From Mueller–Navelet jets to forward J/Ψ-plus-backward jet production

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Quarkonia As Tools

Centre Paul Langevin, Aussois

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HAS QCD
HADRONIC STRUCTURE AND QUANTUM CHROMODYNAMICS

3DSPIN
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Outline

1. Introductory remarks
   - QCD and semi-hard processes
   - High-energy resummation
2. Phenomenology
   - Mueller-Navelet jets
   - Towards more exclusive final states
   - J/Ψ-plus-backward jet production
   - From heavy-quark open states to quarkonia
3. Conclusions and Outlook

QCD and the semi-hard sector

High energies reachable at the LHC and at future colliders:

- ◇ great opportunity in the search for long-waited signals of New Physics...
- ◇ ...faultless chance to test Standard Model in unprecedent kinematic ranges
- ◇ only 5% of Universe visible, but most of this described by **strong interactions**

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Semi-hard processes

Collision processes with a stringent **scale hierarchy**: $s \gg \{Q^2\} \gg \Lambda^2_{\rm QCD}$

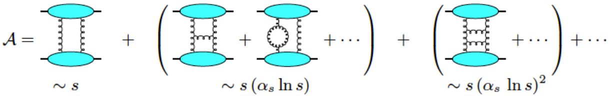
- ◇ $\{Q\}$ is (a set of) process-specific **hard scale**(s) (*e.g.* photon virtuality, heavy quark mass, jet/hadron transverse momentum, t, *etc.*)
- ◇ large $Q \;\Rightarrow\; \alpha_s(Q) \ll 1 \;\Rightarrow\;$ perturbative QCD
- ◇ large $s \;\Rightarrow\;$ large energy single logs
- ¡! Convergence of perturbative series is spoiled when $\alpha_s(Q)\log s \sim 1 \;\Rightarrow$ *all-order* **resummation** needed

The high-energy resummation (BFKL)

- **BFKL resummation**: [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977); Y.Y. Balitskii, L.N. Lipatov (1978)]

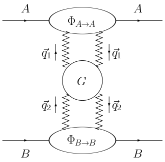
  $\xrightarrow{\text{based on}}$ **gluon Reggeization**

  leading logarithmic approximation (LLA): $\alpha_s^n (\ln s)^n$



  next-to-leading logarithmic approximation (NLA): $\alpha_s^{n+1} (\ln s)^n$

Total cross section for $A + B \to X$: $\sigma_{AB}(s) = \frac{\mathcal{I}m_s\{\mathcal{A}_{AB}^{AB}\}}{s}$ $\Leftarrow$ ***optical theorem***



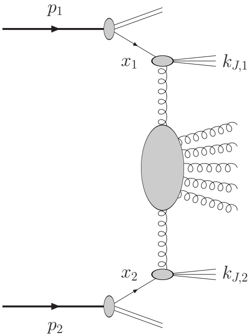
▶ $\mathcal{I}m_s\left\{\mathcal{A}_{AB}^{AB}\right\}$ factorization:

convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles

Green's function is **process-independent**, describes energy dependence and obeys BFKL equation; impact factors are known in the **NLA just for few processes**

Mueller–Navelet jets

$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow \text{jet}_1(k_{J,1}) + X + \text{jet}_2(k_{J,2})$$

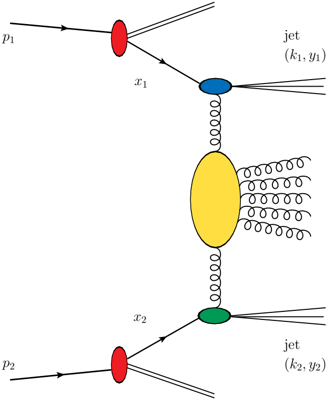


- two hadroproduced jets together with an undetected gluon system, X
- large jet transverse momenta (hard scales): $\vec{k}_{J,1}^2 \sim \vec{k}_{J,2}^2 \gg \Lambda_{\rm QCD}^2$
- large rapidity gap between jets, $\Delta y \equiv Y = y_{J_1} - y_{J_2}$,
  which requires large c.m. energy of the proton collisions, $s = 2\,p_1 \cdot p_2 \gg \vec{k}_{1,2}^2$
- large parton long. fractions (*collinear PDFs*), but non negligible t-channel exchanged momenta (k_T*-factorization*) $\Rightarrow$ **hybrid** approach (next slide)

Factorization of the cross section

$$\frac{d\sigma}{dy_1\, dy_1\, d^2\vec{k}_1\, d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2\, f_r(x_1, \mu_F)\, f_s(x_2, \mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2}$$

The expression for the **partonic cross section** in the BFKL approach reads:



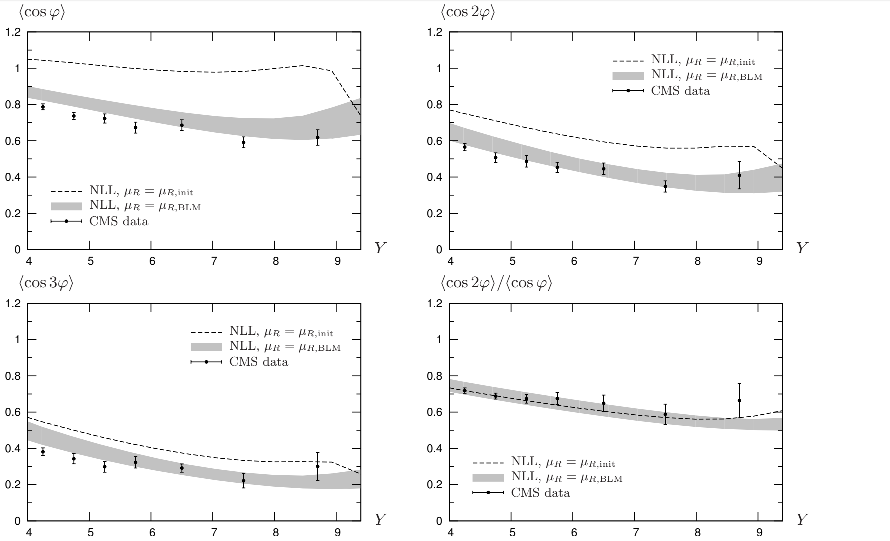
$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \frac{1}{(2\pi)^2}$$

$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_J^{(r)}(\vec{q}_1, s_0, x_1, \vec{k}_1)$$

$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$

$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\, \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{k}_2)$$

Theory vs experiment [7TeV, *CMS-jet*]



(figures in this slide; **7 TeV BFKL + sym.**) [B. Ducloué, L. Szymanowski, S. Wallon (2014)]
(similar analysis; **7 TeV BFKL + sym.**) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

Progress in high-energy phenomenology

Mueller–Navelet jets

- inclusive hadroproduction of two jets featuring high transverse momenta and well separated in rapidity
- possibility to define *infrared-safe* observables and constrain PDFs
- theory vs experiment (CMS @7 TeV with symmetric p_T-ranges, **only!**)

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]
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"[...] the reasonable data-theory agreement shown by the NLL BFKL analytical calculations at large Δy, may be considered as indications that the kinematical domain of the present study lies in between the regions described by the DGLAP and BFKL approaches."

