

# HET Perspective on Lattice QCD

---

Andreas S. Kronfeld  
Fermilab & IAS TU München

LPC Workshop on  
Physics Connections between the LHC and EIC  
Fermilab | November 14, 2019



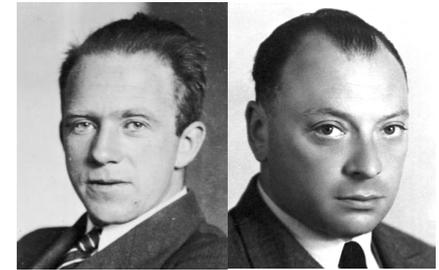
# Basic Idea

- Discretize space to have Kronecker  $\delta_{xy}$  instead of Dirac  $\delta(\mathbf{x}-\mathbf{y})$  so the degrees of freedom are countable [[Z. Phys. 56 \(1929\) 1](#)].
- Set up path integral for these degrees of freedom, which introduces discrete time steps [[RMP 20 \(1948\) 367](#)].
- Figure out how to enforce gauge invariance [[PRD 10 \(1974\) 2445](#)].
- End up with mathematically well-posed definition of QFT:

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(-S) [\bullet]$$

MC hand

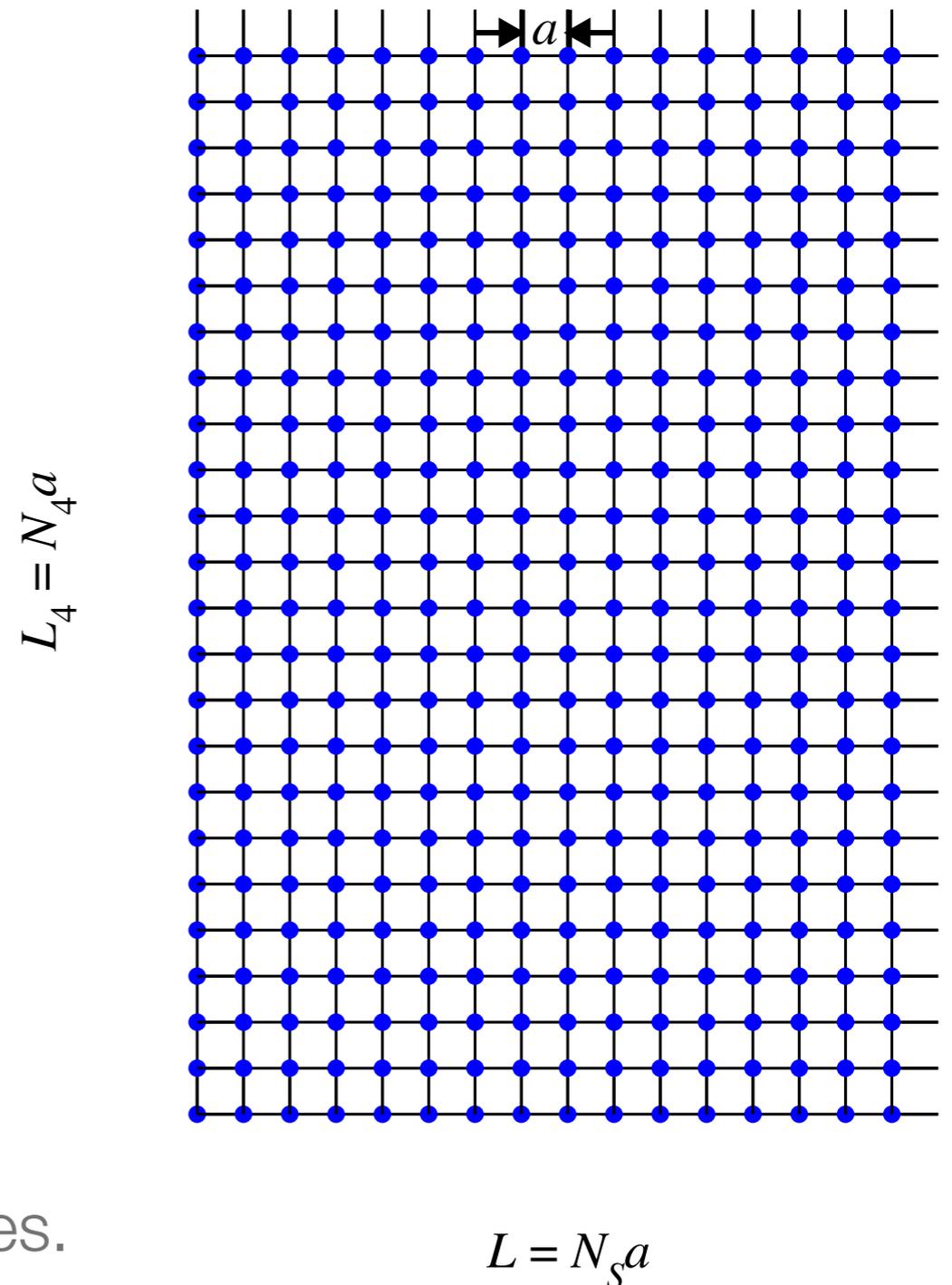
- In finite volume, the number of degrees of freedom is finite (albeit large).



- With a finite number of degrees of freedom, a computer can integrate.
- Random number generator (Markov chain Monte Carlo) produces **lattice data** varying

$$(a, L, L_4, \{m_{\text{sea}}\}, m_q, m_Q)$$

- Extract masses, matrix elements, etc., from various correlation functions.
- Use **effective field theories** to fit the lattice data to obtain results with
  - $a \rightarrow 0,$
  - $L, L_4 \rightarrow \infty,$
  - quark masses tuned to  $n_f$  hadron masses.



- This technique works best when  $e^{-iHt}$  vs  $e^{-Hx_4}$  is irrelevant, in particular,
  - (low-lying) energy levels, i.e., masses;
  - matrix elements with 1 hadron in the initial state and 0 or 1 hadrons in the final state:
    - decay and scattering mediated by electroweak currents;
    - local QCD observables; ....  includes PDFs
- QCD scattering and other two-body (even three-body) amplitudes can be obtained from precise calculations of the volume dependence of (not-so-low-lying) energy levels.
- Real time quantities, e.g., fragmentation functions, require new ideas or even new technologies such as quantum computers.

# Resources

---

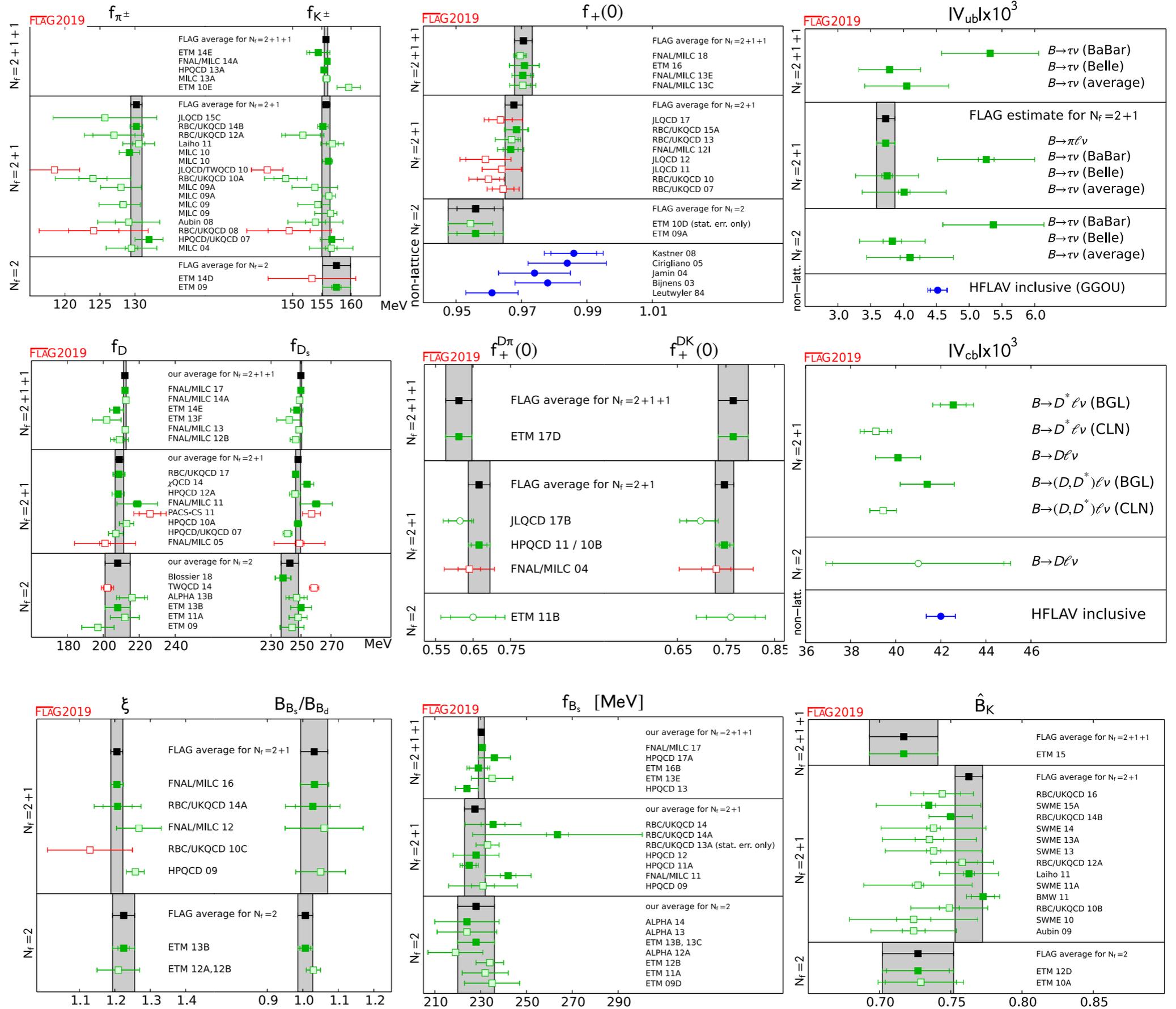
- USQCD Whitepapers (published in the [November 2019 issue of EPJA](#)):
  - Quark and lepton flavor, Lehner, Meinel *et al.* [arXiv:1904.09479](#);
  - Beyond the standard model, Neil *et al.* [arXiv:1904.09964](#);
  - Neutrino-nucleus scattering, ASK, Richards *et al.* [arXiv:1904.09931](#);
  - Fundamental symmetries, Cirigliano, Davoudi *et al.* [arXiv:1904.09704](#);
  - Hadrons and nuclei, Detmold, Edwards *et al.* [arXiv:1904.09512](#);
  - Hot, dense QCD, Karsch, Mukherjee *et al.*, [arXiv:1904.09951](#);
  - Calculations at the exascale, Joó, Jung *et al.*, [arXiv:1904.09725](#);
- Flavor Lattice Averaging Group, [arXiv:1902.08191](#), updates at [FLAG2019](#).

