

# Lattice QCD for Hadron Structure in the EIC Era

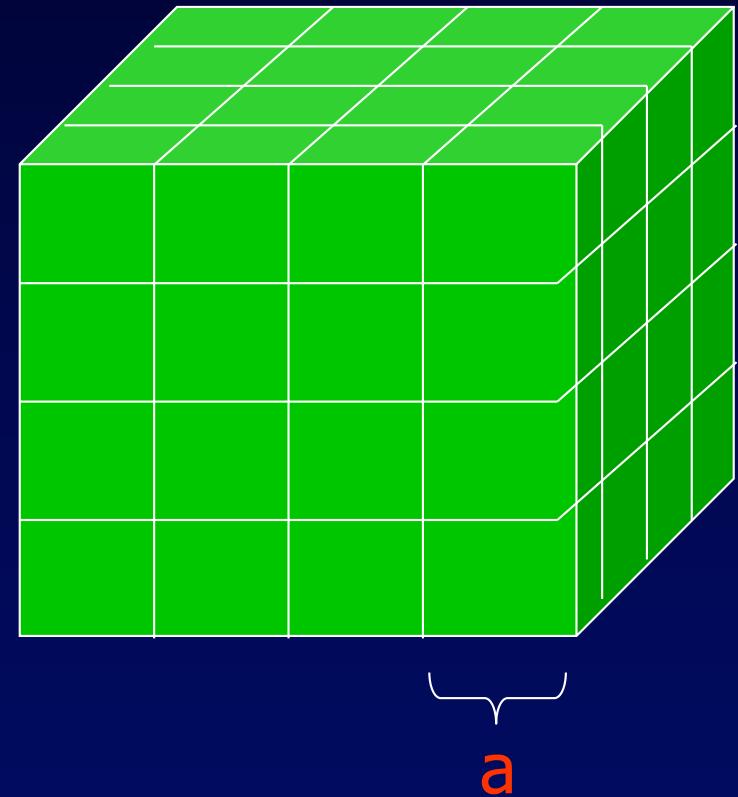
- Synopsis of Lattice QCD
- Hadronic Structure – Parton Moments, Quark and Glue Spins, Proton Spin and Mass Decomposition, Form Factors.
- Parton Distribution Functions
  - Hadronic Tensor
  - Quasi PDF, Pseudo PDF, Lattice Cross Section
- Valence and Sea Partons in NNLO Evolution

Fermilab, Nov. 14, 2019

# Lattice QCD

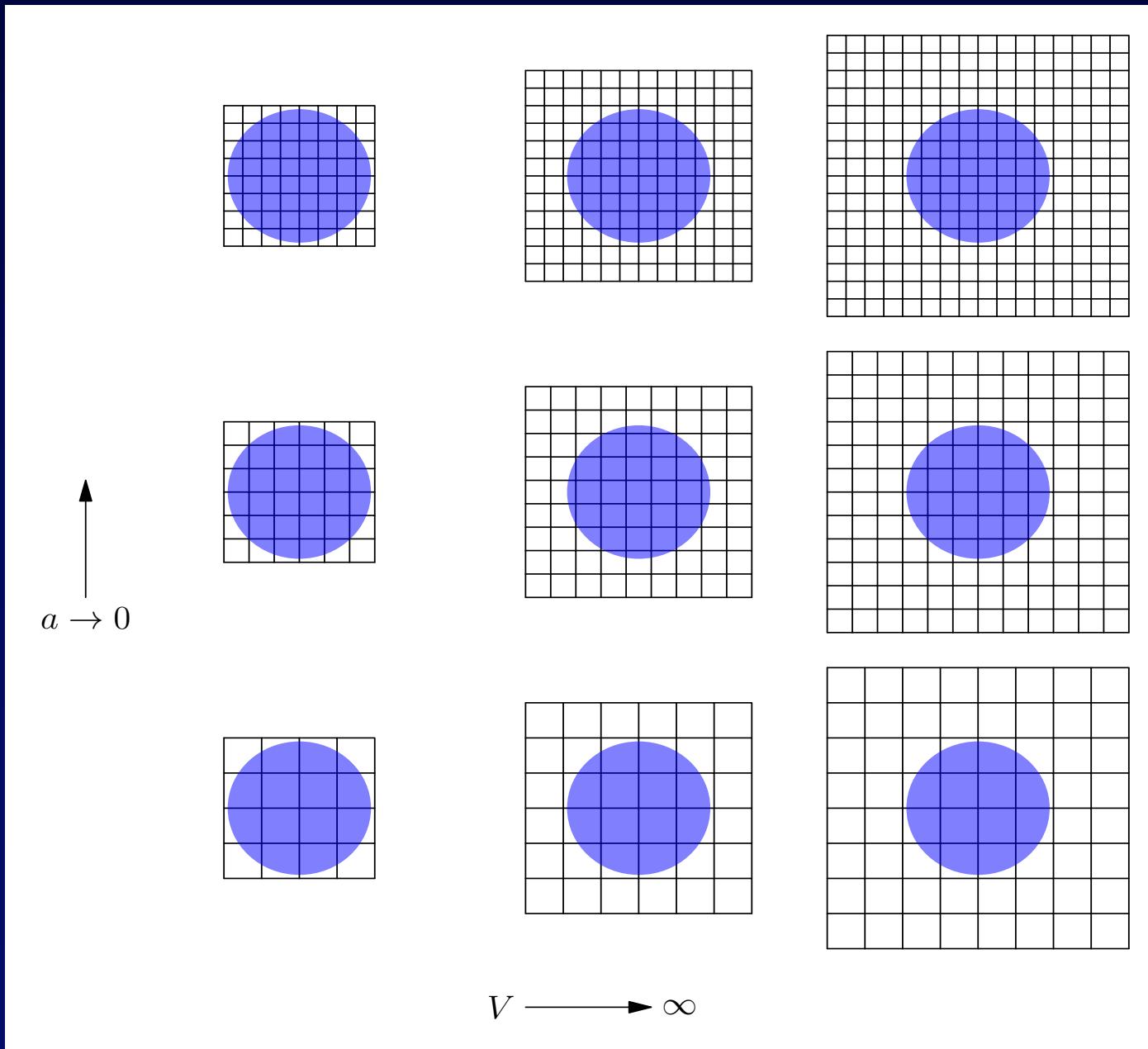
## Why Lattice?

- Regularization
  - Lattice spacing  $a$
  - Hard cutoff,  $p \leq \pi/a$
  - Scale introduced (dimensional transmutation)
- Renormalization
  - Perturbative
  - Non-perturbative
- Regularization independent Scheme  
Schroedinger functional  
Current algebra relations
- Numerical Simulation
  - Quantum field theory  $\rightarrow$  classical statistical mechanics
  - Monte Carlo simulation (importance sampling)



$$e^{-S_G} \det M \geq 0$$

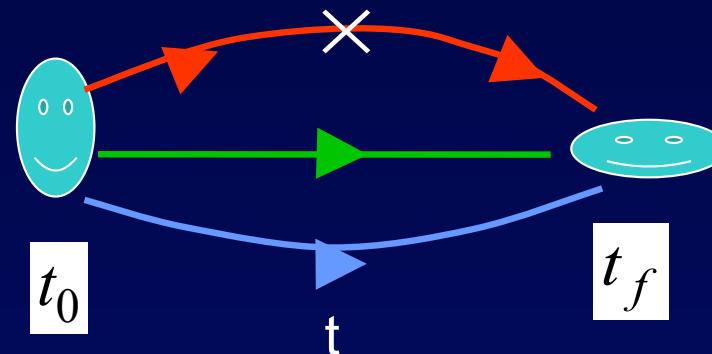
# Continuum and Infinite Volume Limits at Physical Pion Mass (Systemic Errors)



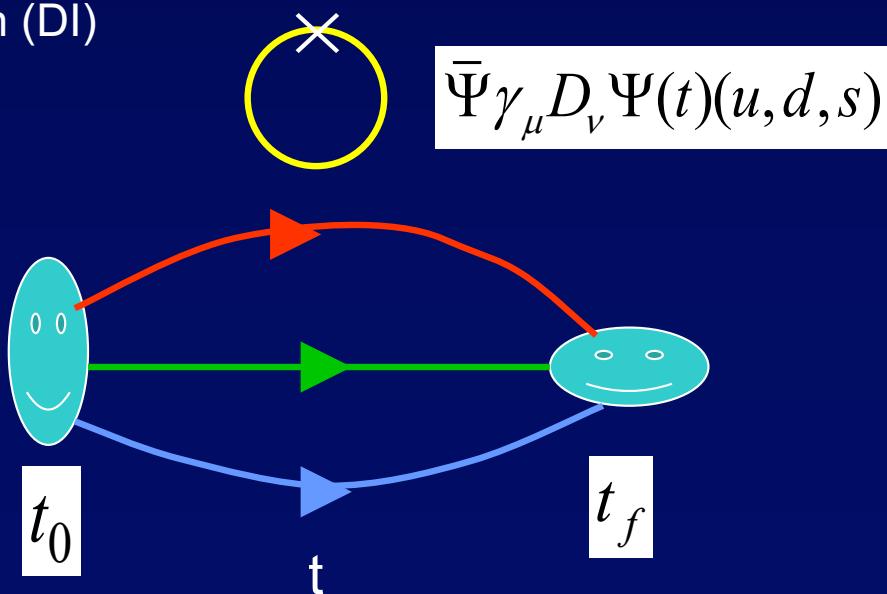
# Lattice Calculations of Quark and Glue Spins