[CMS Collaboration (2016)]

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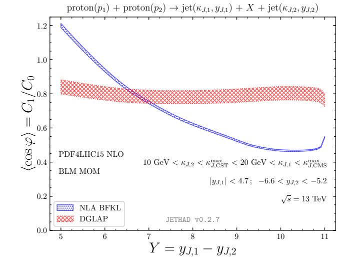
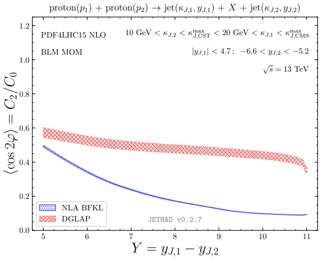
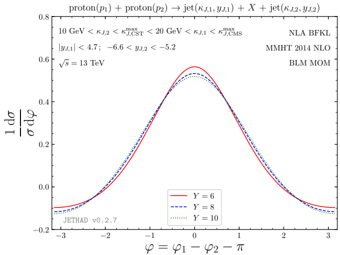
What's next?

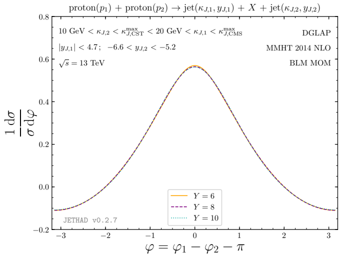
- BFKL vs fixed-order DGLAP adopting **asymmetric** p_T-ranges (next slide)
- need for *more exclusive* final states as well as *more sensitive* observables

BFKL vs fixed-order DGLAP [13TeV, *CASTOR-jet*]









(Mueller–Navelet; **7 TeV BFKL vs DGLAP + asym.**) [F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]
(figures in this slide; Mueller–Navelet, hadron-jet and di-hadron; **13 TeV BFKL vs DGLAP + asym. windows**) [F.G. C. (in preparation)]

More exclusive semi-hard reactions

Inclusive multi-jet production

[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]
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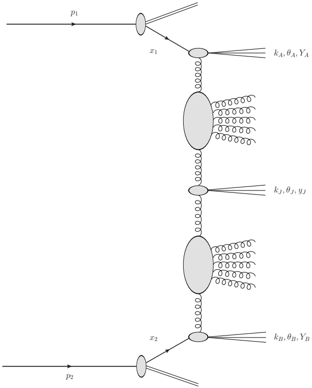

Inclusive hadron-jet production

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]
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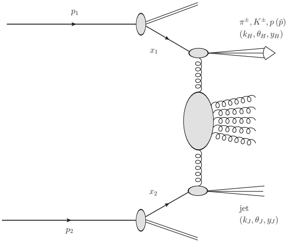

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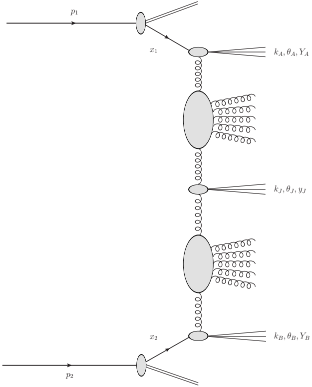

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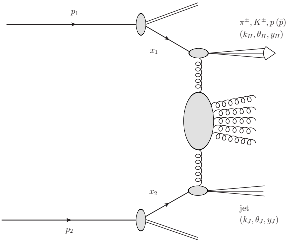
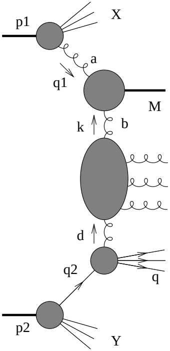


- ◇ new BFKL-sensitive observables (multi-jet), PDFs + FFs at work (hadron-jet)
- ◇ collinear contaminations (multi-jet), minimum-bias effects (hadron-jet)

An inclusive J/Ψ-jet final-state reaction

Process: $\text{proton}(p_1) + \text{proton}(p_2) \to J/\Psi(p_M) + (X, Y) + \text{jet}(p_J)$



- high-energy hadroproduction of a $M \equiv J/\Psi$ meson, via two gluon fusion, with a remnant X, and a jet, with a remnant Y
- both the J/Ψ and the jet emitted with large transvere momenta and well separated in rapidiy
- **hybrid** *collinear and BFKL* description (as for Mueller–Navelet jet production)

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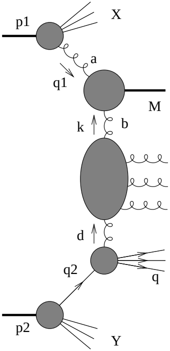

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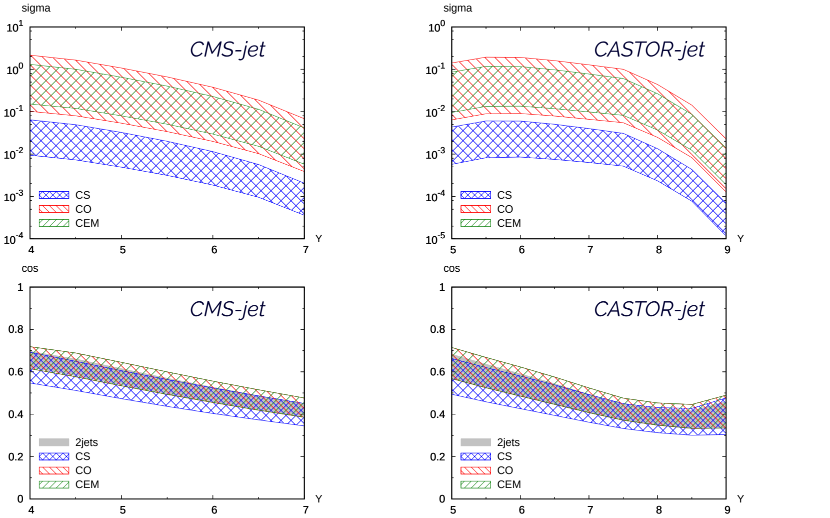
A pioneering study...

[R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

- ◇ NLA BFKL + LO J/Ψ IF + NLO jet IF
- ◇ Different approaches at work: LO J/Ψ IF calculated in **NRQCD singlet and octet** and in **color evaporation model** (**CEM**)
- ◇ realistic CMS and CASTOR rapidity ranges, fixed-p_T final states

Cross section and azimuthal correlations [13TeV]

$0 < y_V < 2.5,\ -4.5 < y_J < 0,\ p_\perp = 10$ GeV $0 < y_V < 2.5,\ -6.5 < y_J < -5,\ p_\perp = 10$ GeV



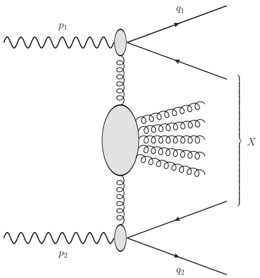
(figures in this slide; ***CMS*** and ***CASTOR*** with $p_{V\perp} = p_{J\perp} \equiv p_\perp = 10$ GeV) [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

