Perturbative QCD: status and opportunities

John Campbell, Fermilab
Making the bulk of LHC predictions is “simplified” by properties of QCD: asymptotic freedom and factorization.

Collinear factorization works well (enough) for colliding protons.

Perturbation theory is a powerful tool!
Higher orders

- Systematic improvement in the prediction at each order of perturbation theory:
  - better description (partons initiating hard process, radiation in final state)
  - reduced dependence on unphysical renormalization and factorization scales.
- But still hard work; primary emphasis on best bang-for-buck!

<table>
<thead>
<tr>
<th>Year</th>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
<th>N^3LO</th>
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Anastasiou, Dulat, Mistlberger et al (2016-)
LO and NLO

- Any process can (in principle) be computed up to NLO using off-the-shelf tools
- For producing cross-sections, observables and events
  - Madgraph5/aMC@NLO, SHERPA, Herwig, …
- Accessing all necessary matrix elements (further assembly required!)
  - OpenLoops, RECOLA, GoSam, NLOX, …

- All thanks to advances in understanding multiple elements of the calculation
  - importance of recursion (recycling)
  - universal and efficient methods to handle infrared singularities (subtraction)
  - knowledge of analytic behavior of amplitudes (unitarity methods)
  - structure of one-loop integrals
  - efficient phase-space integration
  - ….

- Of course, all of the above applies not just to hadron-hadron colliders.
Beyond NLO

- Two main areas requiring attention:
  - calculation of multi-loop diagrams beyond 2→2 topology
  - cancellation of infrared singularities: multiple strategies currently in use, all computationally challenging, no clear consensus on best approach

Example of infrared complications: X+jet @ NNLO

- "Pure virtual", e.g. 2-loop diagrams (Born topology)
- "Real-virtual", 1-loop with an additional parton
- "Real-real", two additional partons
NNLO progress

Explosion of calculations starting in 2014

[slide by L. Cieri]
NNLO: hot topics and prospects

• Pushing beyond the current $2 \rightarrow 2$ frontier desirable for many reasons:
  • Higgs: $t\bar{t}H$ and Higgs+2 jets
  • Precision SM: 3 jets, W/Z/photon + 2 jets
• Requires deeper understanding of two-loop amplitudes: analytic structure, new (elliptic) integrals, numerical techniques for handling integrals.

### numerical evaluation of planar 2-loop W+4 parton amplitudes

Hartanto, Badger, Bronnum-Hansen, Peraro (2019)

### analytic 2-loop leading-color 5-parton amplitudes

Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov (2019)

### full analytic 5-parton $+++++$ amplitude

Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zola (2019)
NNLO: hot topics and prospects

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\[
H_{1}^{(2,0)} = \sum_{S_{T1}} \left\{-\kappa \frac{[45]^2}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} I_{123;45} + \kappa^2 \frac{1}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} \left[ 5 s_{12} s_{23} + s_{12} s_{34} + \frac{\text{tr}_+^2(1245)}{s_{12} s_{45}} \right] \right\}
\]

\[
H_{13}^{(2,1)} = \sum_{S_{T13}} \left\{ \kappa \frac{[15]^2}{\langle 23 \rangle \langle 34 \rangle \langle 42 \rangle} \left[ I_{234;15} + I_{243;15} - I_{324;15} - 4 I_{345;12} - 4 I_{354;12} - 4 I_{435;12} \right] \right. \\
- 6 \kappa^2 \left[ \frac{s_{23} \text{tr}_- (1345)}{s_{34} \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} - \frac{3}{2} \frac{[12]^2}{\langle 34 \rangle \langle 45 \rangle \langle 53 \rangle} \right] \right\},
\]

full analytic 5-parton \\
+++++ amplitude

Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zola (2019)
The increasing availability of calculations at NNLO is essential to properly describe much of the data taken at the LHC.
• Much-reduced scale dependence yields percent-level theory uncertainties that can be competitive with experimental ones
• new opportunities for measurements and constraints
• at this level, often need to consider electroweak effects as well (especially in tails of distributions)

Ellis, Williams, JC (2016)
Differential jet cross-sections

Gehrmann-de Ridder, Gehrmann, Glover, Huss, Pires (2019)

\[ x_{1,2} = \frac{2p_{T,\text{avg}}}{\sqrt{s}} e^{\pm y_b} \cosh y^* \]

**dijet rapidity separation**

\[ y^* = \frac{1}{2} |y_1 - y_2| \]

- Smaller uncertainties, better agreement with data (especially low boost)
- At large boost (and jet \( p_T \)) disagreement an opportunity to refine high-\( x \) PDF

