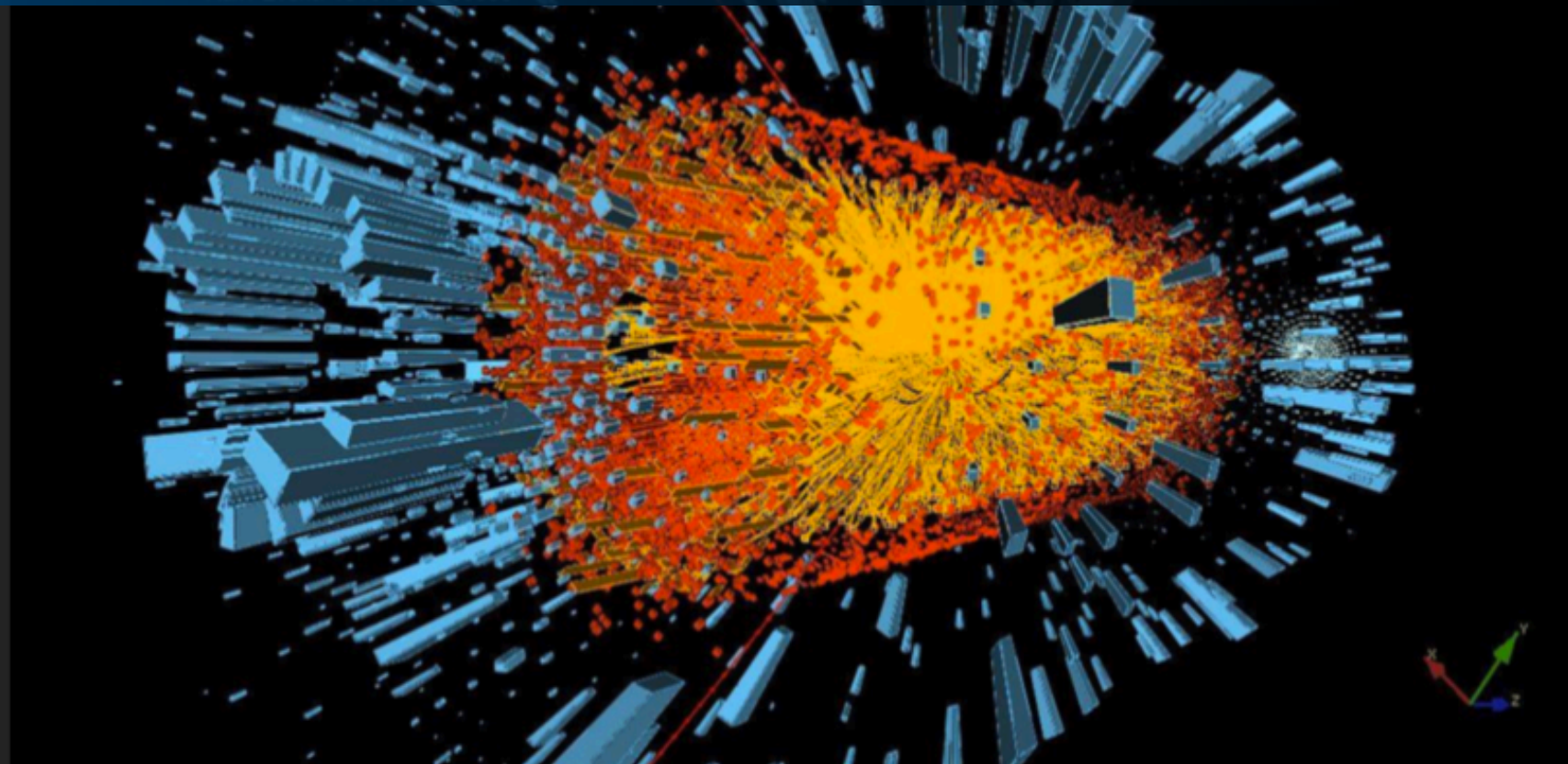


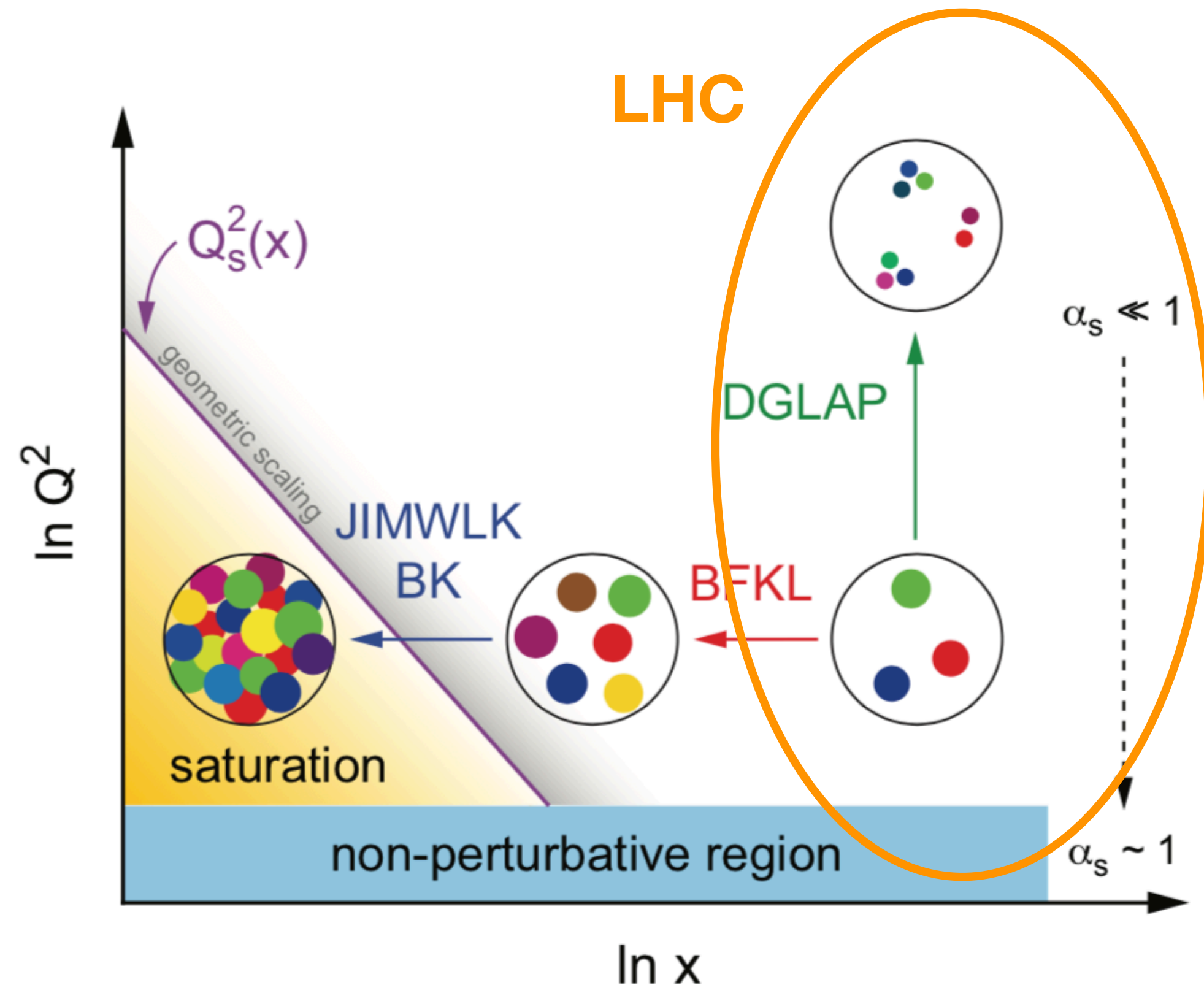
Perturbative QCD: status and opportunities

John Campbell, Fermilab

Workshop on Physics Connections Between the LHC and EIC



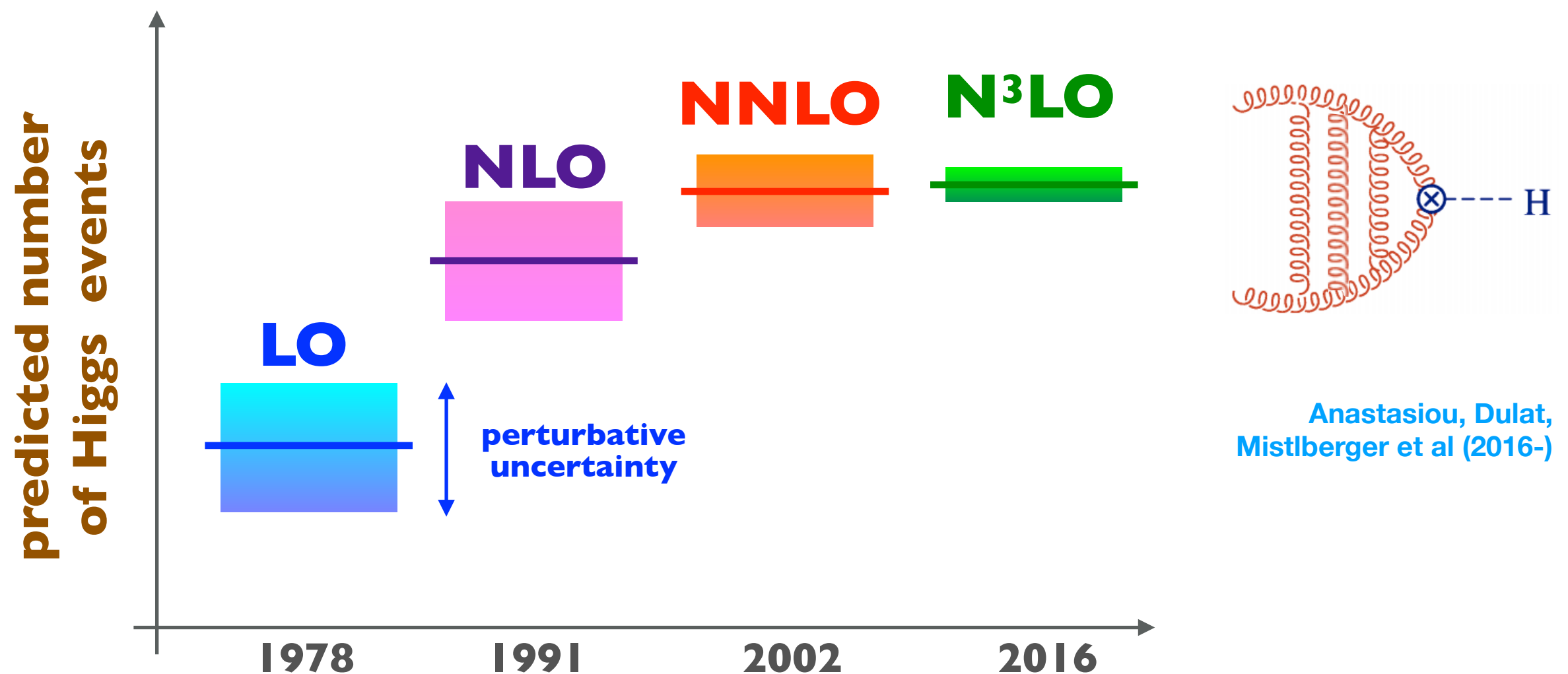
The landscape



- Making the bulk of LHC predictions is “simplified” by properties of QCD: asymptotic freedom and factorization.
- Collinear factorization works well (enough) for colliding protons.
- Perturbation theory is a powerful tool!

Higher orders

- Systematic improvement in the prediction at each order of perturbation theory:
 - better description (partons initiating hard process, radiation in final state)
 - reduced dependence on unphysical renormalization and factorization scales.
- But still hard work; primary emphasis on best bang-for-buck!



LO and NLO

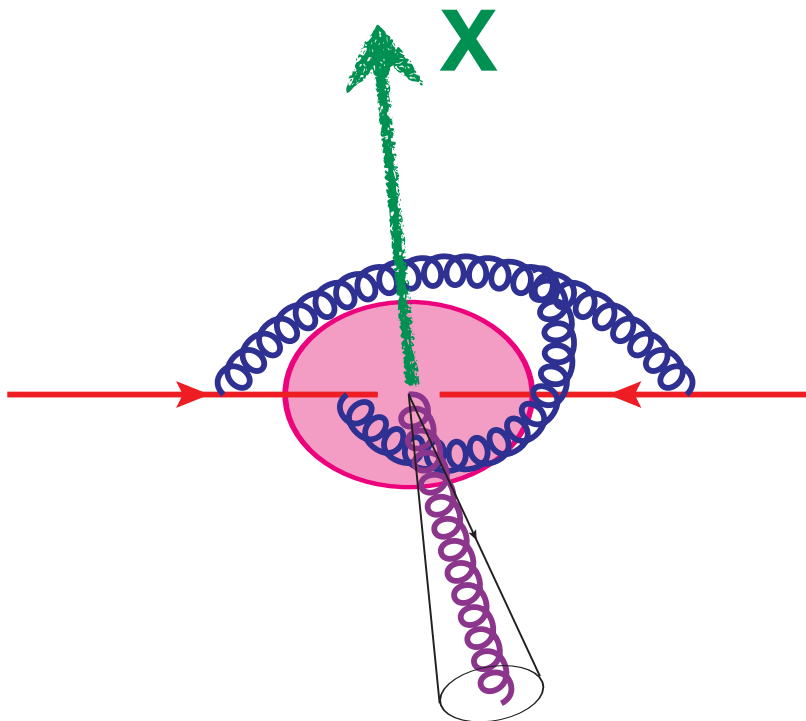
- Any process can (in principle) be computed up to NLO using off-the-shelf tools
- For producing cross-sections, observables and events
 - Madgraph5/aMC@NLO, SHERPA, Herwig, ...
- Accessing all necessary matrix elements (further assembly required!)
 - OpenLoops, RECOLA, GoSam, NLOX, ...
- All thanks to advances in understanding multiple elements of the calculation
 - importance of recursion (recycling)
 - universal and efficient methods to handle infrared singularities (subtraction)
 - knowledge of analytic behavior of amplitudes (unitarity methods)
 - structure of one-loop integrals
 - efficient phase-space integration
 -
- Of course, all of the above applies not just to hadron-hadron colliders.

