Proton Structure from A Light-front Hamiltonian Approach

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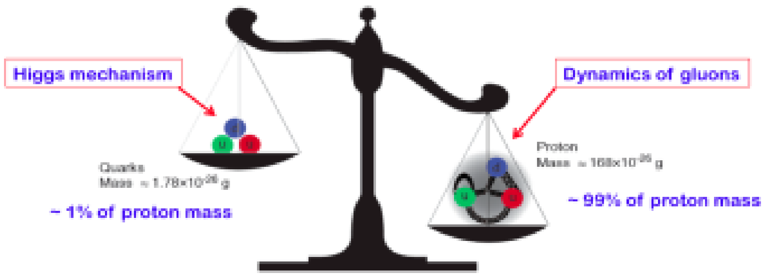
Outline

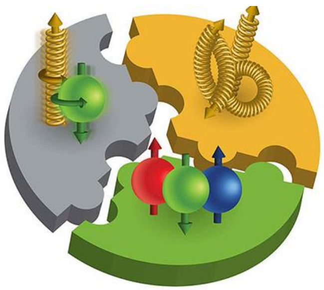
- Light-front Hamiltonian approach

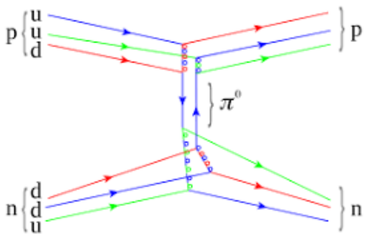
- Application to the nucleon
  - $|qqq\rangle$ [PRD 104,094036 (2021), PLB 833 137306 (2022)]
  - $|qqq\rangle + |qqqg\rangle$ [arXiv:2209.08584 [hep-ph]]

- Conclusion

Fundamental Questions: Emergence





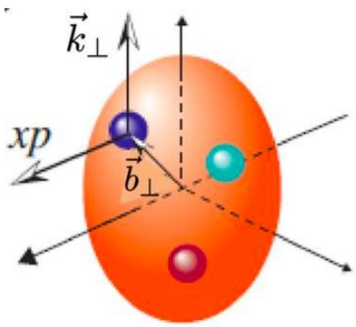


Nucleon mass

Nucleon spin

Nuclear force

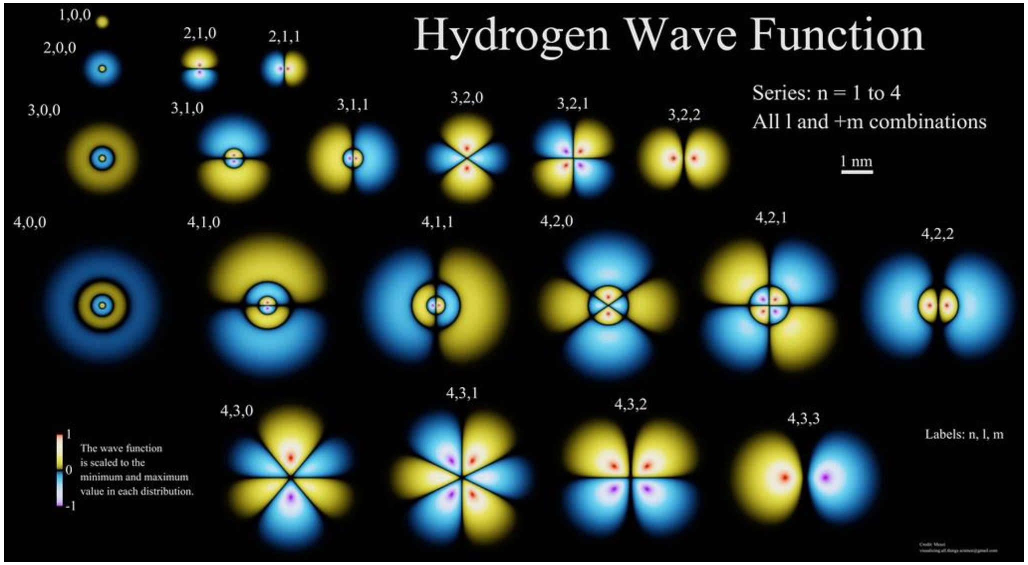
3D image in terms of quarks and gluons in the EIC era



Hamiltonian Formalism

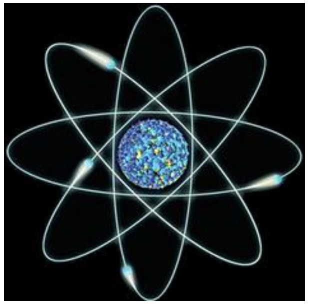
- Schrödinger equation universally describes different physics：

$$H|\psi\rangle = E|\psi\rangle$$



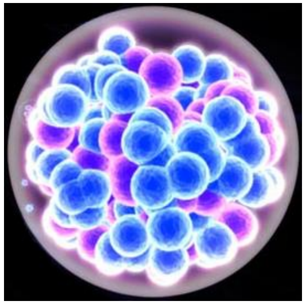
- Wave functions encode full information of the system

Nonrelativistic



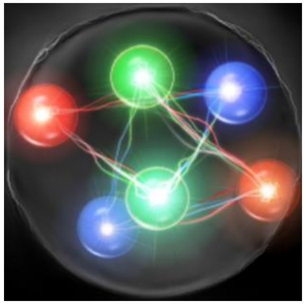
atom

Nonrelativistic



nucleus

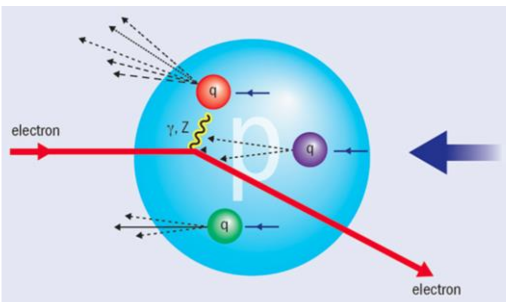
Relativistic

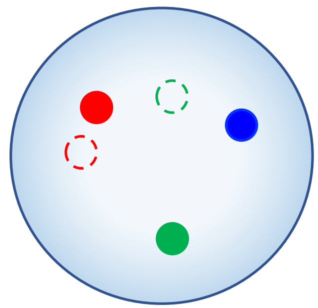


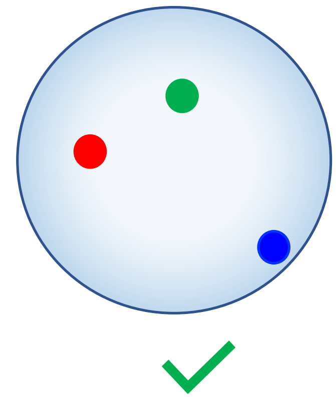
hadron

Light-front Time

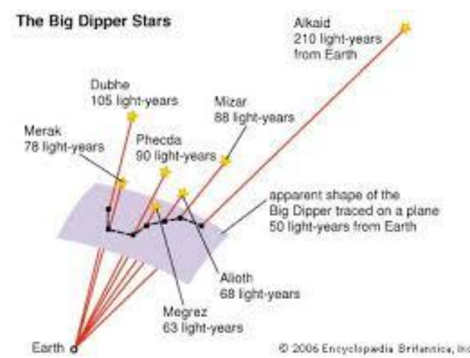
- Hadron structure is measured by (virtual) photon





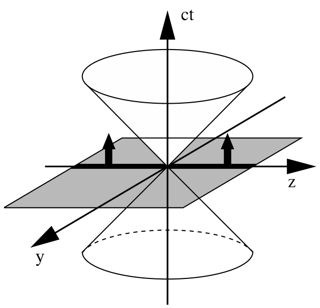
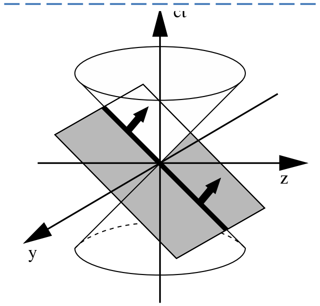


- We "see” the world at fixed light-front time ($t = x^0 + x^3$)



Light-front Quantization

[Dirac, 1949]

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3$, $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3$, $P^+ = P^0 + P^3$, $P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

Advantage:

- Frame-independent light-front wave functions
- Convenience in evaluating observables defined on the light-front
- Light-front wave functions carry parton interpretation
- Hamiltonian formalism
- ...

Basis Light-front Quantization

[Vary et al, 2008]

- Nonperturbative eigenvalue problem

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

  - P^-: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P^-_β: eigenvalue for $|\beta\rangle$

- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

- Fock sector expansion
  - Eg. $|\text{proton}\rangle = a|qqq\rangle + b|qqqg\rangle + c|qqqq\bar{q}\rangle + d|qqqgg\rangle + \ldots$

- Discretized basis
  - Transverse: 2D harmonic oscillator basis: $\Phi^b_{n,m}(\vec{p}_\perp)$.
  - Longitudinal: plane-wave basis, labeled by k.
  - Basis truncation:
    $\sum_i(2n_i + |m_i| + 1) \leq N_{max}$, $\sum_i k_i = K$.
    N_{max}, K are basis truncation parameters
  - Color degrees of freedom

Light-front QCD Hamiltonian (First Principle)

$$
\begin{aligned}
P^-_{LFQCD} = & \frac{1}{2}\int d^3x \overline{\widetilde{\psi}}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\widetilde{\psi} - A^i_a(\mathrm{i}\partial^\perp)^2 A_{ia} \\
& - \frac{1}{2}g^2 \int d^3x \mathbf{Tr}\left[\widetilde{A}^\mu, \widetilde{A}^\nu\right]\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right] \\
& + \frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi}\frac{1}{(\mathrm{i}\partial^+)^2}\overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi} \\
& - g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ \left(\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right)\widetilde{\psi} \\
& + g^2 \int d^3x \mathbf{Tr}\left(\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right) \\
& + \frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\frac{\gamma^+}{\mathrm{i}\partial^+}\widetilde{\not{A}}\widetilde{\psi} \\
& + g \int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\widetilde{\psi} \\
& + 2g \int d^3x \mathbf{Tr}\left(\mathrm{i}\partial^\mu \widetilde{A}^\nu\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right]\right)
\end{aligned}
$$