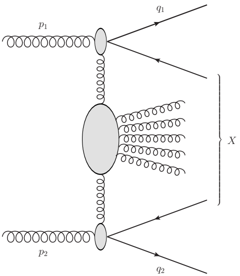
Heavy-flavored emissions and quarkonia

Inclusive heavy-flavored jet photo- and hadroproduction

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018)] [A.D. Bolognino, F.G. C., M. Fucilla, D.Yu. Ivanov, A. Papa (2019)]

LO impact factors; phenomenological analysis: realistic LEP2, CLIC (photo) and LHC cuts (hadro)





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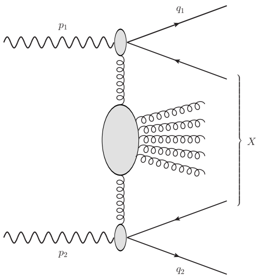
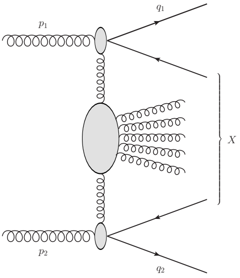




◇ ...convolution with FFs to describe *heavy-light meson* emissions

[A.D. Bolognino, F.G. C., M. Fucilla, D.Yu. Ivanov, A. Papa (in progress)]

◇ ...extension of our formalism $\xrightarrow{\text{to calculate}}$ $(q,\bar{q})$ *bound-state* impact factors

◇ ...**single forward** emissions to probe *small-x gluon-TMD PDFs* (**UGDs**)

Closing statements

- **Semi-hard** reactions: an intriguing class of processes where effects of the **high-energy resummation** emerge and can be effectively disengaged from (pure) fixed-order, DGLAP ones

- Successful tests with NLA accuracy in the **Mueller–Navelet** configuration; nevertheless, *new sensitive observables* as well as *more exclusive final states* are needed

- **Inclusive forward J/Ψ-plus-backward jet** production as a novel channel to hunt for high-energy effects and, *at the same time*, compare different **production mechanisms/models for quarkonia**

  [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

- **From open to bound heavy-quark states**: an *ongoing program* (theory + pheno) on the description of heavy-flavored jets, heavy-light mesons and quarkonia

  [Cosenza Collaboration (2018, 2019, in progress)]

BACKUP slides

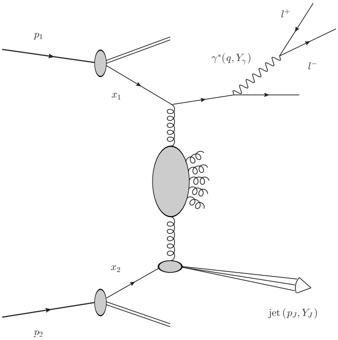
Semi-hard reactions

BACKUP slides

More exclusive semi-hard reactions (II)

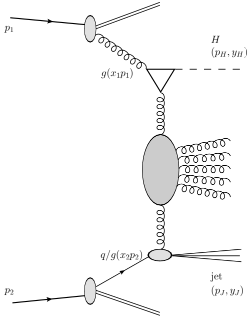
Drell–Yan-plus-jet production

[K.J. Golec-Biernat, L. Motyka, T. Stebel (2018)]
[F.G. C., D. Gordo Gómez, M. Hentschinski, A. Sabio Vera (in progress)]



Higgs-plus-jet production

[B. Xiao, F. Yuan (2018)]
[F.G. C., M.M.A. Mohammed, A. Papa (in preparation)]



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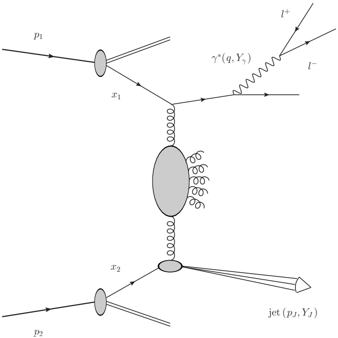

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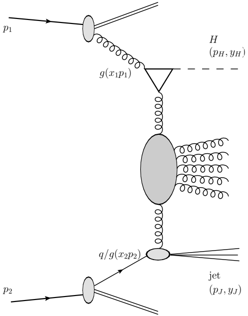


- ◇ large invariant masses stabilize the resummation series
- ◇ p_T-distributions probe kinematic ranges sensitive also to other resummations

High-energy resummation

Gluon Reggeization in perturbative QCD

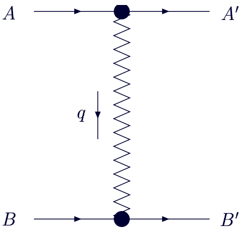
- ◇ Gluon quantum numbers in the t-channel: 8^- representation
- ◇ Regge limit: $s \simeq -u \to \infty$, t not growing with s

$\to$ amplitudes governed by **gluon Reggeization** $\to$ $D_{\mu\nu} = -i\frac{g_{\mu\nu}}{q^2}\left(\frac{s}{s_0}\right)^{\alpha_g(q^2)-1}$

$\xrightarrow{\text{feature}}$ all-order resummation: **LLA** $[\alpha_s^n(\ln s)^n]$ + **NLA** $[\alpha_s^{n+1}(\ln s)^n]$

$\xrightarrow{\text{consequence}}$ factorization of elastic and real part of inelastic amplitudes

$\xrightarrow{\text{example}}$ Elastic scattering process: $A + B \longrightarrow A' + B'$



$$(\mathcal{A}_8^-)_{AB}^{A'B'} = \Gamma_{A'A}^c\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma_{B'B}^c$$

$$j(t) = 1 + \omega(t)\,, \quad j(0) = 1$$

$\omega(t) \to$ Reggeized gluon trajectory

$\Gamma_{A'A}^c = g\langle A'|T^c|A\rangle\Gamma_{A'A} \to$ PPR vertex

$T^c \to$ fundamental (q) or adjoint (g)

- QCD is the unique SM theory where all elementary particles reggeize
- Possible extensions: N=4 SYM, AdS/CFT,...

BACKUP slides

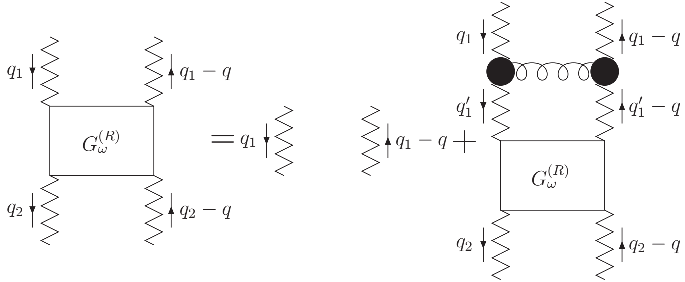
$$\mathcal{I}m_s\{\mathcal{A}\} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}} \Phi_A(\vec{q}_1, s_0) \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}} \Phi_B(-\vec{q}_2, s_0) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

- **Green's function** is **process-independent** and takes care of the **energy dependence**

$\longrightarrow$ determined through the **BFKL equation**

[Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}q\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_1)\,.$$



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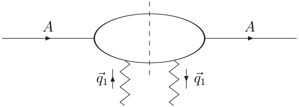
- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy $\longrightarrow$ known in the NLA just for few processes



◇ **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni, G. Rodrigo (2000)]