- Quark and Glue Momentum and Angular Momentum in the Nucleon

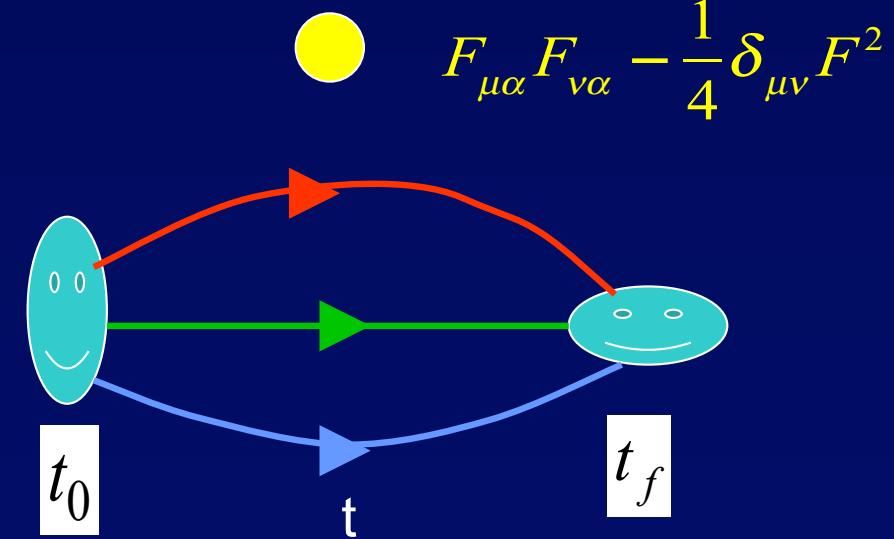
$$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$$



Disconnected insertion (DI)



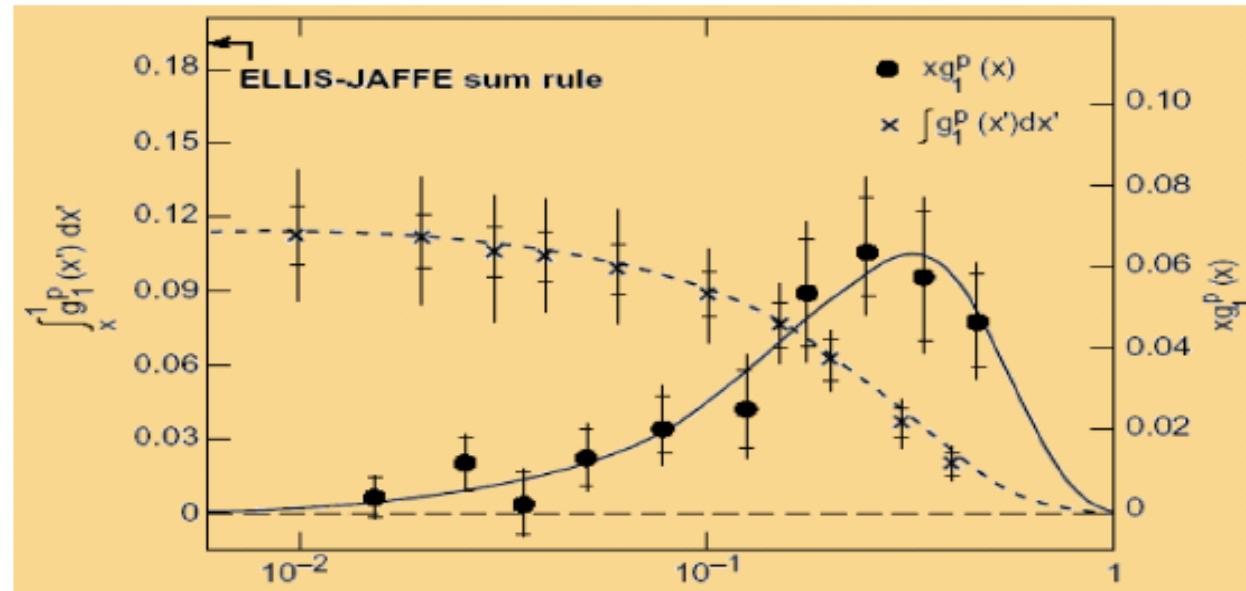
$$\bar{\Psi} \gamma_\mu D_\nu \Psi(t)(u, d, s)$$



$$F_{\mu\alpha} F_{\nu\alpha} - \frac{1}{4} \delta_{\mu\nu} F^2$$

## 30 ~~Twenty~~ years since the “spin crisis”

- EMC experiment in 1988/1989 – “the plot”:



$$g_1(x) = \frac{1}{2} \sum q e_q^2 [\Delta q(x) + \Delta \bar{q}(x)] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_{\parallel} | \bar{\psi}_q(0) \gamma^+ \gamma_5 \psi_q(0) | P, s_{\parallel} \rangle$$

- “Spin crisis” or puzzle:  $\Delta \Sigma = \sum_q \Delta q + \Delta \bar{q} \sim 0.3$

# Quark Spin Components $\overline{\text{MS}}$ (2 GeV)

$g_A$	$\Delta(u+d)$ CI	$\Delta(u/d)$ DI	$\Delta s$	$\Delta u$	$\Delta d$	$\Delta u-\Delta d$ ( $g_A^3$ )	$\Delta \Sigma$
PNDME			-0.053 (8)	0.777 (25)(30)	-0.438 (18)(30)	1.128 (27)(30)	0.286 (62)(72)
C. Alexandrou	0.598 (24)(6)	-0.077 (15)(5)	-0.042 (10)(2)	0.830 (26)(4)	-0.386 (16)(6)	1.216 (31)(7)	0.402 (34)(10)
$\chi$ QCD	0.580 (16)(30)	-0.070 (12)(15)	-0.035 (6)(7)	0.847 (18)(32)	-0.407 (16)(18)	1.254 (16)(30)	0.405 (25)(37)
NPPDFpol1.1 ( $Q^2=10$ GeV $^2$ )			-0.10 (8)	0.76 (4)	-0.41 (4)	1.17 (6)	0.25 (10)
DSSV ( $Q^2=10$ GeV $^2$ )			-0.012 +(56)-(62)	0.793 +(28)-(34)	-0.416 +(35)-(25)	1.209 +(45)-(42)	0.366 +(62)-(42)

PNDME,  $N_F=2+1$ , Clover fermion, multiple ensembles,  $m_\pi = 315 - 135$  MeV

C. Alexandrou et al.,  $N_F=2$ , twisted mass fermion, ,  $m_\pi = 131$  MeV, one lattice

$\chi$  QCD ,  $N_F=2+1$ , Overlap fermion, ,  $m_\pi = 170, 290, 330$  MeV, 5 - 6 valence quarks for each of the three lattices → non-perturbative renormalization and normalization with anomalous Ward identity

Expt.  $g_A^3 = 1.2723(23)$ ; CalLat:  $g_A^3 = 1.271(13)$

# Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T_{\mu\nu}^q = \frac{i}{4} \left[ \bar{\psi} \gamma_\mu \vec{D}_\nu \psi + (\mu \leftrightarrow \nu) \right] \rightarrow \vec{J}_q = \int d^3x \left[ \frac{1}{2} \bar{\psi} \gamma_5 \psi + \vec{x} \times \bar{\psi} \gamma_4 (-i \vec{D}) \psi \right]$$

$$T_{\mu\nu}^g = F_{\mu\lambda} F_{\lambda\nu} - \frac{1}{4} \delta_{\mu\nu} F^2 \rightarrow \vec{J}_g = \int d^3x \left[ \vec{x} \times (\vec{E} \times \vec{B}) \right]$$

- Nucleon form factors

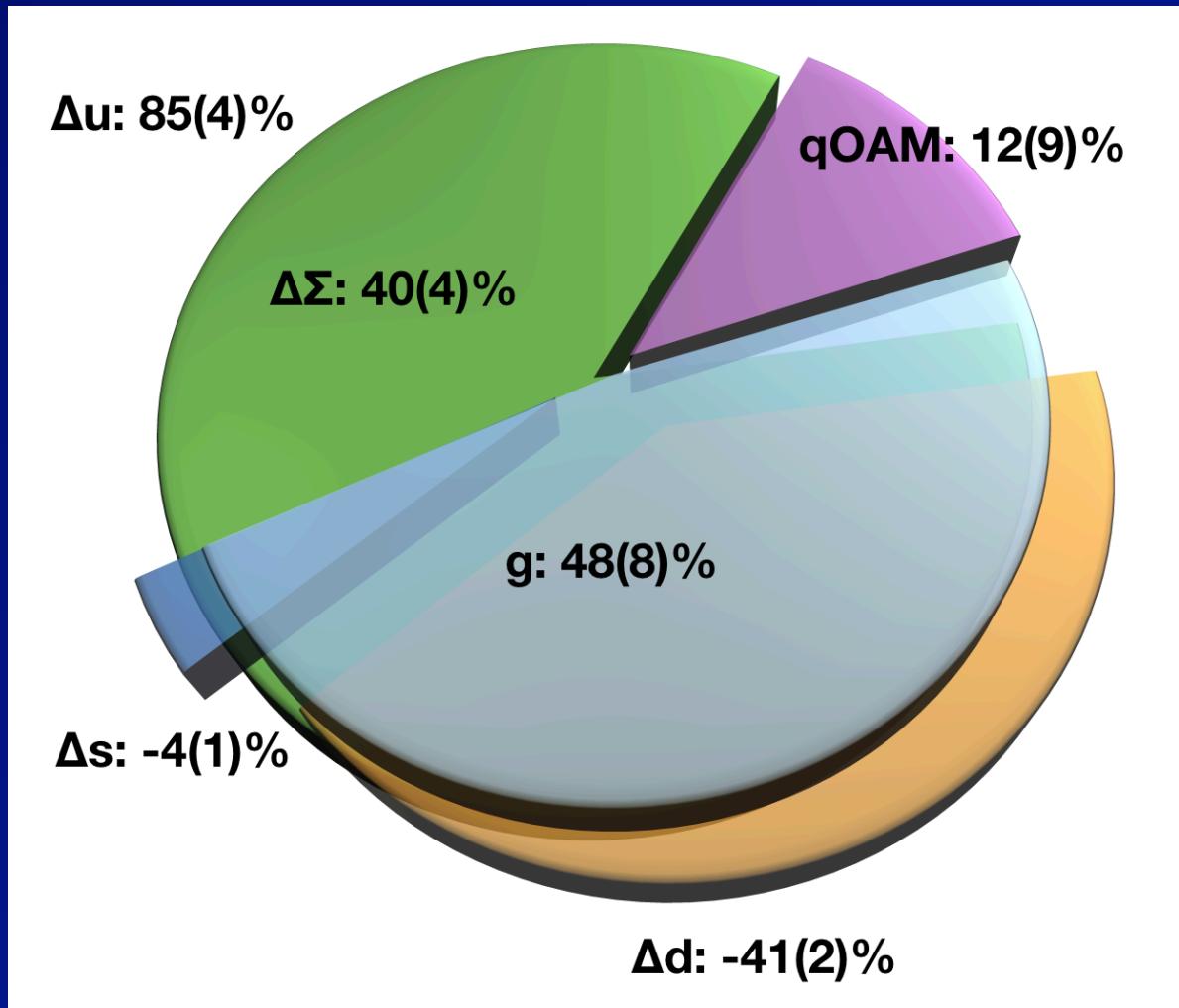
$$\begin{aligned} \langle p, s | T_{\mu\nu} | p' s' \rangle &= \bar{u}(p, s) [T_1(q^2) \gamma_\mu \bar{p}_\nu - T_2(q^2) \bar{p}_\mu \sigma_{\nu\alpha} q_\alpha / 2m \\ &\quad - i T_3(q^2) (q_\mu q_\nu - \delta_{\mu\nu} q^2) / m + T_4(q^2) \delta_{\mu\nu} m / 2] u(p' s') \end{aligned}$$