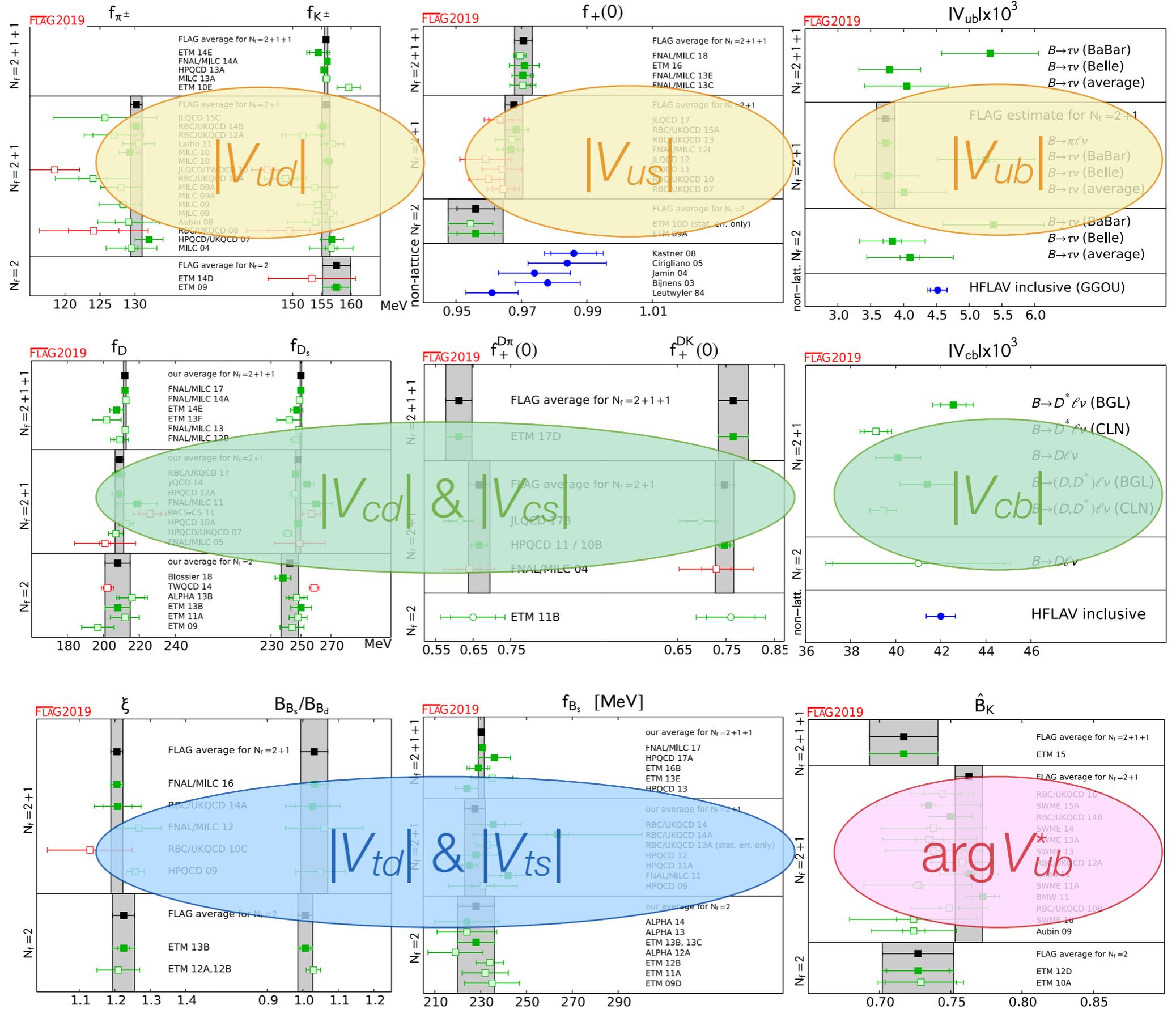
# Outline

---

- Introduction
- Quark Flavor Physics: Form Factors and Decay Constants
- Higgs Physics: Quark Masses;  $\alpha_s$
- Lepton-Flavor Physics: Nucleonic Ingredients for  $\nu A$  Scattering
- HI-LHC+EIC Physics: Parton Distribution Functions
- Outlook

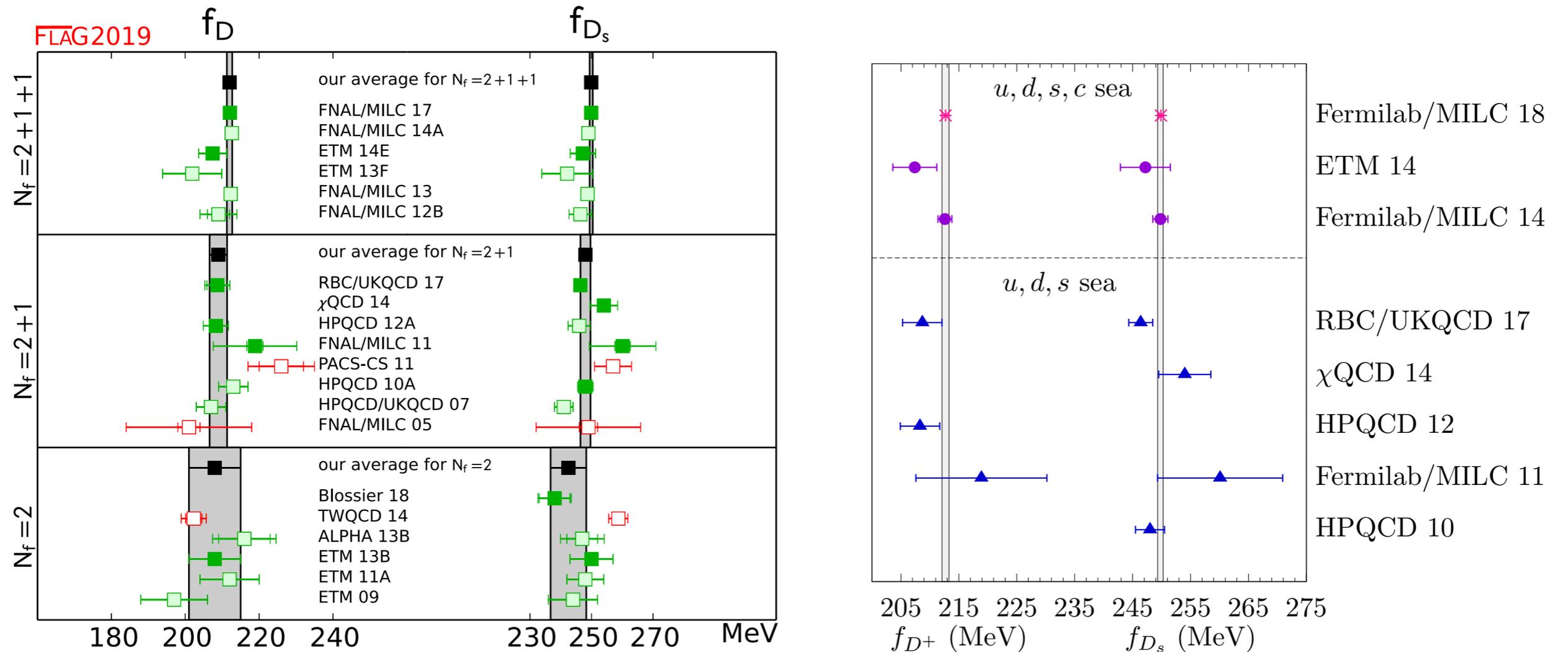
# Quark-Flavor Physics





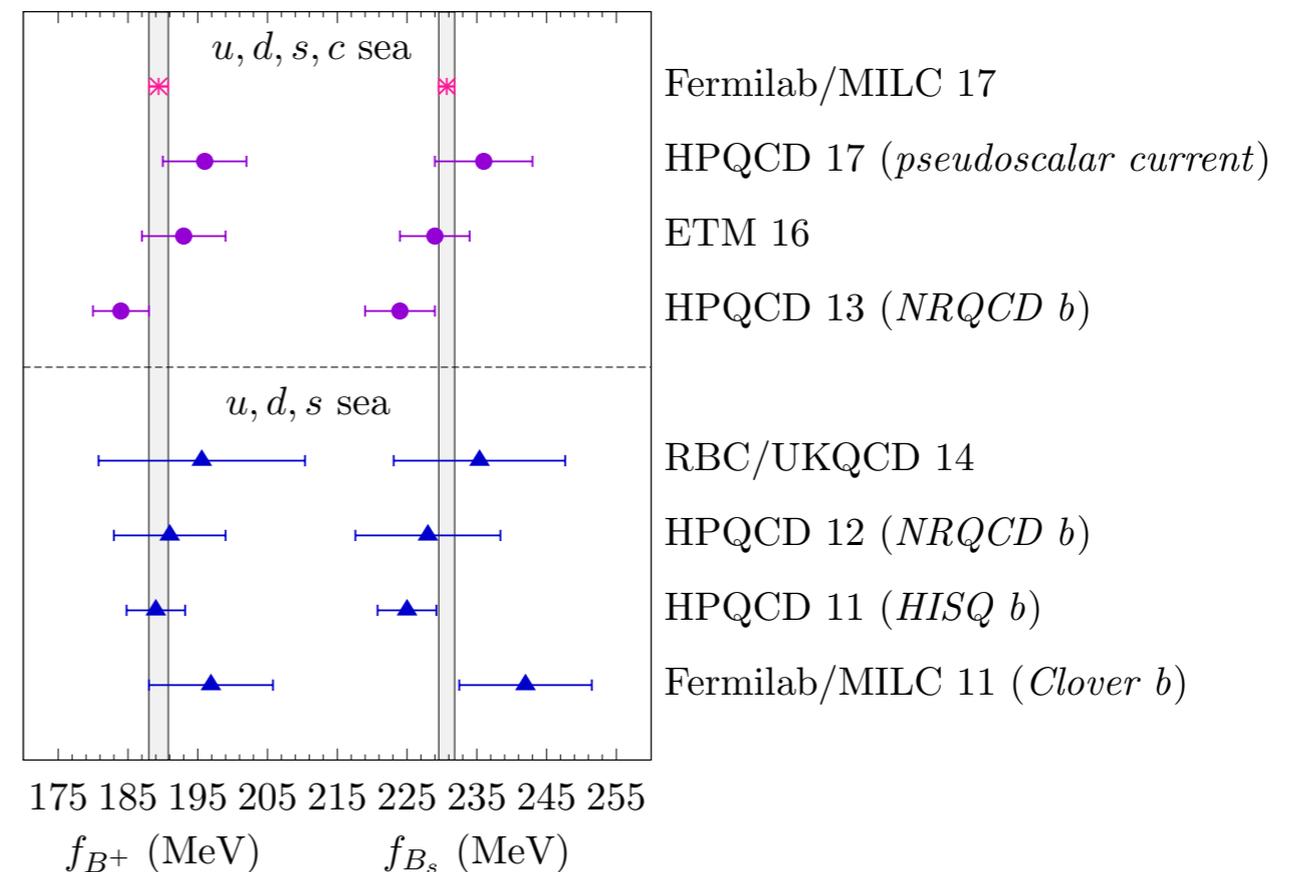
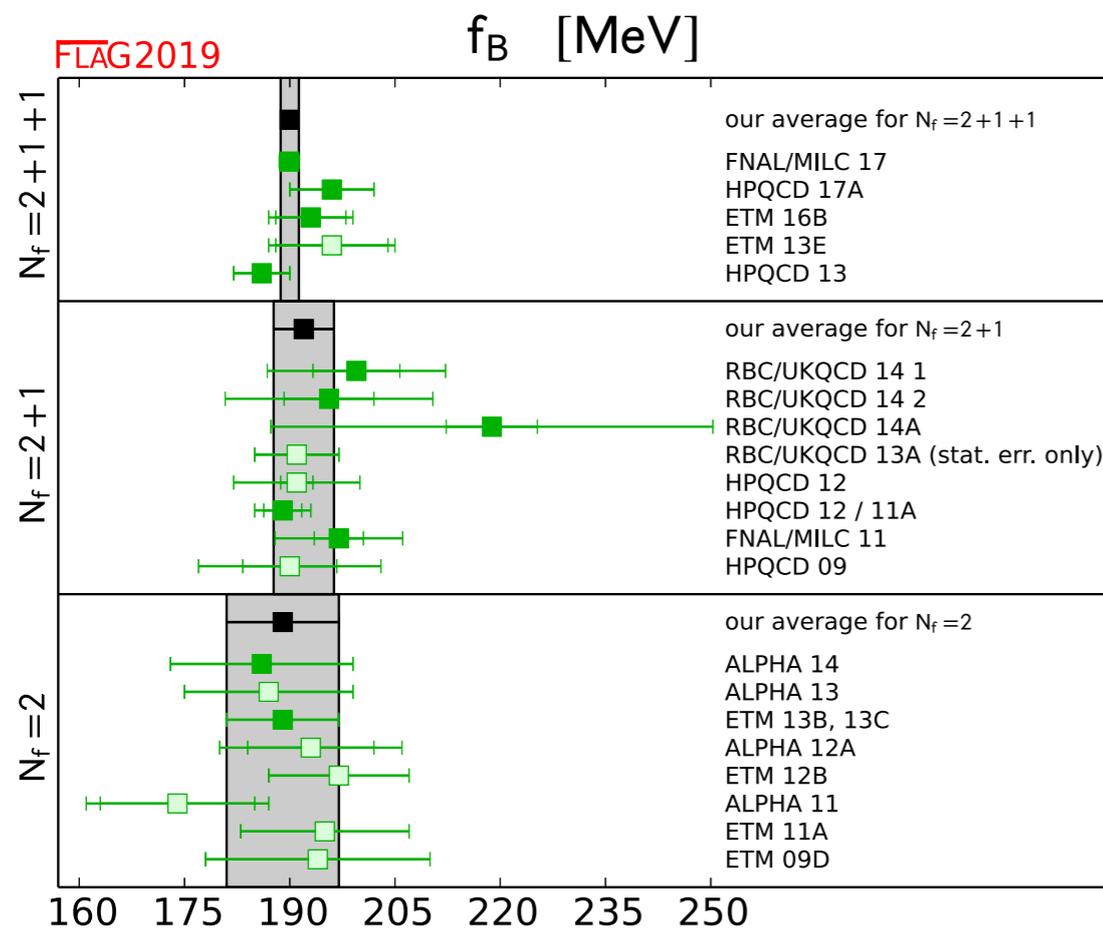
# $|V_{cd}|$ and $|V_{cs}|$ : Leptonic

- Major update from Fermilab Lattice/MILC [[arXiv:1712.09262](https://arxiv.org/abs/1712.09262)]:  $\sim 0.2\%$
- Also RBC/UKQCD [[arXiv:1701.02644](https://arxiv.org/abs/1701.02644)].



