO result start to deviate from LO at large $\sqrt{s}$

Quarkonia & PDFs: Results

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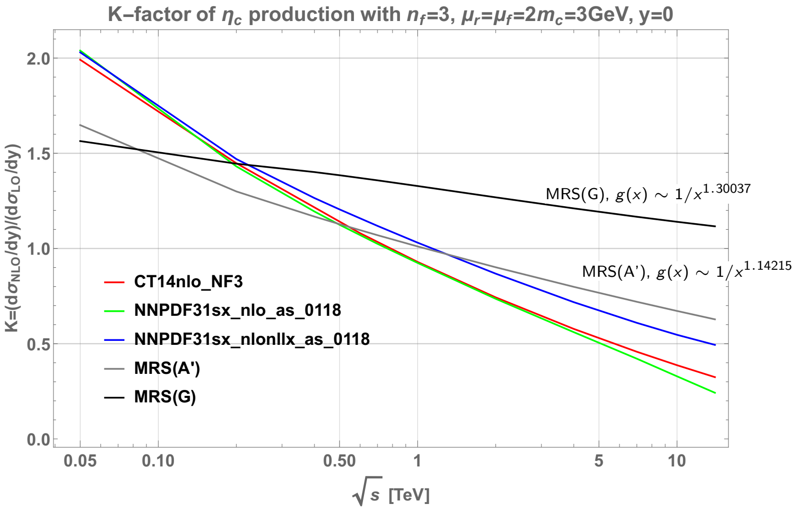
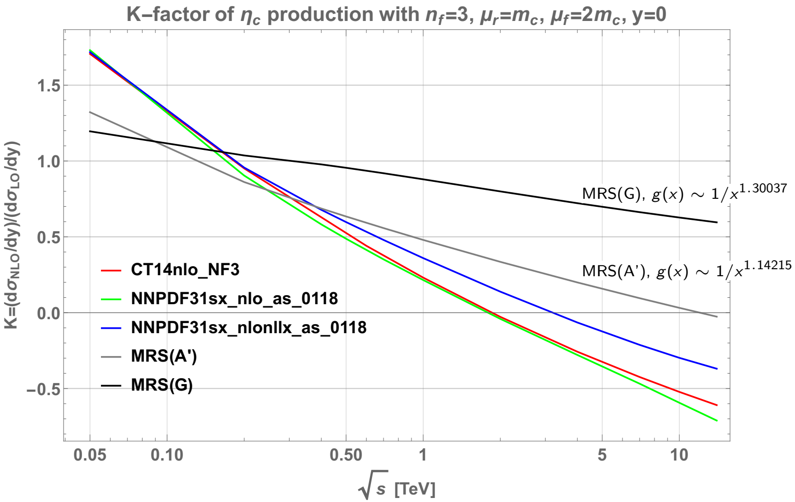
  - $\mu_R = m_c = 1.5$GeV, $\mu_F = 2m_c = 3$GeV
    - lower renormalisation choice leads to larger $\alpha_s \rightarrow$ real emission contributions become more important; the objective is to see the impact of the PDFs on the real corrections

K-factor at $y = 0$ - $\mu_R = \mu_F = 2m_c = 3$GeV



K-factor at y=0 as a function of energy and with different PDF choices. Default scale choice used $\mu_R = \mu_F = 2m_c = 3$GeV.

K-factor at $y = 0$ - $\mu_R = m_c = 1.5\text{GeV}, \mu_F = 2m_c = 3\text{GeV}$



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NNPDF31 - sxNLL extension

- NNPDF31NLL gives slightly better results than NNPDF31

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- this is expected from the previous analysis because NNPDF31NLL has steeper gluon PDFs than NNPDF31

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[comment by Nestor Armesto]

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    [comment by Nestor Armesto]
  - PDFs are 'pointlike' objects/functions without any spatial extensions. Strong saturation effects are expected to happen only in finite-size objects. **[comment by Renaud Boussarie]**

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- How? Let's now generate Replicas with the CT14nlo PDFs and try to identify the gluon PDF shapes of those Replicas that yielded positive cross-section results.