- Momentum and Angular Momentum

$$Z_{q,g} T_1(0)_{q,g} \left[ \text{OPE} \right] \rightarrow \langle x \rangle_{q/g}(\mu, \bar{MS}), \quad Z_{q,g} \left[ \frac{T_1(0) + T_2(0)}{2} \right]_{q,g} \rightarrow J_{q/g}(\mu, \bar{MS})$$

# Proton Spin Decomposition (2+1 Flavor)

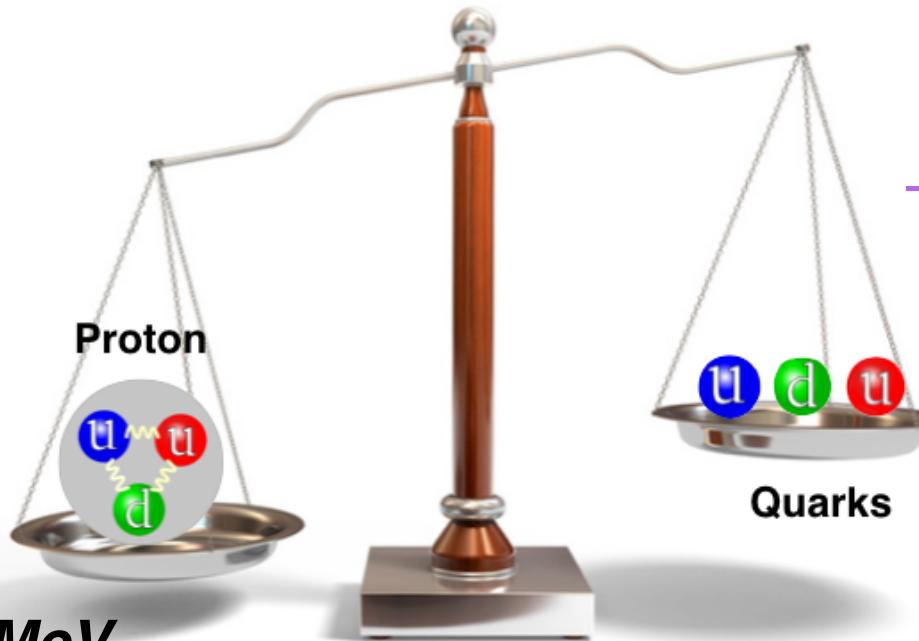
*$\chi$ QCD Preliminary*



Approximate by setting  $T_2 = 0$

# Motivation

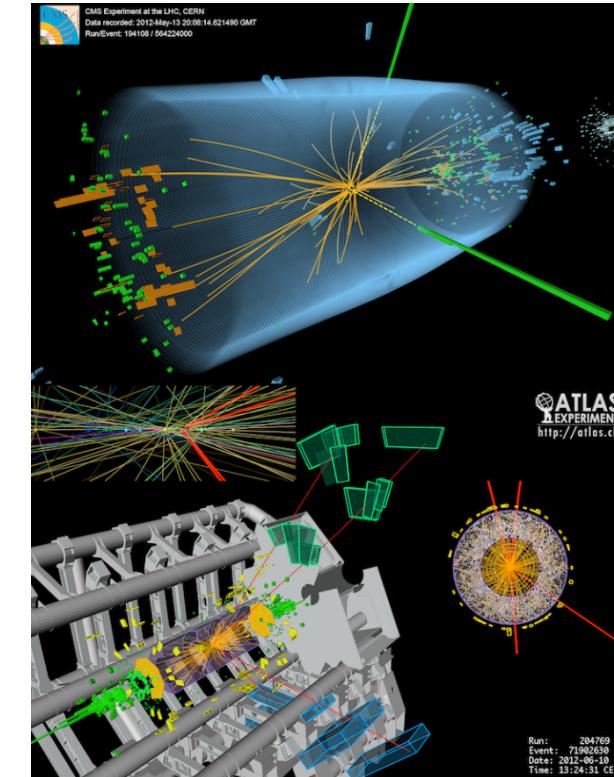
Where does the proton mass come from, and how ?



But the mass of the proton is

**938.272046(21) MeV.**

~100 times of the sum of the quark masses!



The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$$m_u = 2.08(9) \text{ MeV}$$
$$m_d = 4.73(12) \text{ MeV}$$

Laiho, Lunghi, & Van de Water,  
Phys.Rev.D81:034503,2010

# Quark and Glue Components of Hadron Mass

- Energy momentum tensor

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\vec{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2 \quad \langle P | T_{\mu\nu} | P \rangle = P_\mu P_\nu / M$$

- Trace anomaly

$$T_{\mu\mu} = -m(1+\gamma_m)\bar{\psi}\psi + \frac{\beta(g)}{2g}G^2$$

- Separate into traceless part  $\bar{T}_{\mu\nu}$  and trace part  $\hat{T}_{\mu\nu}$

$$\langle P | \bar{T}_{\mu\nu}^{q,g} | P \rangle = \langle x \rangle_{q,g}(\mu^2)(P_\mu P_\nu - \frac{1}{4}\delta_{\mu\nu}P^2) / M, \quad \langle x \rangle_q(\mu^2) + \langle x \rangle_g(\mu^2) = 1$$

$$\langle \bar{T}_{44} \rangle = -3/4M; \quad \langle \hat{T}_{\mu\mu} \rangle = -M$$