# $|V_{ub}|$ : Leptonic

- Updates from Fermilab Lattice/MILC [[arXiv:1712.09262](https://arxiv.org/abs/1712.09262)]:  $\sim 0.7\%$
- Other updates from HPQCD [[arXiv:1711.09981](https://arxiv.org/abs/1711.09981)], ETM [[arXiv:1603.04306](https://arxiv.org/abs/1603.04306)].



# Semileptonic Decays

---

- Semileptonic decays are a bit harder for lattice QCD: function of  $q^2$  and final-state hadron. But experimental rates are higher.
- For  $|V_{cb}|$  and  $|V_{ub}|$ :
  - $B \rightarrow D^{(*)}l\nu$  [[arXiv:1501.05373](#), [arXiv:1503.07839](#), earlier, underway];
  - $B \rightarrow \pi l\nu$  [[arXiv:1501.05373](#), [arXiv:1503.07839](#), earlier, underway];
  - $B_s \rightarrow K l\nu$  [[arXiv:1808.09285](#), [arXiv:1901.02561](#), underway];
  - $\Lambda_b \rightarrow \Lambda_c/p l\nu$ : Detmold, Lehner, Meinel [[arXiv:1503.01421](#)].
- Explore prospects for, e.g.,  $B \rightarrow K^* l l$  (two-hadron final state).
  - ↳  $K\pi$

# Semileptonic Decays

---

- Semileptonic decays are a bit harder for lattice QCD: function of  $q^2$  and final-state hadron. But experimental rates are higher.
- For  $|V_{cb}|$  and  $|V_{ub}|$ :

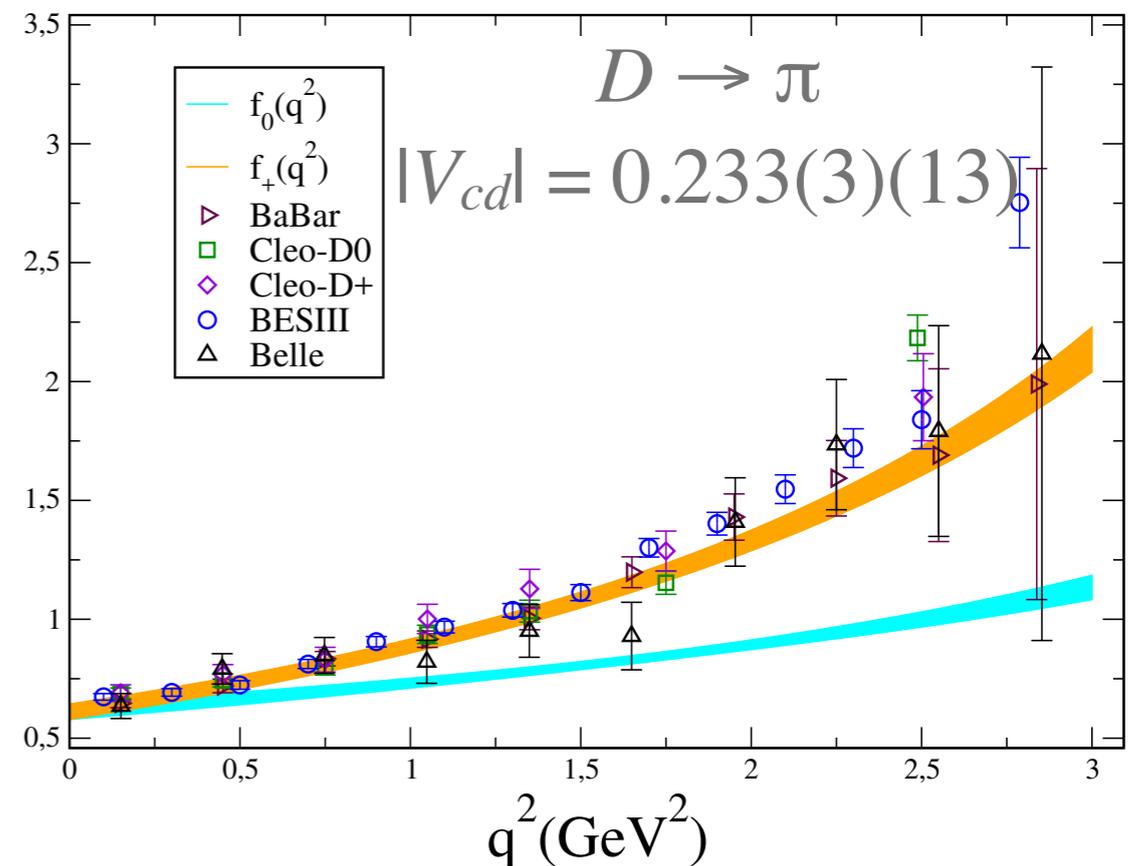
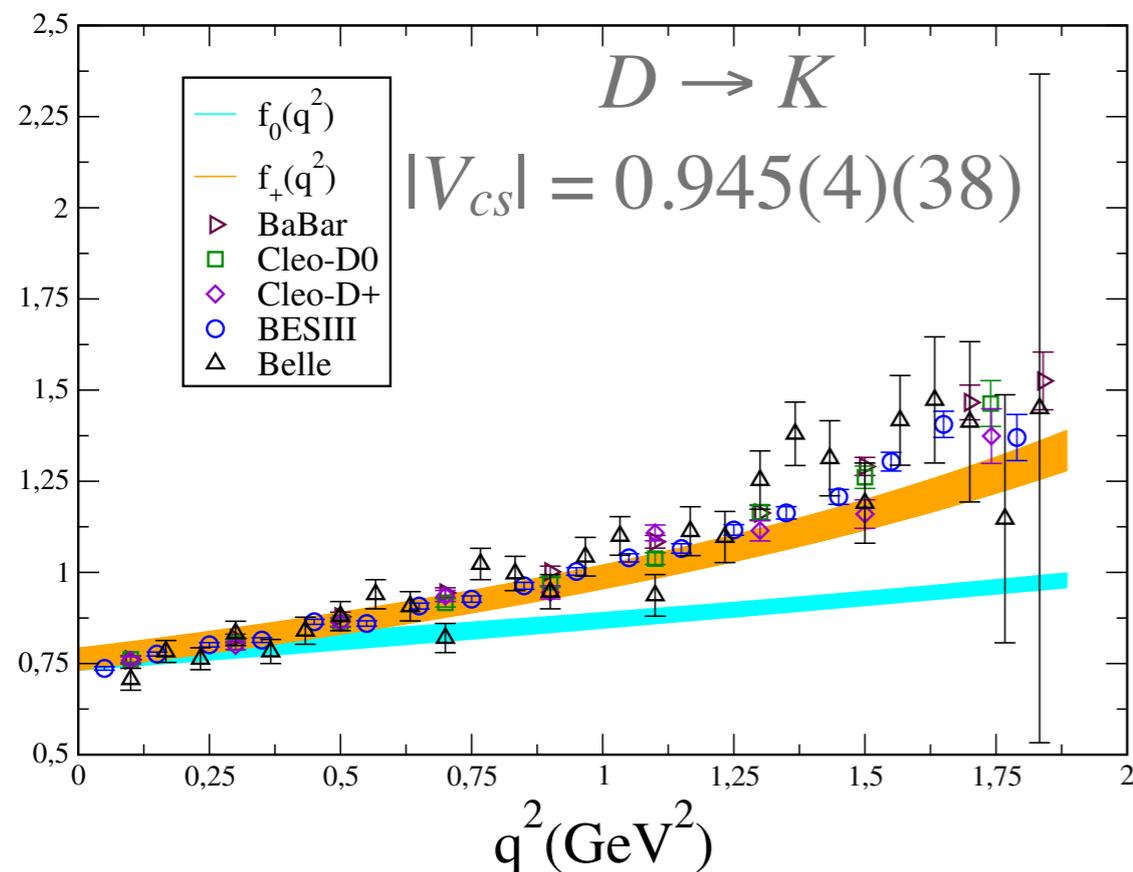
- $B \rightarrow D^{(*)}l\nu$  [[arXiv:1501.05373](#), [arXiv:1503.07830](#), earlier, underway].

Aim to keep QCD uncertainties similar to Belle 2 and LHCb uncertainties.

- $B_s \rightarrow Kl\nu$  [[arXiv:1808.09285](#), [arXiv:1901.02561](#), underway];
- $\Lambda_b \rightarrow \Lambda_c/pl\nu$ : Detmold, Lehner, Meinel [[arXiv:1503.01421](#)].
- Explore prospects for, e.g.,  $B \rightarrow K^*ll$  (two-hadron final state).  
↳  $K\pi$

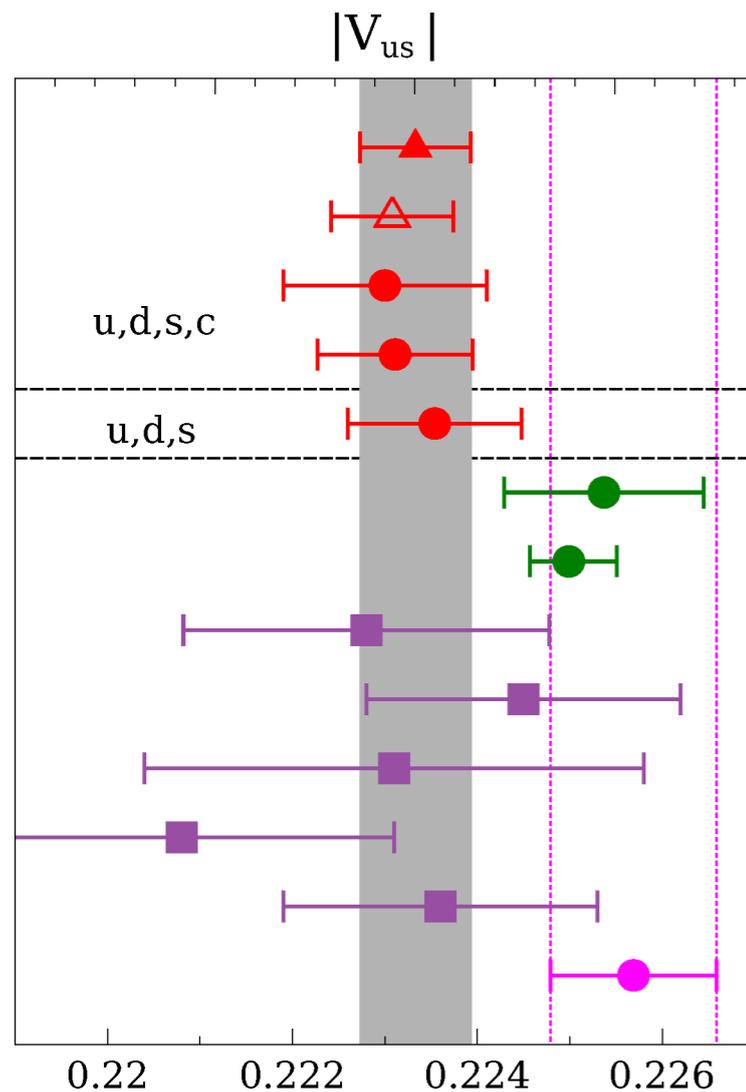
# $|V_{cd}|$ and $|V_{cs}|$ : Semileptonic

- Most recent entry in FLAG 2016 was HPQCD 2010 ( $D \rightarrow K$ ), 2011 ( $D \rightarrow \pi$ ); see also [arXiv:1305.1462](https://arxiv.org/abs/1305.1462) (unpublished).
- Newer results from ETM removes hypercubic artifacts [[arXiv:1706.03017](https://arxiv.org/abs/1706.03017)] and determines CKM [[arXiv:1706.03657](https://arxiv.org/abs/1706.03657)].



# $|V_{us}|$ : Semileptonic

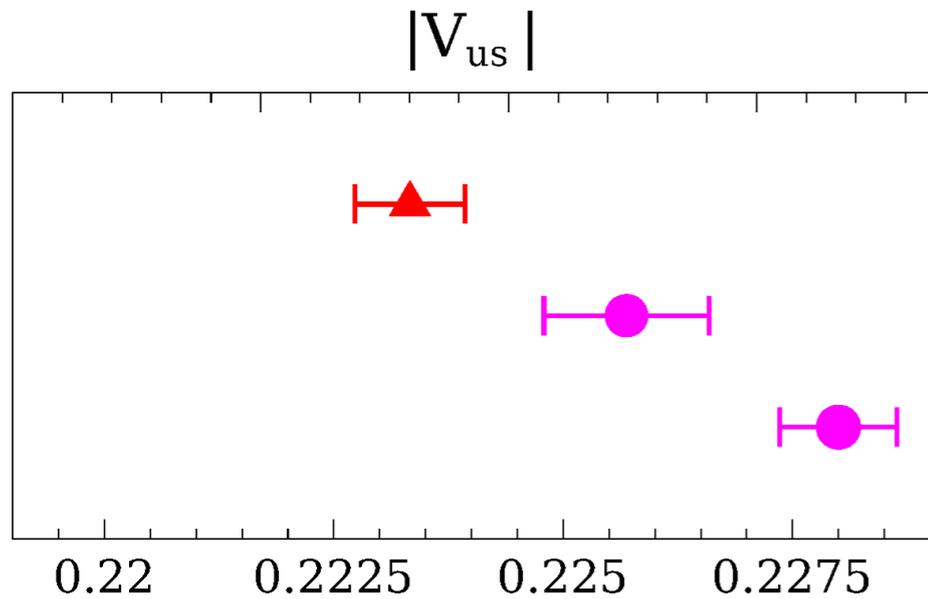
- QCD uncertainty in  $|V_{us}|$  now the same as experiment [[arXiv:1809.02827](https://arxiv.org/abs/1809.02827)].