Beyond NLO

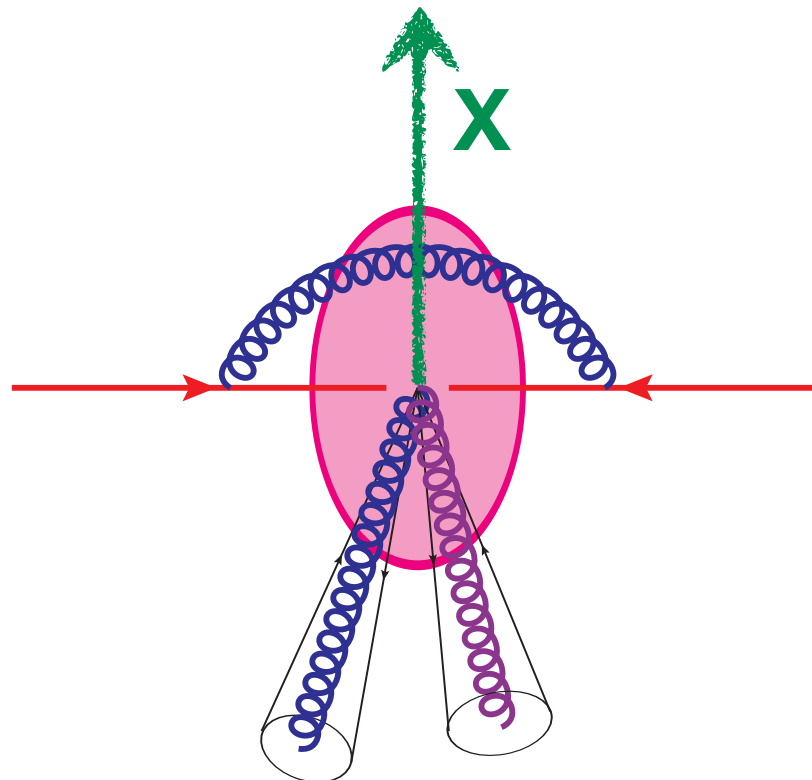
- Two main areas requiring attention:
 - calculation of multi-loop diagrams beyond $2 \rightarrow 2$ topology
 - cancellation of infrared singularities: multiple strategies currently in use, all computationally challenging, no clear consensus on best approach

Example of infrared complications: $X + \text{jet}$ @ NNLO

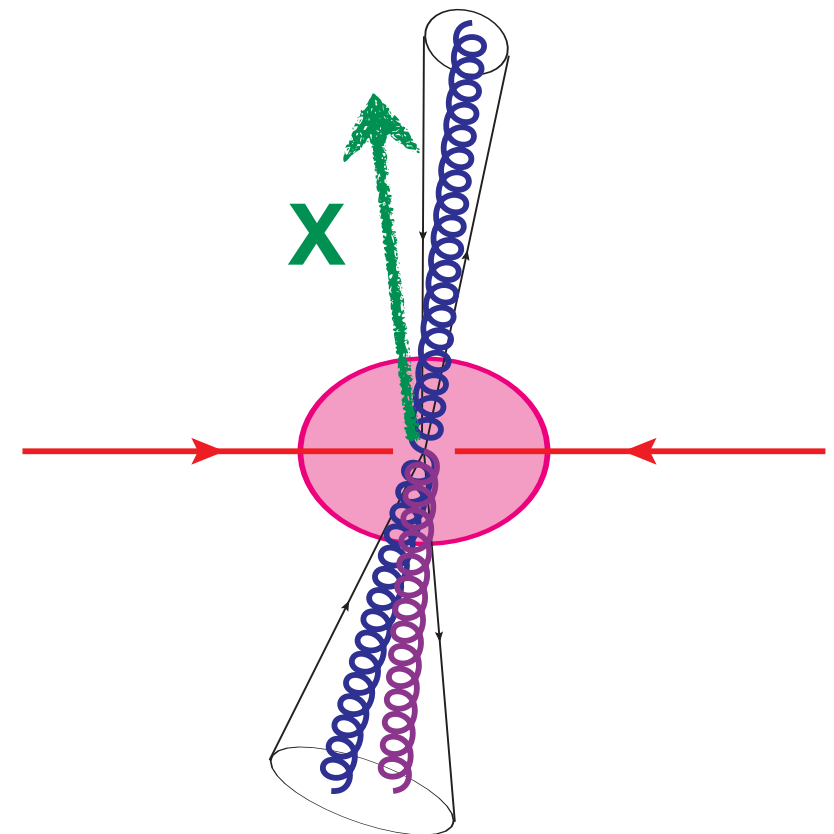
“Pure virtual”, e.g. 2-loop diagrams (Born topology)



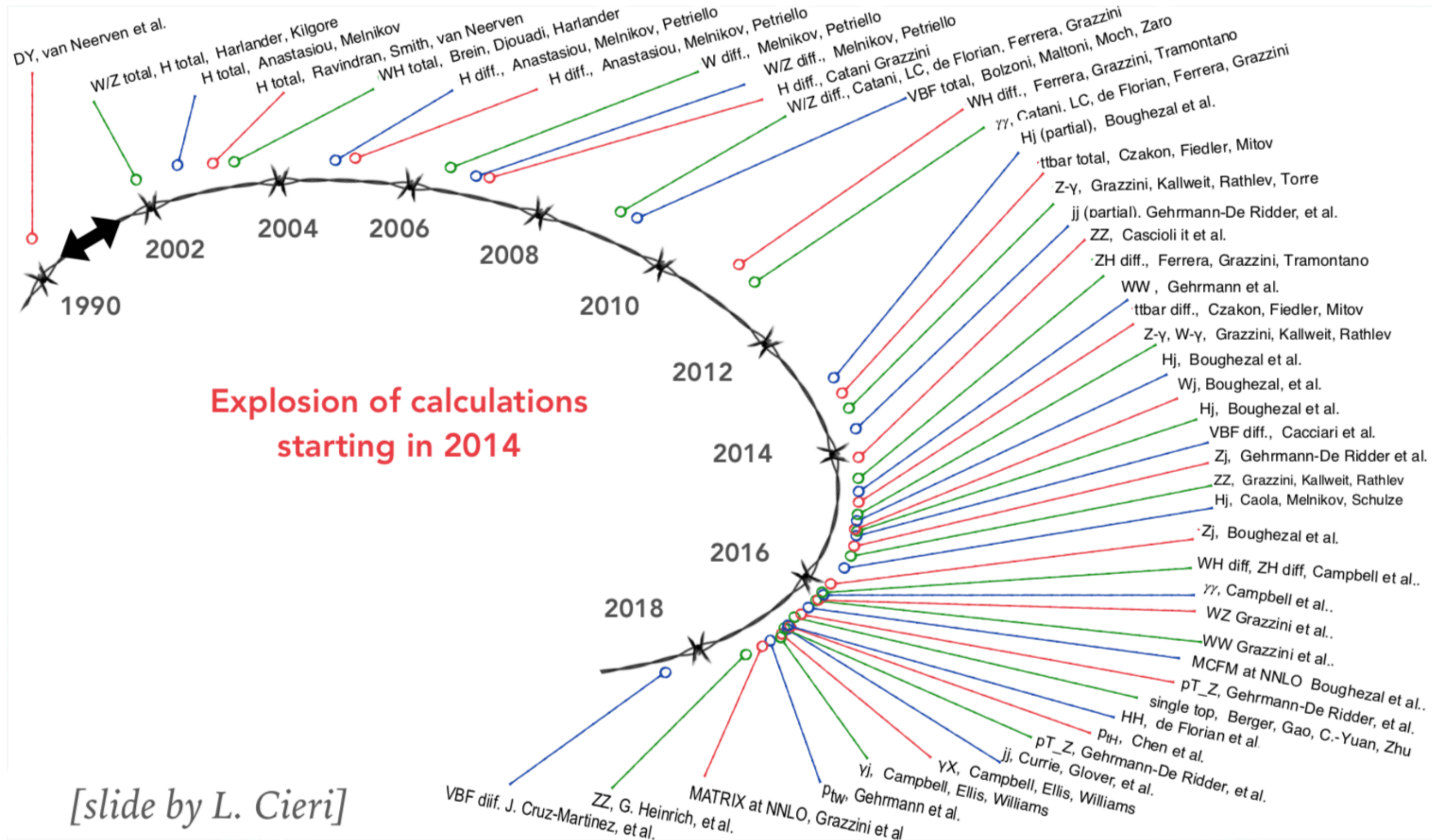
“Real-virtual”, 1-loop with an additional parton



“Real-real”, two additional partons



NNLO progress

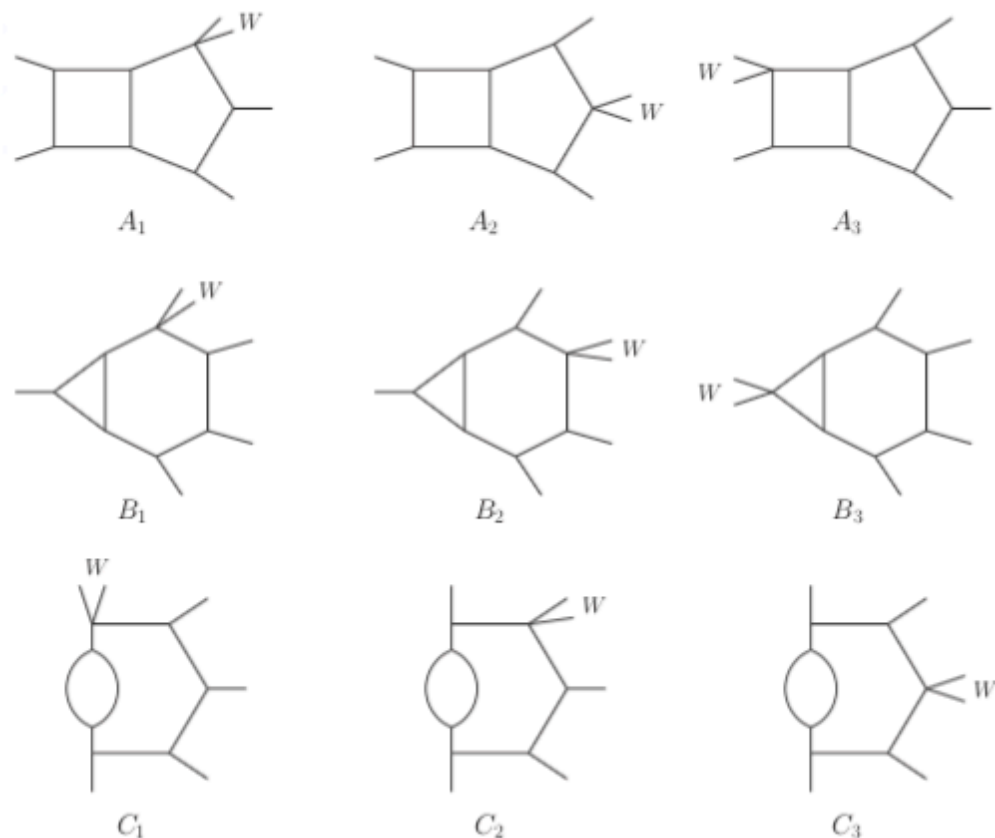


NNLO: hot topics and prospects

- Pushing beyond the current $2 \rightarrow 2$ frontier desirable for many reasons:
 - Higgs: ttH and Higgs+2 jets
 - Precision SM: 3 jets, W/Z/photon + 2 jets
- Requires deeper understanding of two-loop amplitudes: analytic structure, new (elliptic) integrals, numerical techniques for handling integrals.