From QCD Lagrangian with $A^+ = 0$

Light-Front Hamiltonian (Model I)

$$|P_{baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$$

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

[S. Xu et al, PRD 104 094036(2021)]

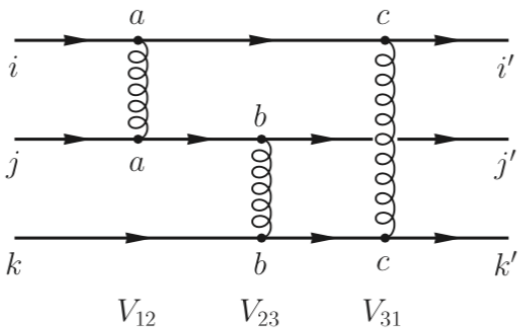
$$H_{trans} \sim \kappa_T^4 r^2$$

[S. J. Brodsky, G. de Teramond arXiv: 1203.4025]

$$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i} \left(x_i x_j \partial_{x_j} \right)$$

[Y. Li, X. Zhao , P Maris , J. P. Vary, PLB 758(2016)]

$$H_{Interact} = -\frac{C_F 4\pi\alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_\mu u_{s_j}(k_j)$$



Parameters are determined through fitting nucleon mass and EMFFs.

Nucleon Form Factor

[C. Mondal, *et al.* PRD. 102. 016008 (2020)]

- Form factors: spatial distributions of charge and magnetization in the transverse plane

$$hP^{0};\," \,|\frac{J^{+}(0)}{2P^{+}}|P;"i \;=\; F_1(q^2)$$

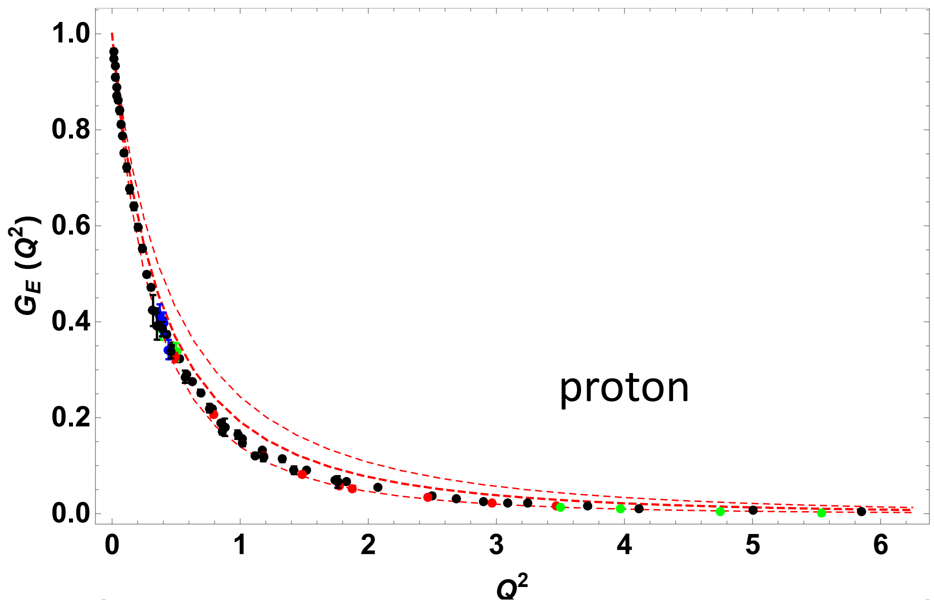
$$hP^{0};\," \,|\frac{J^{+}(0)}{2P^{+}}|P;\#i \;=\; -(q^1 - iq^2)\,\frac{F_2(q^2)}{2M}$$

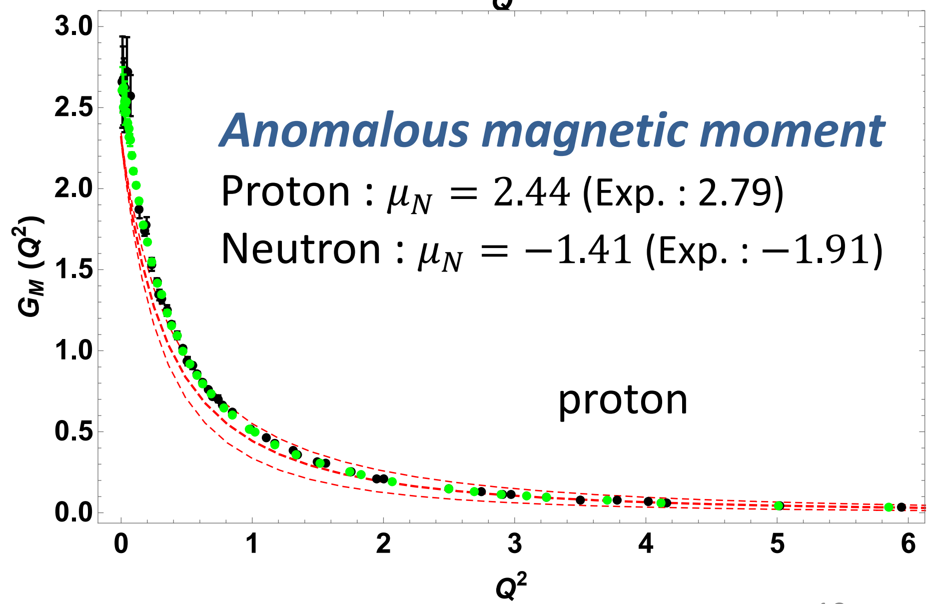
$$G_E(Q^2) = \sum_q e_q F_1^q(Q^2) - \frac{Q^2}{4M^2}\sum_q e_q F_2^q(Q^2)\,,$$

$$G_M(Q^2) = \sum_q e_q F_1^q(Q^2) + \sum_q e_q F_2^q(Q^2).$$

$m_{q/k}$	$m_{q/g}$	κ	α_s
0.3 GeV	0.2 GeV	0.34 GeV	1.1 ± 0.1

Truncation parameters: N_{max}=10 K =16.5

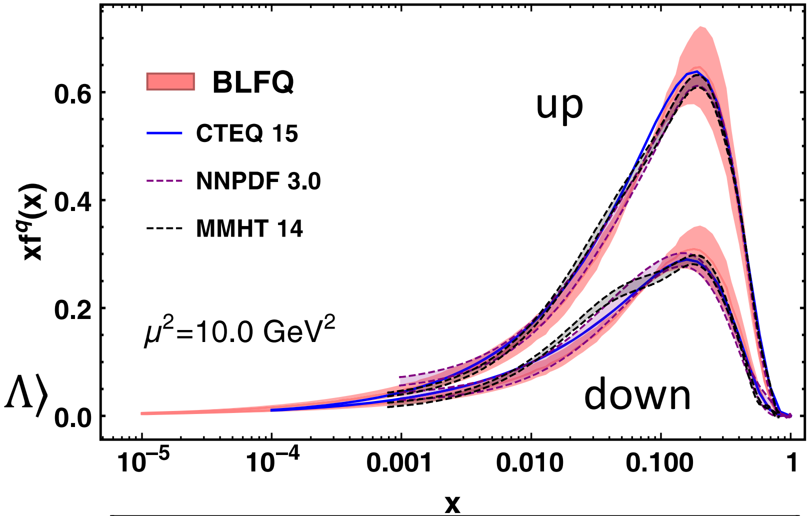




Parton Distribution Functions

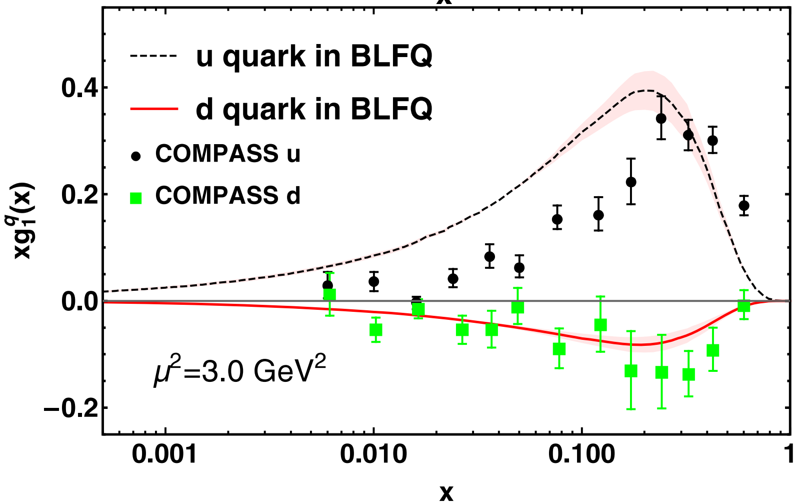
- PDFs: longitudinal distribution of partons

$$\Phi^{[\gamma^+]}(x, Q^2) = \int \frac{dz^-}{8\pi} e^{ixP^+z^-/2} \langle P, \Lambda | \bar{\psi}(x)\gamma^+\psi(0) | P, \Lambda \rangle$$



- Initial scale obtained from first moments $\mu_0^2 = 0.19 \pm 0.02\ \text{GeV}^2$