- This work  $|V_{us}| = 0.22333(42)_{\text{expt}}(43)_{f_+(0)}$
- This work (only neutral kaon exp. data)
- $K_{l3}$  ETMC 2016
- $K_{l3}$  Fermilab Lattice/MILC 2014
- $K_{l3}$  RBC/UKQCD 2014
- $K_{l2}$  FLAG 2016 +  $f_K$  FLAG  $N_f=2+1$
- $K_{l2}$  +  $f_K/f_\pi$  Fermilab Lattice/MILC 2017
- $\tau \rightarrow s$  inclusive, Boyle et al. 2018
- $\tau \rightarrow s$  inclusive +  $K_{l2}$  input, Boyle et al. 2018
- $\tau \rightarrow s$  inclusive, Hudspith et al. 2017
- $\tau \rightarrow s$  inclusive, Hudspith et al. 2017 + HFLAV 2016 exp. input
- $\tau \rightarrow K \ell \nu / \tau \rightarrow \pi \ell \nu$  HFLAV2017 +  $f_K/f_\pi$  Fermilab Lattice/MILC 2017
- Unitarity with  $|V_{ud}|=0.97420(21)$ , RC from Marciano & Sirlin 2005

# $|V_{us}|$ : Semileptonic

- QCD uncertainty in  $|V_{us}|$  now the same as experiment [[arXiv:1809.02827](https://arxiv.org/abs/1809.02827)].

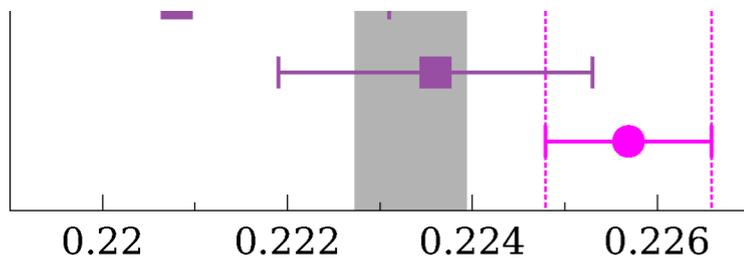


This work

Unitarity with  $|V_{ud}|=0.97420(21)$ , RC from Marciano & Sirlin 2005

Unitarity with  $|V_{ud}|=0.97366(15)$ , RC from Seng et al. 2018

$\sim 2\sigma$  violation of unitarity becomes  $>5\sigma$  with new RC



$\tau \rightarrow \pi \ell \nu$  inclusive, [Hadjiportis et al. 2017](https://arxiv.org/abs/1708.07401) + [HFLAV 2018](https://arxiv.org/abs/1708.07401) exp. input

$\tau \rightarrow K \ell \nu / \tau \rightarrow \pi \ell \nu$  HFLAV2017 +  $f_K/f_\pi$  Fermilab Lattice/MILC 2017

Unitarity with  $|V_{ud}|=0.97420(21)$ , RC from Marciano & Sirlin 2005

# Quark Masses



# Four Methods

---

- Mass renormalization [ $\bar{m} = m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$ ]:
  - $\bar{m} = Z_m(m_0 - m_{\text{cr}})$ , with  $Z_m$  either  $\leq 2$  loops or **nonperturbative**. 
- Ward identities:
  - $m_{\text{AW}} Z_P \langle P \rangle = Z_A \langle \partial \cdot A \rangle$ , run  $m_{\text{AW}}$  to high scale  $\mu$  & convert to  $m_{\overline{\text{MS}}}(\mu)$ .
- Continuum limit  $\otimes$  continuum pQCD:
  - $\lim_{a \rightarrow 0} G_n(Qa) = 1 + \sum_{k=1}^{K \geq 3} G_n^{(k)} \alpha_s^k(Q)$ , e.g., quarkonium moments. 
- Continuum limit  $\otimes$  HQET:
  - $M = m_{\text{MRS}} + \bar{\Lambda}_{\text{MRS}} + \dots$ , e.g., heavy-light meson masses. 

# Charmonium Correlators

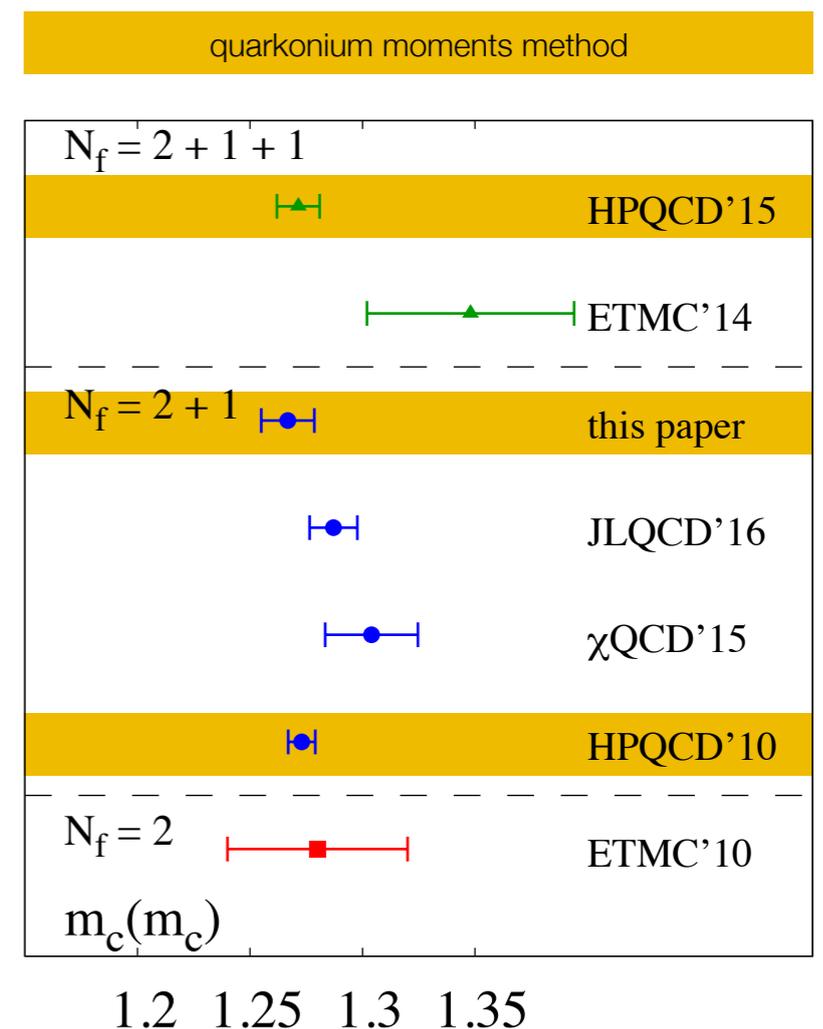
- Very precise determinations of heavy-quarks come from:

$$\lim_{a \rightarrow 0} \frac{G_n}{G_n^{\text{tree}}} = g_n(\alpha_s(\mu), m_{c, \overline{\text{MS}}}/\mu)$$

$$G_n = \sum_t t^n m_0^2 \left\langle \bar{c} \gamma^5 c(t) \bar{c} \gamma^5 c(0) \right\rangle, \quad \text{even } n \geq 4$$

Bochkarev & de Forcrand [[hep-lat/9505025](#)],  
Allison et al. [HPQCD, [arXiv:0805.2999](#)].

- New analysis on 11 hotQCD ensembles ( $n_f = 2+1$  HISQ) by Maezawa & Petreczky [[arXiv:1606.08798](#)] compatible with earlier HPQCD results [[arXiv:1408.4169](#)].



# RI-SMOM Method

arXiv:1805.06225

- Fix gauge and use quark vertex function:

$$\Lambda_P = \langle S^{-1}(p_1) \rangle \langle \bar{\psi} \gamma^5 \psi(p_1, p_2) \rangle \langle S^{-1}(p_2) \rangle$$

$$|p_1 + p_2| = |p_1| = |p_2| \equiv \mu$$

$$Z_P^{\text{SMOM}}(\mu) \equiv \frac{\Lambda_P(\mu)}{\Lambda_P^{(0)}}$$

$$Z_m^{\text{SMOM}}(\mu) = 1/Z_P^{\text{SMOM}}(\mu)$$

- Trickiest part of the analysis is that this gauge-fixed quantity has “gluon-mass condensate” contribution  $\sim \langle A^2 \rangle / \mu^2$ .
- Run  $m^{\text{SMOM}}(\mu) = Z_m^{\text{SMOM}}(\mu) m_0$  to a high scale and convert to  $m_{\overline{\text{MS}}}(\mu)$ .

# Heavy-light Meson Masses

---

- From HQET (or other approaches to the  $1/m_h$  expansion):

$$M_{H_J} = m_h + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_h} - d_J \frac{\mu_G^2(m_h)}{2m_h}$$

- This formula used to determine  $\bar{\Lambda}$ ,  $\mu_\pi^2$ , and  $\mu_G^2(m_b)$  from lattice QCD in [hep-ph/0006345](https://arxiv.org/abs/hep-ph/0006345); new, improved theory [[arXiv:1712.04983](https://arxiv.org/abs/1712.04983)].

# Heavy-light Meson Masses

- From HQET (or other approaches to the  $1/m_h$  expansion):

The diagram illustrates the heavy-light meson mass formula  $M_{H_J} = m_h + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_h} - d_J \frac{\mu_G^2(m_h)}{2m_h}$ . Each term is annotated with a callout box:

- mass of spin- $J$  meson**: Points to the entire formula.
- mass of heavy quark**: Points to  $m_h$ .
- energy of gluons and light quarks**: Points to  $\bar{\Lambda}$ .
- kinetic energy of heavy quark**: Points to  $\frac{\mu_\pi^2}{2m_h}$ .
- spin-orbit interaction**: Points to  $-d_J \frac{\mu_G^2(m_h)}{2m_h}$ .
- 1 for  $B$ ,  $-\frac{1}{3}$  for  $B^*$** : Points to the coefficient  $d_J$ .

- This formula used to determine  $\bar{\Lambda}$ ,  $\mu_\pi^2$ , and  $\mu_G^2(m_b)$  from lattice QCD in [hep-ph/0006345](https://arxiv.org/abs/hep-ph/0006345); new, improved theory [[arXiv:1712.04983](https://arxiv.org/abs/1712.04983)].

# Minimal Renormalon Subtraction

arXiv:1701.00347, arXiv:1712.04983

---

- New idea how to isolate the leading  $O(\Lambda)$  ambiguity in the pole mass, yielding an unambiguous definition of  $\bar{\Lambda}$ .
- For the bottom quark (tree, 1-loop, 2-loop, 3-loop, 4-loop):

$$m_{b,\text{pole}}/\bar{m}_b = (1, 1.093, 1.143, 1.183, 1.224)$$

$$m_{b,\text{MRS}}/\bar{m}_b = (1.157, 1.133, 1.131, 1.132, 1.132)$$

- For the top quark (tree, 1-loop, 2-loop, 3-loop, 4-loop):

$$m_{t,\text{MRS}}/\bar{m}_t = (1.0687, 1.0576, 1.0573, 1.0574, 1.0574)$$

could have implications for top physics at LHC.