# Non-perturbative Renormalization and Mixing

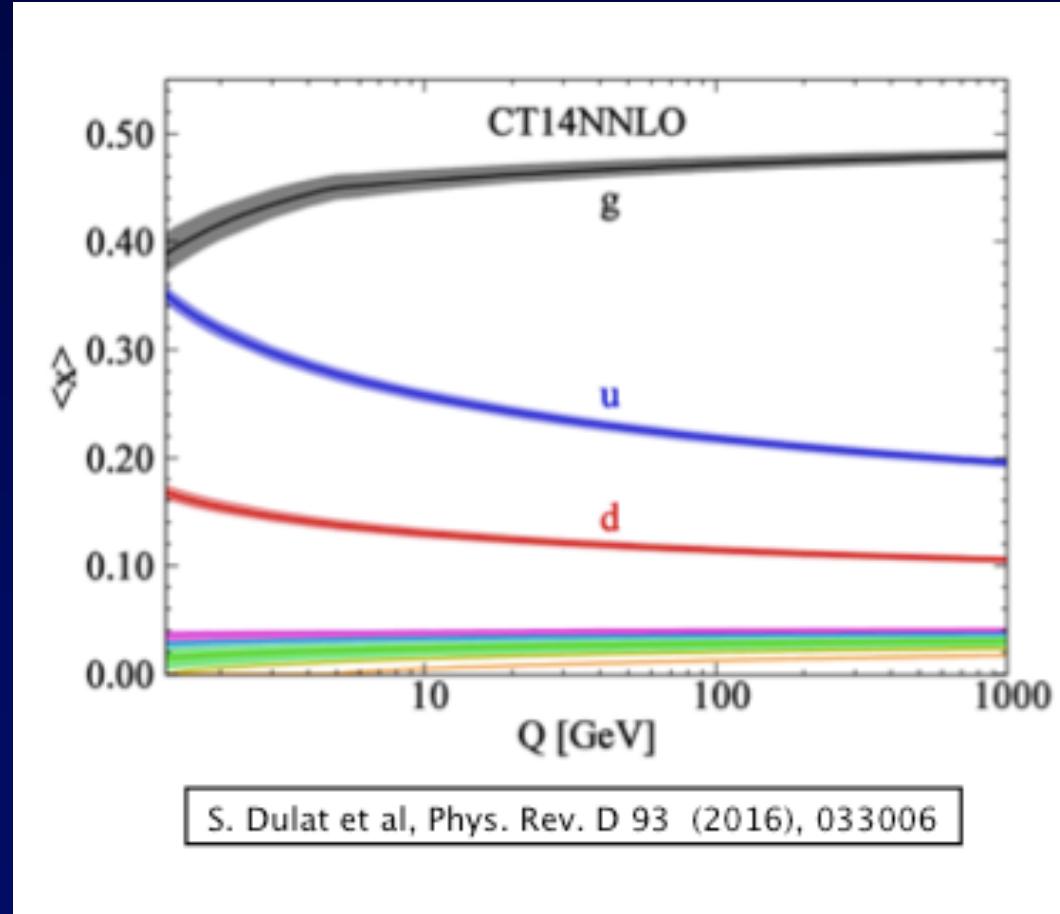
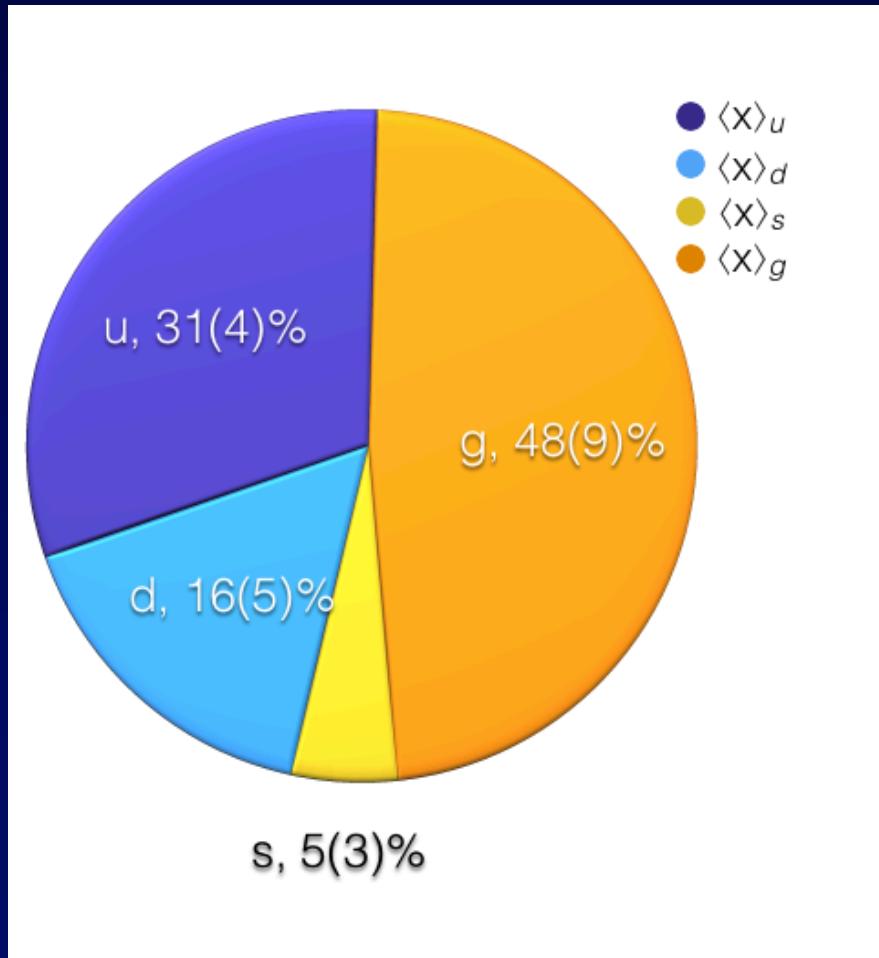
- Renormalized  $\langle x \rangle_q$  and  $\langle x \rangle_g$  in MS-bar at  $\mu$

$$\langle x \rangle_{u,d,s}^R = Z_{QQ}^{\overline{\text{MS}}(\mu)} \langle x \rangle_{u,d,s} + \delta Z_{QQ}^{\overline{\text{MS}}}(\mu) \sum_{q=u,d,s} \langle x \rangle_q + Z_{QG}^{\overline{\text{MS}}}(\mu) \langle x \rangle_g, \quad \langle x \rangle_g^R = Z_{GQ}^{\overline{\text{MS}}}(\mu) \sum_{q=u,d,s} \langle x \rangle_q + Z_{GG}^{\overline{\text{MS}}} \langle x \rangle_g,$$

$$\begin{pmatrix} Z_{QQ}^{\overline{\text{MS}}}(\mu) + N_f \delta Z_{QQ}^{\overline{\text{MS}}}(\mu) & N_f Z_{QG}^{\overline{\text{MS}}}(\mu) \\ Z_{GQ}^{\overline{\text{MS}}}(\mu) & Z_{GG}^{\overline{\text{MS}}}(\mu) \end{pmatrix} = \left\{ \left[ \begin{pmatrix} Z_{QQ}(\mu_R) + N_f \delta Z_{QQ}(\mu_R) & N_f Z_{QG}(\mu_R) \\ Z_{GQ}(\mu_R) & Z_{GG}(\mu_R) \end{pmatrix} \right. \right. \\ \left. \left. \begin{pmatrix} R_{QQ}(\frac{\mu}{\mu_R}) + \mathcal{O}(N_f \alpha_s^2) & N_f R_{QG}(\frac{\mu}{\mu_R}) \\ R_{GQ}(\frac{\mu}{\mu_R}) & R_{GG}(\frac{\mu}{\mu_R}) \end{pmatrix} \right] |_{a^2 \mu_R^2 \rightarrow 0} \right\}^{-1}$$

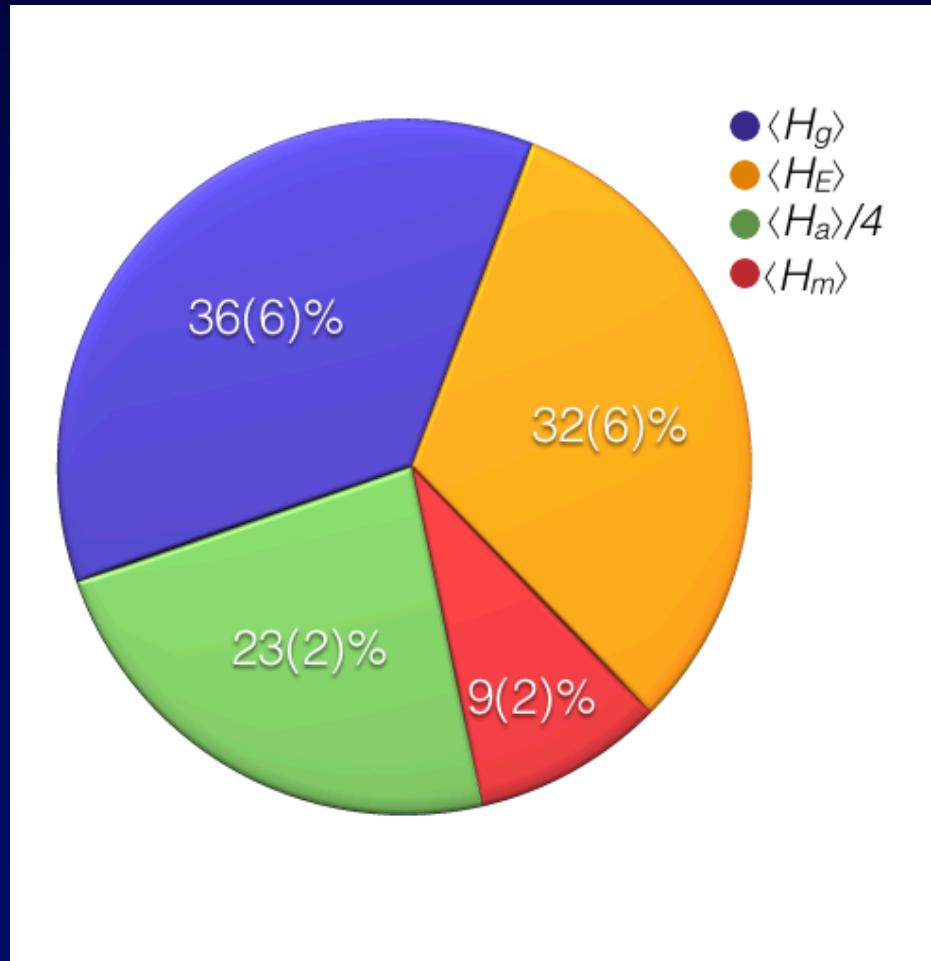
- Renormalization of glue operator in gluon propagator is very noisy  $\rightarrow$  Cluster decomposition error reduction (CDER)

# Comparison with Global Fitting of $\langle x \rangle$ MS-bar at 2 GeV



$\chi$ QCD, preliminary

# Proton Mass Decomposition



Y.B. Yang et al ( $\chi$ QCD), PRL 121,212001 (2018)