- Qualitative agreement with global fits

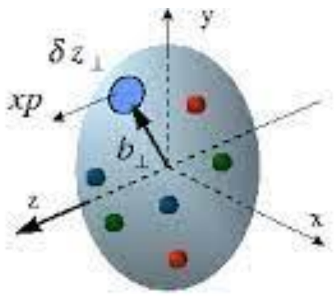


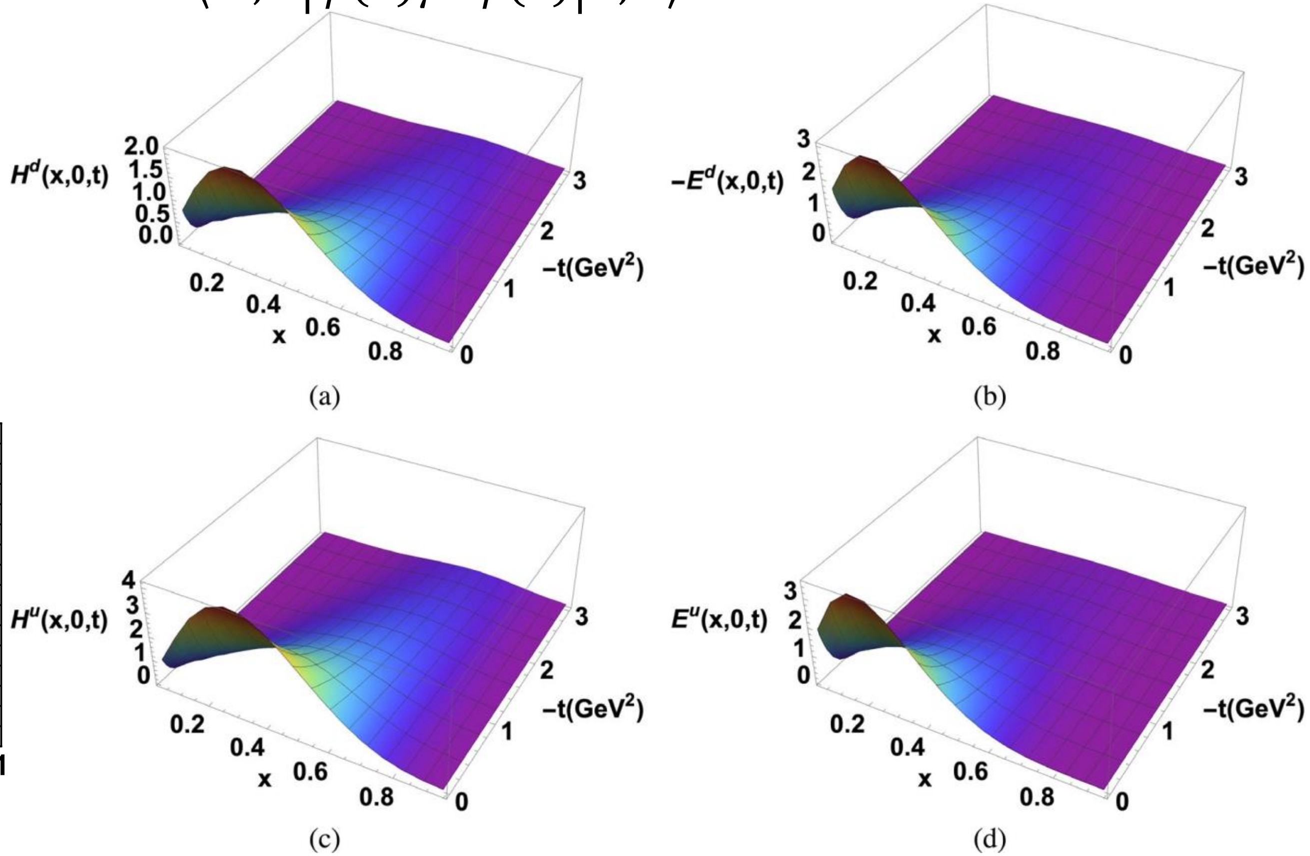
[S. Xu et al, PRD 104, 094036 (2021)]

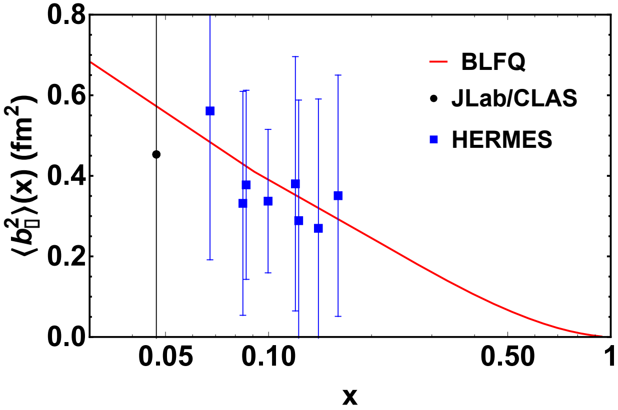
Generalized Parton Distribution Functions (GPD)

- GPD: 3D distribution of partons in coordinate space

$$\Phi^{[\gamma^+]}(x,Q^2) = \int \frac{dz^-}{8\pi}\, e^{ixP^+z^-/2}\langle P',\Lambda\big|\bar{\psi}(x)\gamma^+\psi(0)\big|P,\Lambda\rangle$$







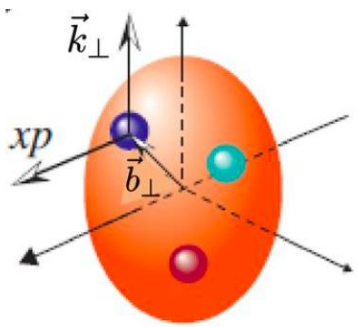
- x-dependent radius qualitatively agree with experimental data

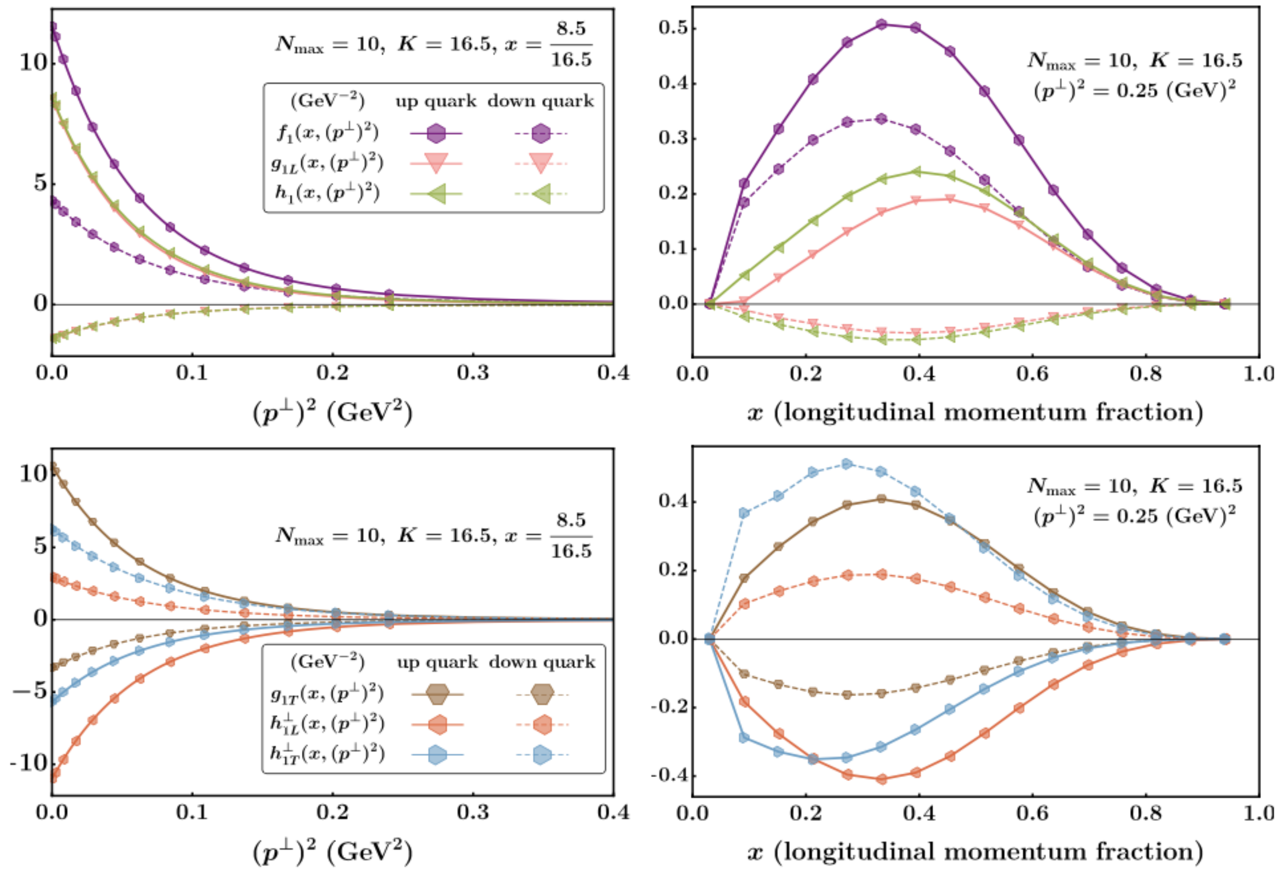
[S. Xu et al, PRD 104, 094036 (2021)]

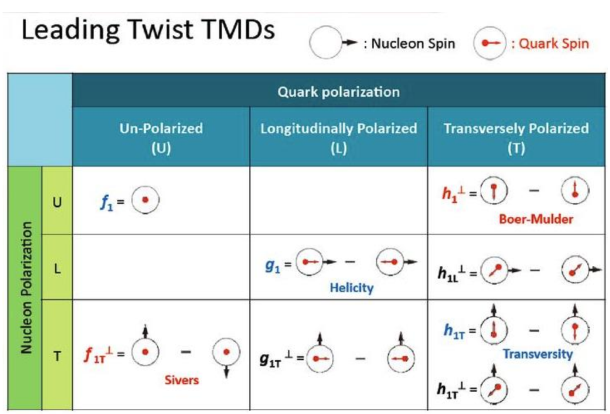
Transverse Moment Dependent Distributions (TMD)