# Results from MRS/HQET

$\alpha_s$  parametric not  
PT truncation

- Masses in numerical form:

$$m_{l,\overline{\text{MS}}}(2 \text{ GeV}) = 3.404(14)_{\text{stat}}(08)_{\text{syst}}(19)_{\alpha_s}(04)_{f_{\pi,\text{PDG}}} \text{ MeV}$$

$$m_{u,\overline{\text{MS}}}(2 \text{ GeV}) = 2.118(17)_{\text{stat}}(32)_{\text{syst}}(12)_{\alpha_s}(03)_{f_{\pi,\text{PDG}}} \text{ MeV}$$

$$m_{d,\overline{\text{MS}}}(2 \text{ GeV}) = 4.690(30)_{\text{stat}}(36)_{\text{syst}}(26)_{\alpha_s}(06)_{f_{\pi,\text{PDG}}} \text{ MeV}$$

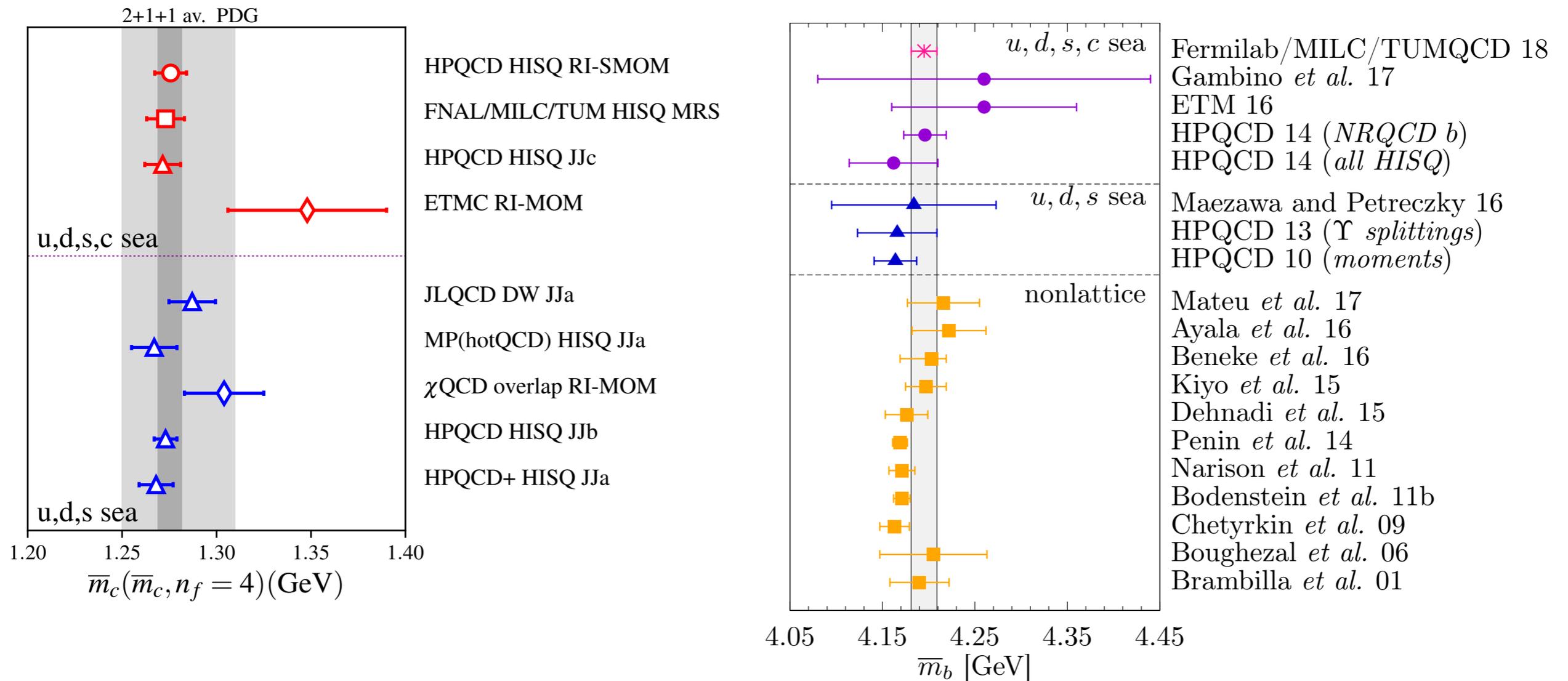
$$m_{s,\overline{\text{MS}}}(2 \text{ GeV}) = 92.52(40)_{\text{stat}}(18)_{\text{syst}}(52)_{\alpha_s}(12)_{f_{\pi,\text{PDG}}} \text{ MeV}$$

$$m_{c,\overline{\text{MS}}}(3 \text{ GeV}) = 984.3(4.2)_{\text{stat}}(1.6)_{\text{syst}}(3.2)_{\alpha_s}(0.6)_{f_{\pi,\text{PDG}}} \text{ MeV}$$

$$m_{b,\overline{\text{MS}}}(m_{b,\overline{\text{MS}}}) = 4203(12)_{\text{stat}}(1)_{\text{syst}}(8)_{\alpha_s}(1)_{f_{\pi,\text{PDG}}} \text{ MeV}$$

- Mass ratios:  
 $m_c/m_s = 11.784(11)_{\text{stat}}(17)_{\text{syst}}(00)_{\alpha_s}(08)_{f_{\pi,\text{PDG}}}$   
 $m_b/m_s = 53.93(7)_{\text{stat}}(8)_{\text{syst}}(1)_{\alpha_s}(5)_{f_{\pi,\text{PDG}}}$   
 $m_b/m_c = 4.577(5)_{\text{stat}}(7)_{\text{syst}}(0)_{\alpha_s}(1)_{f_{\pi,\text{PDG}}}$

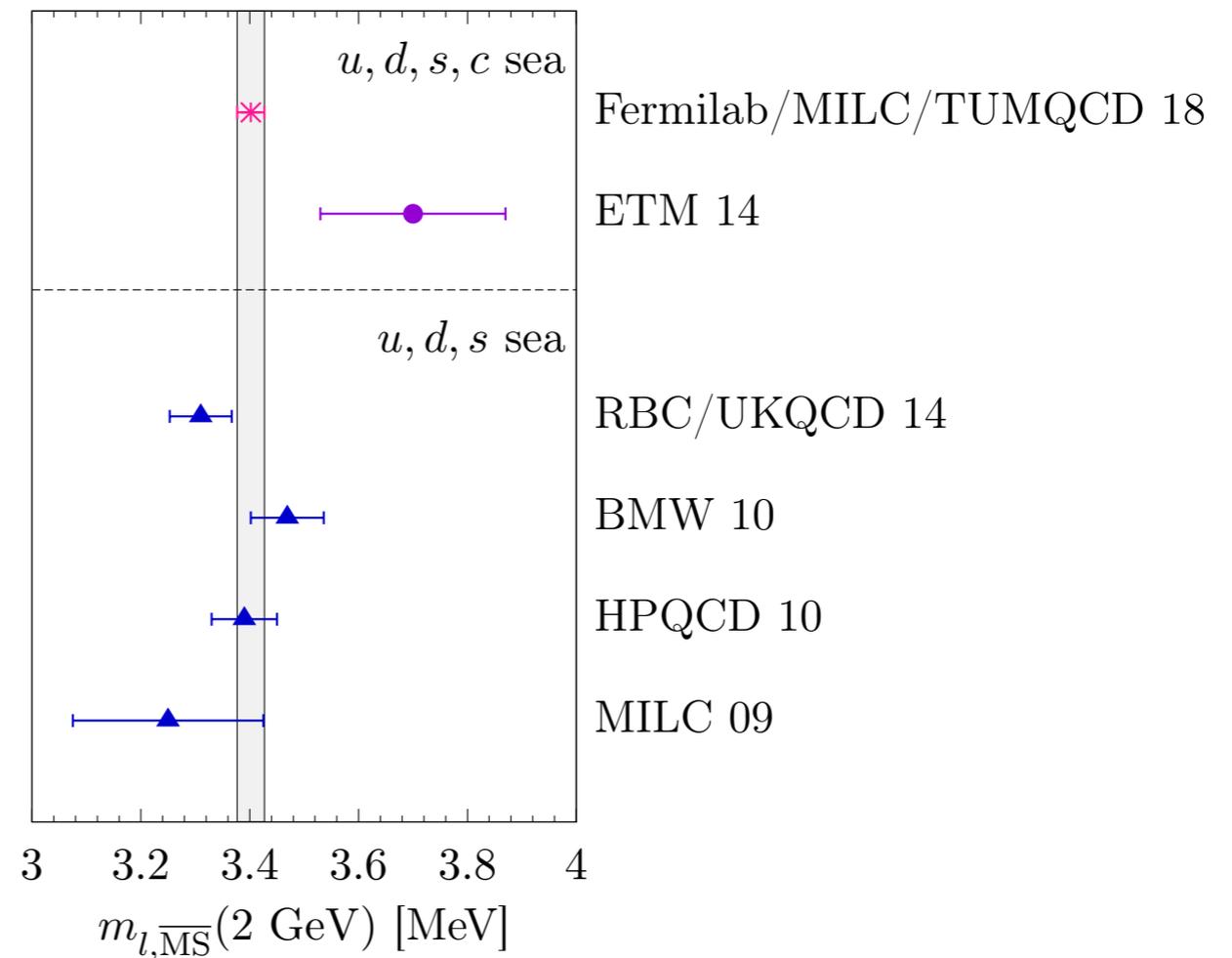
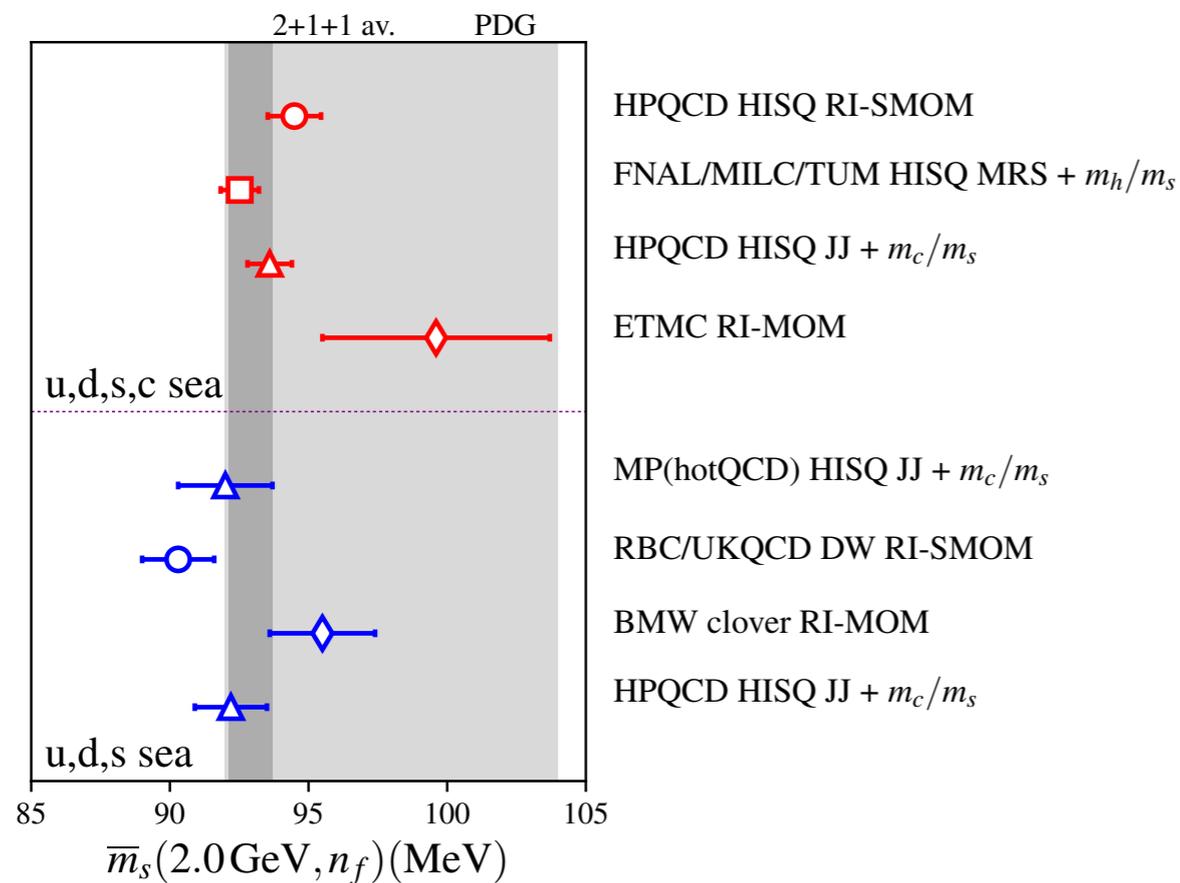
# Heavy Comparisons



- Precision: 0.3% for bottom to 0.5% for charm.

plots from [arXiv:1802.04248](https://arxiv.org/abs/1802.04248), [arXiv:1805.06225](https://arxiv.org/abs/1805.06225)

# Light Comparisons

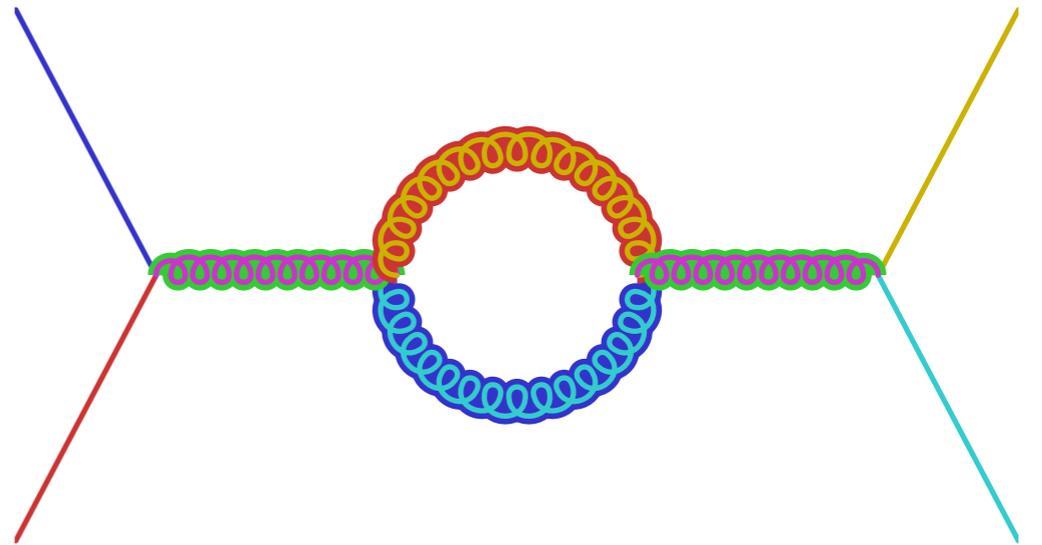


plots from [arXiv:1802.04248](https://arxiv.org/abs/1802.04248)  
[arXiv:1805.06225](https://arxiv.org/abs/1805.06225)

- Precision: 2% for up quark.