◇ $\gamma^* \longrightarrow V$, with $V = \rho^0,\ \omega,\ \phi$, forward case

[D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

◇ forward jet production

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
(exact IF) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2012)]
(small-cone IF) [D.Yu. Ivanov, A. Papa (2012)]
(several jet algorithms discussed) [D. Colferai, A. Niccoli (2015)]

◇ forward identified hadron production

[D.Yu. Ivanov, A. Papa (2012)]

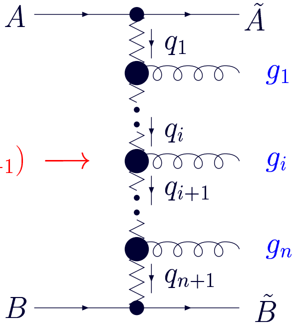
◇ $\gamma^* \longrightarrow \gamma^*$

[J. Bartels *et al.* (2001), I. Balitsky, G.A. Chirilli (2011, 2013)]

BFKL in the LLA (I)

Inelastic scattering process $A + B \;\; \rightarrow \;\; \tilde{A} + \tilde{B} + n$ in the LLA

$\gamma^{G_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \longrightarrow$



multi-Regge kinematics

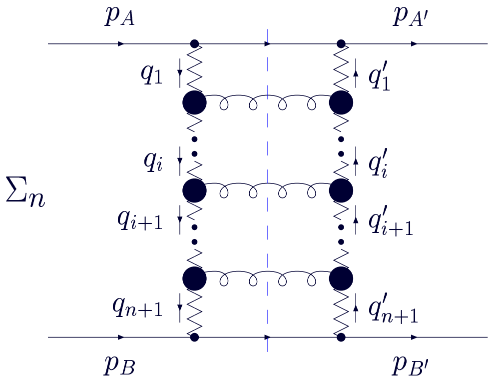
$$\text{Re}\mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\,\Gamma^{c_1}_{\tilde{A}A} \left(\prod_{i=1}^{n} \gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \left(\frac{s_i}{s_R} \right)^{\omega(t_i)} \frac{1}{t_i} \right) \frac{1}{t_{n+1}} \left(\frac{s_{n+1}}{s_R} \right)^{\omega(t_{n+1})} \Gamma^{c_{n+1}}_{\tilde{B}B}$$

$\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \;\; \rightarrow$ RRG vertex

$s_R \;\; \rightarrow$ energy scale, irrelevant in the LLA

BFKL in the LLA (II)

Elastic amplitude $A + B \longrightarrow A' + B'$ in the LLA via s-channel unitarity



$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} \,, \qquad \mathcal{R} = 1\,(\text{singlet}),\, 8^-\,(\text{octet}), \ldots$$

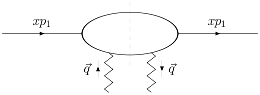
The 8^- color representation is important for the bootstrap, i.e. the consistency between the above amplitude and that with one Reggeized gluon exchange

Mueller–Navelet jets

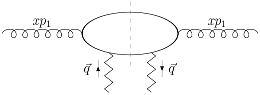
Forward jet impact factor

- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni and G. Rodrigo (2000)]

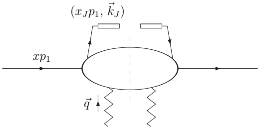


quark vertex

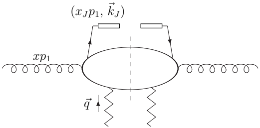


gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



quark jet vertex

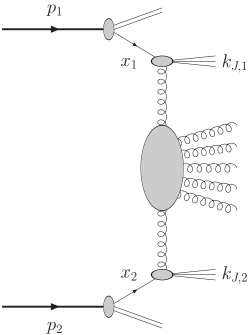


gluon jet vertex

- use QCD collinear factoriz.: $\sum_{s=q,\bar{q}} f_s \otimes$ [quark vertex] $+ f_g \otimes$ [gluon vertex]

BFKL cross section (MN jets)...

$$\frac{d\sigma}{dx_{J_1} dx_{J_2} d^2k_{J_1} d^2k_{J_2}} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1 \int_0^1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}_{i,j}(x_1 x_2 s,\mu)}{dx_{J_1} dx_{J_2} d^2k_{J_1} d^2k_{J_2}}$$



- slight change of variable in the final state
- project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer from the reggeized gluon momenta to the (n,ν)-representation
- suitable definition of the **azimuthal coefficients**

$$\frac{d\sigma}{dx_{J_1} dx_{J_2}\, d|\vec{k}_{J_1}|\, d|\vec{k}_{J_2}| d\phi_{J_1} d\phi_{J_2}} = \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\, \mathcal{C}_n \right]$$

with $\phi = \phi_{J_1} - \phi_{J_2} - \pi$

...useful definitions:

$$Y = \ln \frac{x_{J_1} x_{J_2} s}{|\vec{k}_{J_1}||\vec{k}_{J_2}|}, \qquad Y_0 = \ln \frac{s_0}{|\vec{k}_{J_1}||\vec{k}_{J_2}|}$$

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...and azimuthal coefficients (MN jets)

$$\mathcal{C}_n = \int_{-\infty}^{+\infty} d\nu \, e^{(Y-Y_0)\left[\bar{\alpha}_s(\mu_R)\chi(n,\nu)+\bar{\alpha}_s^2(\mu_R)\mathcal{K}^{(1)}(n,\nu)\right]} \alpha_s^2(\mu_R)$$
$$\times c_1(n,\nu)\, c_2(n,\nu) \left[1 + \alpha_s(\mu_R)\left(\frac{c_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{c_2^{(1)}(n,\nu)}{c_2(n,\nu)}\right)\right]$$

where

$$\chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2}+\frac{1}{2}+i\nu\right) - \psi\left(\frac{n}{2}+\frac{1}{2}-i\nu\right)$$

$$\mathcal{K}^{(1)}(n,\nu) = \bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c}\chi(n,\nu)\left(-\chi(n,\nu) + \frac{10}{3} + i\frac{d}{d\nu}\ln\left(\frac{c_1(n,\nu)}{c_2(n,\nu)}\right) + 2\ln\left(\mu_R^2\right)\right)$$

$$c_1(n,\nu,|\vec{k}|,x) = 2\sqrt{\frac{C_F}{C_A}}(\vec{k}^2)^{i\nu-1/2}\left(\frac{C_A}{C_F}f_g(x,\mu_F) + \sum_{a=q,\bar{q}} f_a(x,\mu_F)\right)$$

...several NLA-equivalent expressions can be adopted for $\mathcal{C}_n$!
$\longrightarrow$ ...we use the *exponentiated* one

[F. Caporale, D.Yu Ivanov, B. Murdaca, A. Papa (2014)]

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Observables and kinematics (MN jets)

- **Observables:**

ϕ-averaged cross section $\mathcal{C}_0$, $\langle\cos[n(\phi_{J_1}-\phi_{J_2}-\pi)]\rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0}$, with $n = 1, 2, 3$

$$\frac{\langle\cos[2(\phi_1-\phi_2-\pi)]\rangle}{\langle\cos(\phi_1-\phi_2-\pi)\rangle} \equiv \frac{\mathcal{C}_2}{\mathcal{C}_1} \equiv R_{21}\,, \quad \frac{\langle\cos[3(\phi_1-\phi_2-\pi)]\rangle}{\langle\cos[2(\phi_1-\phi_2-\pi)]\rangle} \equiv \frac{\mathcal{C}_3}{\mathcal{C}_2} \equiv R_{32}\,.$$