CT14nlo 57 members - Replicas

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- the uncertainty is determined in the Hessian way w.r.t. the central value
- generate Replicas f_k from eigensets

$$f_k = f_0 + \sum_i^N \frac{f^+ - f^-}{2} R_{ki}, \quad (4)$$

where R_{ki} are random numbers from a normalised Gaussian distribution with uniform variance centered at the origin

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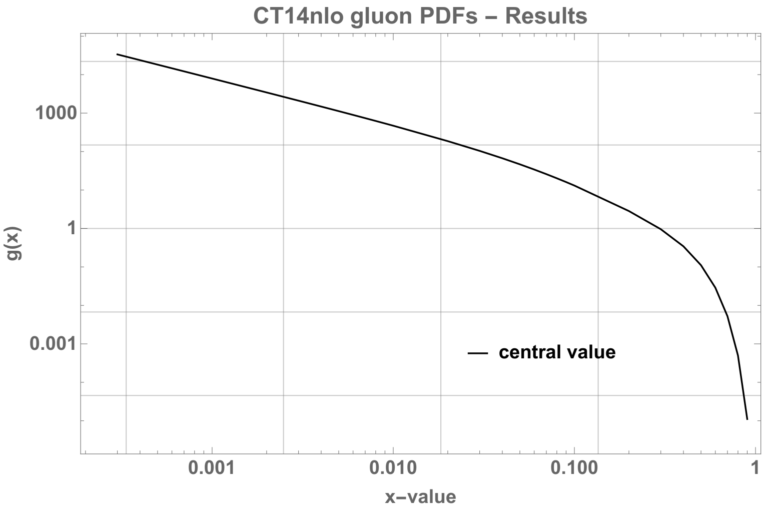
- in order to see this, let's imagine that $f_0(x) = 1/x$
- now if $\frac{f^+ - f^-}{2}(x) = 1/x^{1.5}$ and $R_{ki} > 0$, the resulting PDF becomes naturally steeper
- now if $\frac{f^+ - f^-}{2}(x) = 1/x^{0.5}$ and $R_{ki} < 0$, then the resulting PDF is pulled down more at large x than low x, therefore becoming steeper in the intermediate x-region
- let's generate 10000 Replicas...

CT14nlo 57 members - Replicas



shape of gluon PDF at $\mu_F = 2m_c = 3\text{GeV}$.

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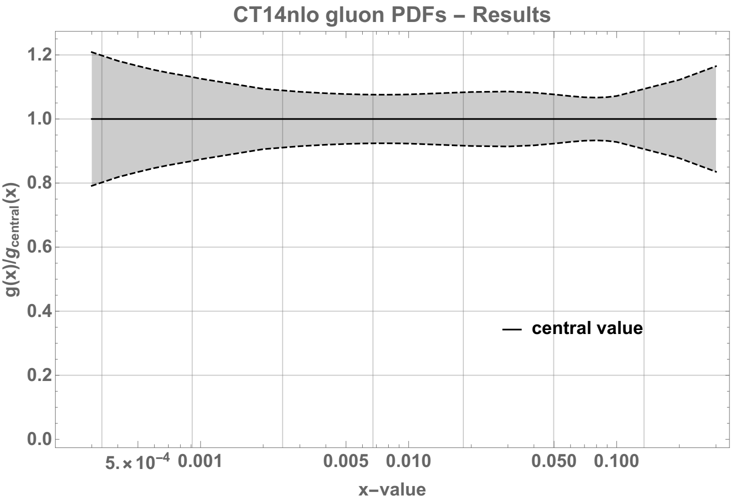

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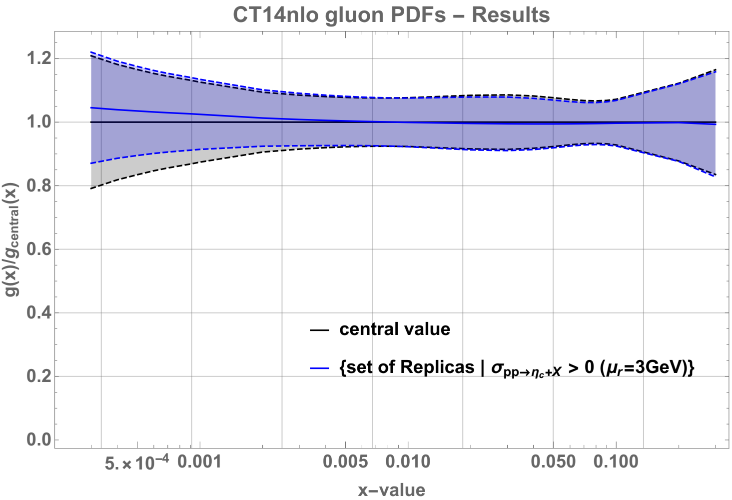