Obtained from charmed quark mass and mass ratios.

$\alpha_s$



# Four Methods

---

- Small Wilson loops:
  - $\square = 1 + \alpha_s z_1 + \alpha_s^2 z_2 + \dots$ ; isolate sides, corners, area.
- Vertex functions:
  - Ratio of dressed and bare ghost-gluon vertex.
- Continuum limit  $\otimes$  continuum pQCD:
  - $\lim_{a \rightarrow 0} G_n(Qa) = 1 + \sum_{k=1}^{K \geq 3} G_n^{(k)} \alpha_s^k(Q)$ , e.g., quarkonium moments.
- Continuum limit  $\otimes$  step scaling:
  - QCD in a can.

# Small Wilson Loops

HPQCD, arXiv:0807.1687; Maltman, arXiv:0807.2020

- Wilson loop  $W(\mathcal{P}) = \text{Re} \langle \text{tr} \exp \oint_{\mathcal{P}} dz \cdot A \rangle$  has UV singularities.

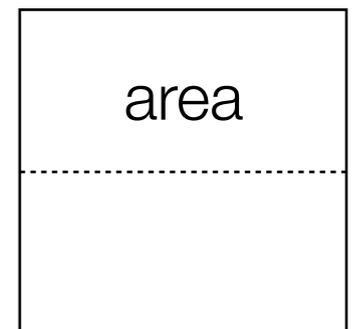
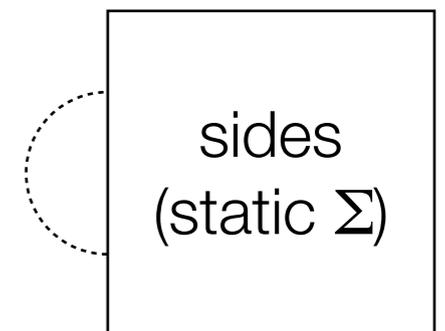
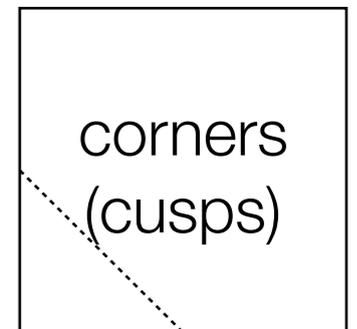
- Creutz ratios  $\chi(N_1, M_1, N_2, M_2) = \frac{W(N_1 \times M_1)W(N_2 \times M_2)}{W(N_2 \times M_1)W(N_1 \times M_2)}$  cancels these, but still has UV behavior.

- Tadpole-improved Wilson loops: cancel sides.

- OPE/Symanzik:  $W_{N \times M} = Z_{N \times M}(\alpha_s) + a^4 \mathcal{K}_{N \times M}(\alpha_s) \alpha_s \text{tr} G_{\mu\nu} G^{\mu\nu}$ ; condensate information must be accounted for.

- $R = -\ln W$ ,  $-\ln \chi$ ;  $Q = d/a$ , with  $d$  estimated via BLM.

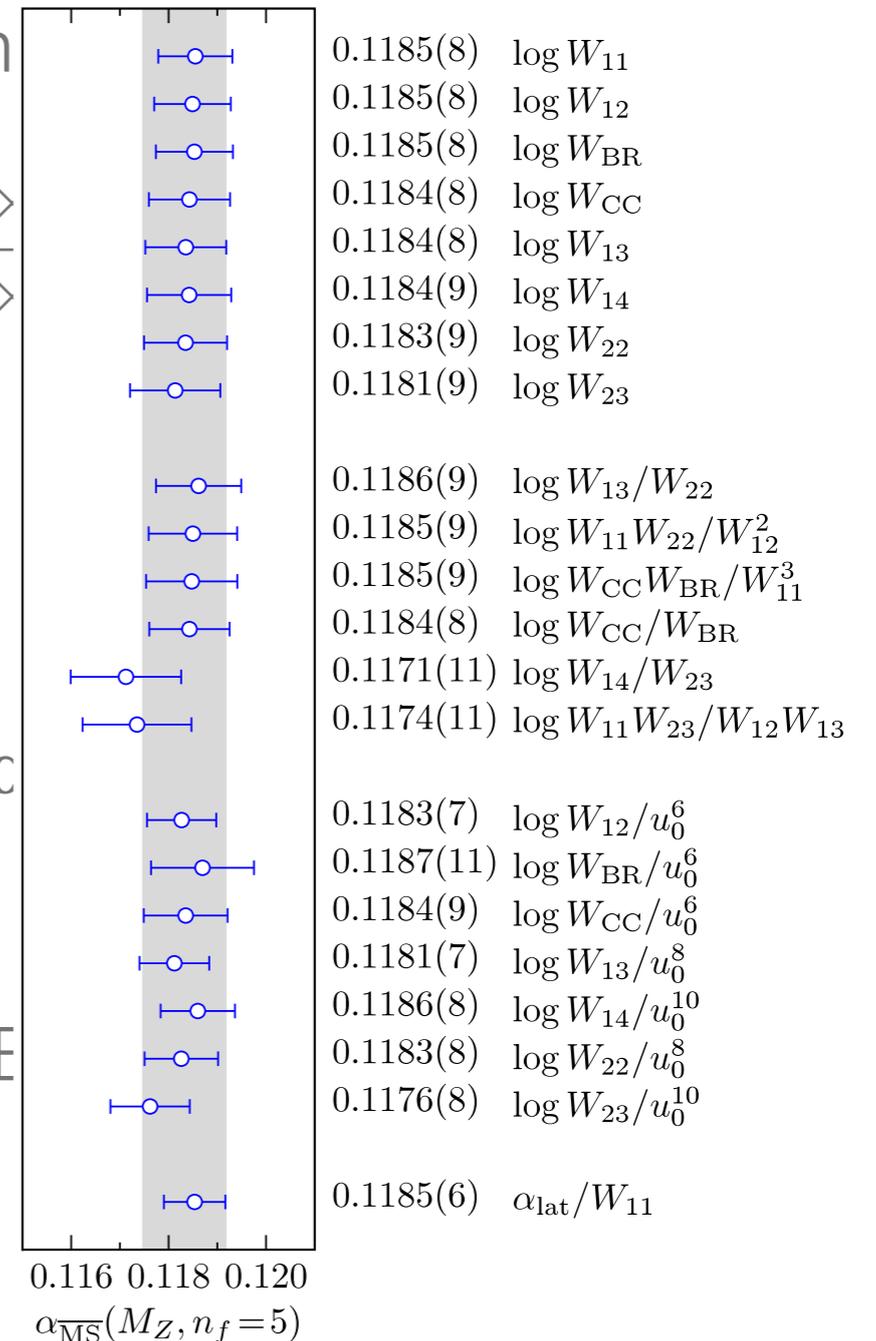
- Range of  $Q$  at present is  $\sim 6 = 1.8/0.3$ .



# Small Wilson Loops

HPQCD, arXiv:0807.1687; Maltman, arXiv:0807.2020

- Wilson loop  $W(\mathcal{P}) = \text{Re}\langle \text{tr exp} \oint_{\mathcal{P}} dz \cdot A \rangle$  has UV sin
- Creutz ratios  $\chi(N_1, M_1, N_2, M_2) = \frac{W(N_1 \times M_1)W(N_2 \times M_2)}{W(N_2 \times M_1)W(N_1 \times M_2)}$  cancels these, but still has UV behavior.
- Tadpole-improved Wilson loops: cancel sides.
- OPE/Symanzik:  $W_{N \times M} = Z_{N \times M}(\alpha_s) + a^4 \mathcal{K}_{N \times M}(\alpha_s) c$  condensate information must be accounted for.
- $R = -\ln W, -\ln \chi; Q = d/a$ , with  $d$  estimated via E
- Range of  $Q$  at present is  $\sim 6 = 1.8/0.3$ .



# Vertex Functions

---

- Fix gauge and use gluon-ghost vertex function—an RI-MOM definition.
- Same basic idea as the RI-MOM schemes for quark masses, including gauge-variant condensates.
- Run  $\alpha_s^{\text{RI-MOM}}(\mu) = \tilde{Z}_1^{\text{RI-MOM}}(\mu) \alpha_0$  to a high scale and convert to  $\alpha_{\overline{\text{MS}}}(\mu)$ .
- Not as advanced as other methods at present.
- References: [hep-lat/9605033](#), [arXiv:1201.5770](#), [arXiv:1702.00612](#).

# Quarkonium Correlators

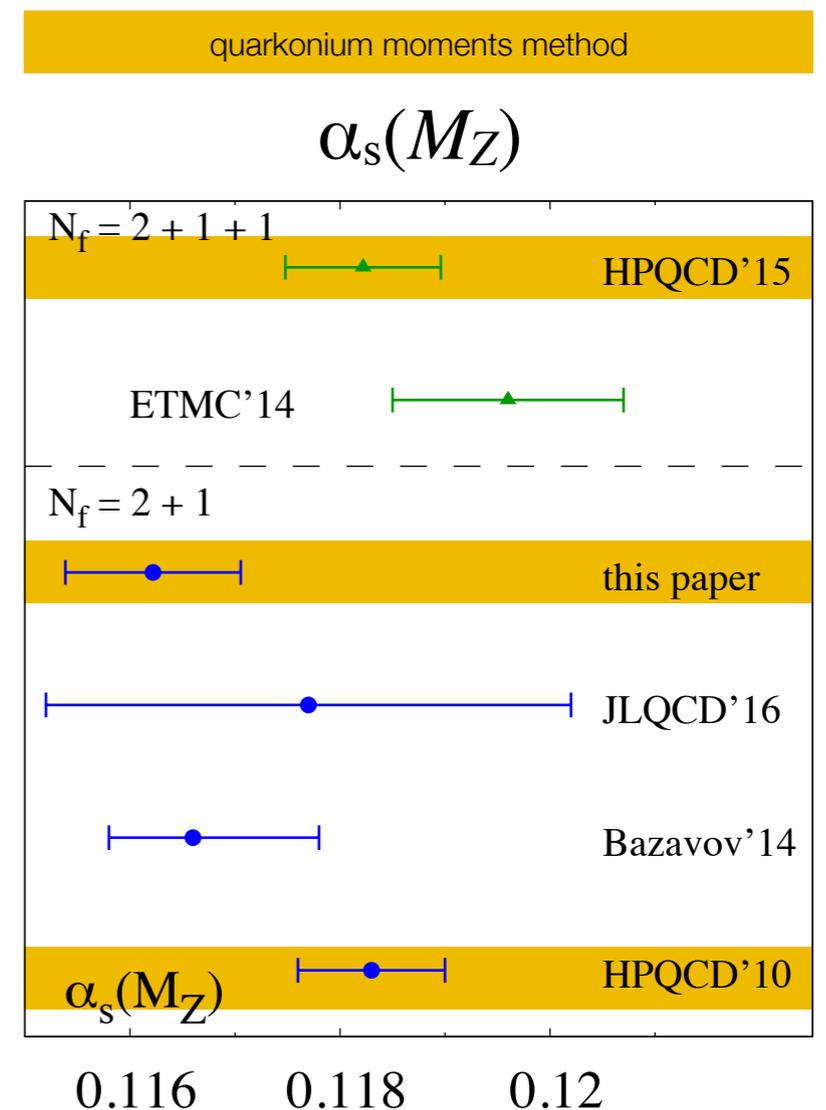
- Many precise determinations of heavy-quarks come from:

$$\lim_{a \rightarrow 0} \frac{G_n}{G_n^{\text{tree}}} = g_n(\alpha_s(\mu), m_{c, \overline{\text{MS}}}/\mu)$$

$$G_n = \sum_t t^n m_0^2 \left\langle \bar{c} \gamma^5 c(t) \bar{c} \gamma^5 c(0) \right\rangle, \quad \text{even } n \geq 4$$

Bochkarev & de Forcrand [[hep-lat/9505025](#)],  
Allison et al. [HPQCD, [arXiv:0805.2999](#)].