◇ *Integrated coefficients:*

$$C_n = \int_{y_1^{\min}}^{y_1^{\max}} dy_1 \int_{y_2^{\min}}^{y_2^{\max}} dy_2 \int_{k_{J_1}^{\min}}^{k_{J_1}^{\max}} dk_{J_1} \int_{k_{J_2}^{\min}}^{k_{J_2}^{\max}} dk_{J_2}\, \delta\left(y_1 - y_2 - Y\right) \mathcal{C}_n\left(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2}\right)$$

- **Kinematic settings:**
  - ◇ $R = 0.5$ and $\sqrt{s} = 7, 13$ TeV
  - ◇ $y^{\rm C}_{\max} \leqslant |y_{J_{1,2}}| \leqslant 4.7$
  - ◇ symmetric and asymmetric choices for k_{J_1} and k_{J_2} ranges

- **Numerical tools:** JETHAD (HEP@WORK, FORTRAN08/PYTHON3) + LHAPDF

[A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (under development)]

High-energy DGLAP

◇ NLA BFKL expressions for the observables truncated to $\mathcal{O}\left(\alpha_s^3\right)$ **!**

Why asymmetric cuts?

▶ suppress Born contribution to ϕ-averaged cross section C_0 (back-to-back jets)

◇ avoid instabilities observed in NLO fixed-order calculations

[J.R. Andersen, V. Del Duca, S. Frixione, C.R. Schmidt, W.J. Stirling (2001)]
[M. Fontannaz, J.P. Guillet, G. Heinrich (2001)]

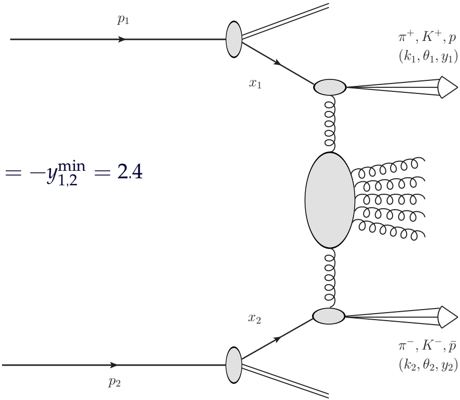
◇ **enhance effects of additional hard gluons** $\xrightarrow{\text{emphasize}}$ **BFKL effects**

di-hadron production

Di-hadron production

Process: $\text{proton}(p_1) + \text{proton}(p_2) \rightarrow \text{hadron}(k_1) + X + \text{hadron}(k_2)$

$k_{1,2}^{\text{min}} = 5$ GeV, $y_{1,2}^{\text{max}} = -y_{1,2}^{\text{min}} = 2.4$



(NLO impact factor) [D.Yu. Ivanov, A. Papa (2012)]
[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]

Di-hadron production

Process: proton(p_1) + proton(p_2) $\to$ hadron(k_1) + X + hadron(k_2)

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2} = \sum_{i,j=q,g} \int_0^1 \int_0^1 dx_1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}(x_1 x_2 s,\mu)}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2}$$

- ◇ large hadron transverse momenta: $\vec{k}_1^2 \sim \vec{k}_2^2 \gg \Lambda_{\rm QCD}^2 \;\Rightarrow$ *pQCD allowed*
- ◇ QCD collinear factorization
- ◇ large rapidity intervals between hadrons (high energies) $\Rightarrow\; \Delta y = \ln \frac{x_1 x_2 s}{|\vec{k}_1||\vec{k}_2|}$

  $\Rightarrow$ BFKL resummation: $\sum_n \left(a_n^{(0)} \; \alpha_s^n \ln^n s + a_n^{(1)} \; \alpha_s^n \ln^{n-1} s \right)$

- ◇ Collinear fragmentation of the parton i into a hadron h
  $\Rightarrow$ convolution of D_i^h with a coefficient function C_i^h

$$d\sigma_i = C_i^h(z) dz \to d\sigma^h = d\alpha_h \int_{\alpha_h}^1 \frac{dz}{z} D_i^h\left(\frac{\alpha_h}{z},\mu\right) C_i^h(z,\mu)$$

where α_h is the momentum fraction carried by the hadron

hadron-jet correlations

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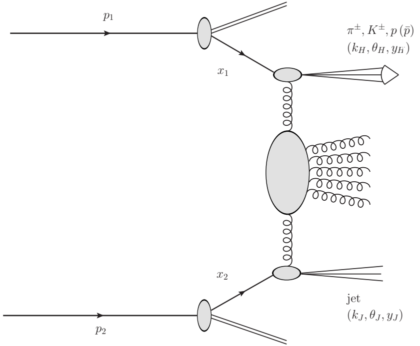
A hadron-jet final-state reaction

Process: $\text{proton}(p_1) + \text{proton}(p_2) \to \text{hadron}(k_H) + X + \text{jet}(k_J)$



$k_H^{\rm min} = 5$ GeV, $|y_H| < 2.4$ [CMS]

$k_J^{\rm min} = 35$ GeV, $|y_J| < 4.7$ [CMS]

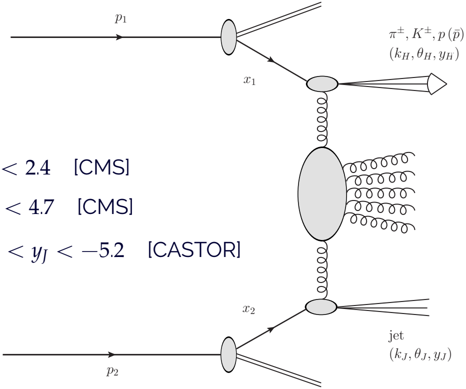
$k_J^{\rm min} = 5$ GeV, $-6.6 < y_J < -5.2$ [CASTOR]

Why hadron-jet correlations?

- ◇ *asymmetric* cuts suppress Born, allowing to discriminate BFKL from DGLAP
- ◇ one-hadron detection quenches "minimum-bias" contaminations
- ◇ *linear* observables facilitate to compare different FF sets and jet algorithms
- ◇ similar analysis: J/Ψ + backward jet [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

A hadron-jet final-state reaction

Process: $\text{proton}(p_1) + \text{proton}(p_2) \to \text{hadron}(k_H) + X + \text{jet}(k_J)$



$k_H^{\min} = 5$ GeV, $|y_H| < 2.4$ [CMS]

$k_J^{\min} = 35$ GeV, $|y_J| < 4.7$ [CMS]

$k_J^{\min} = 5$ GeV, $-6.6 < y_J < -5.2$ [CASTOR]

- ◇ full **NLA** BFKL: NLA gluon Green's function $\circledast$ NLO collinear impact factors
- ◇ **JETHAD** (**HEP@WORK**, FORTRAN08/PYTHON3) + LHAPDF + native FF sets
- ◇ (MMHT14, CT14, NNPDF3.0) $\circledast$ (AKK08, DSS07, HKNS07, NNFF1.0)