- TMD: 3D distribution of partons in momentum space

$$\Phi^{[\Gamma]}(P,S,S';x=\frac{p^+}{P^+},p^\perp)=\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}z^\perp}{2(2\pi)^3N_0}e^{ip\cdot z}\,\langle P,S'|\,\bar{\Psi}(0)\Gamma\Psi(z)\,|P,S\rangle\,|_{z^+=0}$$







- Six leading twist T-even TMDs at initial scale ($\mu_i^2=\zeta_i\ =0.195\ \mathrm{GeV}^2$)

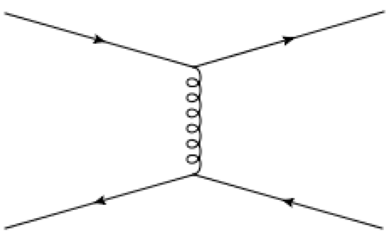
[Z. Hu et al, PLB 833, 137360 (2022)]

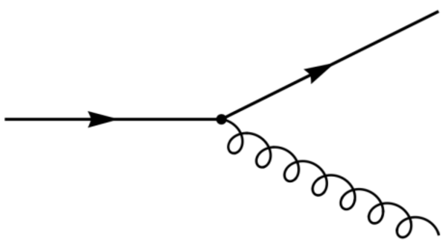
[S. Xu, C. Mondal, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph]]

Light-Front Hamiltonian (Model II)

$$|P_{baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$$

- $H_{Interact} \rightarrow H_{Interact} = H_{Vertex} + H_{inst}$





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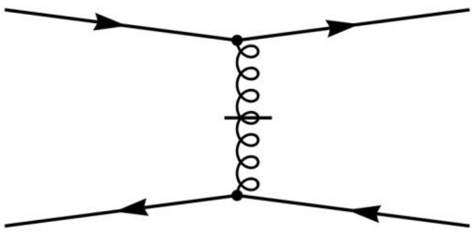


$N_{\max} = 9, K = 16.5$

Confining interaction

Parameters are determined through fitting nucleon mass and EMFFs.

m_u	m_d	κ	m_g	m_{int}	b_{inst}	b	g
0.32 GeV	0.25 GeV	0.54 GeV	0.50 GeV	1.80 GeV	3.00 GeV	0.70 GeV	2.40

Different Mass Asymmetry of u and d

UV Cutoff In Instantaneous term

Light-Front Hamiltonian (Model I)

$$|P_{baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$$

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

[S. Xu et al, PRD 104 094036(2021)]

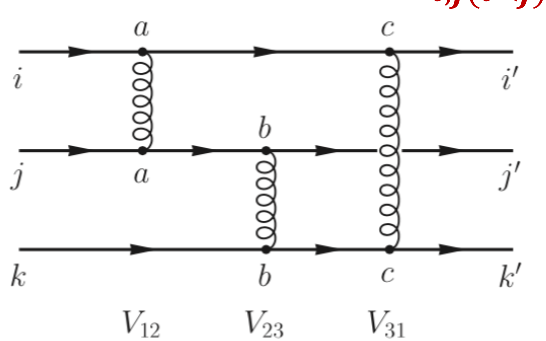
$$H_{trans} \sim \kappa_T^4 r^2$$

[S. J. Brodsky, G. de Teramond arXiv: 1203.4025]

$$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i} \left(x_i x_j \partial_{x_j} \right)$$

[Y. Li, X. Zhao , P Maris , J. P. Vary, PLB 758(2016)]

$$H_{Interact} = -\frac{C_F 4\pi\alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_\mu u_{s_j}(k_j)$$



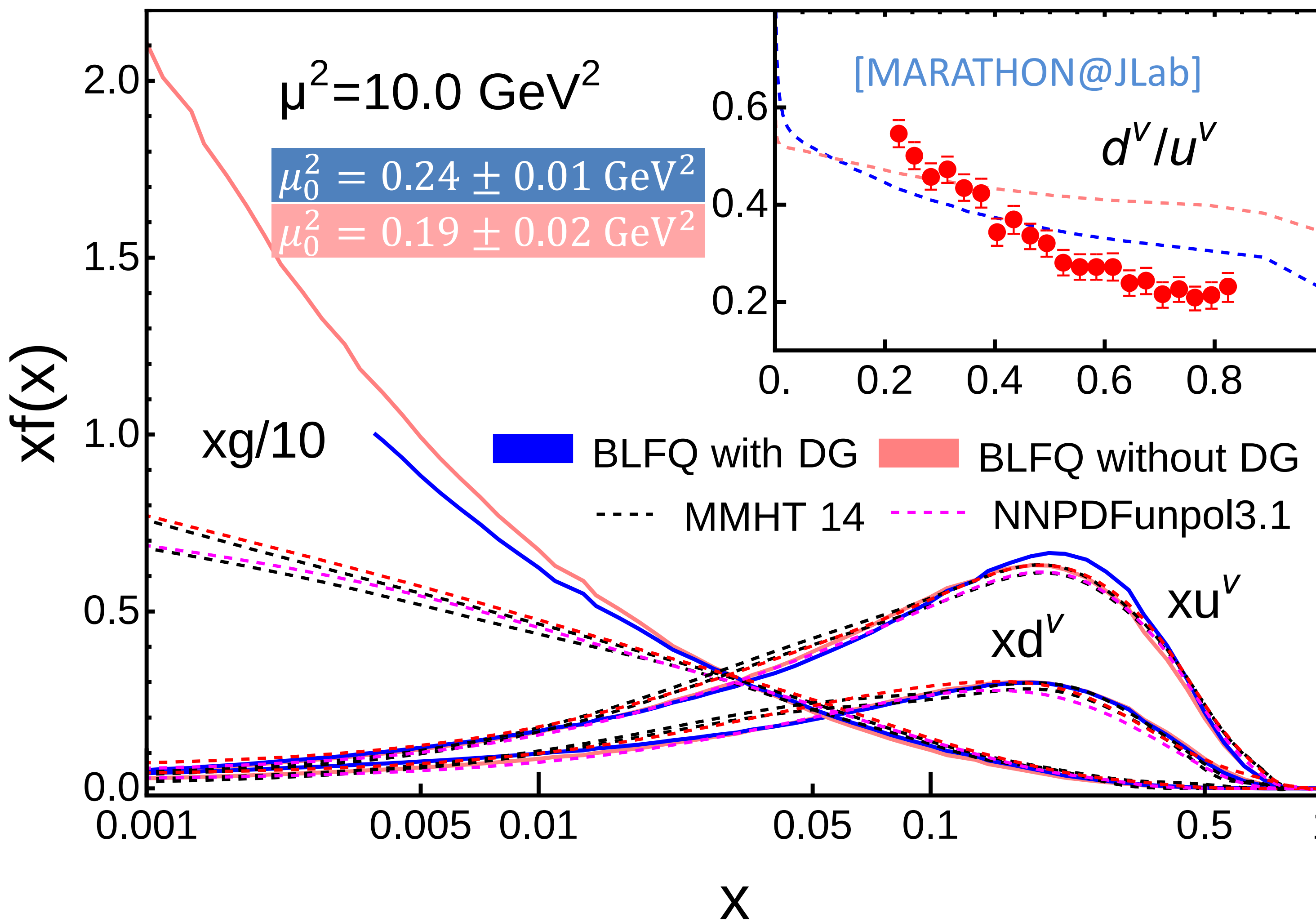
Parameters are determined through fitting nucleon mass and EMFFs.

Connection with Light-front QCD Hamiltonian

$$
\begin{aligned}
P^{-}_{LFQCD} = & \frac{1}{2}\int d^3x \overline{\widetilde{\psi}}\gamma^+ \frac{(i\partial^\perp)^2+m^2}{i\partial^+}\widetilde{\psi} - A^i_a (i\partial^\perp)^2 A_{ia} \\
& - \frac{1}{2}g^2 \int d^3x \, \mathrm{Tr}\left[\widetilde{A}^\mu, \widetilde{A}^\nu\right]\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right] \\
& + \frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi} \frac{1}{(i\partial^+)^2} \overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi} \\
& - g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ \left(\frac{1}{(i\partial^+)^2}\left[i\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right)\widetilde{\psi} \\
& + g^2 \int d^3x \, \mathrm{Tr}\left(\left[i\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\frac{1}{(i\partial^+)^2}\left[i\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right) \\
& + \frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}} \widetilde{\not{A}} \frac{\gamma^+}{i\partial^+} \widetilde{\not{A}} \widetilde{\psi} \\
& + g \int d^3x \overline{\widetilde{\psi}} \widetilde{\not{A}} \widetilde{\psi} \\
& + 2g \int d^3x \, \mathrm{Tr}\left(i\partial^\mu \widetilde{A}^\nu \left[\widetilde{A}_\mu, \widetilde{A}_\nu\right]\right)
\end{aligned}
$$

First-principles interactions are included

Unpolarized Parton Distribution Functions



- Initial scale increases with the inclusion of dynamical gluon
- Overall results improve with the inclusion of dynamical gluon

[S. Xu, C. Mondal, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph]]

Angular Momentum Distributions

- Proton spin decomposition

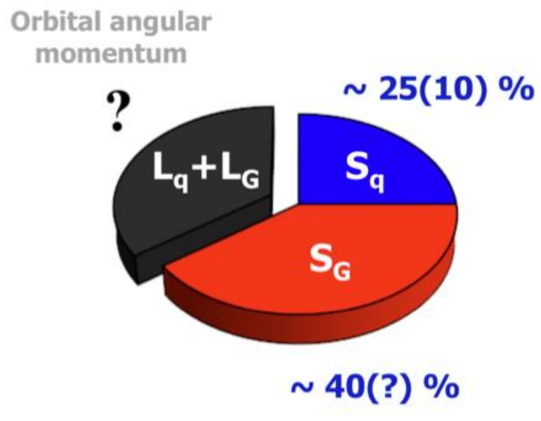
[Jaffe-Manohar 90']