# Hadronic Tensor in Euclidean Path-Integral Formalism

- Deep inelastic scattering  
In Minkowski space

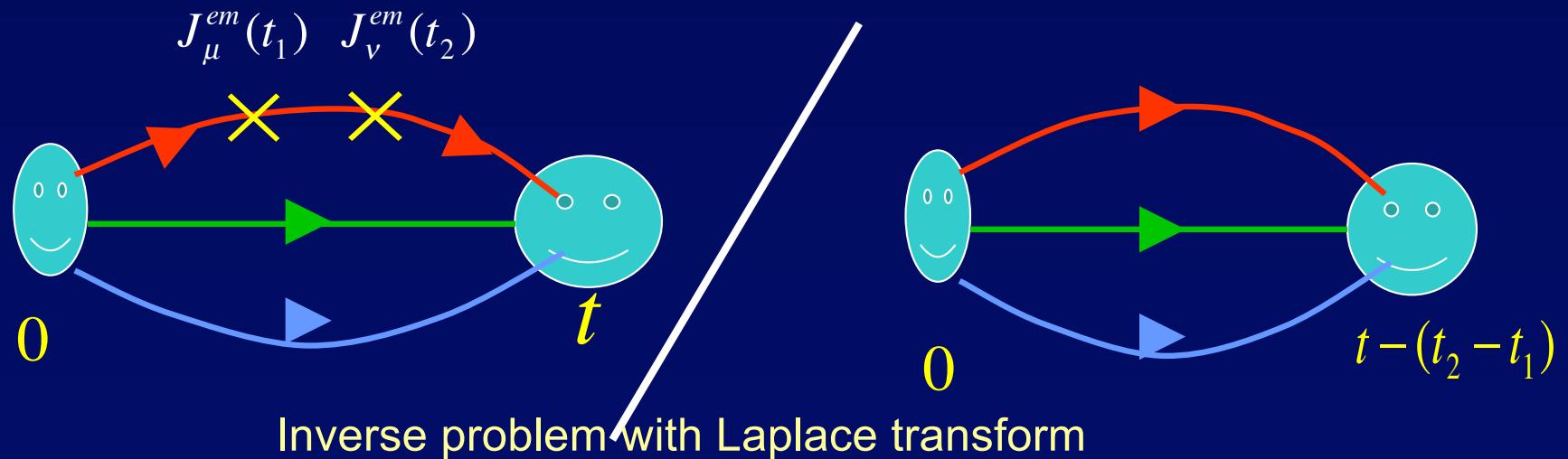
$$\frac{d^2\sigma}{dE'd\Omega} = \frac{\alpha^2}{q^4} \left(\frac{E'}{E}\right) l^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu}(\vec{q}, \vec{p}, v) = \frac{1}{\pi} \text{Im} T_{\mu\nu} = \langle N(\vec{p}) | \int \frac{d^4x}{4\pi} e^{iq\cdot x} J_\mu(x) J_\nu(0) | N(\vec{p}) \rangle_{\text{spin avg}}$$

$$= \frac{1}{2} \sum_n \int \prod_{i=1}^n \left[ \frac{d^3 p_i}{(2\pi)^3 2E_{pi}} \right] (2\pi)^3 \delta^4(p_n - p - q) \langle N(\vec{p}) | J_\mu | n \rangle \langle n | J_\nu | N(\vec{p}) \rangle_{\text{spin avg}}$$

- Euclidean path-integral

KFL and S.J. Dong, PRL 72, 1790 (1994)  
KFL, PRD 62, 074501 (2000)



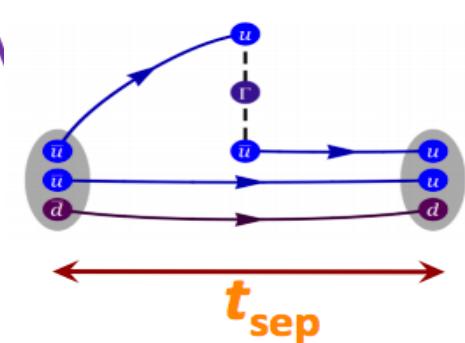
# Large Momentum

## Approach

§ Take the large- $P_z$  limit:

$$q(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P | \bar{\psi}(z) \gamma^i \exp \left( -ig \int_0^z dz' A^z(z') \right) \psi(0) | P \rangle + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

↑      ↑      ↑  
 $x = k^z/P^z$    Lattice z coordinate   Nucleon momentum  $P^\mu = \{P^0, 0, 0, P^z\}$



Product of lattice gauge links

- ☞ At  $P^z \rightarrow \infty$  limit, twist-2 parton distribution is recovered
- ☞ For finite  $P^z$ , corrections are needed

# Inverse problems are everywhere

- ◆ Extracting spectral functions from lattice data
- ◆ Global fittings of PDFs
- ◆ Lattice calculation of Quasi-PDFs

$$\bar{q}_\Gamma(x, P_z, \tilde{\mu}) = \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{ixP_z z} \langle P | O_\Gamma(z) | P \rangle$$

$$\bar{q}(x, P_z, p_z^R, \mu_R) = \int_{-1}^1 \frac{dy}{|y|} C\left(\frac{x}{y}, r, \frac{yP_z}{\mu}, \frac{yP_z}{p_z^R}\right) q(y, \mu)$$

H.-W. Lin et.al., PRL 121, 242003(2018)

- ◆ Lattice calculation of Pseudo-PDFs

$$\mathfrak{M}_R(\nu, \mu^2) \equiv \int_0^1 dx \cos(\nu x) q_v(x, \mu^2)$$

K. Orginos et al., PRD 96, 094503 (2017)

- ◆ Lattice cross sections

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^1 \frac{dx}{x} f_a(x, \mu^2) \times K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + O(\xi^2 \Lambda_{\text{QCD}}^2)$$

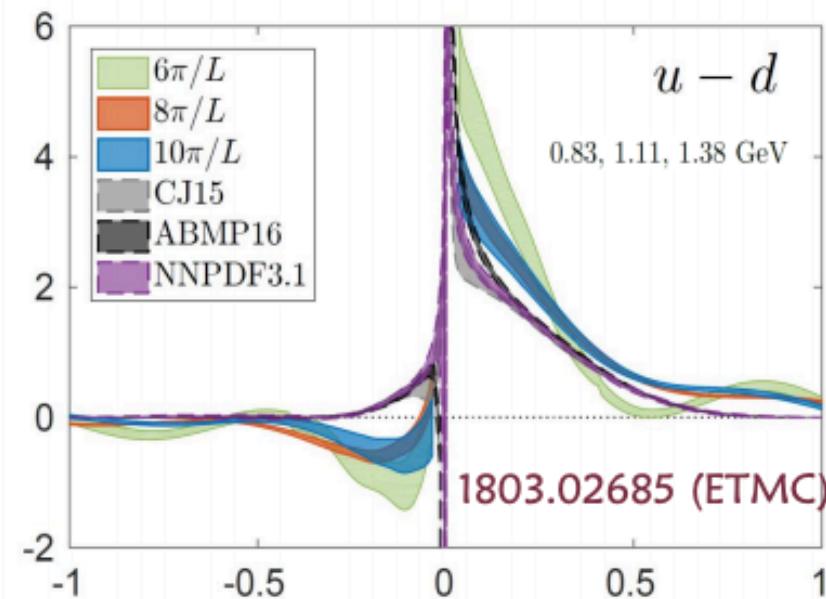
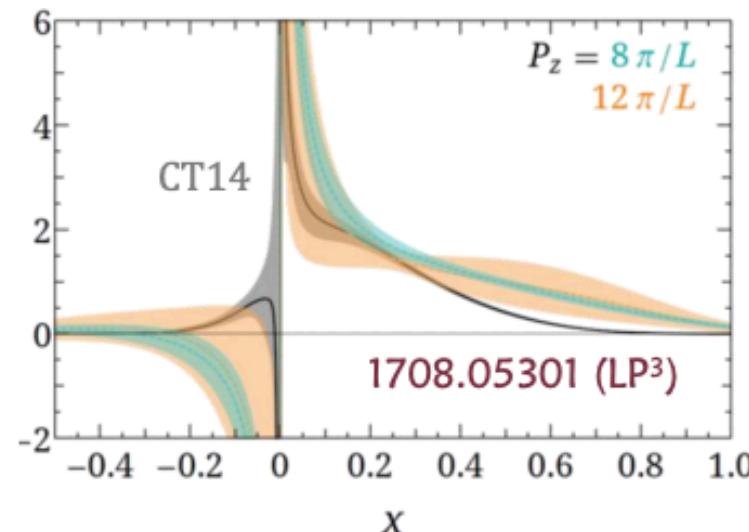
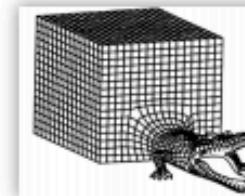
Y.-Q. Ma and J.-W. Qiu, PRL 120, 022003 (2018)

# Quasi PDF results from LP3 and ETMC

## *Physical Pion Mass Results*

§ Exciting! Two collaborations' results at physical pion mass

- ❖ Boost momenta  $P_z \leq 1.4$  GeV
- ❖ Study of systematics still needed



Not use any parametrization form like  $xf(x, \mu_0) = a_0 x^{a_1} (1-x)^{a_2} P(x)$

## Sea Partons from Euclidean Path-integral Formulation and Definition of Valence Patons in NNLO Evolution

- The definition  $\mathbf{q}_i^v \equiv q_i - \bar{q}_i$  for the valence is no longer adequate with NNLO evolution and can be confusing for the concept of valence.
- Strange has valence distribution  $\bar{q}_s^- \equiv q_s^v = s - \bar{s}$  (?)