- New analysis on 11 ensembles from hotQCD ( $n_f = 2+1$  HISQ) by Maezawa & Petreczky [[arXiv:1606.08798](#)] compatible with earlier HPQCD results [[arXiv:1408.4169](#)].



# Schrödinger Functional

e.g., [arXiv:1706.03821](https://arxiv.org/abs/1706.03821)

---

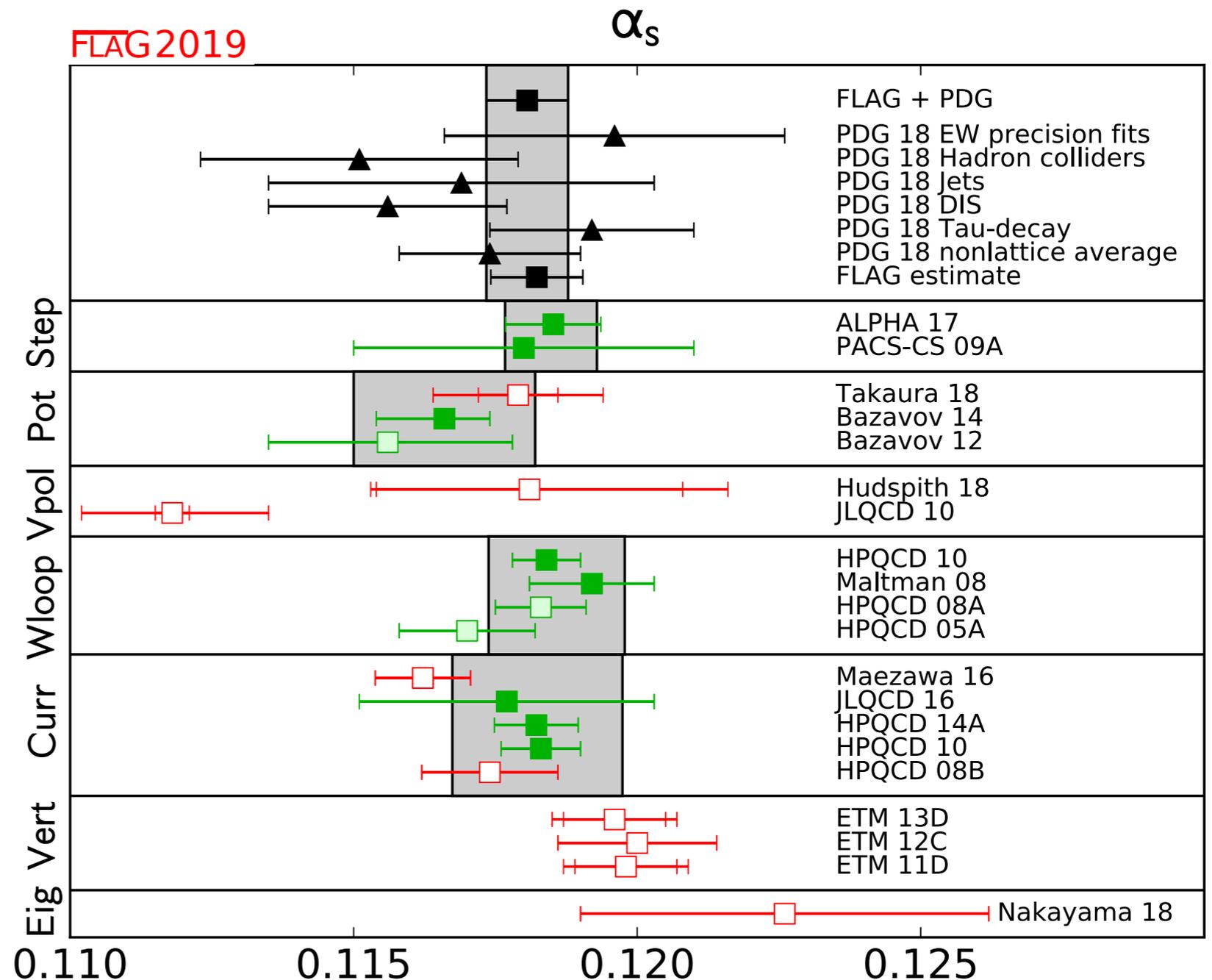
- QCD in a small volume is still QCD, but it is (mostly) perturbative, with the box size  $L$  being the short distance.
- In this way, it is possible to probe a very large range of scales:
  - if  $L$  is small, the continuum limit is feasible.
- Can obtain a nonperturbative calculations of  $\alpha_{\text{SF}}(1/L)$  (in a finite-volume scheme) and verify where perturbative running is observed.
- Units of  $L$  obtained from computing a hadron mass in a large volume  $10L$ :

$$\frac{10L}{a} M_p a = C \Rightarrow L = \frac{C}{10M_p^{\text{PDG}}}$$

# $\alpha_s$ Comparisons

arXiv:1902.08191

- Good consistency
  - w/in lattice QCD;
  - w/ scattering + perturbative QCD.
- Lattice QCD will presumably dominate  $\alpha_s$  determinations in HL-LHC + EIC eras.



# Lepton-Flavor Physics

# Neutrino Physics – $\nu N$ Scattering

---

- Key issue in neutrino oscillations is to reconstruct the neutrino energy, but the nuclear remnant (and possibly other particles) are not detected.
- Therefore, nuclear models are needed.
- The models require nucleonic ingredients which should come from QCD:
- Therefore, aim to get scattering amplitudes from first principles:
  - $\bar{\nu}p$  or  $\nu d$  scattering experiments in the Tokai, NuMI, or LBNF beam;
  - lattice QCD with error budgets as comprehensive as those for CKM.

# Lattice QCD Calculations

---

- Calculate quantities of the form ( $J_\nu =$  electroweak current)

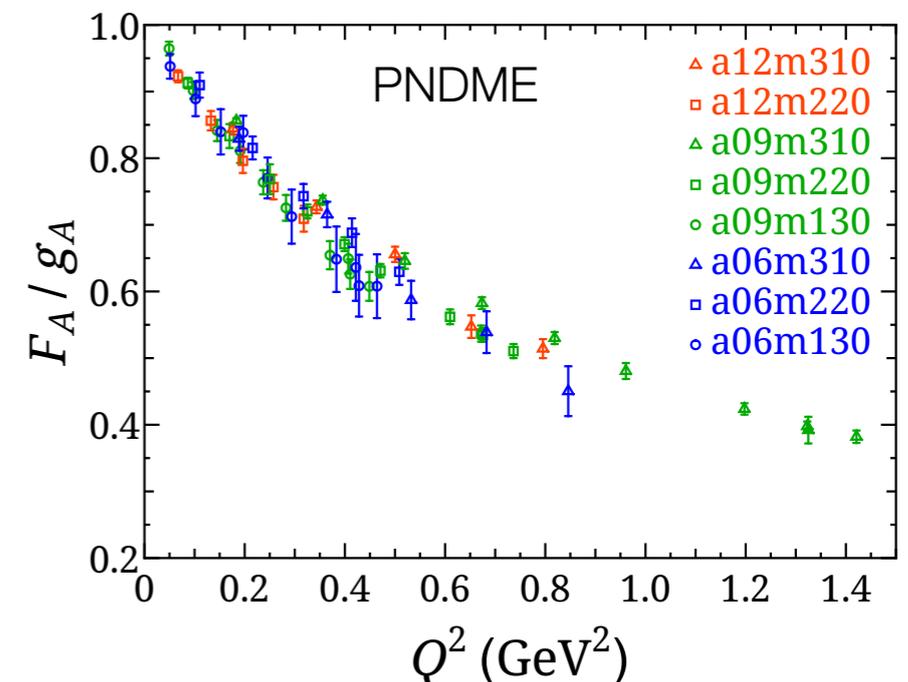
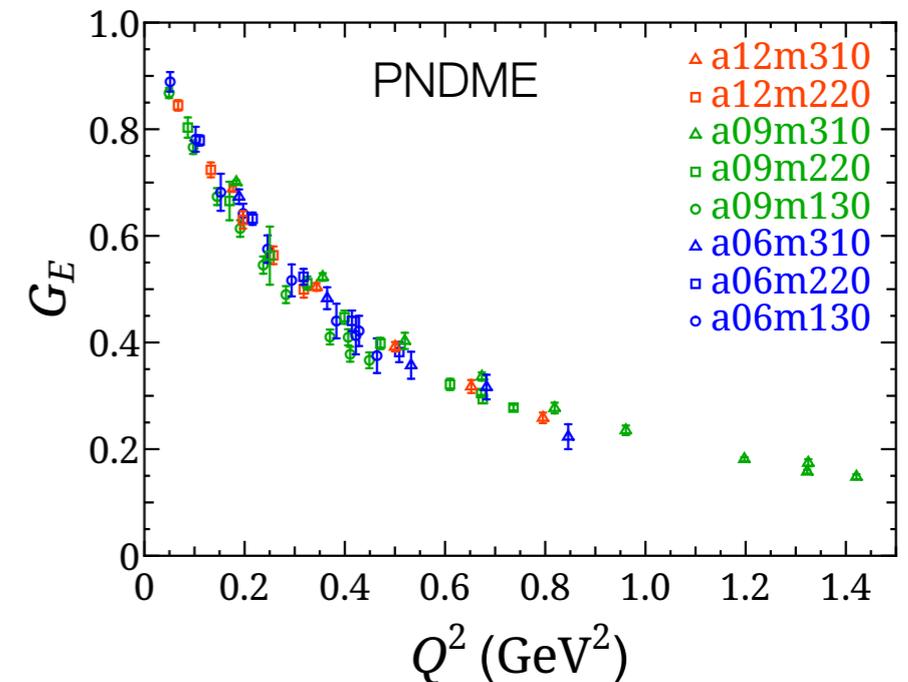
$$\langle N_f | J_\nu | N_i \rangle \quad \langle N_f | J_\mu^\dagger J_\nu | N_i \rangle$$

where  $N_i$  and  $N_f$  are states are single nucleons, two nucleons, nucleons with pions (including resonances), or small nuclei.

- Ingredients for nuclear many-body theory:
  - the matrix elements themselves;
  - chiral-effective-theory couplings, potentials, *etc.*, from EFT analysis of lattice-QCD properties of small (and eventually medium) nuclei.

# Nucleon Isovector Form Factors

- In the quasielastic region, nucleon form factors are key:
  - vector form factors can be obtained from  $eN$  scattering and isospin;
  - axial form factor  $F_A(Q^2)$  is important only in neutrino scattering —
    - elementary target,  $\nu d$  or  $\bar{\nu} p$ ;
    - lattice QCD (e.g., [arXiv:1801.01635](https://arxiv.org/abs/1801.01635));
    - slope of  $F_A$  is controversial (see next).

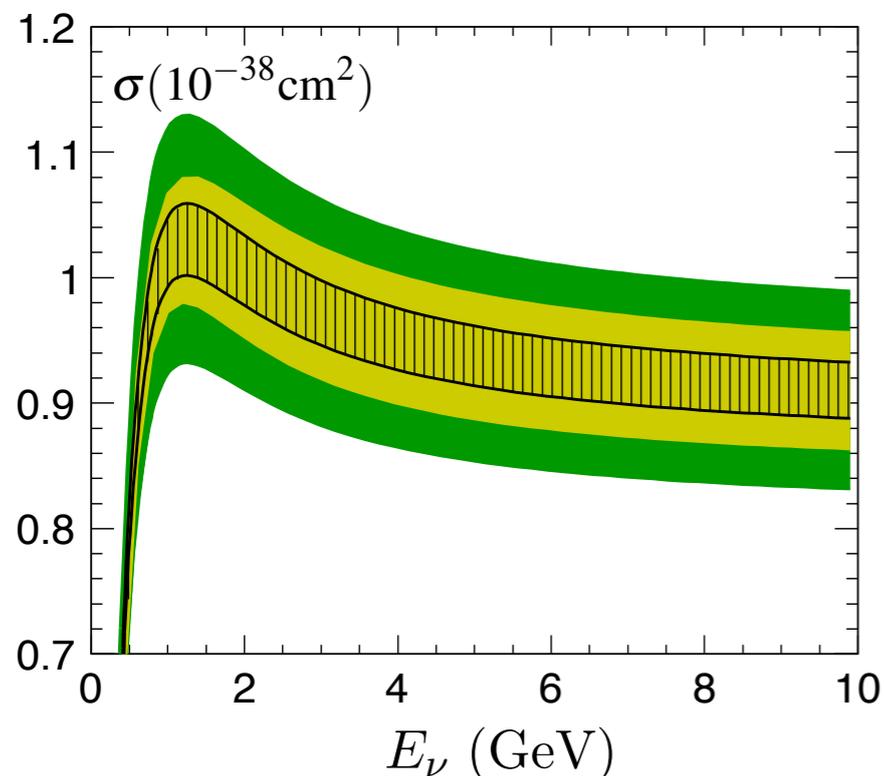


# Axial Form Factor

- Hill, Marciano, Kimmel, Sirlin [[arXiv:1708.08462](https://arxiv.org/abs/1708.08462)]  
argue solid lattice-QCD calculations of

$$r_A^2 = -\frac{6}{g_A} \left. \frac{dF_A}{dQ^2} \right|_{Q^2=0}$$

are urgently needed.