**boost of dijet system**

\[ y_b = \frac{1}{2} |y_1 + y_2| \]
PDF studies

- Need tools able to compute NNLO predictions for multiple PDFs, precisely enough to see differences, both for assessing compatibility and eventually for global fitting.
Higher-order uncertainty in fits

• Attempt to capture uncertainty in fits due to missing higher orders (scale uncertainty)
• so far only to NLO where all calculations are readily available.
• general formalism worked out, also applicable to nuclear & higher-twist corrections

Not just fixed order

- $W$ and $Z$ $p_T$ spectrum important for PDF determination, $W$ mass (also $H$ for BSM effects)
- State-of-the-art combines NNLO fixed order with $N^3LL$ large-log resummation

Bizon, Chen, Gehrmann-de Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli (2018)
Bizon, Gehrmann-de Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Walker (2019)

resummation crucial
NNLO sufficient
significant shape change (up to ~10%) wrt NNLL+NLO, residual uncertainty < 5%
Beyond NNLO

- Only a handful of $N^3LO$ results, focussed on Higgs production
  - inclusive cross-sections for gluon fusion and VBF
  - $W$ and $Z$ production notably absent!
- Aside from experimental considerations, pure theoretical interest
  - first order at which all parton channels are computed to at least NLO
  - how does series converge?
- Latest results: completely-differential calculation of Higgs production at $N^3LO$
  - excellent agreement with earlier (threshold expansion) calculation
  - Open question: how does perturbative stability look after fiducial cuts?

Dulat, Mistlberger, Pelloni (2018)

Cieri, Chen, Gehrmann, Glover, Huss (2018)

Dulat, Mistlberger, Pelloni (2018)
Steps towards the EIC

- $N^3LO$ jet production in DIS in the lab frame
  
  - overlapping uncertainty bands, factor two smaller uncertainties, better description even in regions with lower accuracy or susceptible to large logs.

NNLO calculations for DIS

- NNLO corrections to DIS jet and dijet production in the Breit frame
  Currie, Gehrmann, Huss, Niehues (2017)

- inclusive jet: substantial corrections at low $Q^2$ and $p_T$, up to 60%, much improved description of data

- Corresponding results for event-shape distributions
  Gehrmann, Huss, Mo, Niehues (2019)

In general up to 20% corrections, non-uniform, decreased scale uncertainty but small overlap with NLO
Non-perturbative effects

- Description of data requires the addition of power corrections to account for parton-hadron transition.
- Dispersive model (also used at LEP) shifts differential distribution:
  \[
  \frac{d\sigma_{\text{hadron}}(F)}{dF} = \frac{d\sigma_{\text{parton}}(F - a_F P)}{dF}
  \]
  and mean values correspondingly:
  \[
  \langle F' \rangle = \langle F \rangle_{\text{pert.}} + a_F P.
  \]
  (universal $P$, $a_F$ varies by event shape)
- Precision QCD studies now possible through reanalyses of HERA data
  - opportunity for EIC
Extraction of $\alpha_s$ from HERA data

- Demonstration for APPLfast project
  Britzger et al (2019)
  - uses perturbative input from NNLO (NNLOJET) to produce interpolation tables for a posteriori PDF analyses
Single-inclusive production at an EIC

- Need predictions for single-inclusive hadron production: no lepton observed.
- However, when inclusive of the lepton, must also account for configurations resulting from quasi-real photon (lepton travels down the beam pipe).
  - can capture through Weizsacker-Williams approach (lepton structure function)
  - Recently used to compute NNLO predictions for EIC

\[ lp \rightarrow j + X \]

\[ \sqrt{s} = 100 \text{ GeV} \]
\[ p_T^{l+} > 10 \text{ GeV} \]
\[ |\eta_{l+}| < 2 \]
\[ p_T^{g+} = p_T^{j+} \]
\[ R = 0.5 \]
\[ \text{CT14 pdf} \]

Abelof, Boughezal, Liu, Petriello (2016)
Summary

- Perturbation theory at NLO a workhorse of the LHC.
- Many calculations at NNLO (even $N^3$LO) have emerged over the last 5-10 years that are suited to precision studies:
  - describe data over a wider kinematic range;
  - exhibit uncertainties smaller than, or at least competitive with, data.
- Turning complex calculations into tools for data analysis still a challenge
  - new tools making better use of CPU resources, interpolation techniques
- Some attention from the LHC precision community turning to topics closer to EIC
  - variety of calculations for DIS in particular

- Areas ripe for cross-fertilization:
  - inclusion of higher-order corrections in Monte Carlo tools
  - extraction of PDFs at higher perturbative orders, “ultimate” LHC precision
  - understanding remaining non-perturbative effects, e.g. in event shapes