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

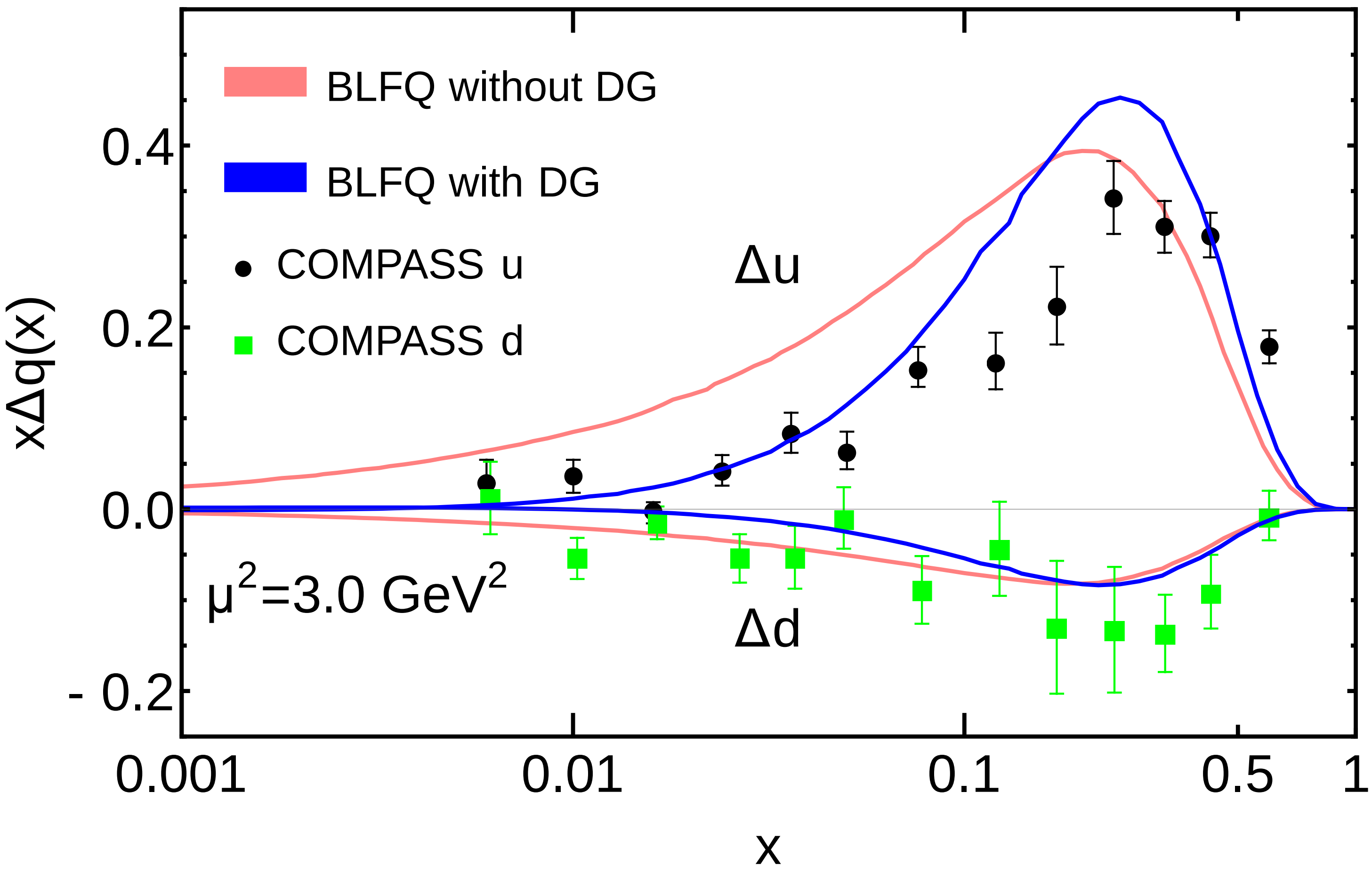
Quark spin, obtained from Δq, in quark model ΔΣ=1

Gluon spin, obtained from Δg

Quark/gluon orbital angular momentum, obtained from GTMD $F_{1,4}$



Helicity Parton Distribution Functions

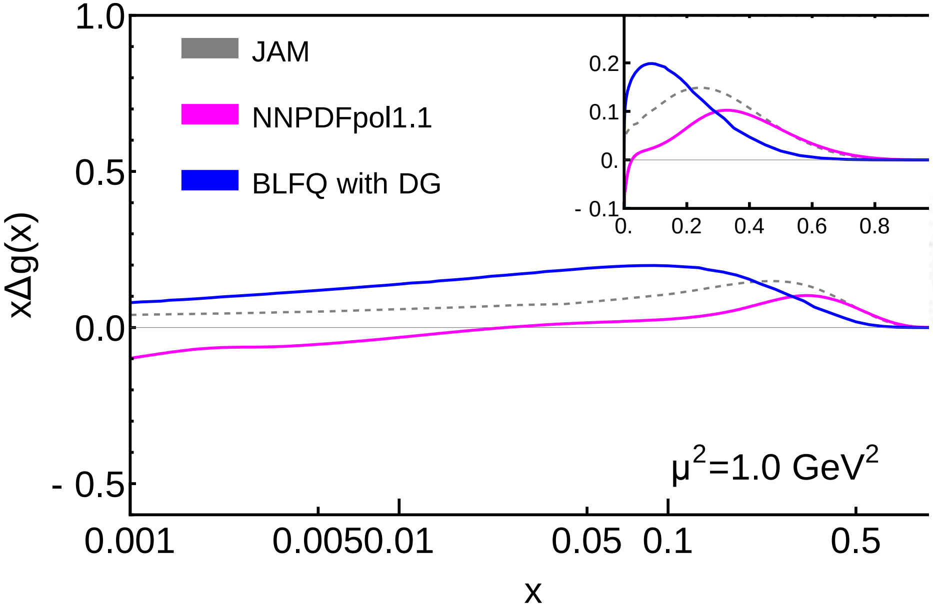


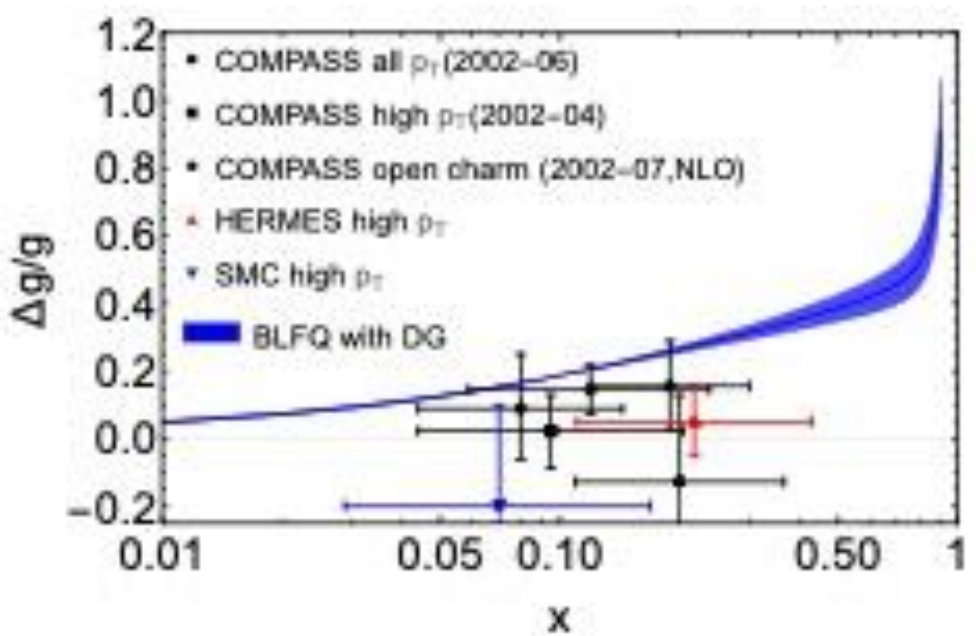
- $\Delta\Sigma_q \approx 0.7 \quad \Delta\Sigma_u \approx 0.86 \quad \Delta\Sigma_d \approx -0.16$
- Valence quark distributions at x<0.1 and x>0.5 regions show improvement with DG

[S. Xu, C. Mondal, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph]]