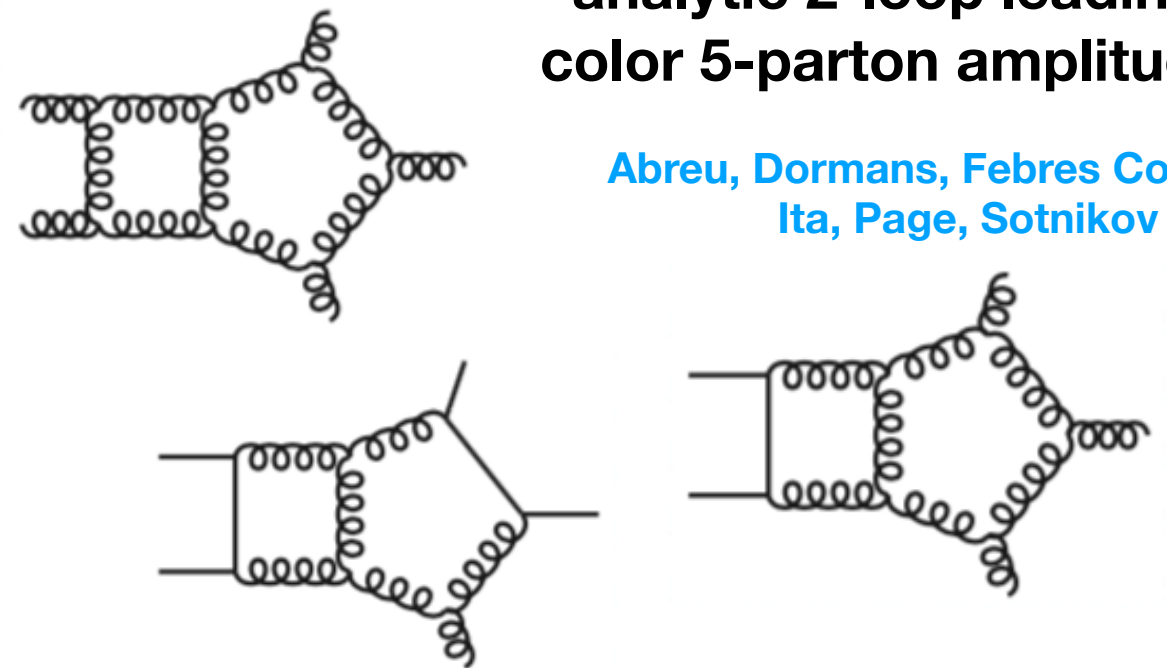
numerical evaluation of planar 2-loop W+4 parton amplitudes

Hartanto, Badger, Bronnum-Hansen, Peraro (2019)



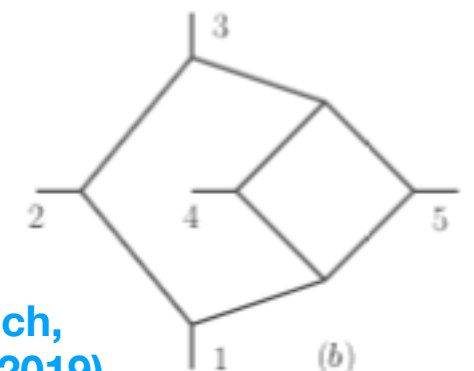
analytic 2-loop leading-color 5-parton amplitudes

Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov (2019)



full analytic 5-parton +++++ amplitude

Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zola (2019)

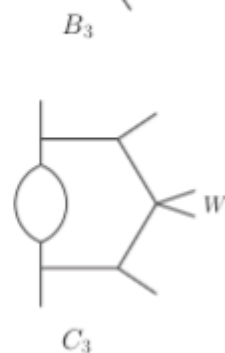
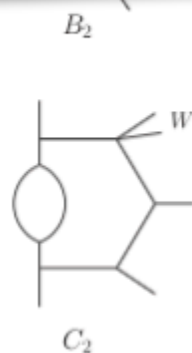
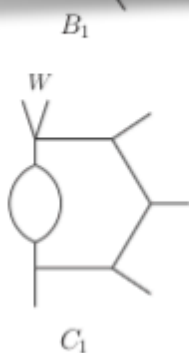


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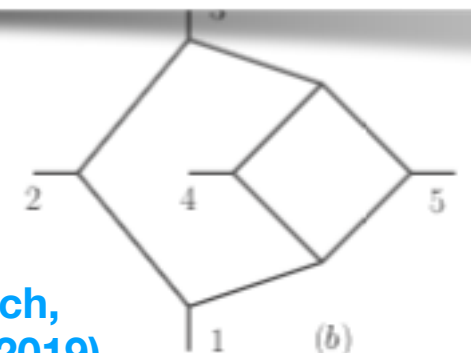
$$\mathcal{H}_1^{(2,0)} = \sum_{S\tau_1} \left\{ -\kappa \frac{[45]^2}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} I_{123;45} + \kappa^2 \frac{1}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} \left[5 s_{12} s_{23} + s_{12} s_{34} + \frac{\text{tr}_+^2(1245)}{s_{12} s_{45}} \right] \right\}$$

$$\mathcal{H}_{13}^{(2,1)} = \sum_{S\tau_{13}} \left\{ \kappa \frac{[15]^2}{\langle 23 \rangle \langle 34 \rangle \langle 42 \rangle} \left[I_{234;15} + I_{243;15} - I_{324;15} - 4 I_{345;12} - 4 I_{354;12} - 4 I_{435;12} \right] \right. \\ \left. - 6 \kappa^2 \left[\frac{s_{23} \text{tr}_-(1345)}{s_{34} \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} - \frac{3}{2} \frac{[12]^2}{\langle 34 \rangle \langle 45 \rangle \langle 53 \rangle} \right] \right\},$$



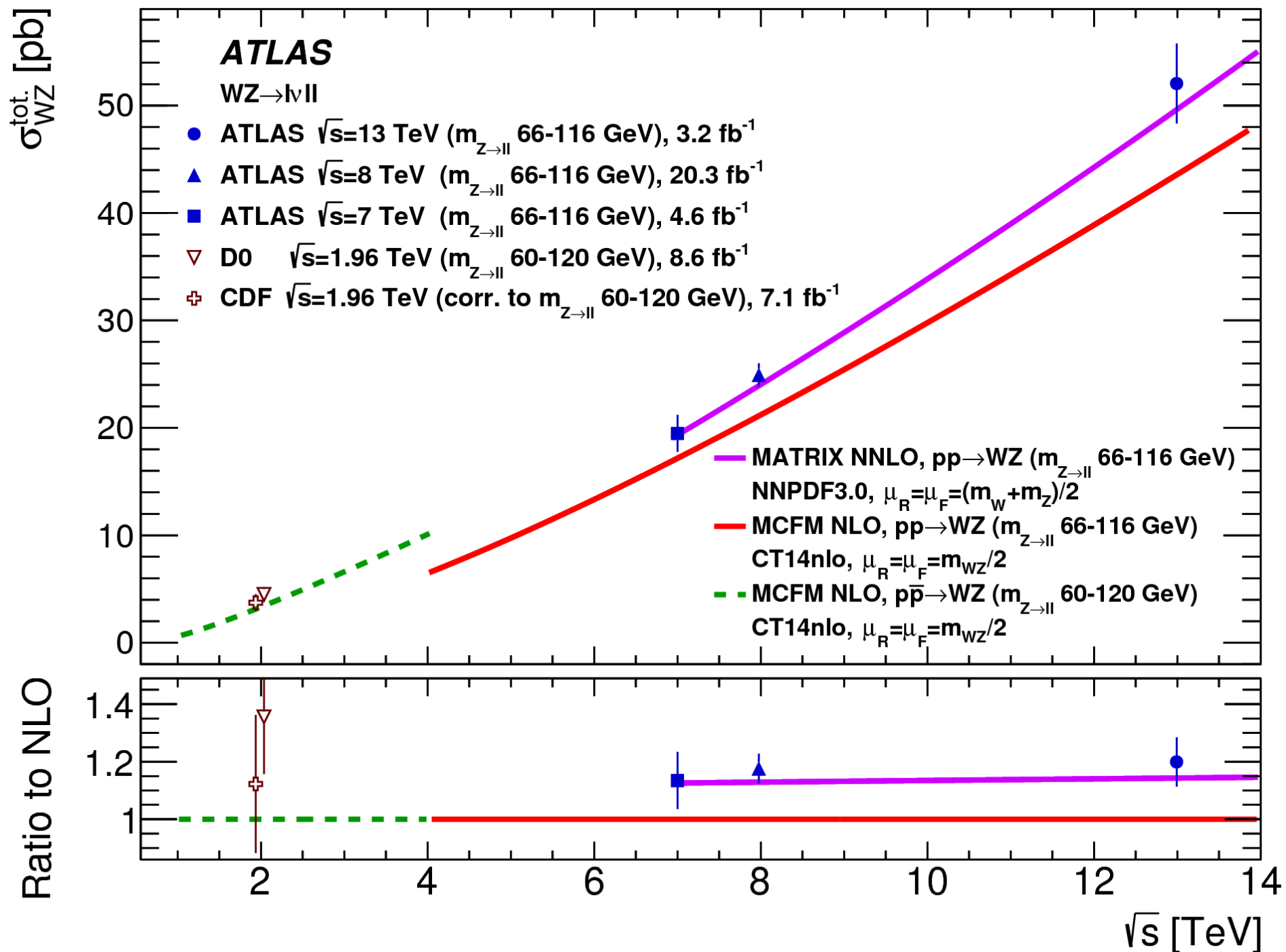
**full analytic 5-parton
+++++ amplitude**

Badger, Chicherin, Gehrmann, Heinrich,
Henn, Peraro, Wasser, Zhang, Zola (2019)



What does it mean?

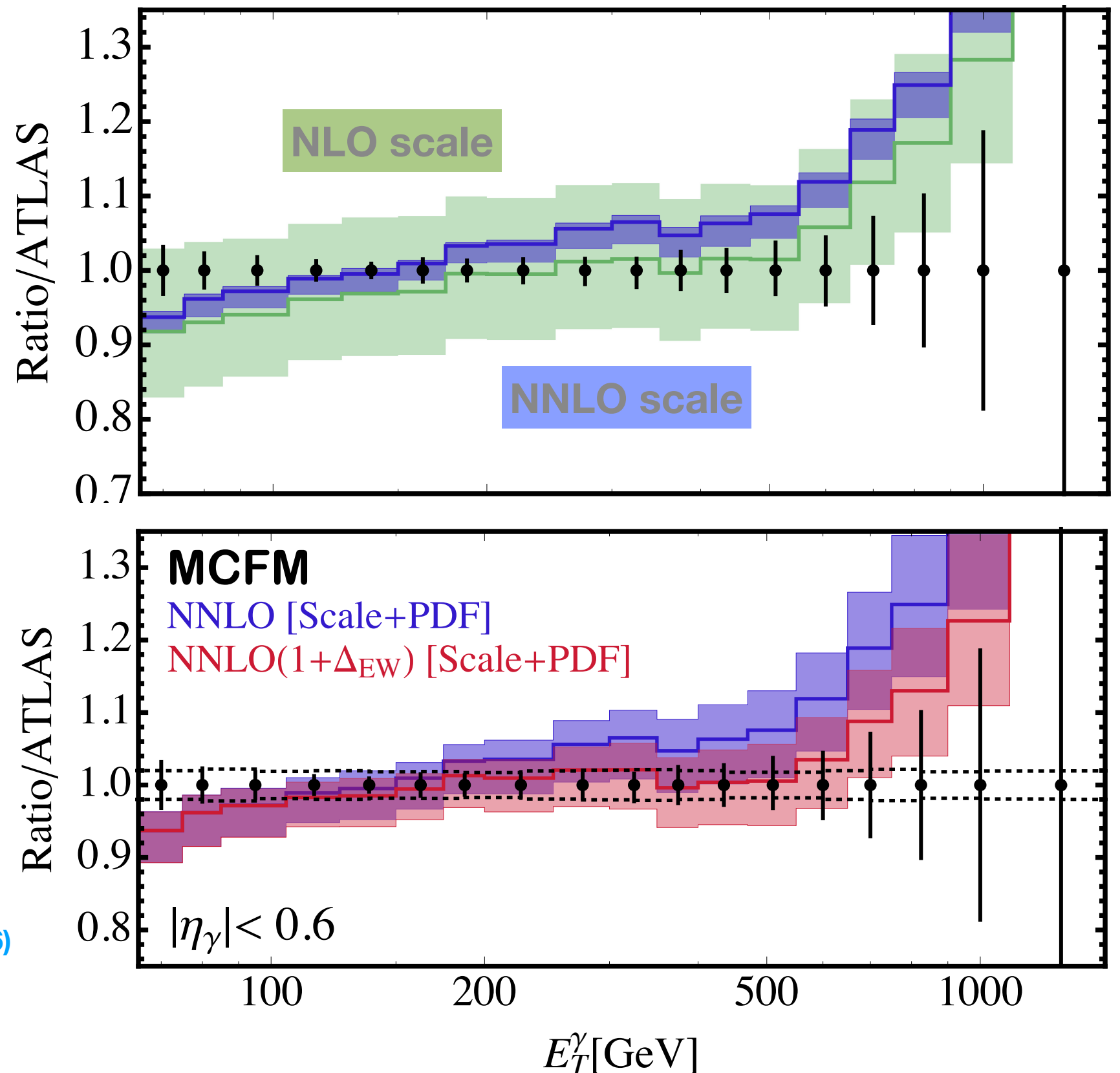
- The increasing availability of calculations at NNLO is essential to properly describe much of the data taken at the LHC.