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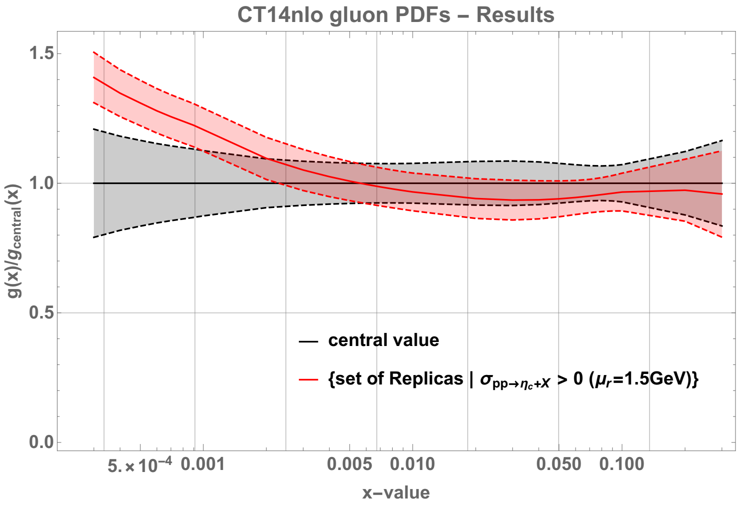

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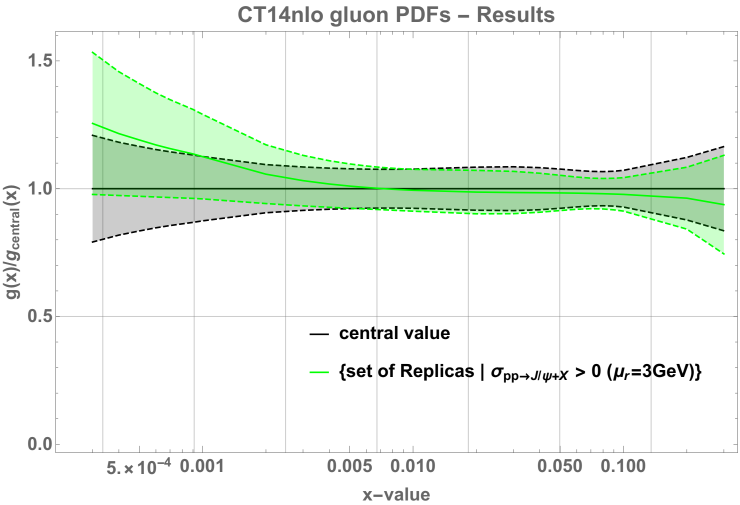

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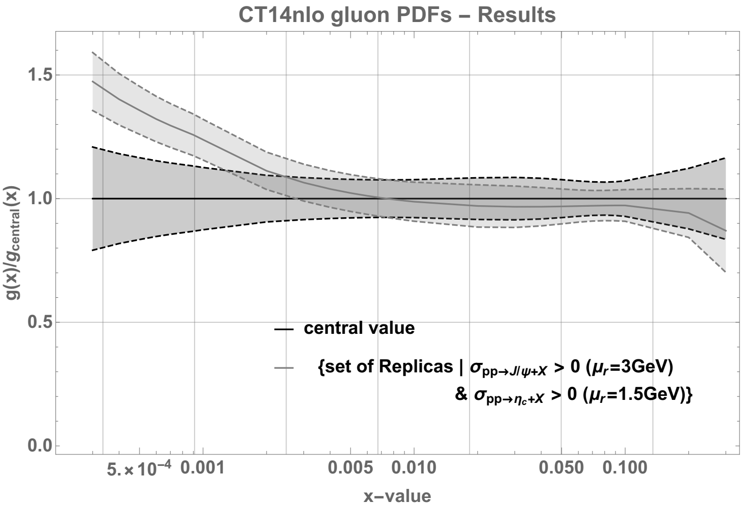