Helicity Parton Distribution Functions

N. Sato et al. [JAM], PRD93 (2016)
E. R. Nocera et al. [NNPDF], NPB 887 (2014)

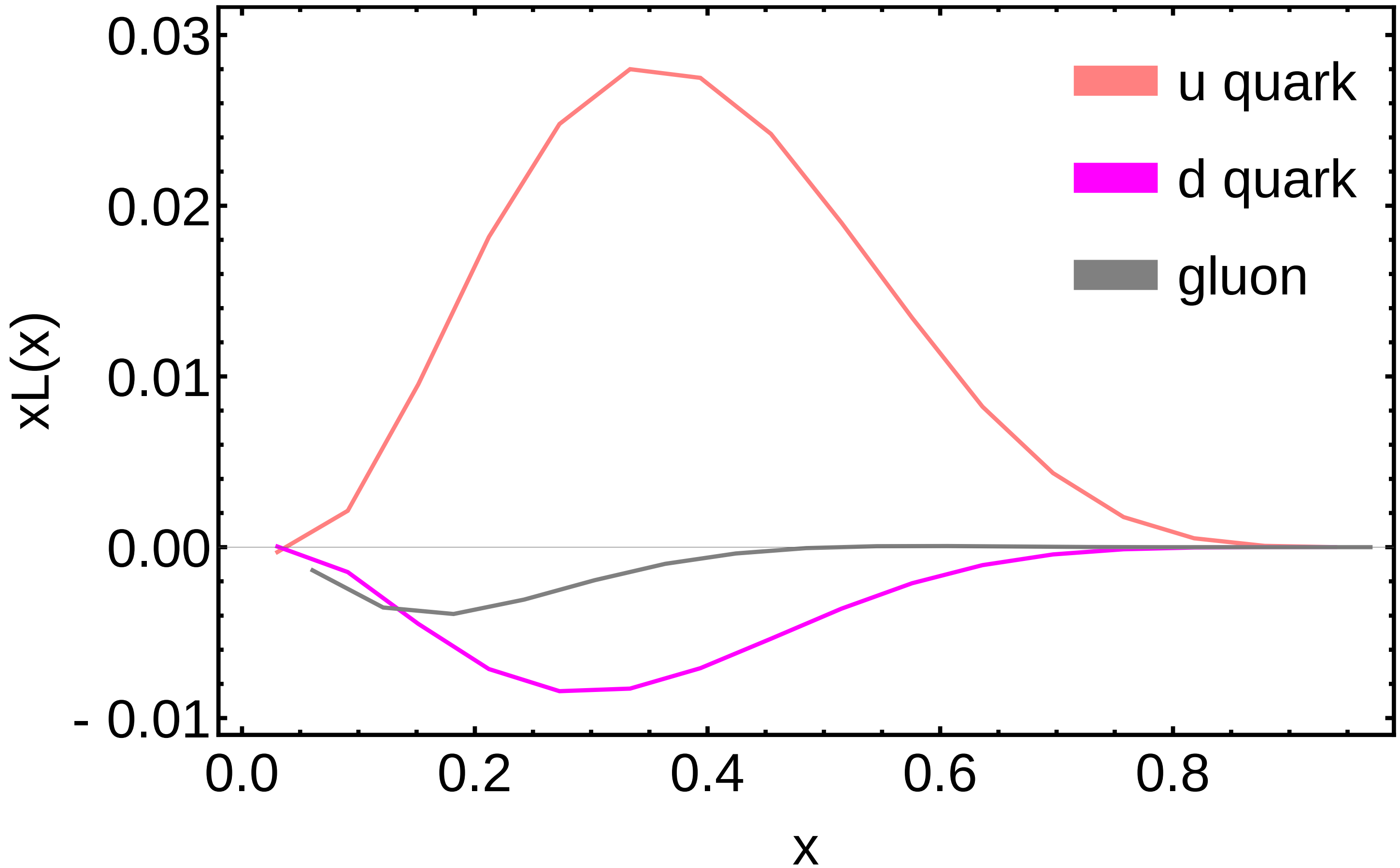




[S. Xu, C. Mondal, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph]]

- Δg is positive over the entire x range, qualitatively agreeing with global fits
- $\Delta g/g$ increases with x, measurements of $\Delta g/g$ at EICs are promising
- $\Delta G = \int_0^1 \Delta g(x) = 0.131 \pm 0.003$, comparable with $\Delta G^{[0.002,0.3]} = 0.2 \pm 0.1$ from PHENIX collaboration [PRL 103 012003 (2009)]

Orbital angular momentum distributions



- $L_d = -0.0114 \pm 0.0004$ $L_u = 0.0327 \pm 0.0013$ $L_g = -0.0065 \pm 0.0005$
- From generalized transverse momentum-dependent parton distribution functions $F_{1,4}$

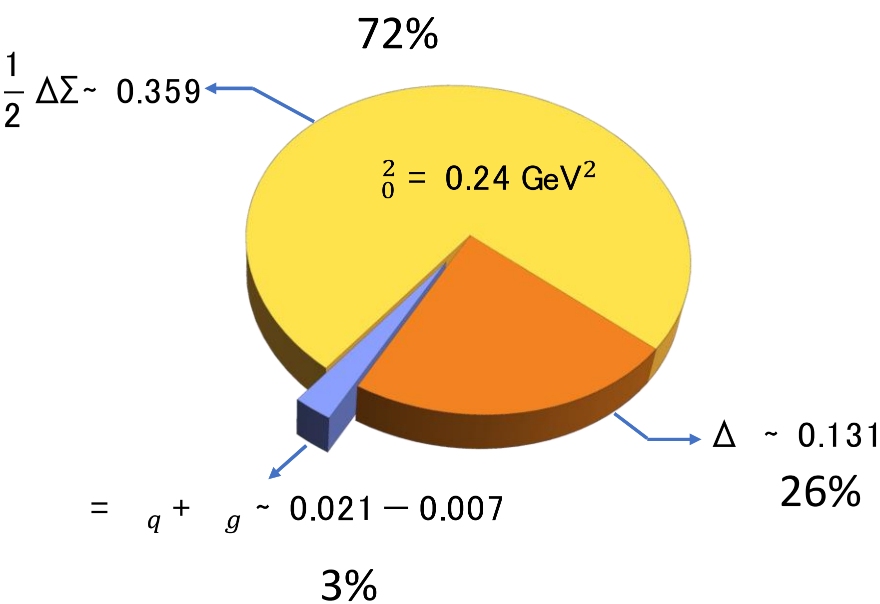
Proton Spin Decomposition

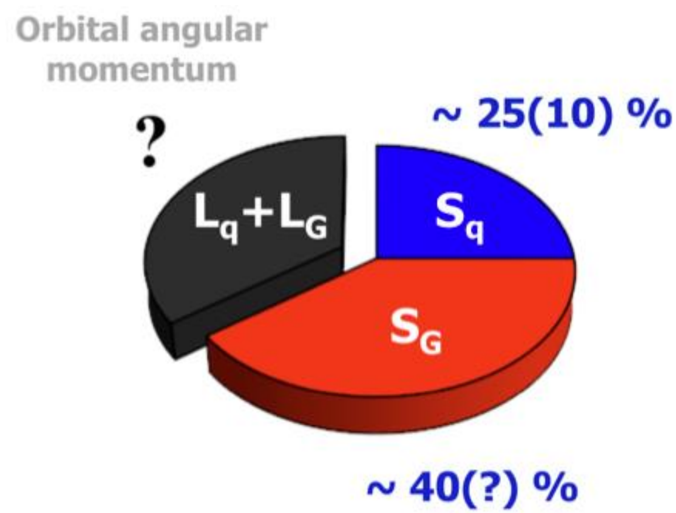
- Fock Sector Expansion

$$|\text{proton}\rangle = |qqq\rangle + |qqq\ g\rangle + |qqq\ q\bar{q}\rangle + |qqq\ gg\rangle + |qqq\ q\bar{q}\ g\rangle + \cdots$$

44% 56%

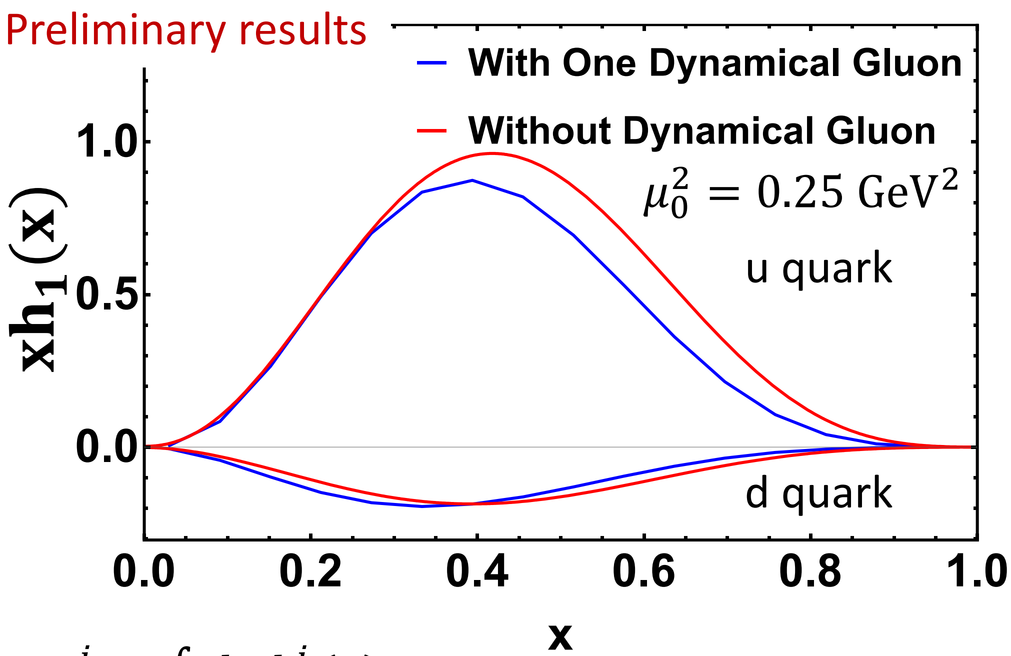
Jaffe-Manohar decomposition: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$





Transversity Parton Distribution Functions

[In preparation, Siqi Xu, C. Mondal *et.al*]