Precision

- Much-reduced scale dependence yields percent-level theory uncertainties that can be competitive with experimental ones
- new opportunities for measurements and constraints
- at this level, often need to consider electroweak effects as well (especially in tails of distributions)

Ellis, Williams, JC (2016)

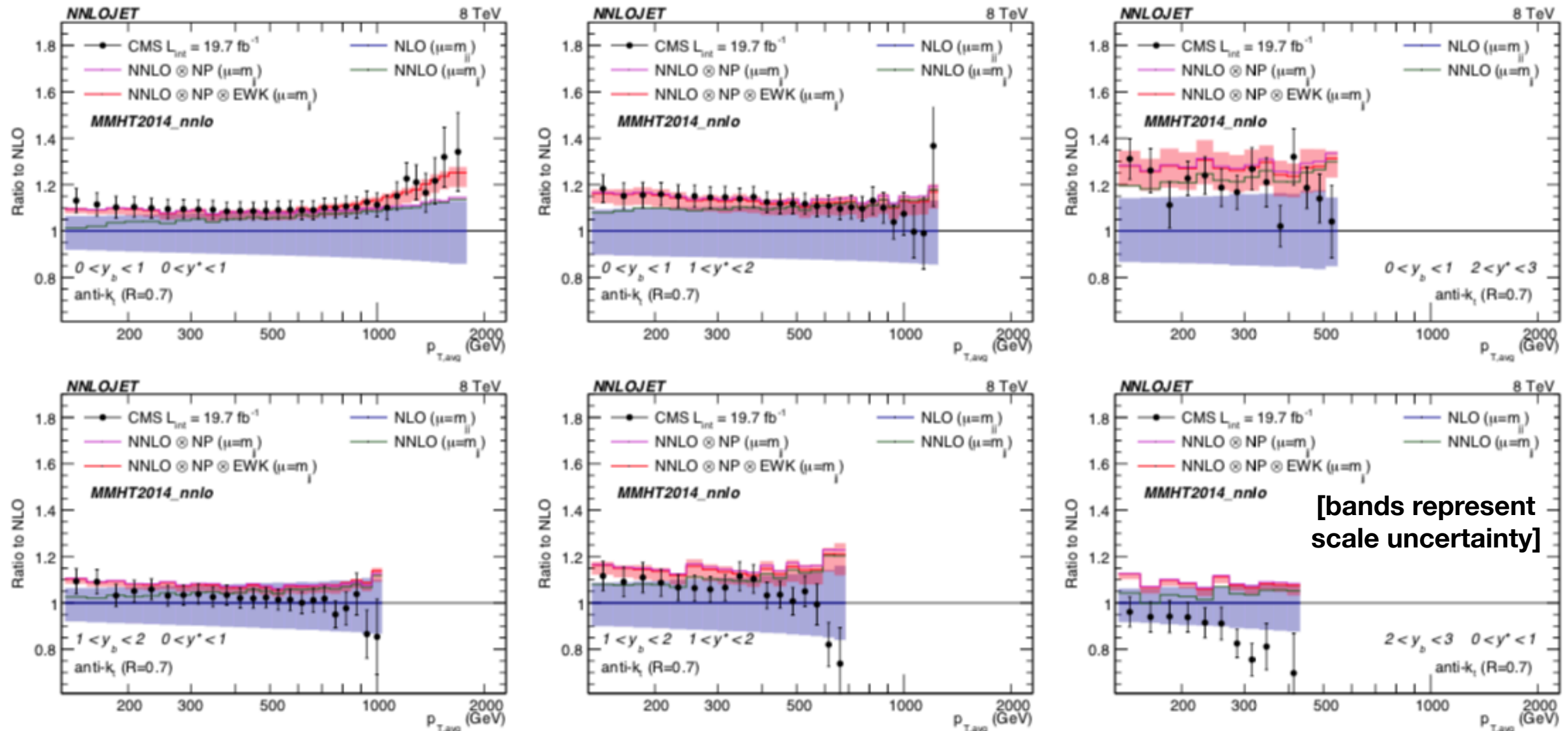


Differential jet cross-sections

Gehrmann-de Ridder, Gehrmann, Glover, Huss, Pires (2019)

$$x_{1,2} = \frac{2p_{T,\text{avg}}}{\sqrt{s}} e^{\pm y_b} \cosh y^*$$

dijet rapidity separation $y^* = \frac{1}{2} |y_1 - y_2|$



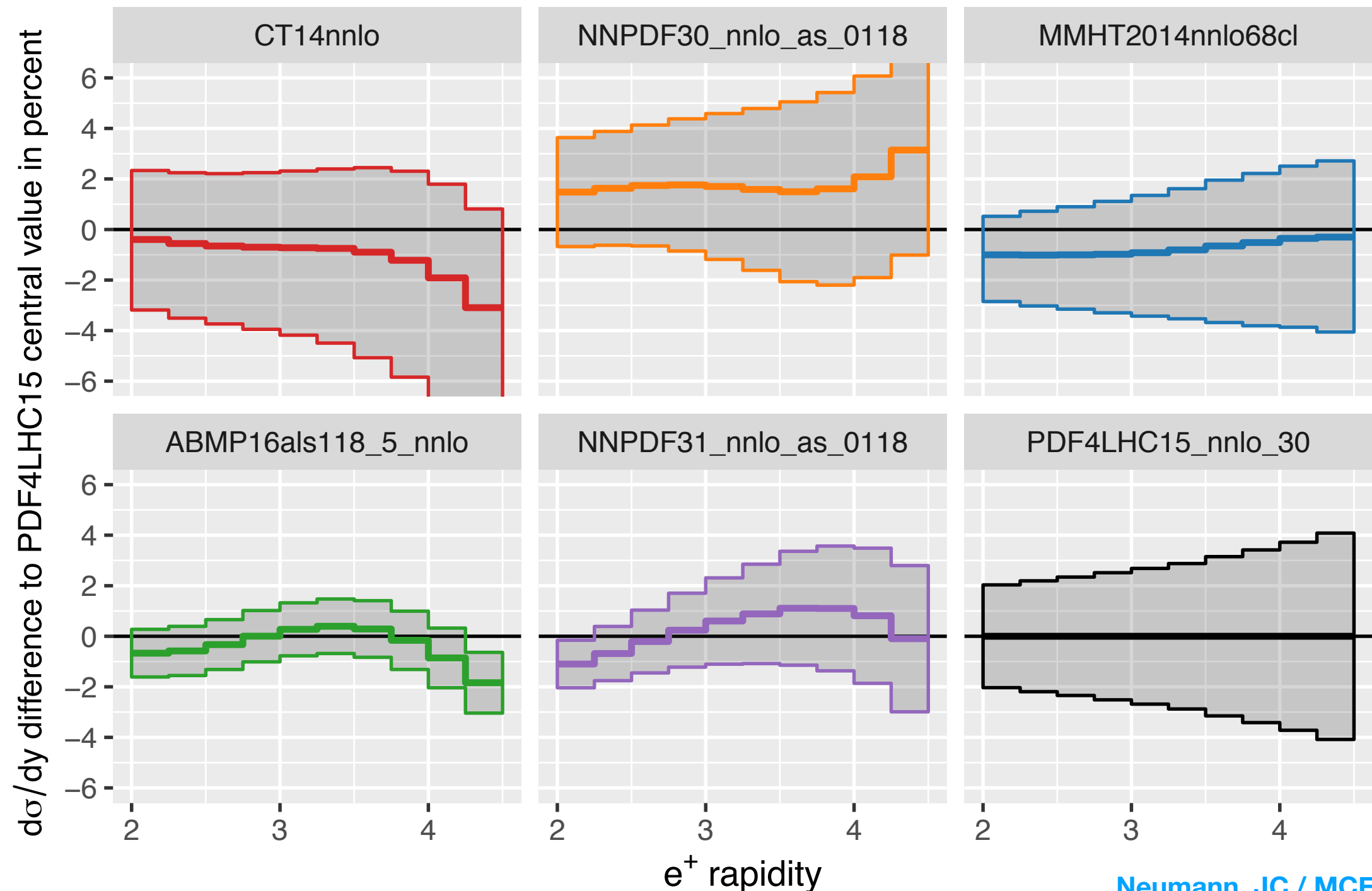
boost of

dijet system
 $y_b = \frac{1}{2} |y_1 + y_2|$

- Smaller uncertainties, better agreement with data (especially low boost)
- At large boost (and jet p_T) disagreement an opportunity to refine high- x PDF

PDF studies

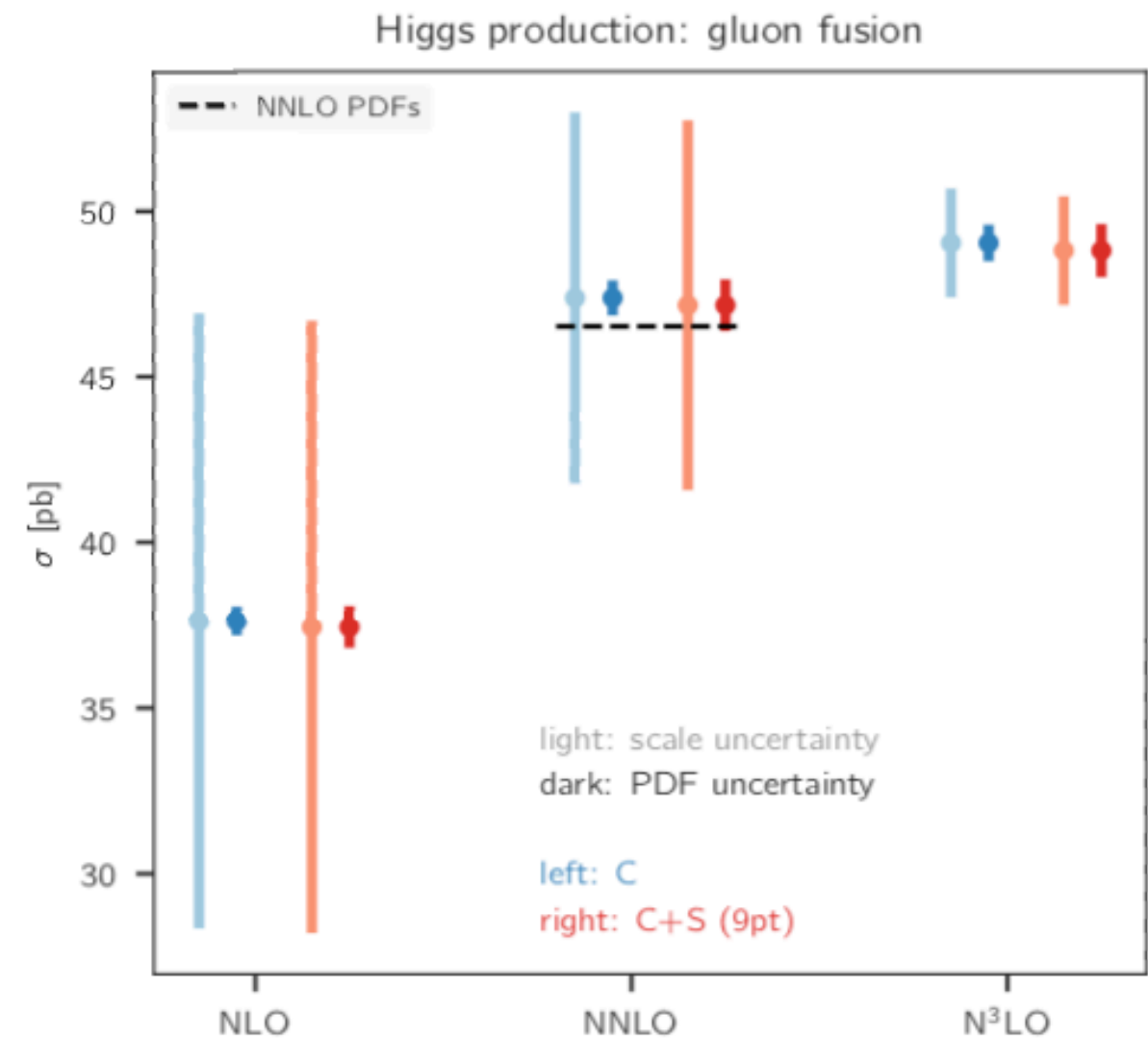
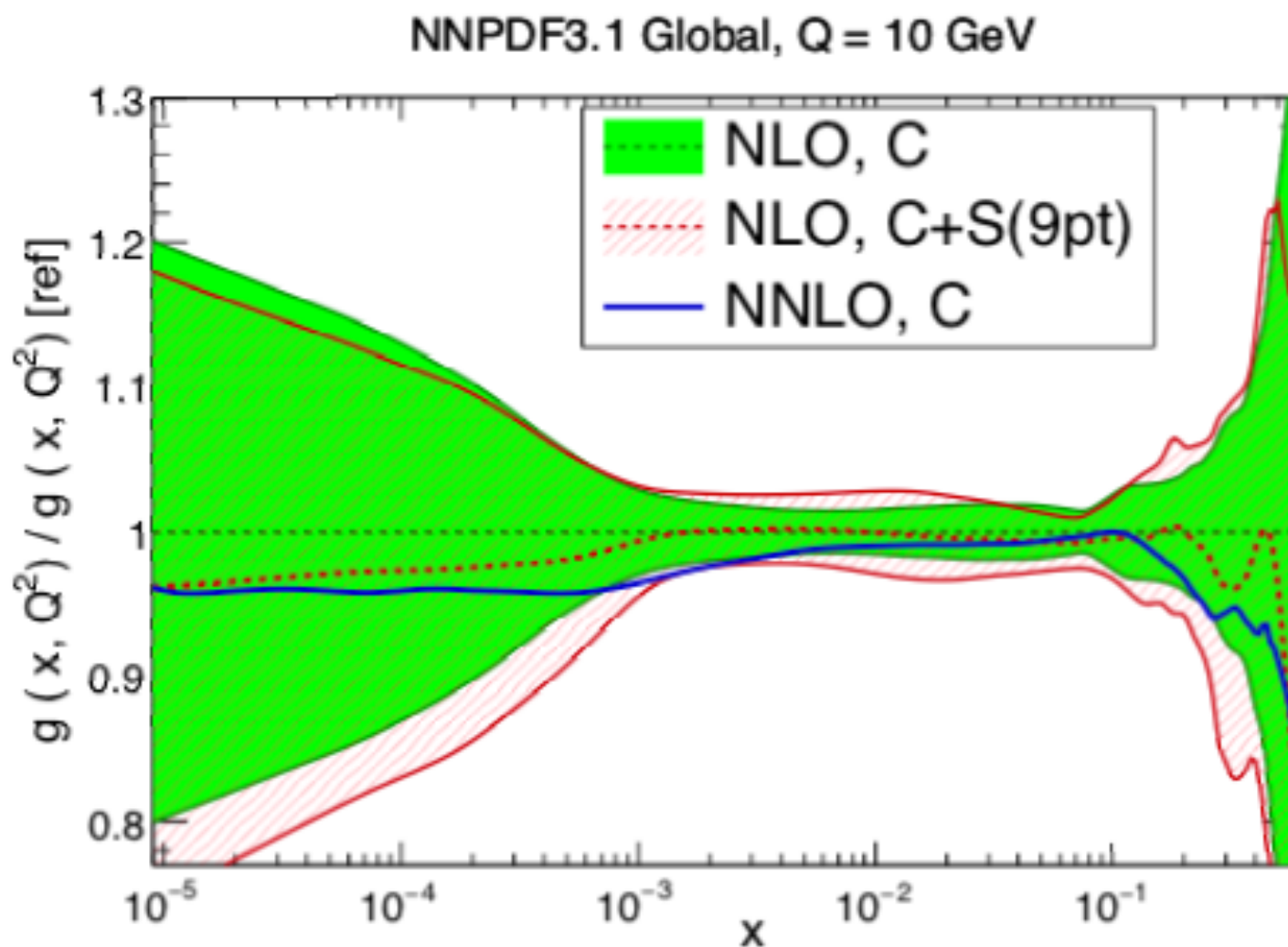
- Need tools able to compute NNLO predictions for multiple PDFs, precisely enough to see differences, both for assessing compatibility and eventually for global fitting.