[M.M.A. Mohammed, MD thesis (2018)]
[A.D. Bolognino, F.G. C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

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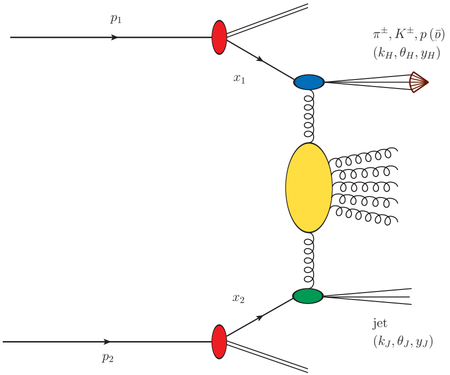
BFKL partonic cross section (hadron-jet)

$$\frac{d\sigma}{dy_H\, dy_J\, d^2\vec{k}_H\, d^2\vec{k}_J} = \sum_{r,s=q,g} \int_0^1 \int_0^1 dx_1\, dx_2\, f_r(x_1, \mu_F)\, f_s(x_2, \mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_H\, dy_J\, d^2\vec{k}_J\, d^2\vec{k}_J}$$

The expression for the **partonic cross section** in the BFKL approach reads:



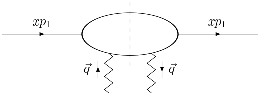
$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H\, dy_J\, d^2\vec{k}_H\, d^2\vec{k}_J} = \frac{1}{(2\pi)^2}$$
$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1; \vec{k}_H, x_H)\, D_r^H\left(\frac{x_H}{x_1}, \mu_F\right)$$
$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$
$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\, \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2; \vec{k}_J, x_J)$$

[A.D. Bolognino, F.G. C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

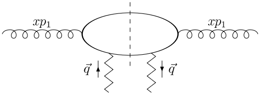
Forward hadron and jet impact factor

- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000); M. Ciafaloni and G. Rodrigo (2000)]

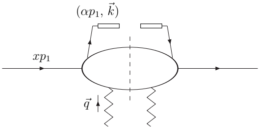


quark vertex

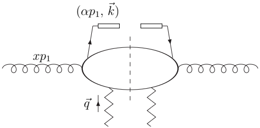


gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



$\mathcal{V}^{(q)}_{H/J}$ (quark hadron/jet vertex)



$\mathcal{V}^{(g)}_{H/J}$ (gluon hadron/jet vertex)

- use QCD collinear factorization

hadron $\rightarrow \sum_{r=q,\bar{q}} f_r \otimes \mathcal{V}^{(r)}_H \otimes D^H_r + f_g \otimes \mathcal{V}^{(g)}_H \otimes D^H_g$

jet $\rightarrow \sum_{s=q,\bar{q}} f_s \otimes \mathcal{V}^{(s)}_J + f_g \otimes \mathcal{V}^{(g)}_J$

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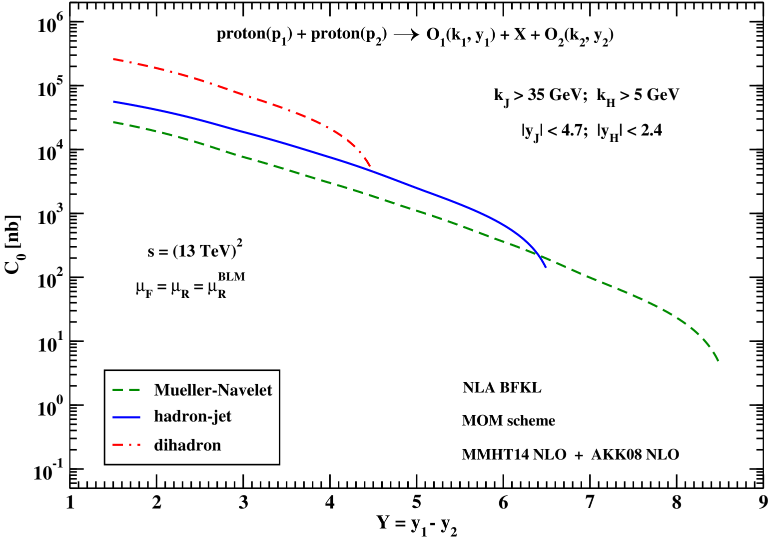
MN, hadron-jet and di-hadron C_0 vs Y, $\sqrt{s} = 13$ TeV



[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

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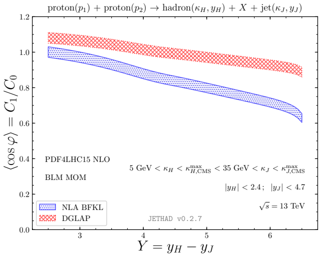
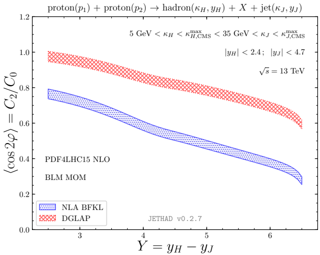
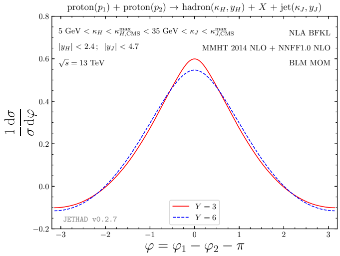
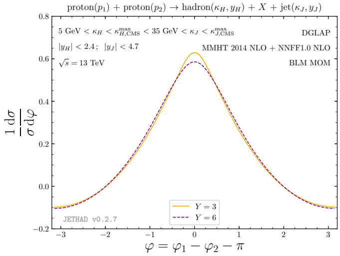
BFKL vs fixed-order DGLAP [13TeV, *CMS-jet*]





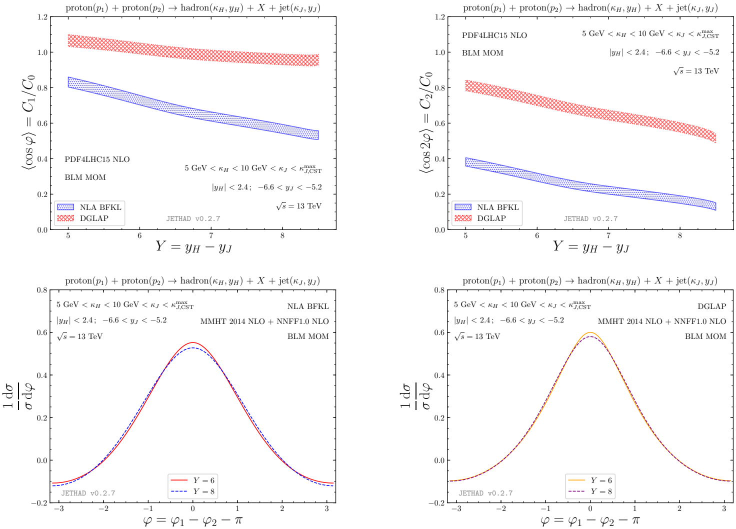




(Mueller–Navelet; **7 TeV BFKL vs DGLAP + asym.**) [F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]
(figures in this slide; Mueller–Navelet, hadron-jet and di-hadron; **13 TeV BFKL vs DGLAP + asym. windows**) [F.G. C. (in preparation)]