## Evolution Equations

S. Moch et al., hep/0403192, 0404111  
A. Cafarella et al., 0803.0462

NNLO

$$\begin{aligned} dq_i / dt &= \sum_k (P_{ik} \otimes q_k + P_{i\bar{k}} \otimes q_{\bar{k}}) + P_{ig} \otimes g; \\ d\bar{q}_i / dt &= \sum_k (P_{\bar{i}k} \otimes q_k + P_{\bar{i}\bar{k}} \otimes q_{\bar{k}}) + P_{\bar{i}g} \otimes g; \\ dg / dt &= \sum_k (P_{gk} \otimes q_k + P_{g\bar{k}} \otimes q_{\bar{k}}) + P_{gg} \otimes g. \end{aligned}$$



$$dq_i^- / dt = P_{qq}^- \otimes q_i^- + \frac{P_{ns}^s}{N_f} \otimes \Sigma_\nu;$$

$$\text{where } q_i^- \equiv q_i - \bar{q}_i, \quad \Sigma_\nu \equiv \sum_k (q_k - \bar{q}_k),$$

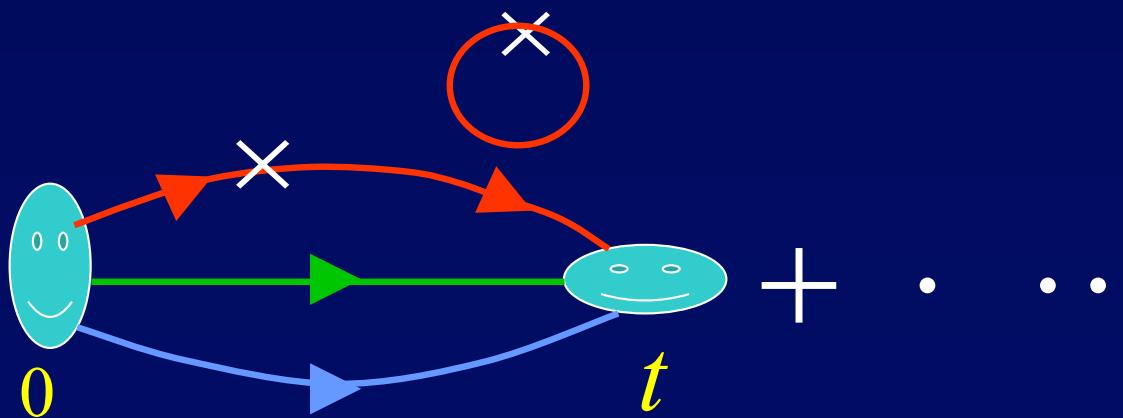
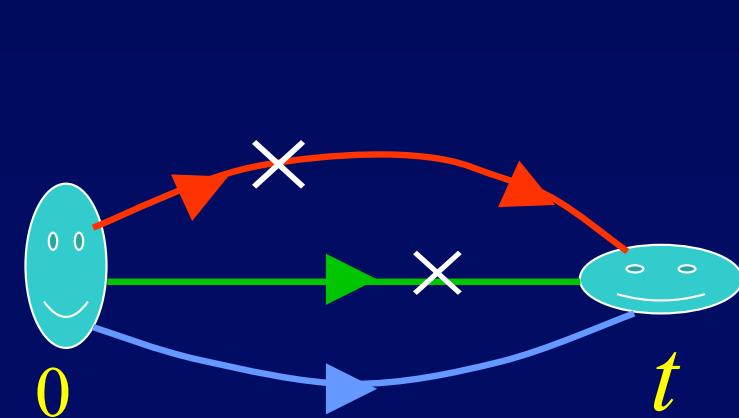
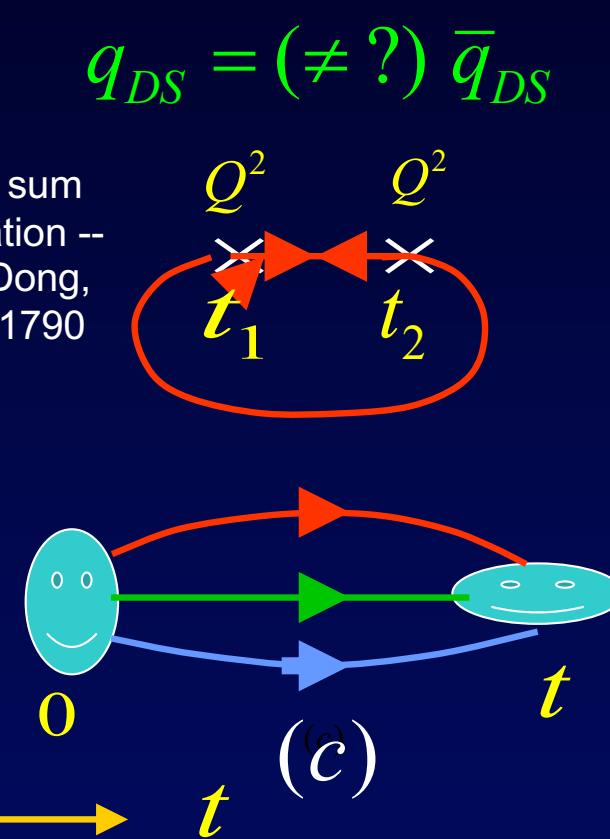
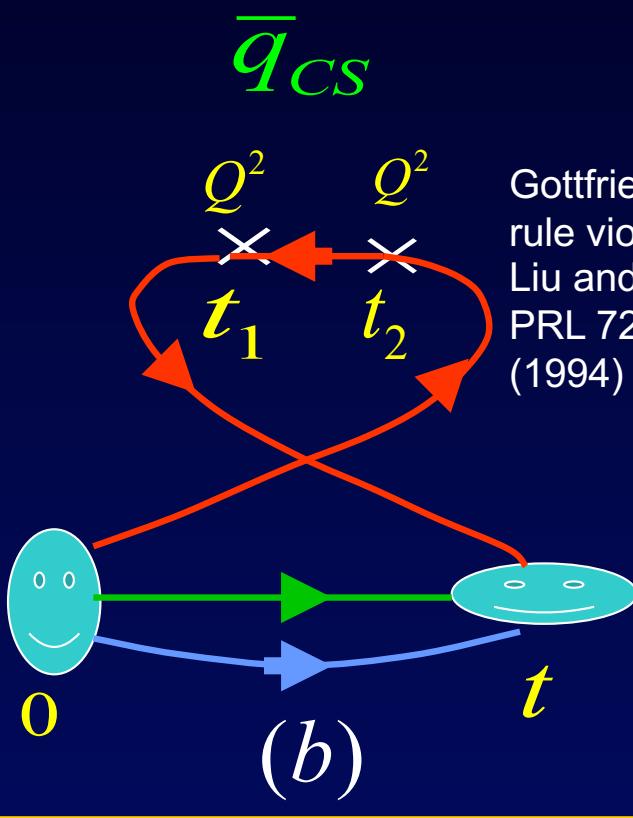
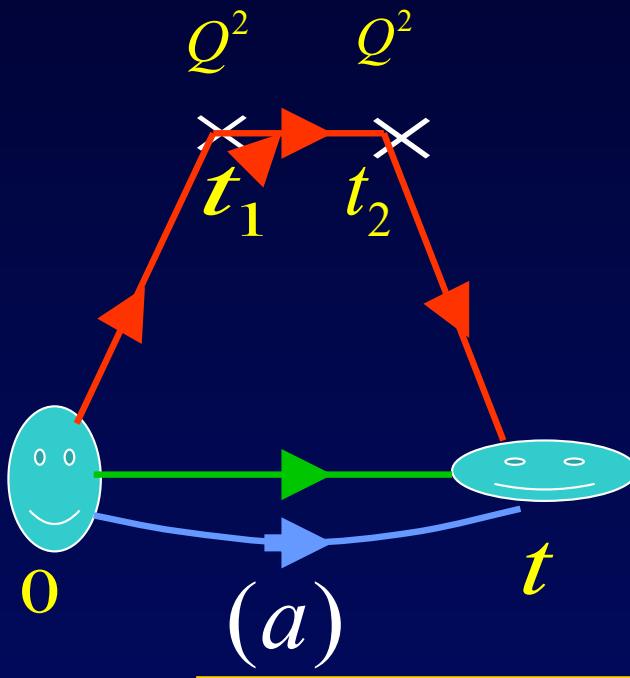
$$\text{and } P_{ns}^s \sim O(\alpha_s^3)$$

Valence u can affect the evolution of valence d ?

Also:  $q_s^- \equiv q_s^\nu = s - \bar{s}$  (?)

## Hadronic Tensor

$$q = q_V + q_{CS}$$



Cat's ears diagrams are suppressed by  $O(1/Q^2)$ .

$$q_i^- = q_i^{v+cs} - \bar{q}_i^{cs} + q_i^{ds} - \bar{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \bar{q}_i^{ds}$$

# NNLO Evolution equations separating CS from the DS partons

K.F. Liu, PRD 96, 033001 (2017),  
arXiv: 1703.04690

$$dq_i^{v+cs} / dt = P_{ii}^c \otimes q_i^{v+cs} + P_{i\bar{i}}^c \otimes \bar{q}_i^{cs};$$

$$d\bar{q}_i^{cs} / dt = P_{i\bar{i}}^c \otimes \bar{q}_i^{cs} + P_{\bar{i}\bar{i}}^c \otimes q_i^{v+cs};$$

$$dq_i^{ds} / dt = \sum_k (P_{ik}^{cd} \otimes q_k^{ds} + P_{i\bar{k}}^{cd} \otimes \bar{q}_k^{ds} + P_{ik}^d \otimes q_k^{v+cs} + P_{i\bar{k}}^d \otimes \bar{q}_k^{cs}) + P_{ig} \otimes g;$$

$$d\bar{q}_i^{ds} / dt = \sum_k (P_{i\bar{k}}^{cd} \otimes \bar{q}_k^{ds} + P_{\bar{i}\bar{k}}^{cd} \otimes q_k^{ds} + P_{i\bar{k}}^d \otimes q_k^{v+cs} + P_{\bar{i}\bar{k}}^d \otimes \bar{q}_k^{cs}) + P_{ig} \otimes g;$$

$$dg / dt = \sum_k [P_{gk} \otimes (q_k^{v+cs} + q_k^{ds}) + P_{g\bar{k}} \otimes (\bar{q}_k^{cs} + \bar{q}_k^{ds}) + P_{gg} \otimes g].$$

$$q_i^- = q_i^{v+cs} - \bar{q}_i^{cs} + q_i^{ds} - \bar{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \bar{q}_i^{ds}$$