Higher-order uncertainty in fits

- Attempt to capture uncertainty in fits due to missing higher orders (scale uncertainty)
 - so far only to NLO where all calculations are readily available.
 - general formalism worked out, also applicable to nuclear & higher-twist corrections

Abdul Khalek, Ball, Carrazza, Forte, Giani, Kassabov, Nocera, Pearson, Rojo, Rottoli, Ubiali, Voisey, Wilson / NNPDF (2019)

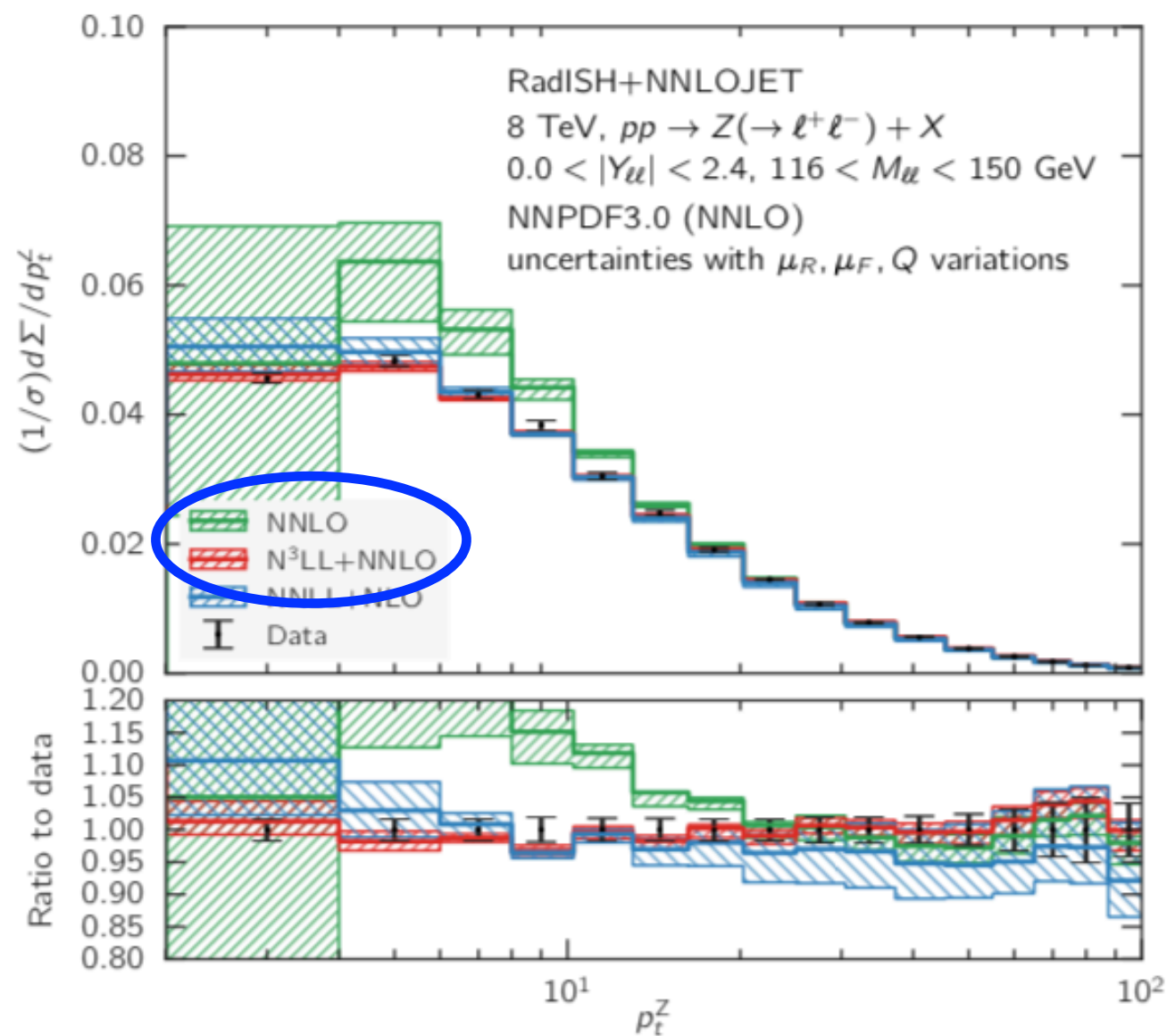


Not just fixed order

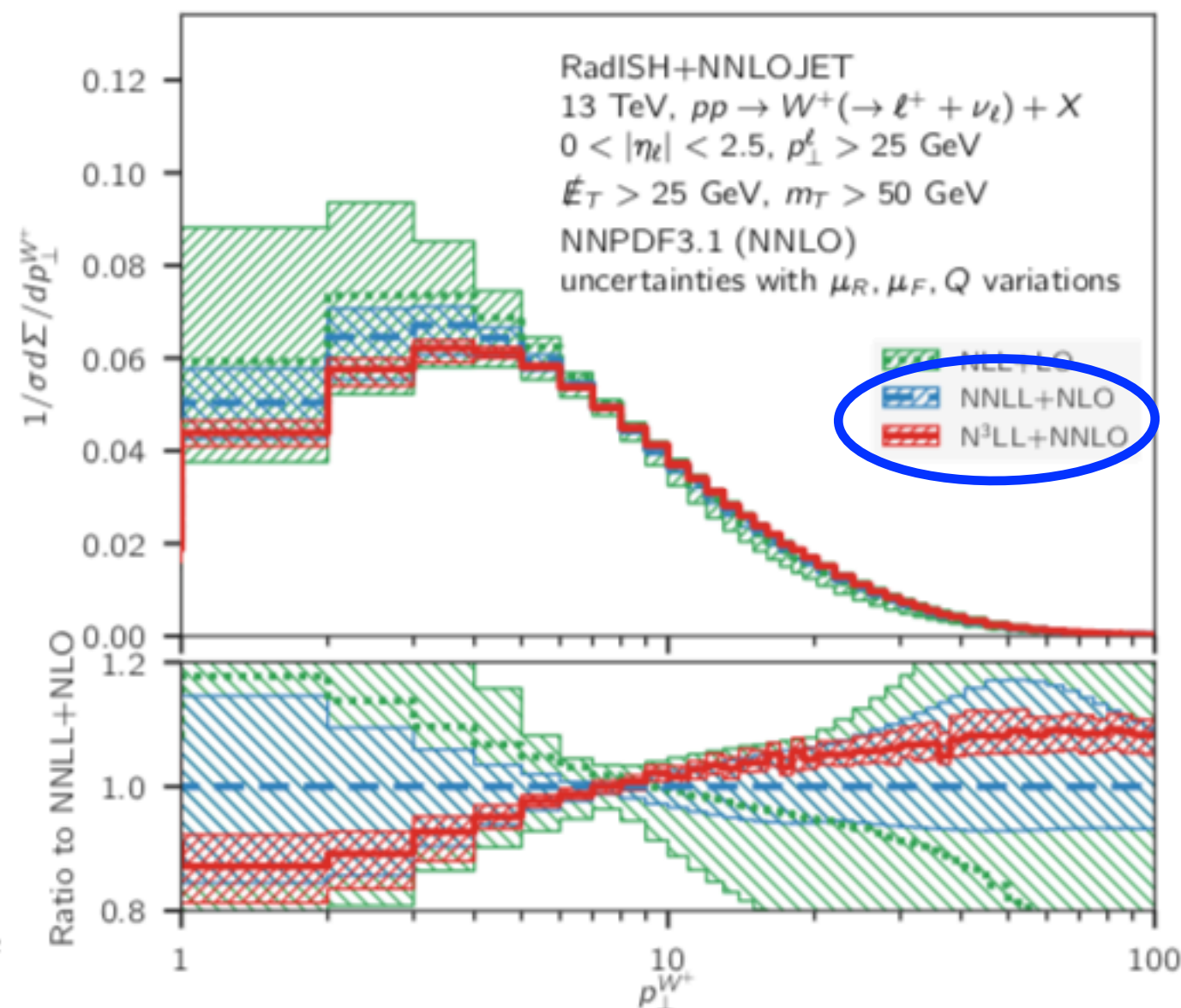
- W and Z p_T spectrum important for PDF determination, W mass (also H for BSM effects)
- State-of-the-art combines NNLO fixed order with N³LL large-log resummation

Bizon, Chen, Gehrmann-de Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli (2018)

Bizon, Gehrmann-de Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Walker (2019)



← resummation crucial →
NNLO sufficient



significant shape change (up to ~10%) wrt NNLL+NLO, residual uncertainty < 5%

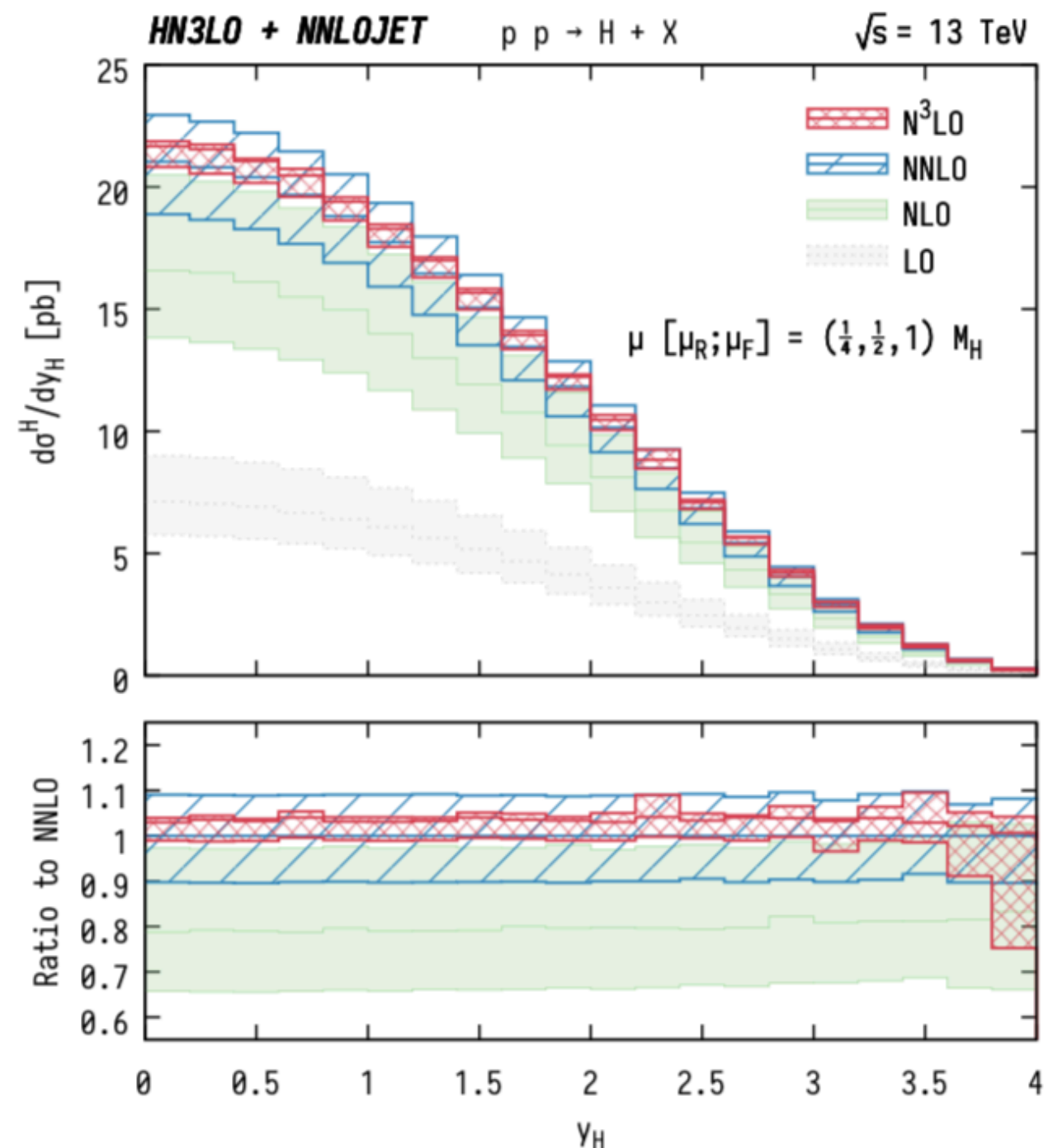
Beyond NNLO

- Only a handful of N³LO results, focussed on Higgs production
 - inclusive cross-sections for gluon fusion and VBF
 - W and Z production notably absent!
- Aside from experimental considerations, pure theoretical interest
 - first order at which all parton channels are computed to at least NLO
 - how does series converge?
- Latest results: completely-differential calculation of Higgs production at N³LO