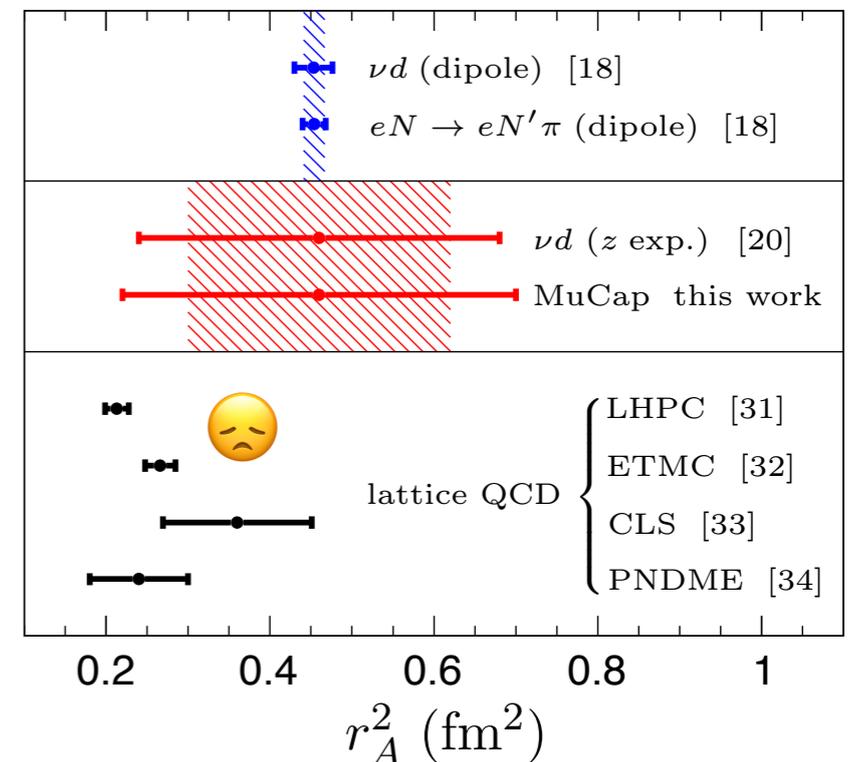


model-independent:  
50% error

subdominant at 20%

20% and 5% feasible  
with lattice QCD

model-dependent estimate:  
5% error debunked



# Future Topics and Challenges

---

- Form factors for  $N\pi$  final states (incorporating resonances)  $\langle N\pi | J_V | N \rangle$ :
  - finite-volume conceptual formalism 2015; calculation with mesons 2017.
- Two-body currents — urgent request from many-body theorists:
  - finite-volume formalism for  $\langle NN | J_V | NN \rangle$  now worked out.
- Parity-violating structure functions for deep inelastic region.
- Axial-current matrix elements in light nuclei:  ${}^3\text{H}$ ,  ${}^3\text{He}$ ,  ${}^4\text{He}$ ,  ${}^6\text{Li}$ , ...
  - growth of Wick contractions factorial is in number of quark lines.

# HL-LHC and EIC Physics

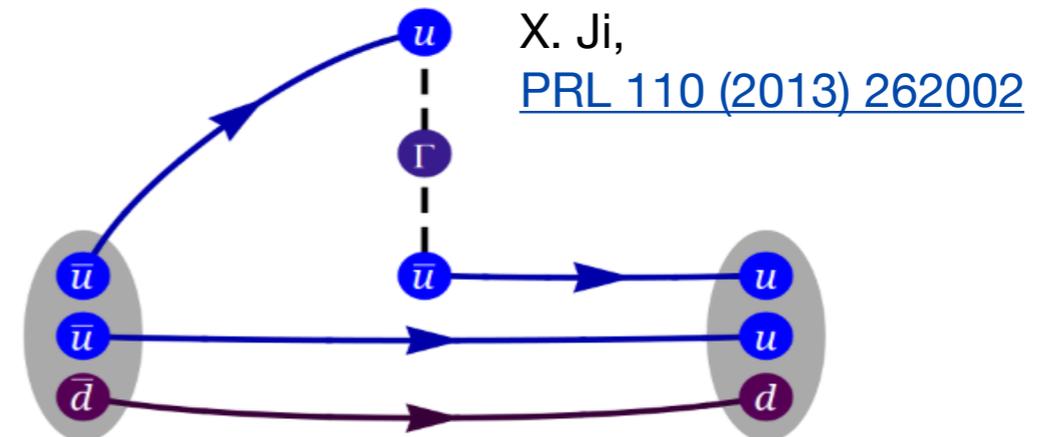
# Parton Distribution Functions

---

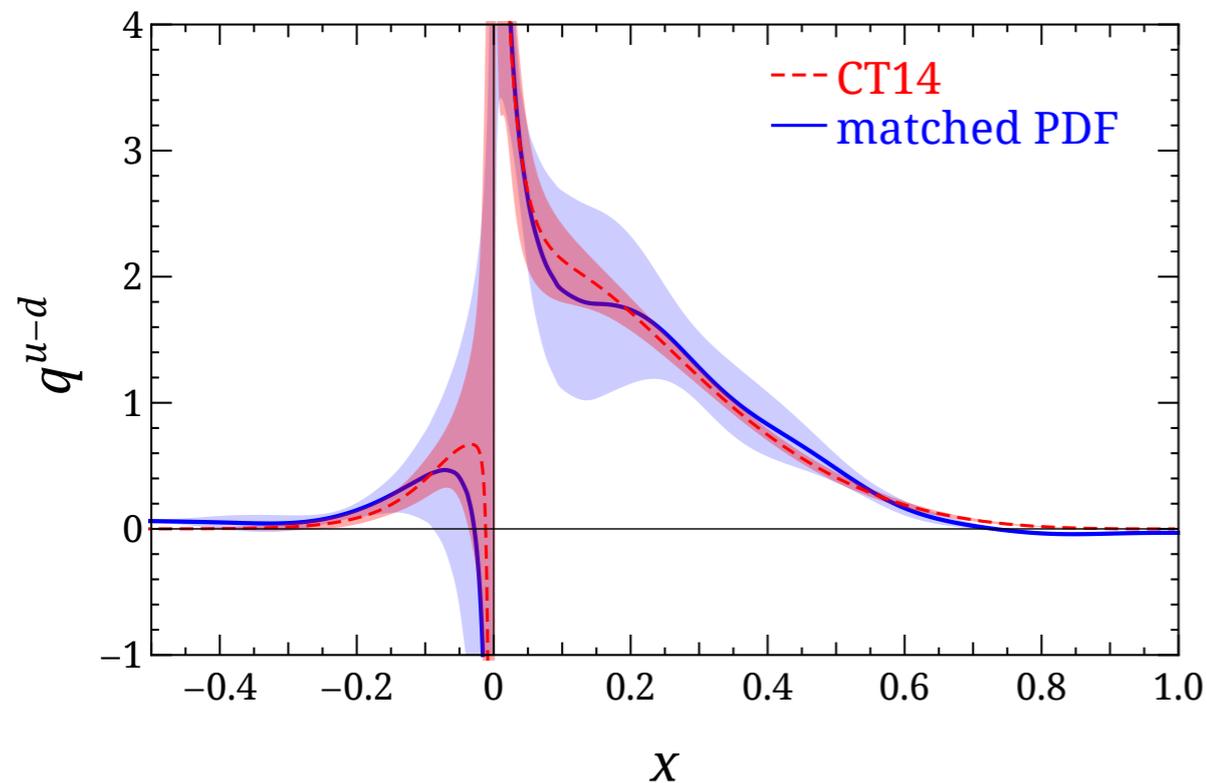
- Any improvement benefits (HL-)LHC searches.
- EIC will provide unprecedented measurements.
- Moments of PDFs:
  - higher moments power-law divergent;
  - gradient flow [[arXiv:1612.01584](https://arxiv.org/abs/1612.01584)] is a theoretically appealing way to smooth a gauge field;
  - fixing a short flow “time” in physical units provides an observable w/o power-law divergence: use continuum pQCD to convert from flow-time scheme to  $\overline{\text{MS}}$  scheme.

# Parton Distribution Functions

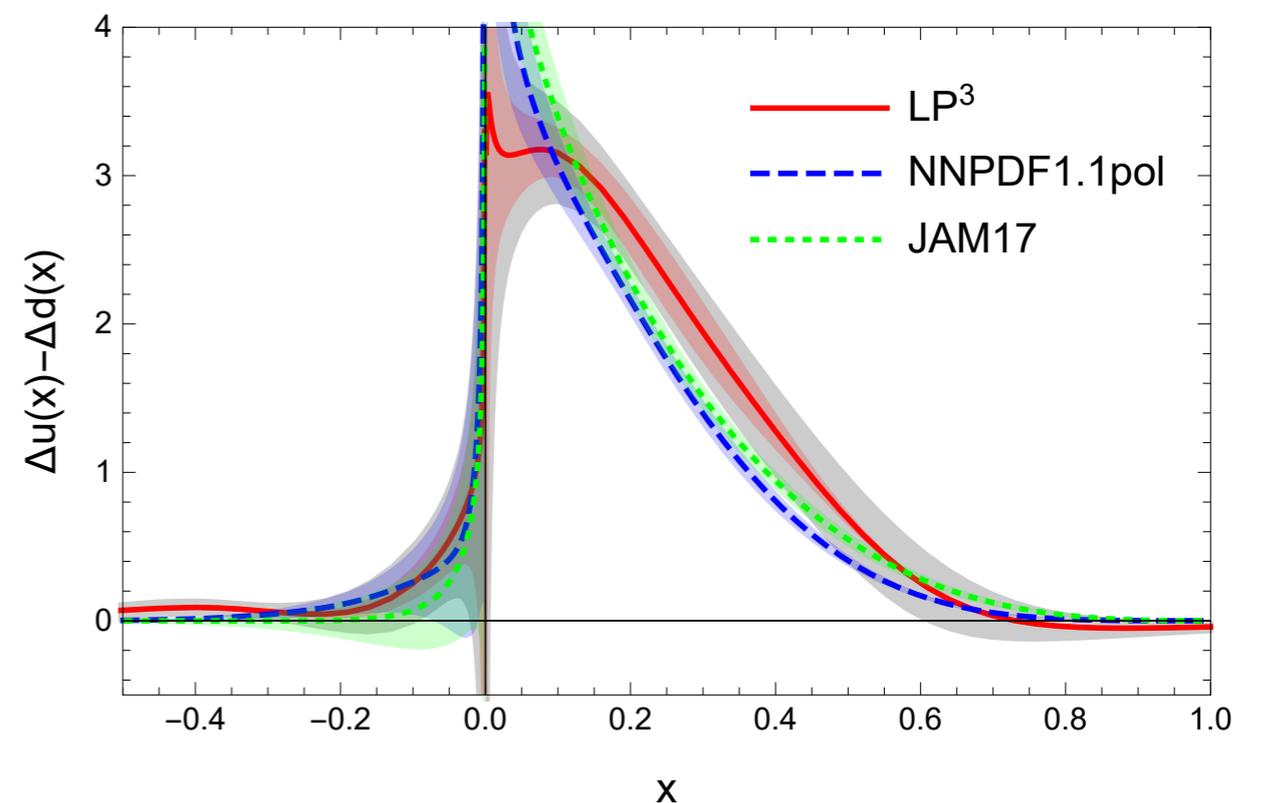
- Dependence on Bjorken  $x$ :
- Explosion of numerical & theoretical work  $\Leftarrow$  first USQCD studies.



Chen *et al.*, [arXiv:1803.04393](#)



Lin *et al.*, [arXiv:1807.07431](#)



# PDF Outlook

---

- Need for dialog between EIC, LHC, and lattice QCD communities!
  - Already begun: “Parton distributions and lattice QCD calculations: A community white paper,” H.-W. Lin *et al.* [arXiv:1711.07916](https://arxiv.org/abs/1711.07916).
- What is the correct way to think about the interplay between experiment and lattice QCD? If PDF uncertainties (at high  $x$ ) are
  - 20%, 10%, or 5%?
  - Results from QCD Lagrangian, with reliable uncertainty estimates.

Outlook

- Lattice QCD now has had huge impact in quark-flavor physics:
  - results are increasingly precise: sub-% threshold has been passed for  $f_{B(s)}$ ,  $f_{D(s)}$ ,  $m_b$ ,  $m_c$  (and good enough for Belle 2, BES III);
  - quark masses already more than good enough for ILC Higgs studies.
- (Lattice QCD is on track to reduce hadronic uncertainties in muon  $g-2$ .)
- Bright future for nucleon matrix elements:
  - form factors for neutrino physics;
  - PDFs for HL-LHC & EIC;
  - many others for the precision frontier.

QCD  
is  
everywhere!

Thank you!