## Comments

$$q_i^- = q_i^{v+cs} - \bar{q}_i^{cs} + q_i^{ds} - \bar{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \bar{q}_i^{ds}$$

- CS and DS are explicitly separated, leading to more equations (11 vs 7) which can accommodate  $s \neq \bar{s}$ ,  $\mathbf{u}^{ds} \neq \bar{\mathbf{u}}^{ds}$
- There is no flavor-changing evolution of the valence partons.

$$dq_i^- / dt = P_{qq}^- \otimes q_i^- + P_{ds}^- \otimes \sum_k (q_k - \bar{q}_k);$$

is the sum of two equations

$$dq_i^v / dt = P_{qq}^- \otimes q_i^v, \quad q^v \equiv q^{v+cs} - \bar{q}^{cs}$$

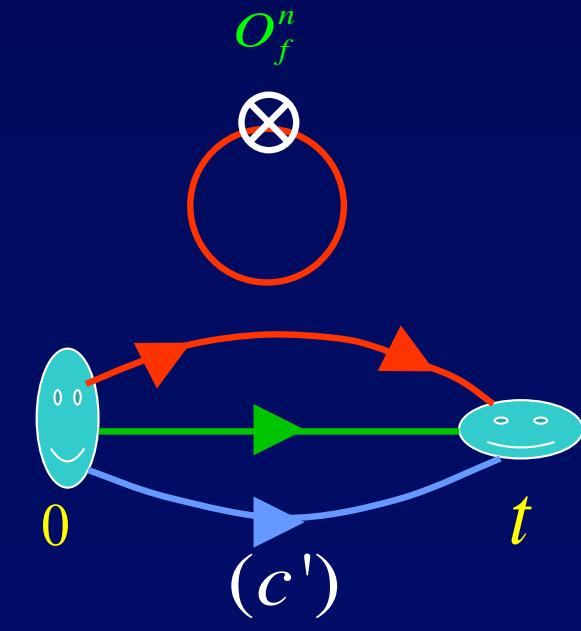
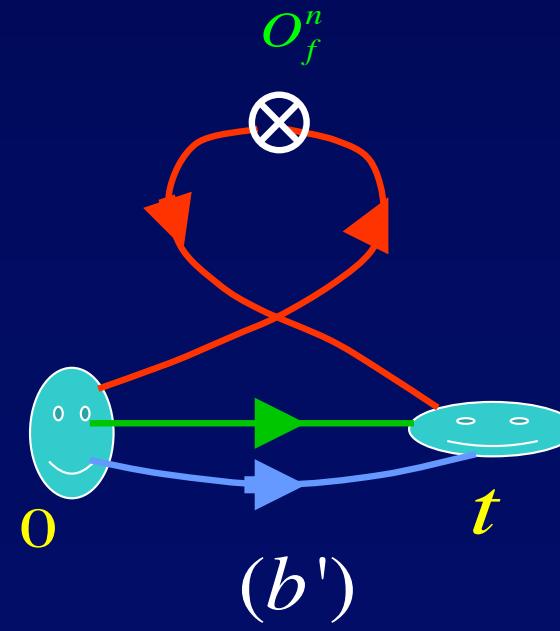
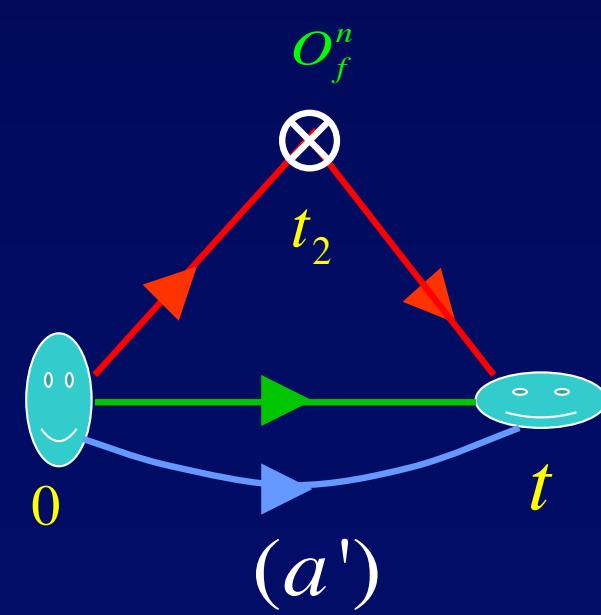
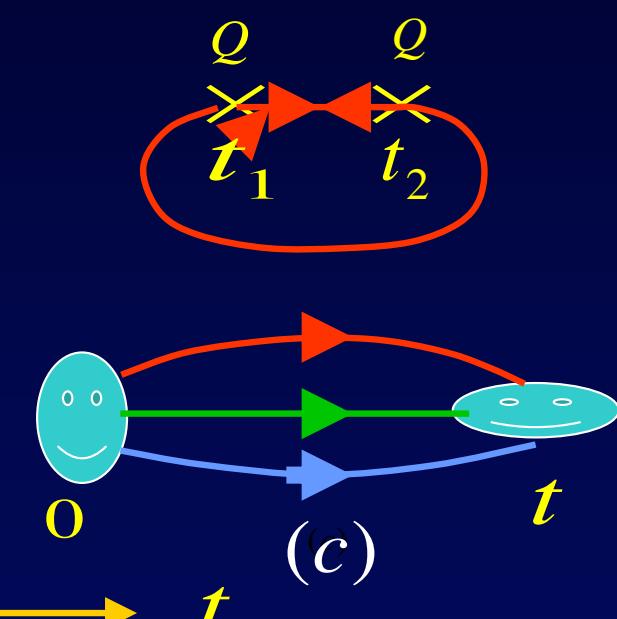
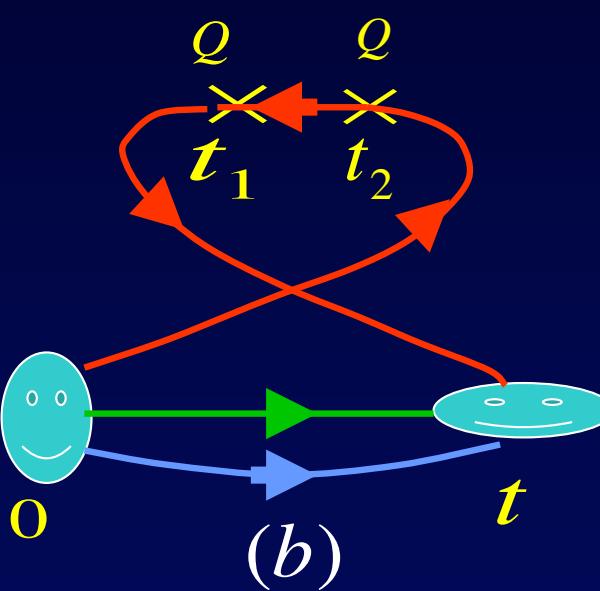
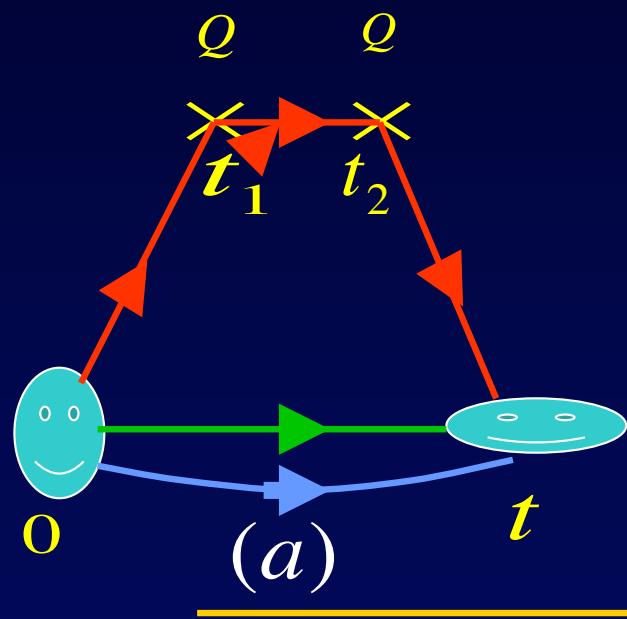
$$d(q_i^{ds} - \bar{q}_i^{ds}) / dt = \sum_k P_{ik}^{cd-} \otimes (q_k^{ds} - \bar{q}_k^{ds}) + \sum_k P_{ds}^{d-} \otimes q_k^v$$

- Once the CS is separated at one  $Q^2$ , it will remain separated at other  $Q^2$ .
- Gluons can split into DS, but not to valence and CS.
- It is necessary to separate out CS from DS when quark and antiquark annihilation (higher twist) is included in the evolution eqs. (Annihilation involves only DS.)