BACKUP slides

BFKL vs fixed-order DGLAP [13TeV, *CASTOR-jet*]

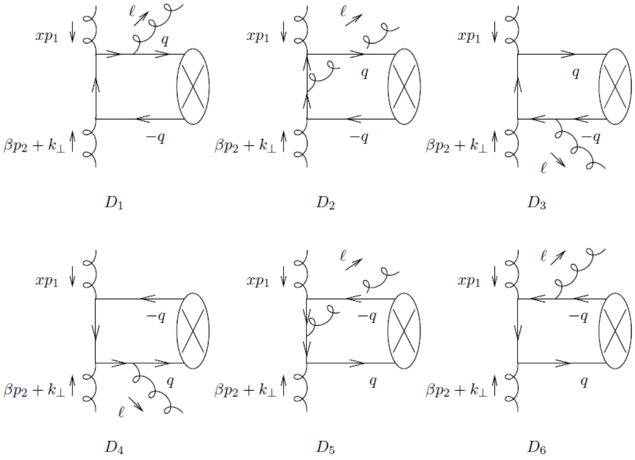


(Mueller–Navelet; **7 TeV BFKL vs DGLAP + asym.**) [F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]
(figures in this slide; Mueller–Navelet, hadron-jet and di-hadron; **13 TeV BFKL vs DGLAP + asym. windows**) [F.G. C. (in preparation)]

J/Ψ-jet correlations

Forward J/Ψ impact factor (I)

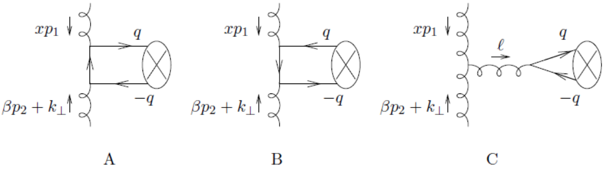
Color singlet NRQCD (6 diagrams)



Crossed blobs indicate Fierz structures

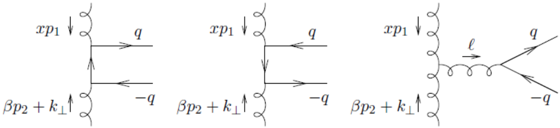
Forward J/Ψ impact factor (II)

Color octet NRQCD (3 diagrams)



Crossed blobs indicate Fierz structures

Color evaporation model (3 diagrams)



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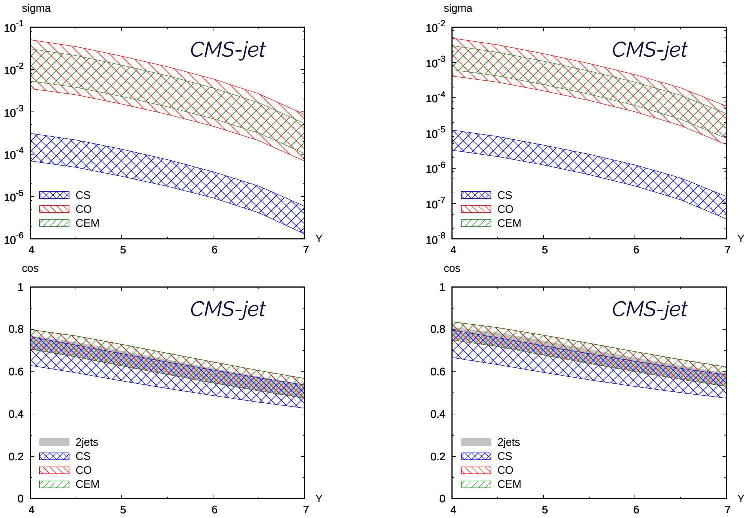
Cross section and azimuthal correlations [13TeV]

$0 < y_V < 2.5,\ -4.5 < y_J < 0,\ p_\perp = 20$ GeV

$0 < y_V < 2.5,\ -4.5 < y_J < 0,\ p_\perp = 30$ GeV



(figures in this slide; ***CMS-jet*** with $p_{V\perp} = p_{J\perp} \equiv p_\perp = 20, 30$ GeV) [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

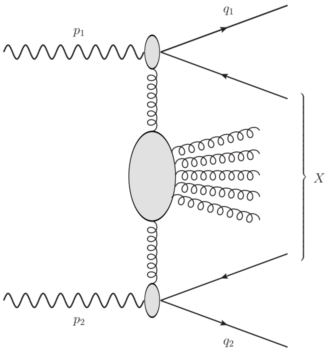
heavy-quark pair photoproduction

BACKUP slides

Heavy-quark pair photoproduction

Process: $\gamma(p_1) + \gamma(p_2) \to Q(q_1) + X + Q(q_2)$

...Q stands for a charm/bottom quark or antiquark



- photoproduction channel
- collision of (quasi-)real photons
- equivalent photon flux approximation
- quark masses play the role of hard scale
- first predictions within partial NLA BFKL (NLA Green's function + LO impact factors)
    - ◇ LEP2 and future e^+e^- colliders

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018)]

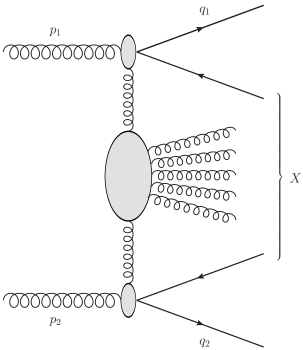
heavy-quark pair hadroproduction

BACKUP slides

Heavy-quark pair hadroproduction

Process: $g(p_1) + g(p_2) \to Q(q_1) + X + Q(q_2)$

...Q stands for a charm/bottom quark or antiquark



- hadroproduction channel
- gluon-initiated subprocess
- BFKL resummation at work
- quark transverse masses play the role of hard scale
- first predictions within partial NLA BFKL (NLA Green's function + LO impact factors)
  - LHC physics

[A.D. Bolognino, F.G. C., M. Fucilla, D.Yu. Ivanov, A. Papa (2019)]]

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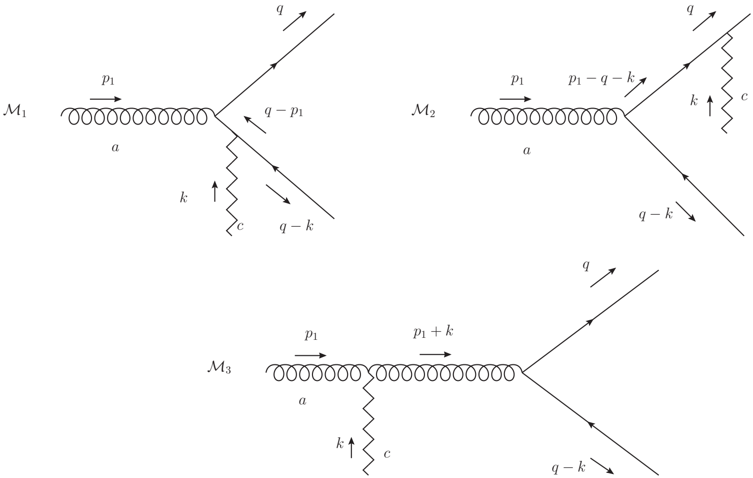
Heavy-quark pair hadroproduction: impact factors



Feynman diagrams relevant for the calculation of the impact factor for the heavy-quark pair hadroproduction. The zigzag line denotes a Reggeized gluon.