- Tensor charge $g_T^i = \int dx\, h_1^i(x)$

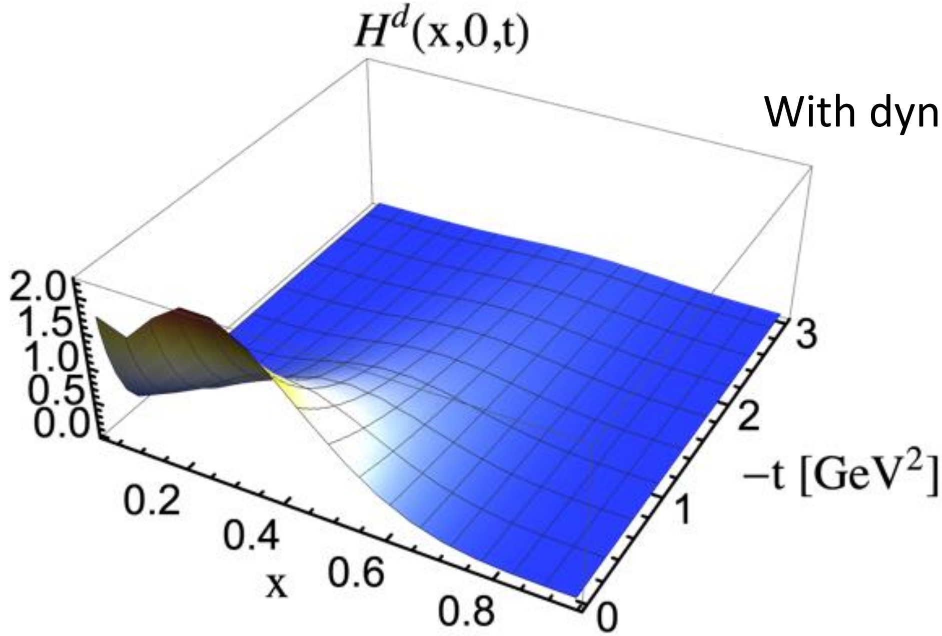
Tensor charge	No dynamical gluon	Dynamical gluon	Extracted from exp. data
g_T^u	0.94	0.55	$0.39^{+0.18}_{-0.12}$
g_T^d	-0.20	-0.29	$-0.25^{+0.30}_{-0.10}$

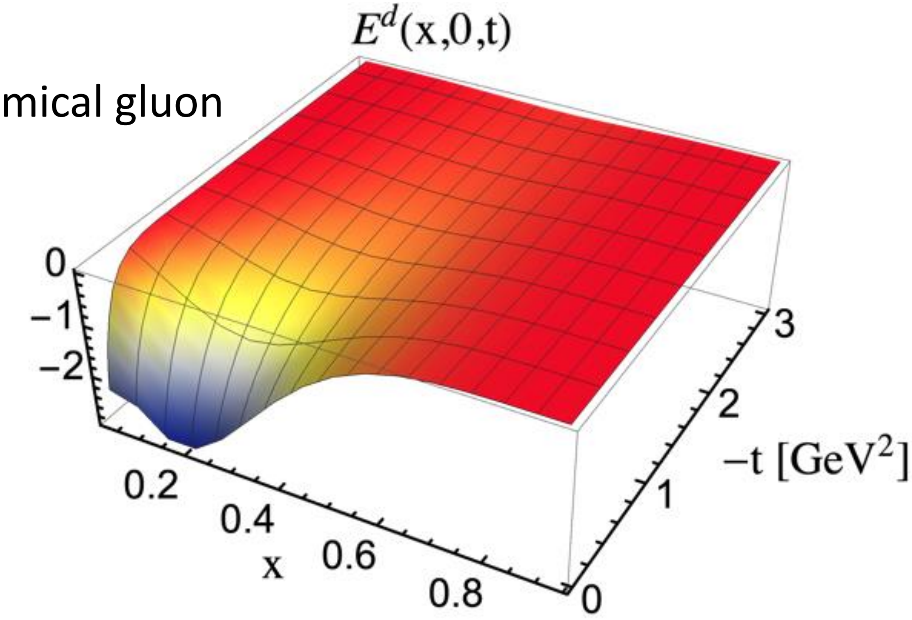
[Phys. Rev. D87, 094019(2013)]

- Tensor charge of up quark with DG show improved agreement with the extracted data

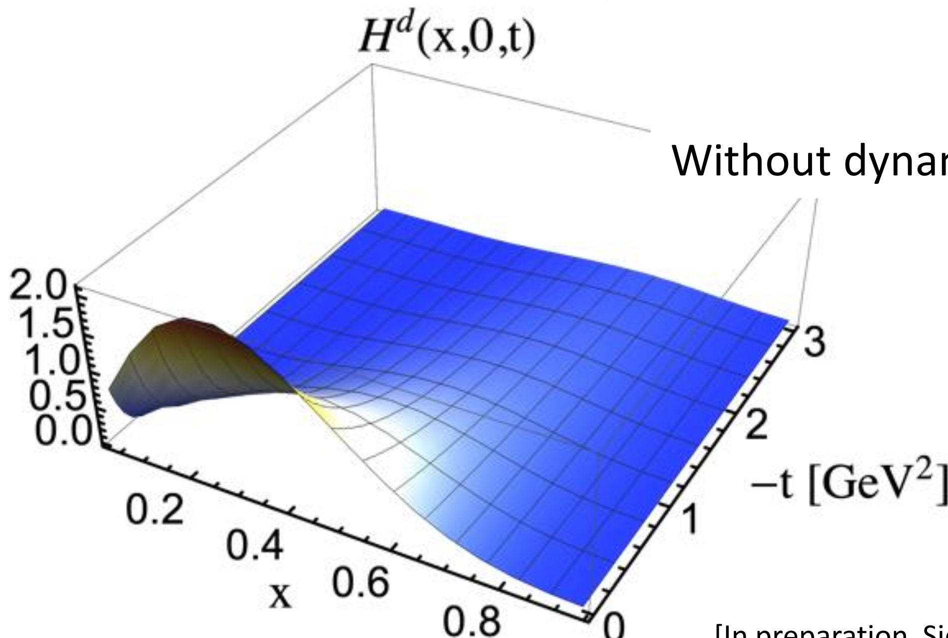
Generalized Parton Distribution Functions (GPD)

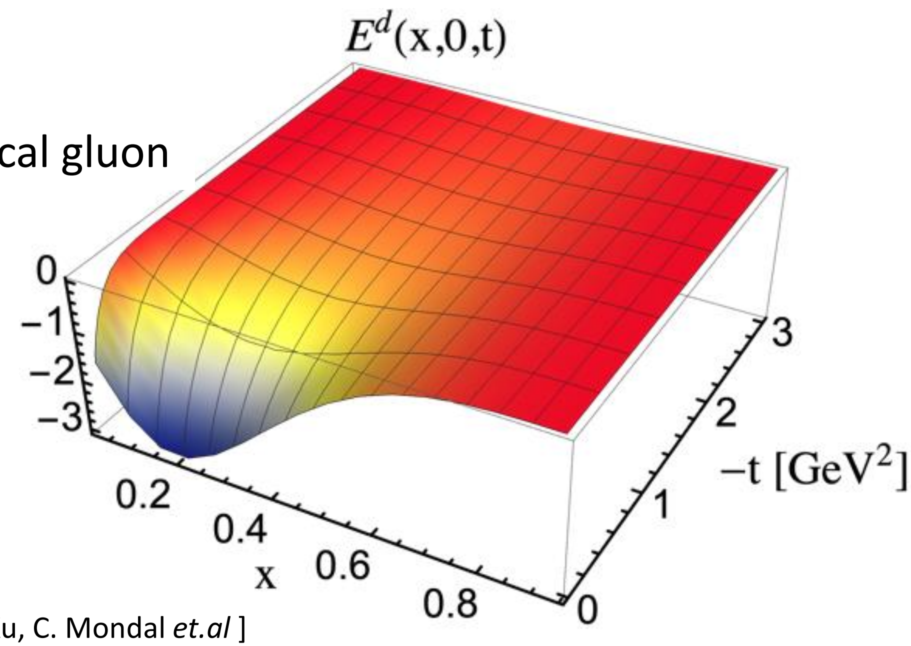
With dynamical gluon





Without dynamical gluon

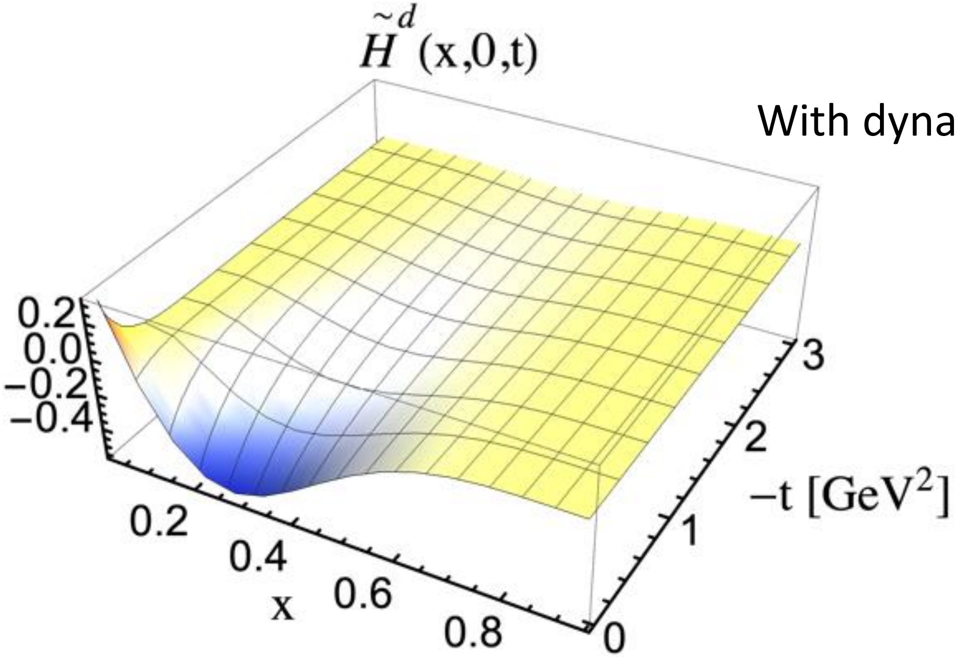


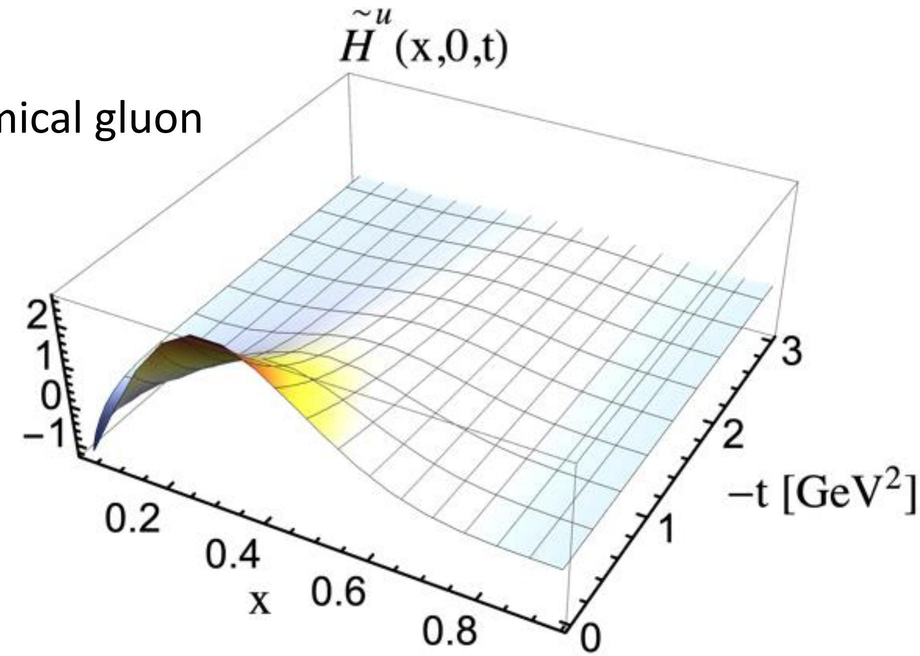


[In preparation, Siqi Xu, C. Mondal *et.al*]