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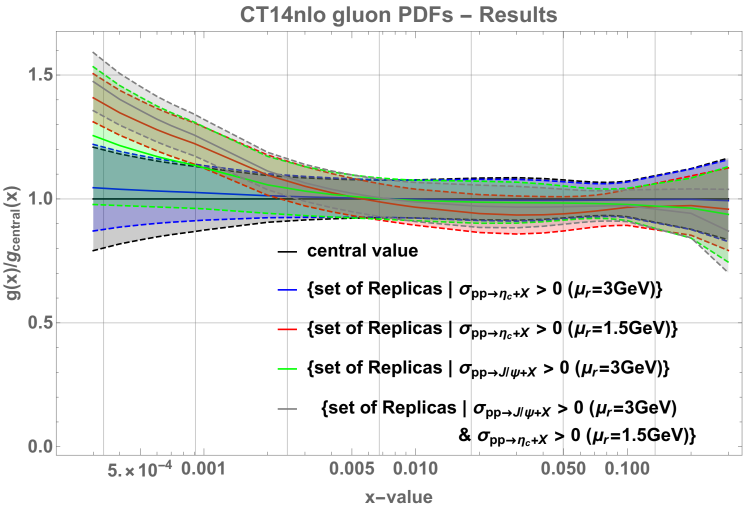

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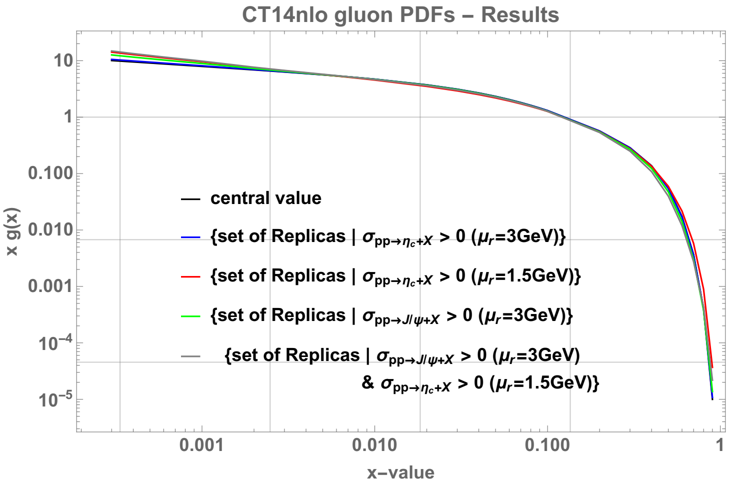

shape of $x\,g(x)$ at $\mu_F = 2m_c = 3\text{GeV}$.

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- need to find some other re-weighting criteria such as reducing the scale sensitivity (μ_R, μ_F), or do a weighting based on the K-factor
- for the moment let's take the set of CT14nlo Replicas that gave postive results for both η_c and J/ψ and compute their K-factors and errors

- with the 10000 Replicas (unweighted) we obtain for the default and alternative scale choices,

	$\sqrt{s} = 7$ TeV			$\sqrt{s} = 14$ TeV
PDF choice	$y = 0$	$y = 1$	$y = 2$	$y = 0$
K_{η_c} = NLO/LO (*default*)	0.4±0.3	0.3±1.0	-1.7±138	0.2±0.5
K_{η_c} = NLO/LO (*alternative*)	-0.6±0.4	-0.6±1.4	-3±188	-0.8±0.7

CT14nlo - 10000 Replicas - Uncertainty of K-factor for η_c

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→ **need to re-weight Replicas!**

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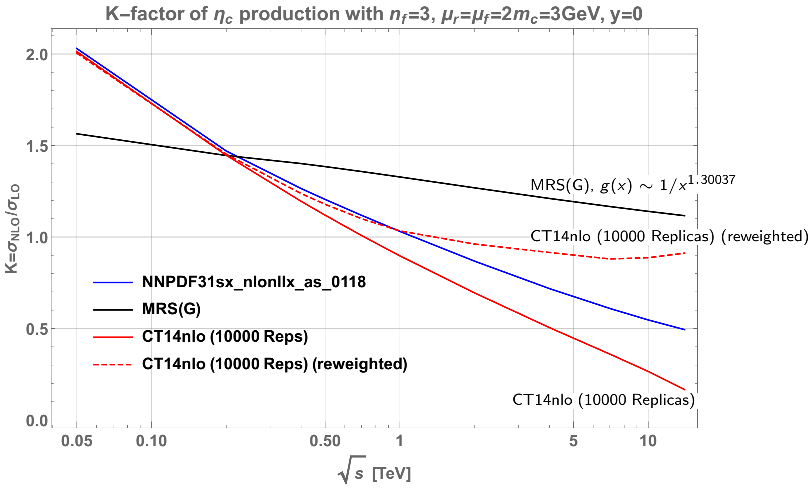
K-factor at $y = 0$ - $\mu_R = \mu_F = 2m_c = 3\text{GeV}$



K-factor at y=0 as a function of energy and with different PDF choices. Default scale choice used $\mu_R = \mu_F = 2m_c = 3\text{GeV}$.