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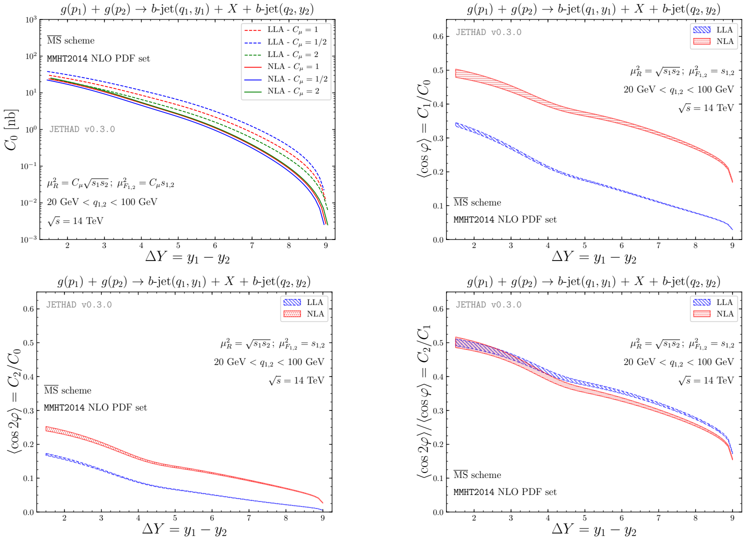
C_0 and R_{nm} vs Y at 13 TeV $(gg \to b\text{-jets})$



$s_{1,2} = m_{1,2}^2 + q_{1,2}^2$

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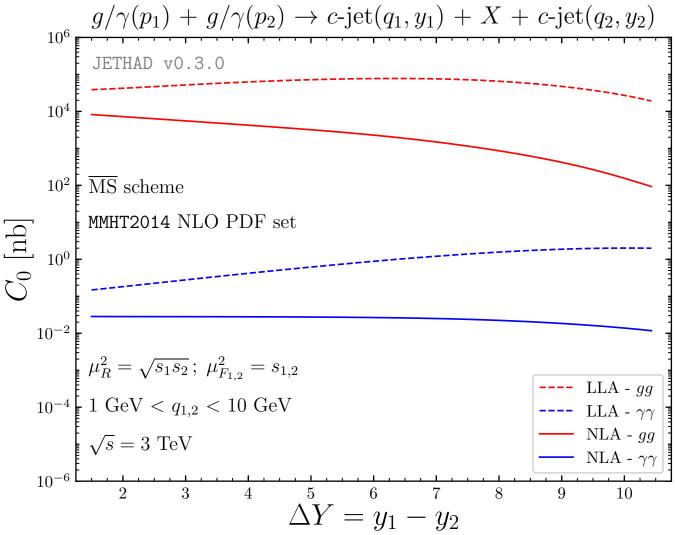
Photoproduction vs hadroproduction $(gg \to c\text{-jets})$



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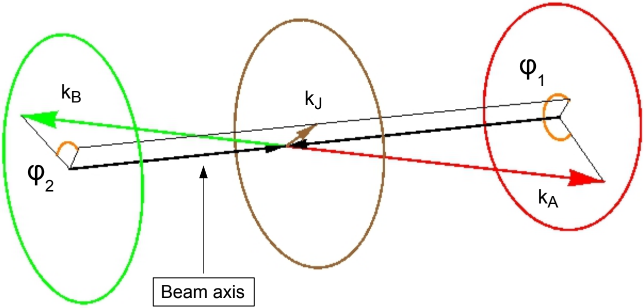
three-jet production

An event with three tagged jets



$Y_B < y_J < Y_A$

four-jet production

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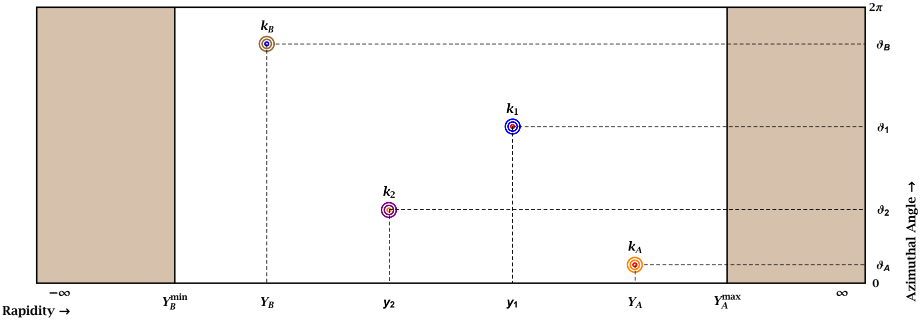
A four-jet primitive lego-plot



$$Y_A^{\max} = -Y_B^{\min} = 4.7$$

The unintegrated gluon distribution (UGD)

Parton densities: hors d'œuvre

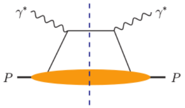
- **Parton densities** are relevant to the search for New Physics...
  - describe the internal structure of the nucleon in terms of its elementary constituents (quarks and gluons)
  - **nonperturbative** objects
  - enter the expression for cross sections
  - can be extracted from experiments through global fits

- Several types of distributions...
  - respect different **factorization theorems**
  - exhibit peculiar **universality properties**
  - obey distinct **evolution equations**

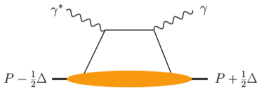
Parton densities: entrée

κ_T-integrated parton densities

- **Collinear PDF factorization**
  - inclusive processes
  - $\kappa_T \sim$ hardest scale

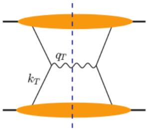


- **GPD factorization**
  - exclusive processes
  - skewness effects

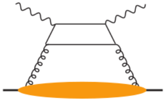


κ_T-unintegrated parton densities

- **TMD factorization**
  - (semi-)inclusive processes
  - $\kappa_T \ll$ hardest scale



- **High-energy (small-x) factorization**
  - inclusive or exclusive processes
  - Unintegrated gluon distribution



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BFKL and the unintegrated gluon density (UGD)

◇ DIS: conventionally described in terms of PDFs

◇ less inclusive processes: need to use distributions unintegrated over the parton κ_T

- example: **virtual photoabsorption** in **high-energy factorization**

$$\sigma_{\rm tot}(\gamma^* p \to X) \;\propto\; \mathcal{I}m_s\{\mathcal{A}(\gamma^* p \to \gamma^* p)\} \;\equiv\; \Phi_{\gamma^* \to \gamma^*} \;\circledast\; \mathcal{F}(x, \kappa^2)$$

◇ $\mathcal{F}(x, \kappa^2)$ is the **unintegrated gluon distribution** (**UGD**) in the proton

▶ small-x limit: UGD $=$ [BFKL gluon ladder] $\circledast$ [proton impact factor]

...UGD has to be modeled!