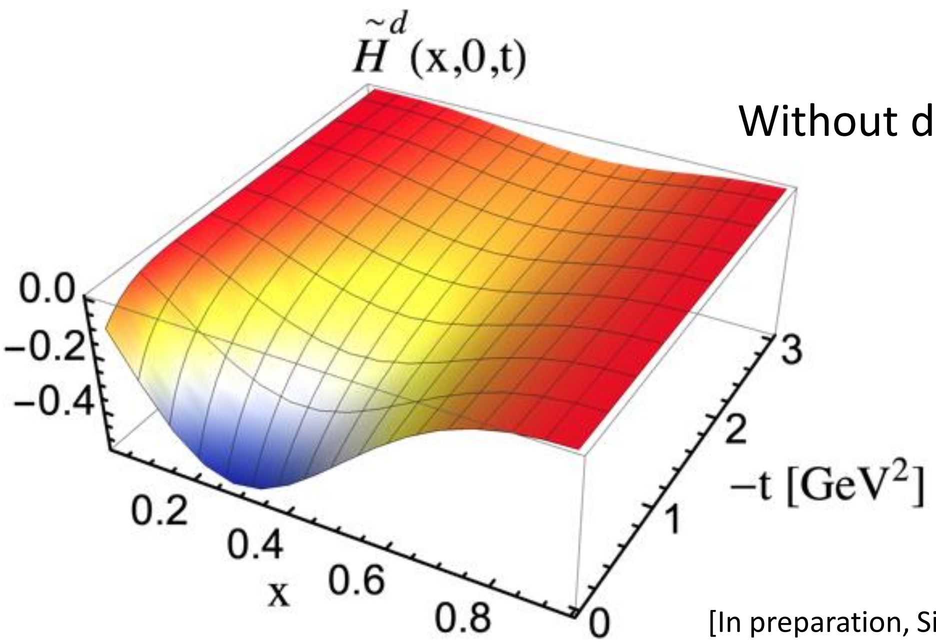
Generalized Parton Distribution Functions (GPD)

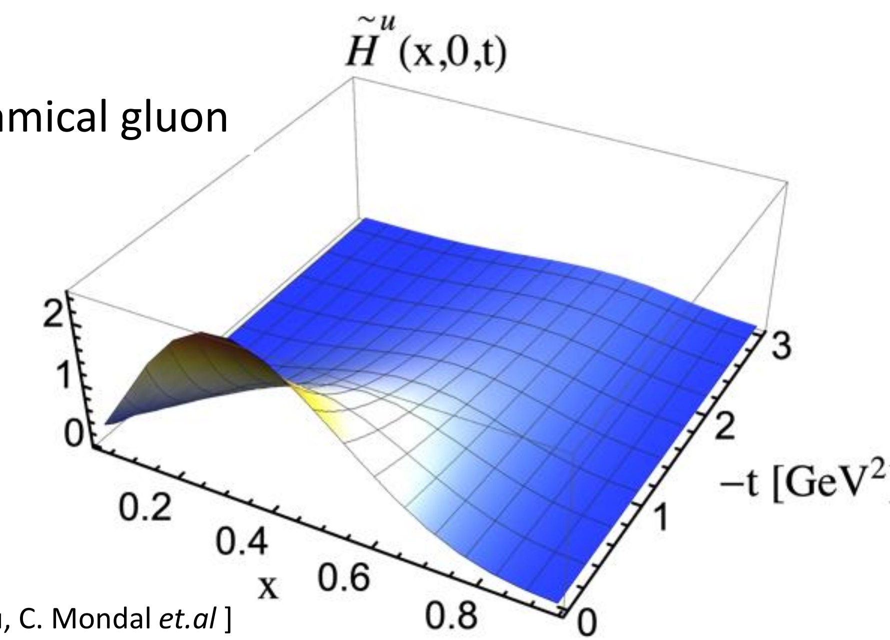
With dynamical gluon





Without dynamical gluon

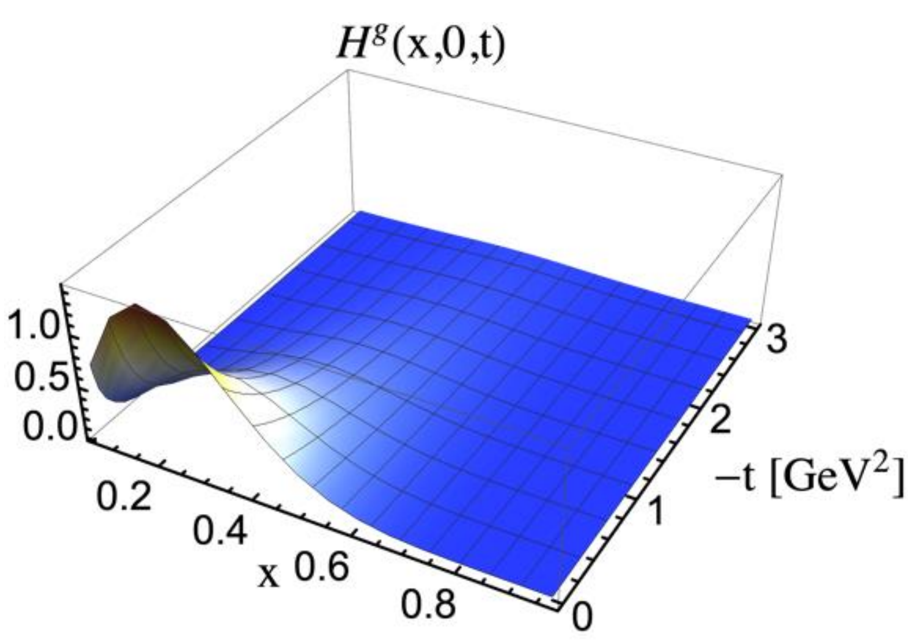


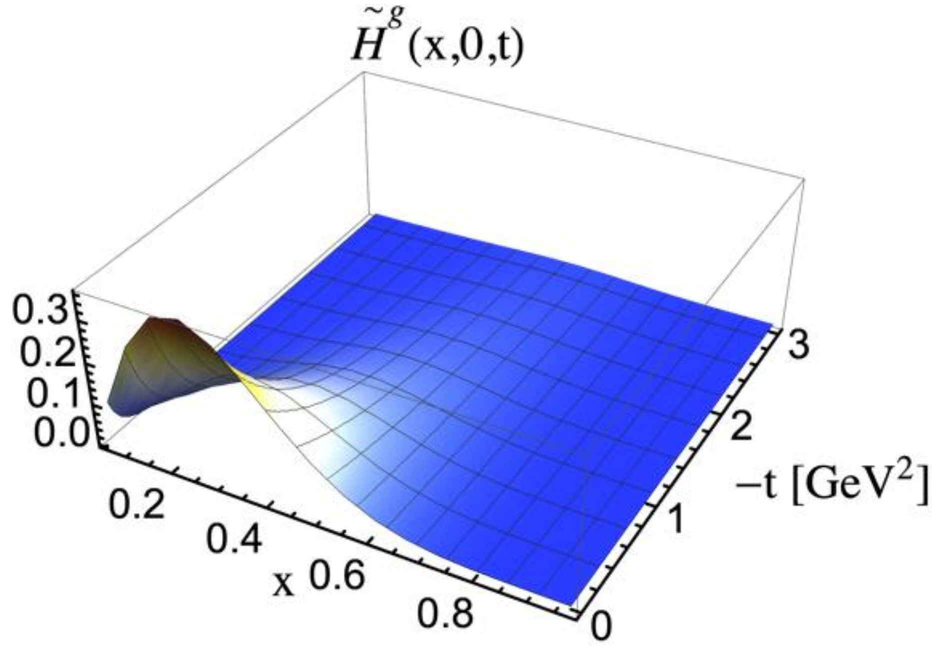


[In preparation, Siqi Xu, C. Mondal *et.al*]

Generalized Parton Distribution Functions (GPD)

➢ Generalized Parton Distribution Functions For Gluon





- Gluon GPD at the initial scale $\mu_0^2 = 0.25\ \mathrm{GeV}^2$

[In preparation, Siqi Xu, C. Mondal *et.al*]

Conclusion

- Light-front Hamiltonian approach
  → 3D image of nucleon with rich details
  - Coordinate and momentum (5D) space
  - Spin degrees of freedom
  - Correlation among partons
- Systematically expandable
  - Sea quarks → multiple gluons → first principles
  - Baryons/mesons/exotic hadrons
  - Light nuclei
- Next generation high performance computers bring immense possibilities

Thank you!