$$q = q_V + q_{CS}$$

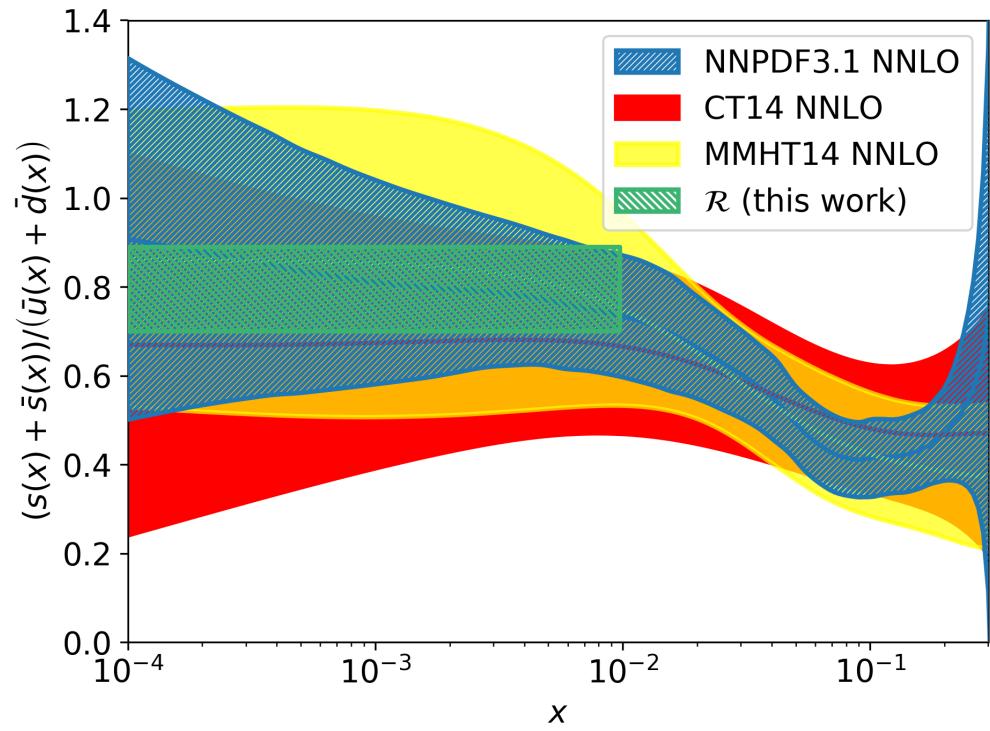
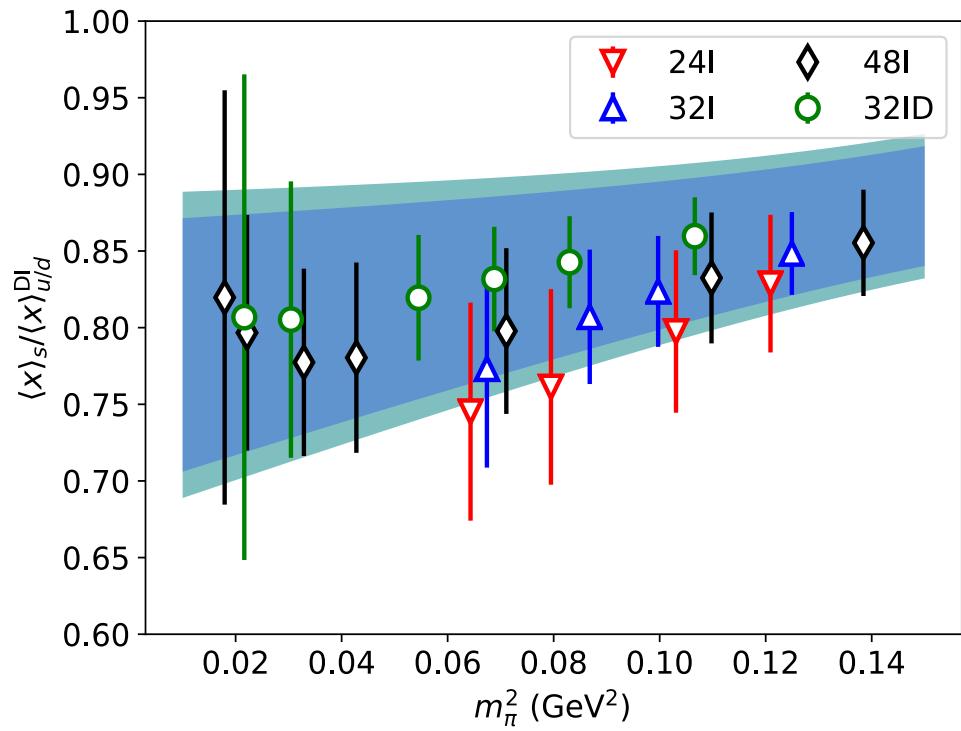
$$\overline{q}_{CS}$$

$$q_{DS} = (\neq ?) \overline{q}_{DS}$$



# Lattice input to global fitting of PDF

Lattice result from overlap on  $N_f = 2+1$  DWF on 4 lattices with one at physical pion mass (1901.07526)



$$\bar{s}(x), \bar{u}^{ds}/\bar{d}^{ds}(x) \sim x^{-1}$$

$$\langle x \rangle_{s+\bar{s}} / \langle x \rangle_{\bar{u}+\bar{d}} \text{ (DI) (2 GeV)} = 0.795(79)(53)$$

$$\bar{u}^{cs}/\bar{d}^{cs}(x) \sim x^{-1/2}$$

# Summary and Challenges

- Together with experiments on LHC and EIC and global fitting of PDF, lattice QCD calculations of hadron structure (proton spin and mass decomposition, moments of PDFs, form factors, etc.) can advance our understanding of the nucleon properties in more detail.
- Path-integral formulation of the hadronic tensor gives a precise definition of the parton degrees of freedom. It would be useful to carry out global fitting with the connected sea and disconnected sea separated so that the direct comparison with lattice calculation can be made.
- The traditional SU(3) classification of nucleon matrix elements in terms of isovector, flavor-octet and flavor-singlet quantities can be extended to CI and DI matrix elements of u and d quarks and strange quark in DI.



# Correlators

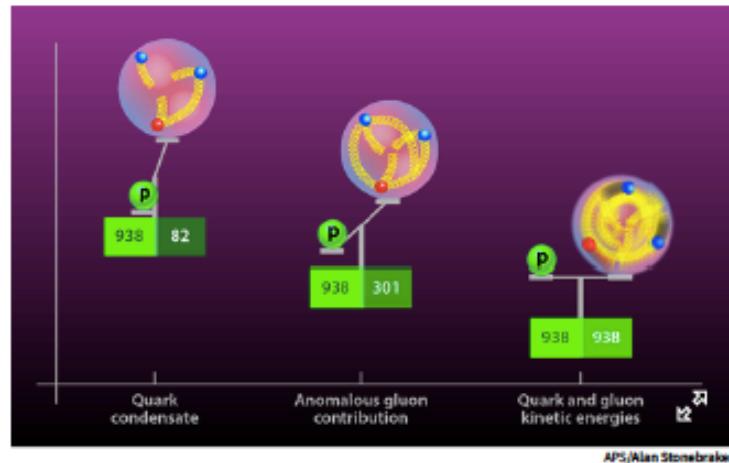
- One point collerators →  $\langle \bar{\psi}\psi \rangle, \langle G_{\mu\nu} G_{\mu\nu} \rangle \dots$
- Two point correlators → hadron masses, decay constants ...
- Three point correlators → parton moments form factors, nucleon matrix elements ...
- Four point correlators → hadronic tensor, PDF, GPD, TMD, nEDM...

## Viewpoint: Dissecting the Mass of the Proton

André Walker-Loud, Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA, USA

November 19, 2018 • Physics 11, 118

A calculation determines four distinct contributions to the proton mass, more than 90% of which arises entirely from the dynamics of quarks and gluons.



APS/Alan Stonebraker

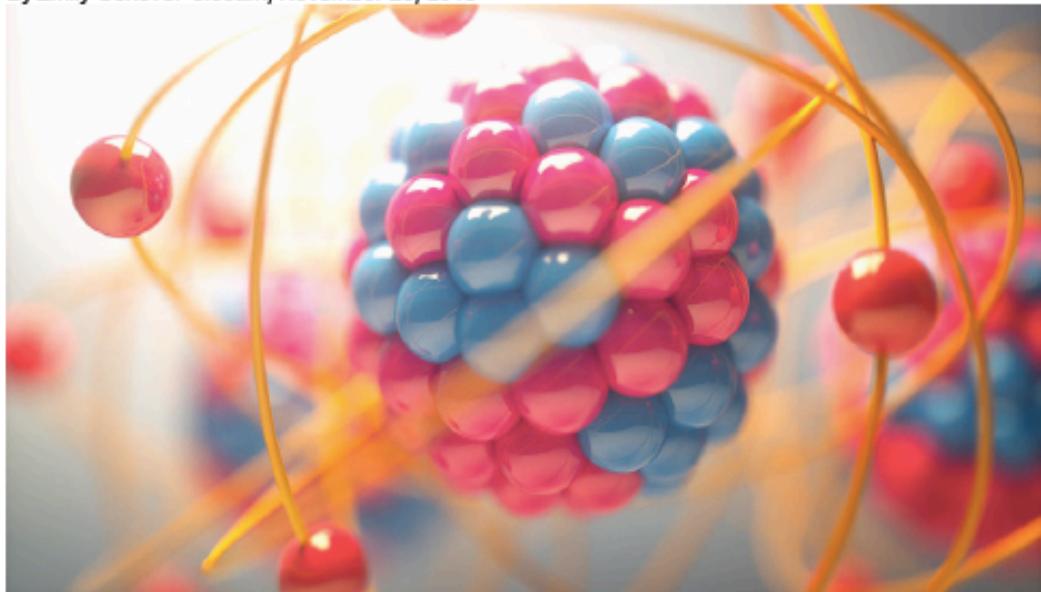
**Figure 1:** The proton is comprised of two up quarks and one down quark, but the sum of these quark masses is a mere 1% of the proton mass. Using lattice QCD, Yang and colleagues determined the relative contributions of the four sources of the proton mass [1]. ... [Show more](#)

News: Particle Physics

## Physicists finally calculated where the proton's mass comes from

*Only 9 percent of the subatomic particle's bulk comes from the mass of its quarks*

By Emily Conover 6:00am, November 26, 2018



**MASSIVE UNDERTAKING** Using a technique called lattice QCD, scientists figured out how protons (illustrated here in the nucleus of an atom) get their mass.

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