Cieri, Chen, Gehrmann, Glover, Huss (2018)

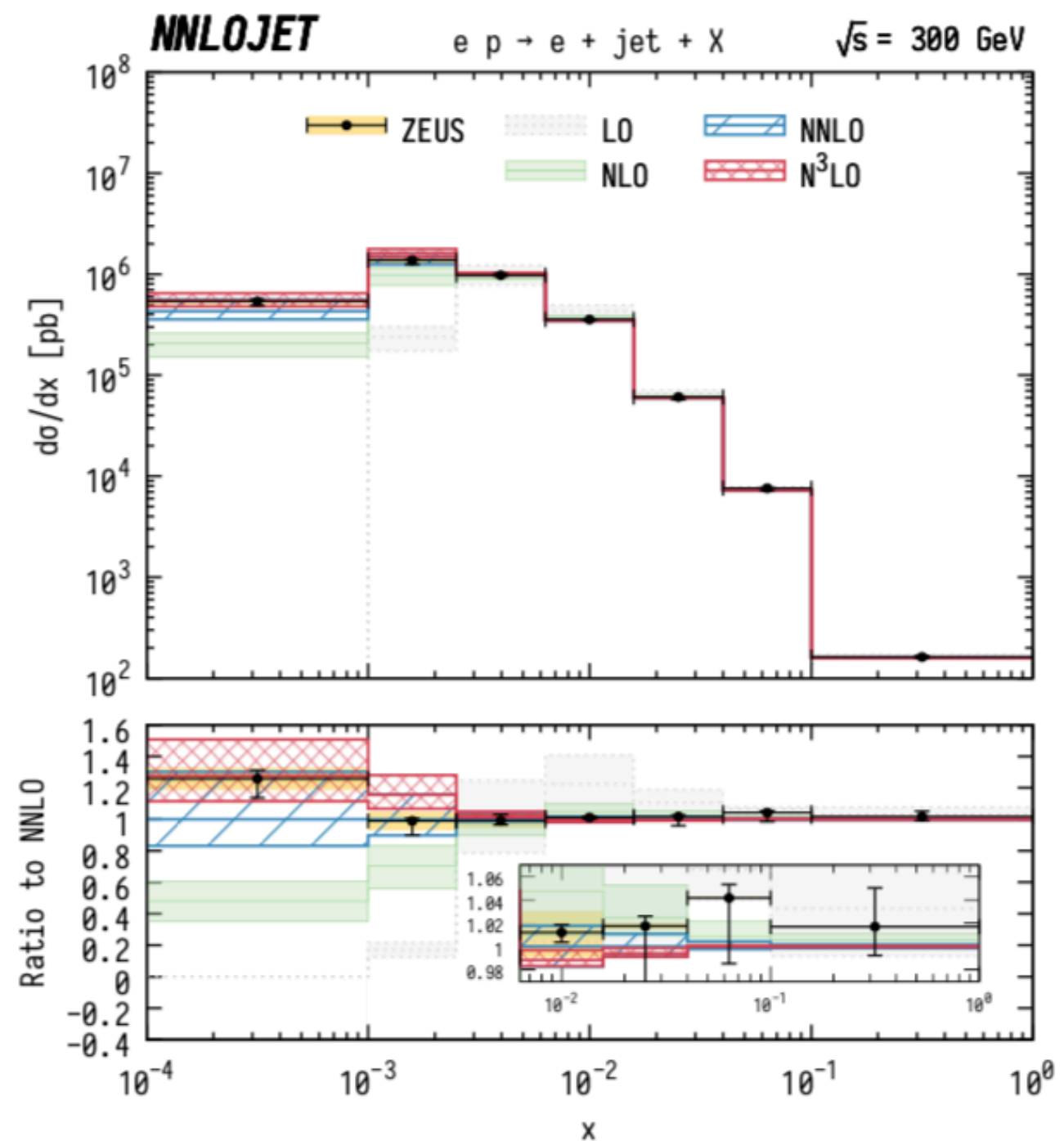
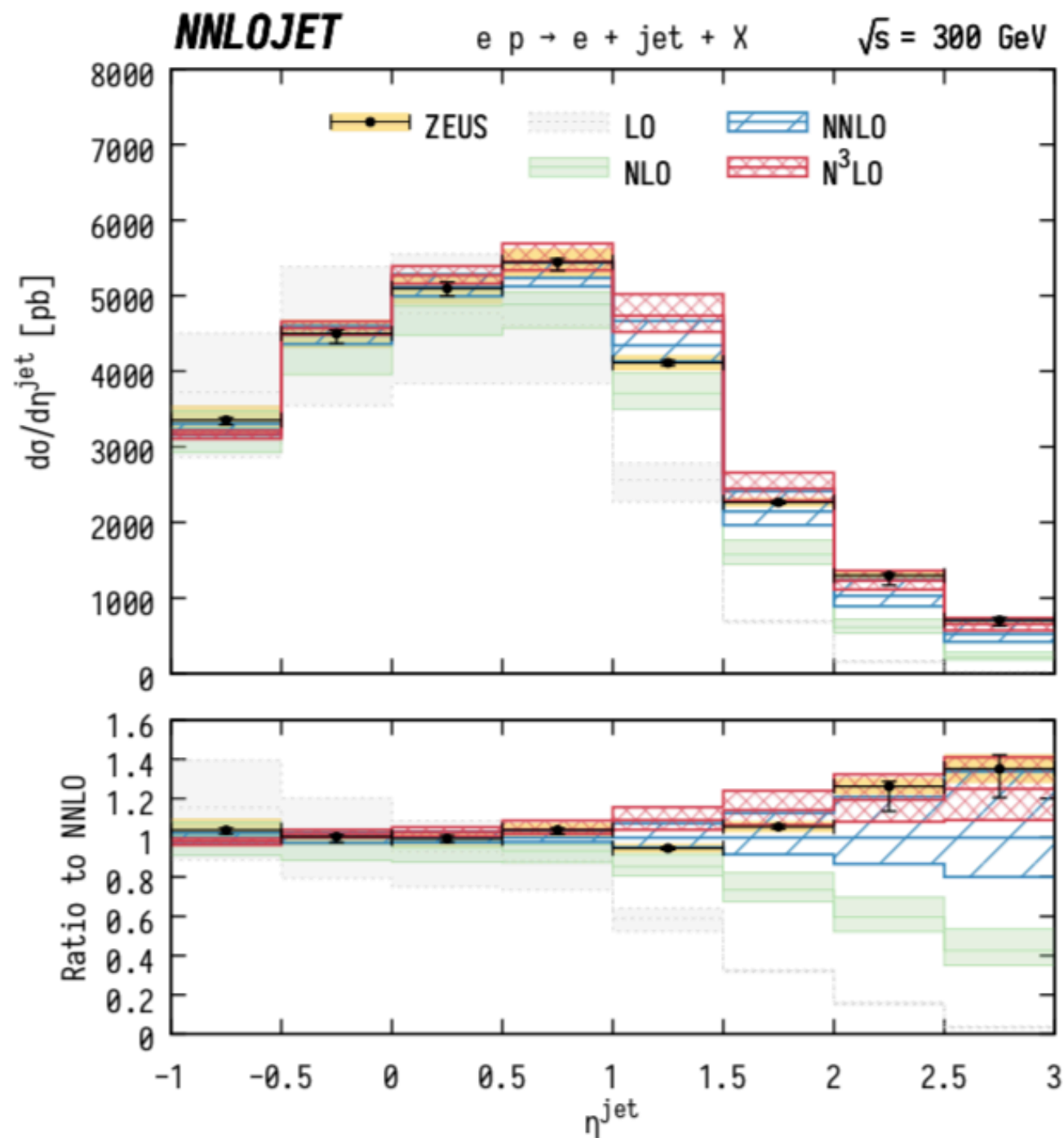
 - excellent agreement with earlier (threshold expansion) calculation

Dulat, Mistlberger, Pelloni (2018)
- Open question: how does perturbative stability look after fiducial cuts?



Steps towards the EIC

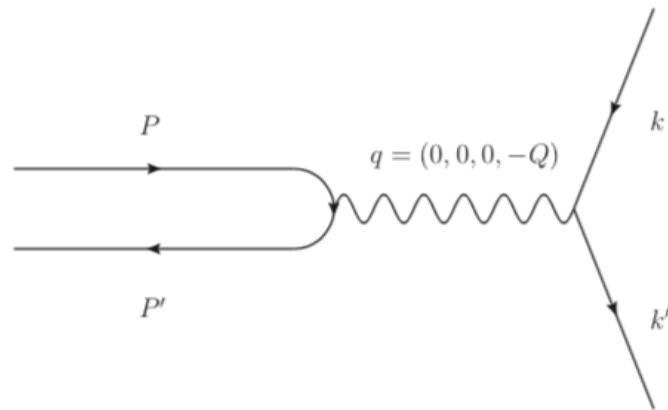
- N³LO jet production in DIS in the lab frame [Currie, Gehrmann, Glover, Huss, Niehues, Vogt \(2017\)](#)
 - overlapping uncertainty bands, factor two smaller uncertainties, better description even in regions with lower accuracy or susceptible to large logs.



NNLO calculations for DIS

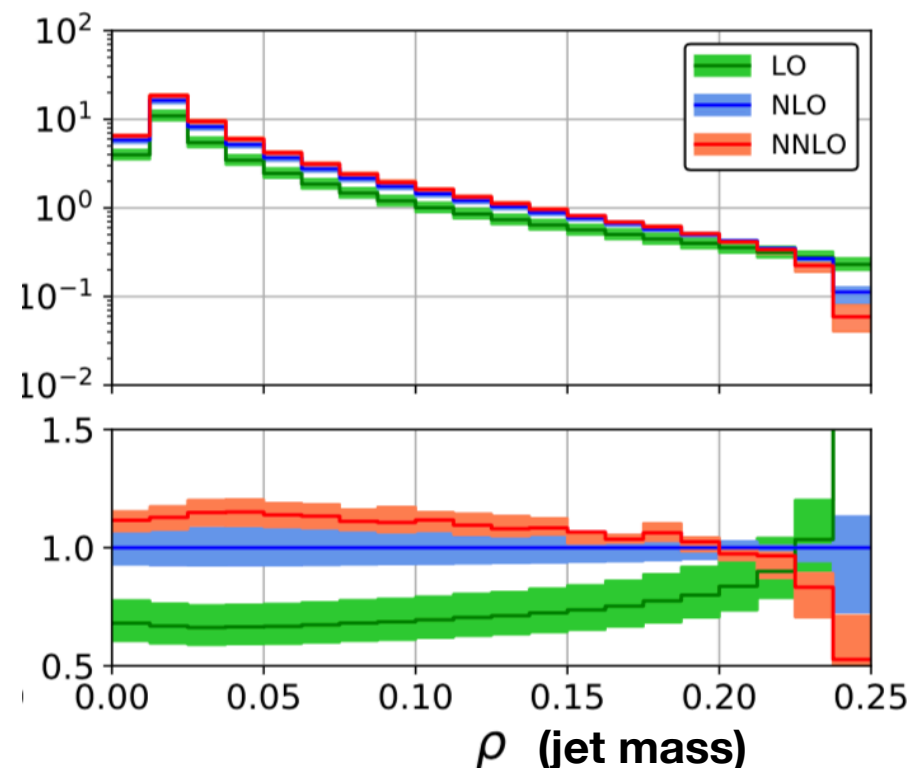
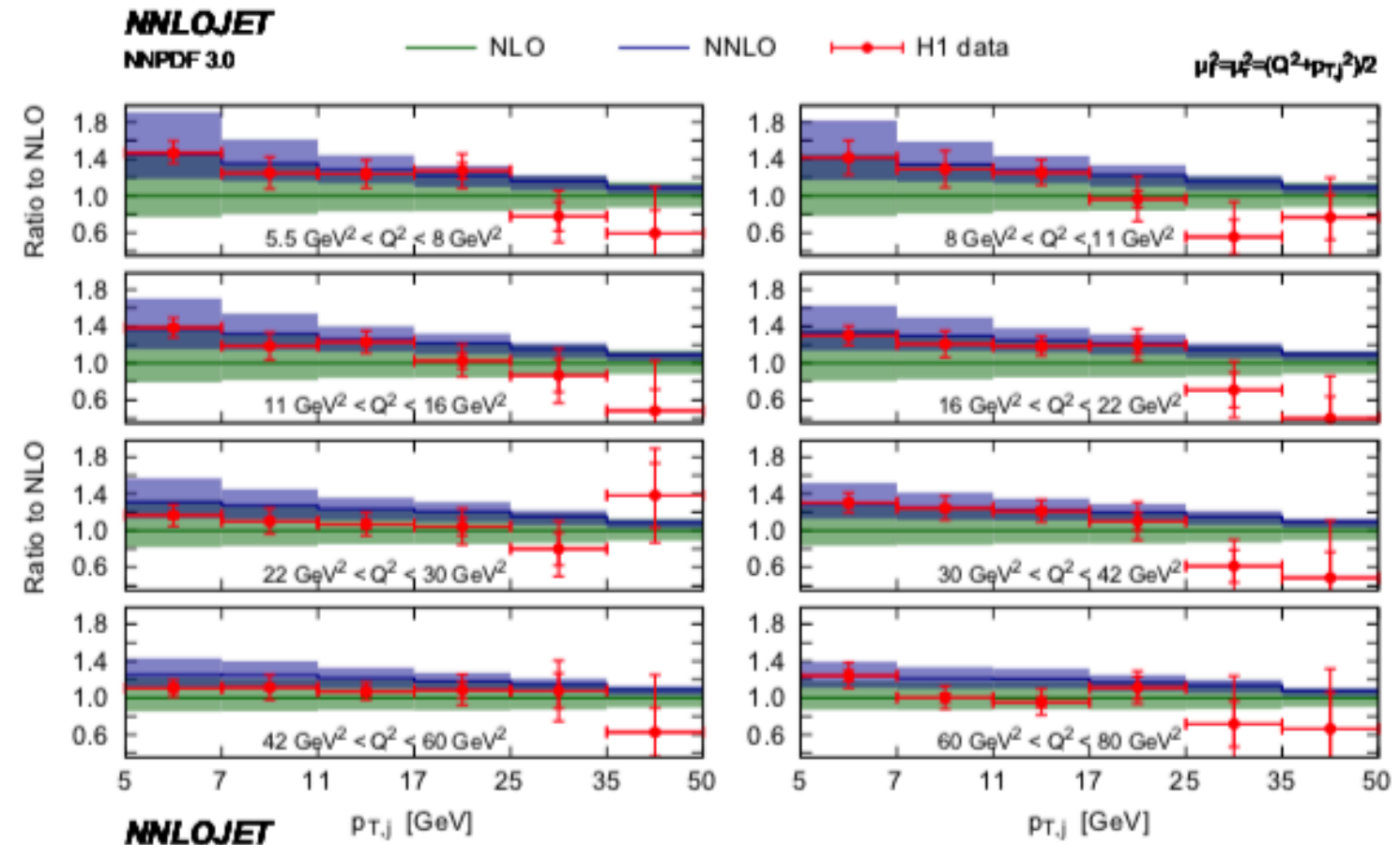
- NNLO corrections to DIS jet and dijet production in the Breit frame