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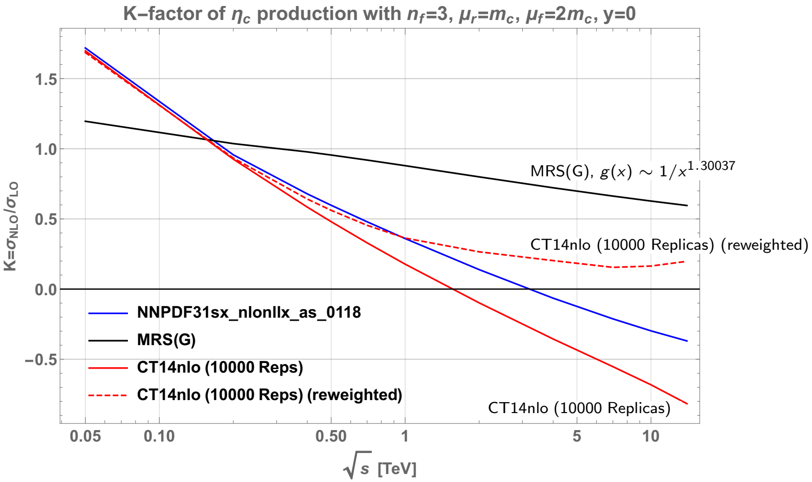
K-factor at $y = 0$ - $\mu_R = m_c = 1.5\text{GeV}, \mu_F = 2m_c = 3\text{GeV}$



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$\rightarrow$ there is room for improvement with PDFs...

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$\rightarrow$ work on-going ...

Thank you for attention!

Backup

Quarkonia - three different models

- Colour-Evaporation Model
  - quark and anti-quark colours are summed up at amplitude squared level (evaporation)
  - no spin-projection
- Colour-Octet Model
  - quark and anti-quark pair are in color-octet state
  - heavy quark spins projected on final bound state
  - higher Fock states in NRQCD, higher v-order
- **Colour-Singlet Model**
  - quark and anti-quark pair are in color-singlet state
  - heavy quark spins projected on final bound state
  - leading Fock state in NRQCD

gluon-gluon channel

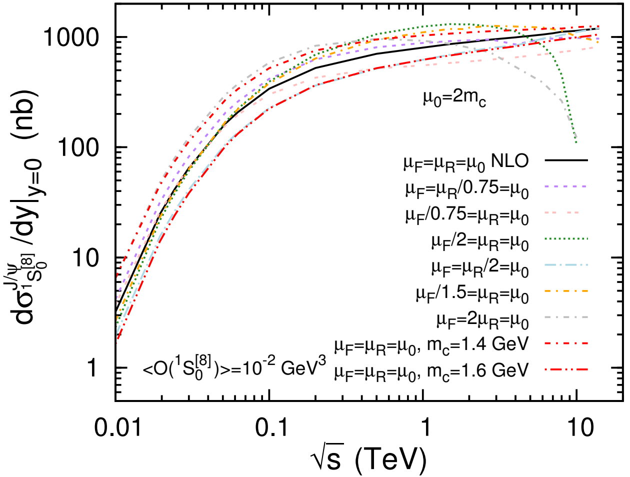
$$
\begin{aligned}
\hat{\sigma}_{gg}(s,\hat{s},\mu_R,\mu_F) &= \frac{\alpha_s^2(\mu_R)\pi^2}{96 m_c^5}|R(0)|^2\delta(1-z) \\
&+ \frac{\alpha_s^3(\mu_R)\pi}{1152 m_c^5}|R(0)|^2\left[\left(-44+7\pi^2+54\log\left(\frac{\mu_R^2}{\mu_F^2}\right)\right.\right. \\
&+72\log\left(1-\frac{4m_c^2}{s}\right)\left(\log\left(1-\frac{4m_c^2}{s}\right)-\log\left(\frac{\mu_F^2}{4m_c^2}\right)\right)\Bigg)\delta(1-z) \\
&+6\left(24\left(\frac{\log(1-z)}{1-z}\right)_\rho(1-(1-z)z)^2\right. \\
&+12\left(\frac{1}{1-z}\right)_\rho\frac{\log(z)}{(1-z)(1+z)^3}\left(1-z^2\left(5+z\left(2+z+3z^3+2z^4\right)\right)\right) \\
&-\left(\frac{1}{1-z}\right)_\rho\frac{1}{(1+z)^2}\left(12+z^2\left(23+z\left(24+2z+11z^3\right)\right)\right) \\
&\left.\left.+12\left(1+z^3\right)^2\log\left(\frac{z\mu_F^2}{4m_c^2}\right)\right)\right], \text{ where } z=4m_c^2/\hat{s} \text{ and } \rho=4m_c^2/s
\end{aligned} \tag{6}
$$

quark-antiquark channel

$$\hat{\sigma}_{q\bar{q}}(\hat{s}, \mu_R) = \frac{16\alpha_s^3(\mu_R)\pi}{81 m_c} |R(0)|^2 \frac{(\hat{s} - 4m_c^2)}{\hat{s}^3} \tag{7}$$

quark-gluon channel

$$\begin{aligned}\hat{\sigma}_{qg}(\hat{s}, \mu_R, \mu_F) = & \frac{\alpha_s^3(\mu_R)\pi}{72 m_c^5 \hat{s}^2} |R(0)|^2 \left(8m_c^4 + 4m_c^2\hat{s} - \hat{s}^2\right. \\ & + 2\left(8m_c^4 - 4m_c^2\hat{s} + \hat{s}^2\right) \log\left(1 - \frac{4m_c^2}{\hat{s}}\right) \\ & + \hat{s}\left(-4m_c^2 + \hat{s}\right) \log\left(\frac{4m_c^2}{\hat{s}}\right) \\ & \left. - \left(8m_c^4 - 4m_c^2\hat{s} + \hat{s}^2\right) \log\left(\frac{\mu_F^2}{\hat{s}}\right)\right)\end{aligned} \tag{8}$$



comparison of J/ψ ${}^1S_0^{[8]}$ differential cross-section at NLO with different choices of μ_R and μ_F with CTEQ6M **[Y. Feng, J.-P. Lansberg, J.X. Wang, Eur.Phys.J. C75 (2015)]**

Schuler 1994 - structure of partonic cross-section

- let's define $z = M^2/\hat{s}$ and $\tau_0 = M^2/s$
- LO partonic cross-section and virtual corrections ($2 \to 1$ process) have $\delta(1-z)$ function while real corrections ($2 \to 2$) are complicated functions of z
- negative contributions come from real corrections
- idea is to use simple toy-models for gluon PDFs and convolute with partonic cross-section; different z-terms will contribute differently at hadronic level

Schuler 1994 - two toymodels - table partonic vs. hadronic

$\hat{\sigma}_{gg}(z, M^2)$	$xg(x) \to 1$	$xg(x) \to 1/\sqrt{x}$
	$\sigma_{pp}(\tau_0, M^2) \overset{\tau_0 \to 0}{\longrightarrow}$	
$\delta(1-z)$	$\ln\left(\frac{1}{\tau_0}\right)$	$\frac{1}{\sqrt{\tau_0}} \ln\left(\frac{1}{\tau_0}\right)$
z^k	$\frac{1}{k} \ln\left(\frac{1}{\tau_0}\right)$	$\frac{2}{(2k+1)\sqrt{\tau_0}} \ln\left(\frac{1}{\tau_0}\right)$
1	$\frac{1}{2} \ln^2\left(\frac{1}{\tau_0}\right)$	$\frac{2}{\sqrt{\tau_0}} \ln\left(\frac{1}{\tau_0}\right)$
$\ln^k\left(\frac{1}{z}\right)$	$\frac{1}{(k+1)(k+2)} \ln^{k+2}\left(\frac{1}{\tau_0}\right)$	$\frac{k!\,2^{k+1}}{\sqrt{\tau_0}} \ln\left(\frac{1}{\tau_0}\right)$

Asymptotic ($\tau_0 = M^2/s \to 0$) behaviour of the proton-proton or proton-antiproton cross section for various forms of the gluon-gluon subprocess ($z = M^2/\hat{s} = \tau_0/\tau$) and two extreme choices of the gluon distribution function. Taken from G. Schuler, Review, 1994

- toymodel $g(x) = 1/x$: real corrections dominate at high energies;
- toymodel $g(x) = 1/x^{1.5}$: all contributions have same energy scaling