Currie, Gehrmann, Huss, Niehues (2017)



- inclusive jet: substantial corrections at low Q^2 and $p_{T,j}$, up to 60%, much improved description of data
- Corresponding results for event-shape distributions

Gehrmann, Huss, Mo, Niehues (2019)



In general up to 20% corrections, non-uniform, decreased scale uncertainty but small overlap with NLO

Non-perturbative effects

- Description of data requires the addition of power corrections to account for parton-hadron transition.
- Dispersive model (also used at LEP) shifts differential distribution:

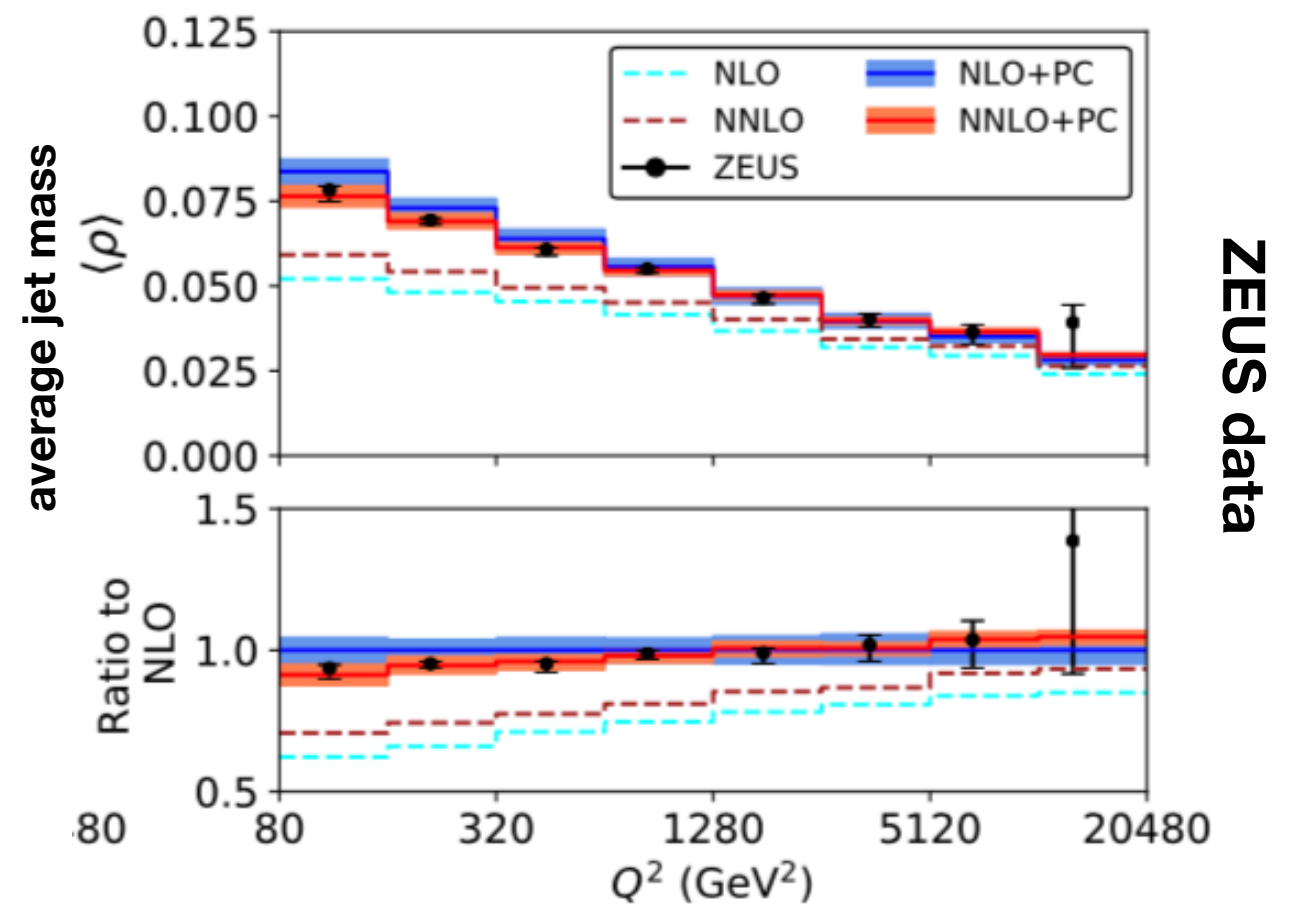
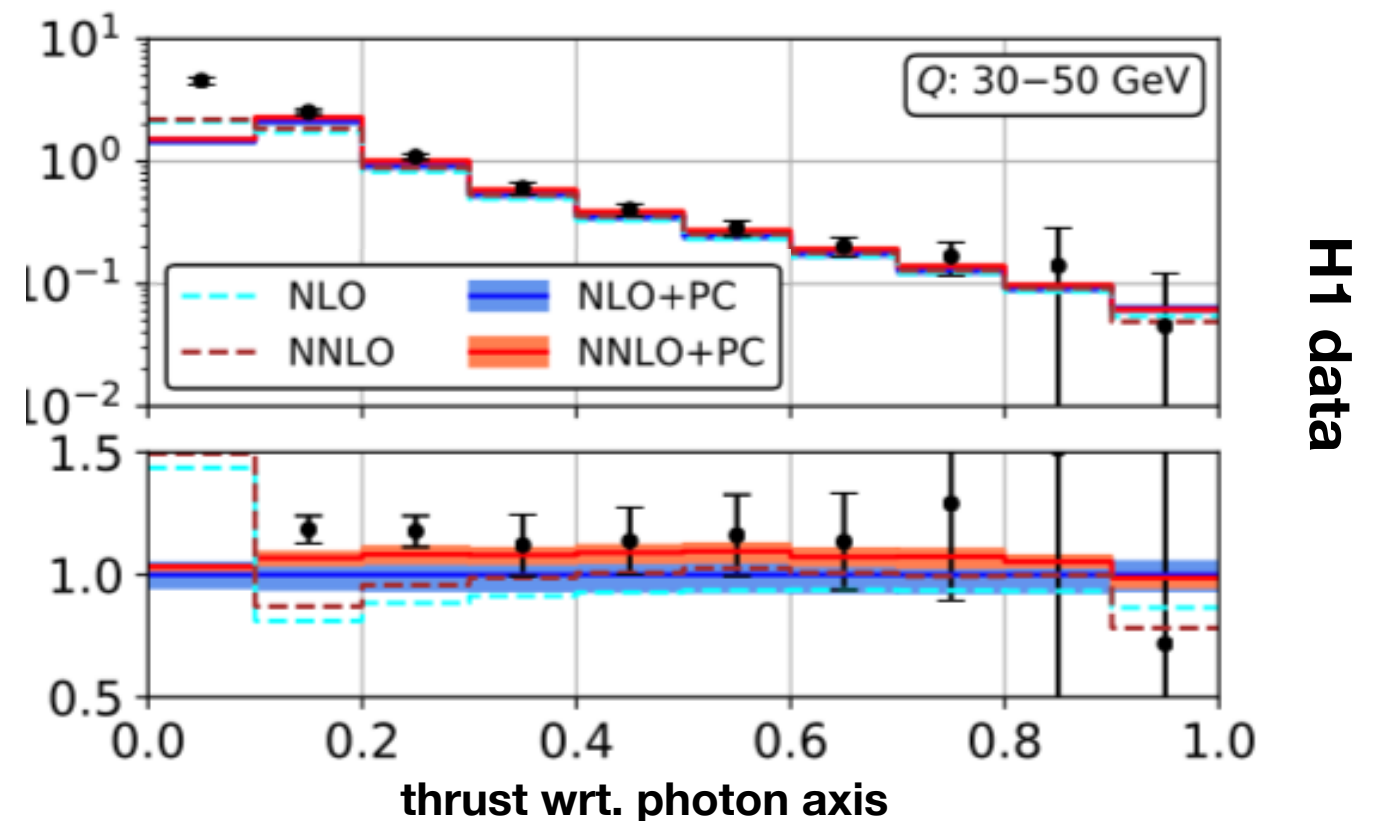
$$\frac{d\sigma^{\text{hadron}}(F)}{dF} = \frac{d\sigma^{\text{parton}}(F - a_F P)}{dF}$$

and mean values correspondingly:

$$\langle F \rangle = \langle F \rangle^{\text{pert.}} + a_F P,$$

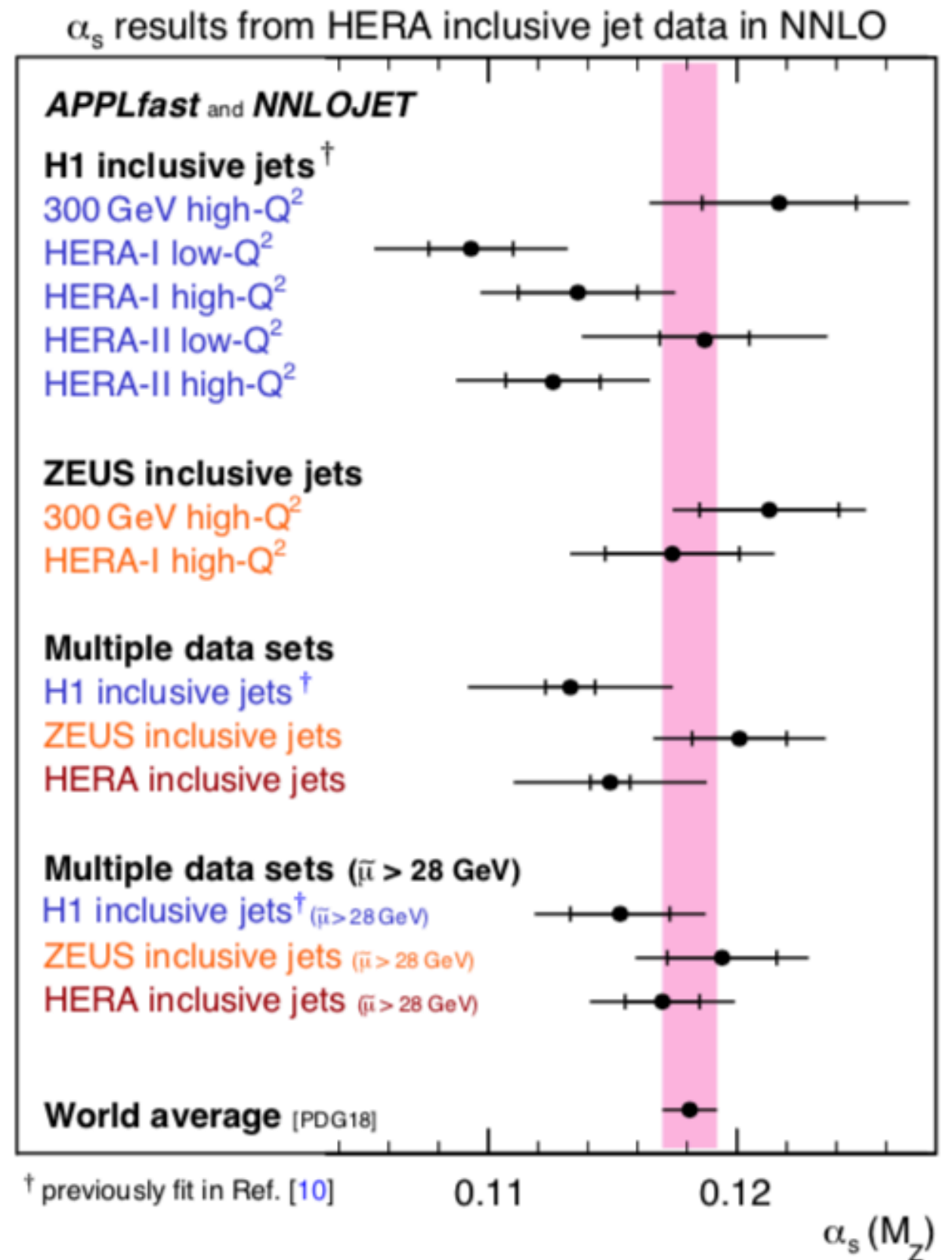
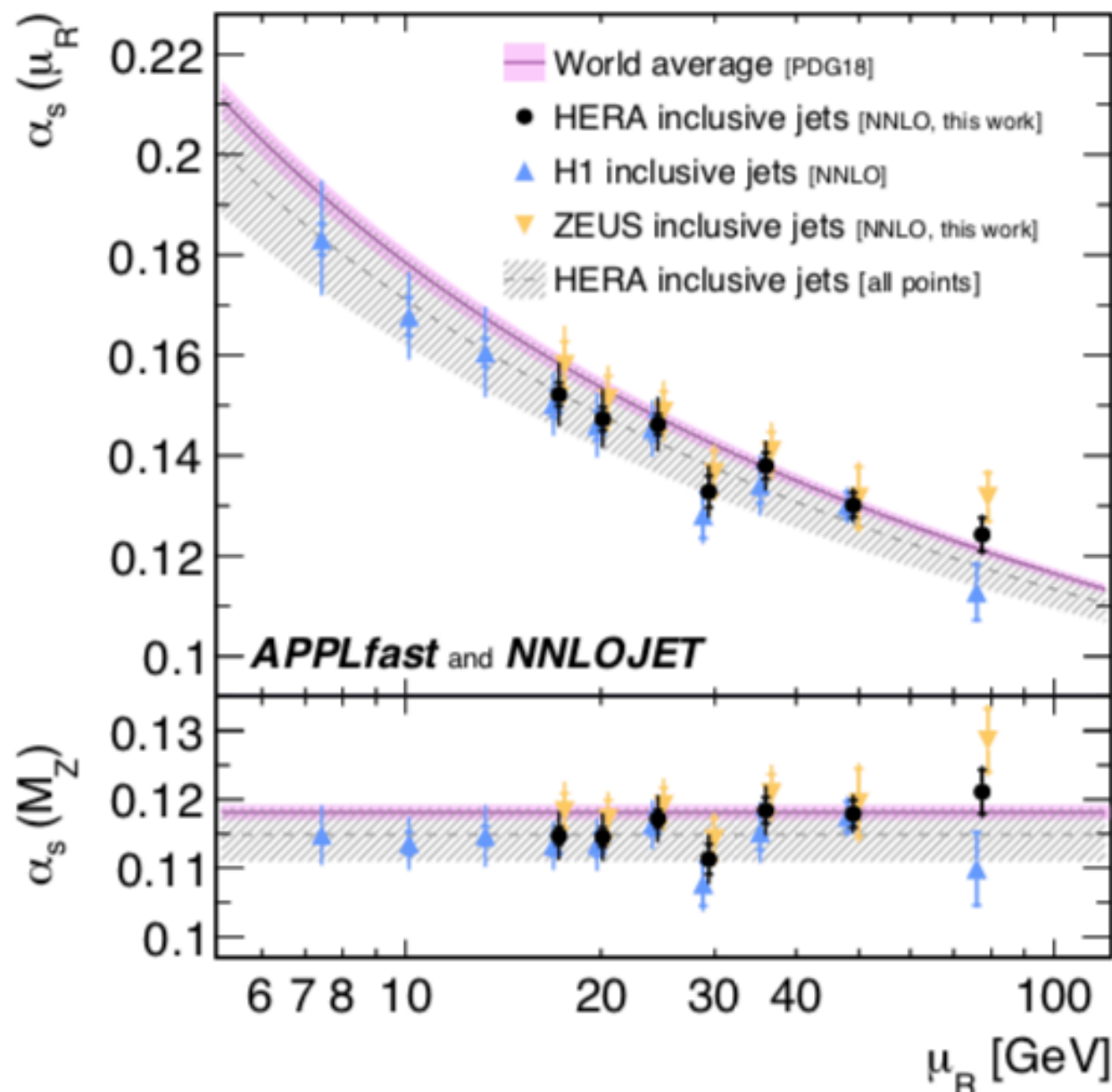
(universal P , a_F varies by event shape)

- Precision QCD studies now possible through reanalyses of HERA data
 - opportunity for EIC



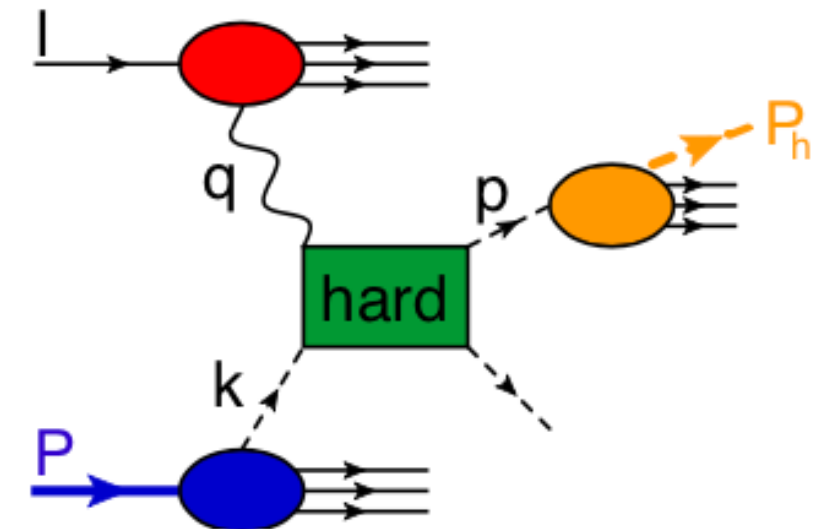
Extraction of α_s from HERA data

- Demonstration for APPLfast project
[Britzger et al \(2019\)](#)
- uses perturbative input from NNLO (NNLOJET) to produce interpolation tables for a posteriori PDF analyses



Single-inclusive production at an EIC

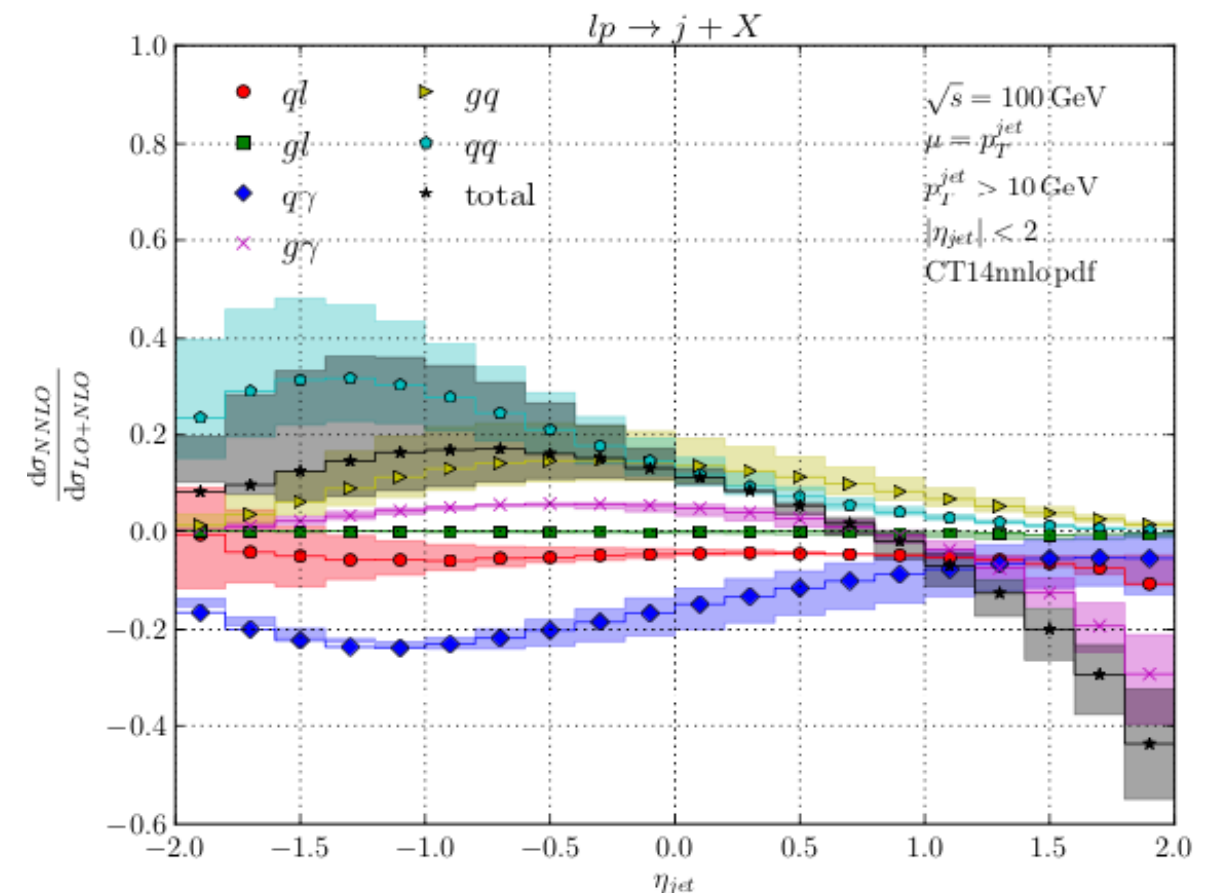
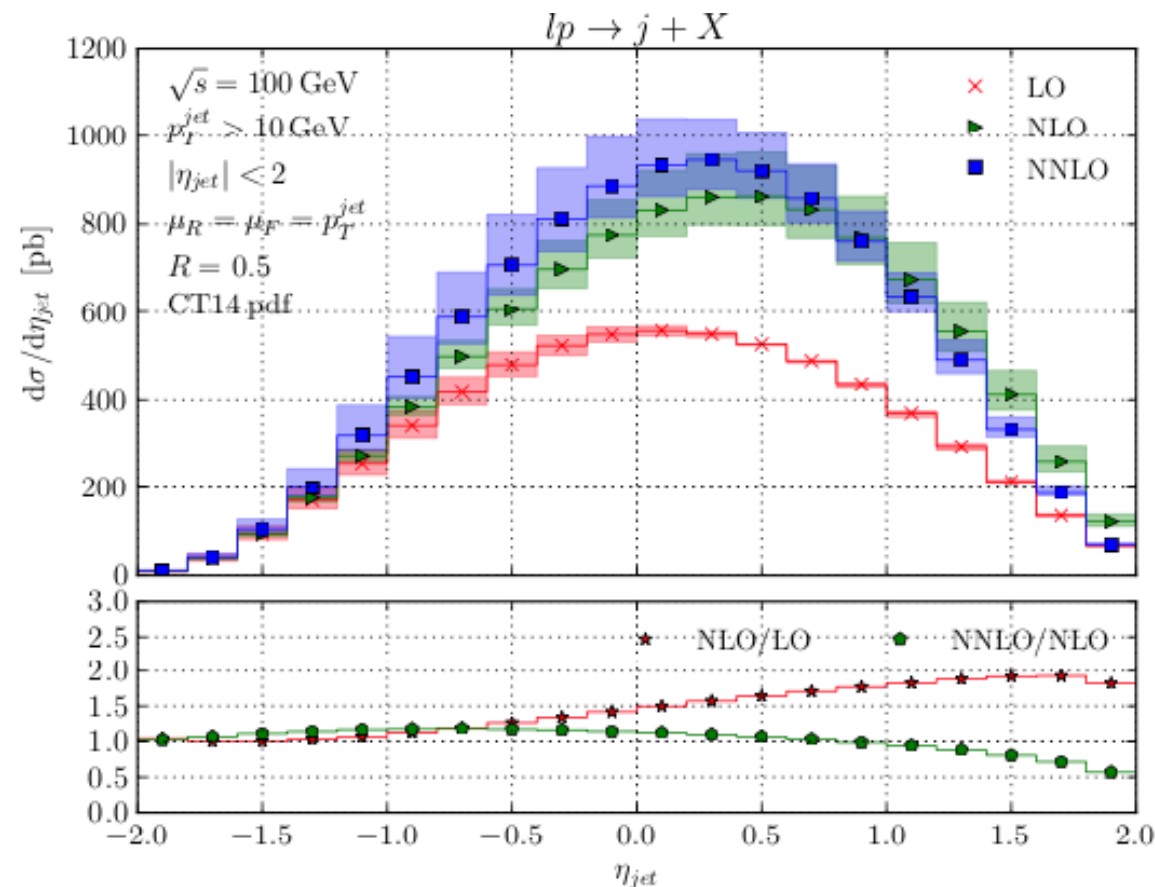
- Need predictions for single-inclusive hadron production: no lepton observed.
- However, when inclusive of the lepton, must also account for configurations resulting from quasi-real photon (lepton travels down the beam pipe).



- can capture through Weizsacker-Williams approach (lepton structure function) [Hinderer, Schlegel, Vogelsang \(2015\)](#)

- Recently used to compute NNLO predictions for EIC

[Abelof, Boughezal, Liu, Petriello \(2016\)](#)



Summary

- Perturbation theory at NLO a workhorse of the LHC.
- Many calculations at NNLO (even N³LO) have emerged over the last 5-10 years that are suited to precision studies:
 - describe data over a wider kinematic range;
 - exhibit uncertainties smaller than, or at least competitive with, data.
- Turning complex calculations into tools for data analysis still a challenge
 - new tools making better use of CPU resources, interpolation techniques
- Some attention from the LHC precision community turning to topics closer to EIC
 - variety of calculations for DIS in particular
- Areas ripe for cross-fertilization:
 - inclusion of higher-order corrections in Monte Carlo tools
 - extraction of PDFs at higher perturbative orders, “ultimate” LHC precision
 - understanding remaining non-perturbative effects, e.g. in event shapes