

# Heavy-light hadrons: old laces and new pieces

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”Solomon saith: There is no new thing upon the earth.  
So that as Plato had an imagination,  
that all knowledge was but remembrance;  
so Solomon giveth his sentence, that  
**all novelty is but oblivion**”

Francis Bacon: Essays, LVIII

- Jorge Luis Borges, *The Aleph and Other Stories*

- **Old Laces** - EFT (see e.g. MAN, Rho, Zahed, *Nuclear Chiral Dynamics*, WS 1995)
- **Two Revolutions** ([ex] and [th])
- **New Pieces** - Holographic approach to Exotic QCD  
Liu, Zahed; PRD95 (2017)056022; PRD95(2017) 116012.  
Liu, MAN, Zahed; PRD100 (2019)126023, PRD105 (2022)054021. (**tetraquarks**)  
Liu, MAN, Zahed; PRD104(2021)114021; PRD104(2021) 114022. (**pentaquarks: masses; decays**)  
Liu, Mamo, MAN, Zahed, PRD104(2021) 114023. (**Photoproduction of pentaquarks**)  
Liu, MAN, Zahed, PRD 105 (2022) 114021. (**Hyperons**)

# Light Baryons

- Inspired by large  $N$ , we treat light baryon as **Skyrme soliton**. Quantization of collective modes (moduli space) from symmetries leads to

$$H_1 = \frac{\vec{I}^2}{2\Omega_{sol}} = \frac{\vec{J}^2}{2\Omega_{sol}}$$

where  $\vec{I}(\vec{J})$  are isospin (angular momentum) respectively.

- Already at the level of strange quarks symmetry breaking is so bad, so collective method fails.
- Proper way is to use Born-Oppenheimer approximation - fast vibration of the kaon in the  $SU(2)$  solitonic background. Then slow rotation of the bound state happens in the presence of non-Abelian Berry phase originating from kaon.

$$H = H_0 + \text{binding} + \frac{(\vec{J} - (1 - c_k) \text{tr}(K\vec{I}K^\dagger))^2}{2\Omega_{sol}}$$

([isospin-spin transmutation in Callan-Klebanov 1985])

- For  $c(b)$  baryons, similar picture, but with **two** Berry phases from  $D$  and  $D^*$ , so one gets

$$H_1 = \frac{[(\vec{J} - \vec{S}_H) - (1 - c_D)tr(D\vec{I}D^\dagger) - (1 - c_{D^*})tr(D^*\vec{I}D^{*\dagger})]^2}{2\Omega_{sol}}$$

- In the infinitely heavy mass Berry phases *exactly* cancel  
Then

$$H_1 = \frac{(\vec{J} - \vec{S}_H)^2}{2\Omega_{sol}} = \frac{\vec{I}^2}{2\Omega_{sol}}$$

(Realization of **Isgur-Wise symmetry** at the baryonic level)

- Also, soliton can capture more than one meson (double heavy baryons, exotica).

# Heavy-Light Effective Action Revisited

Combining chiral symmetry with heavy-spin symmetry leads to novel feature [MAN, Rho, Zahed (1992), Bardeen-Hill (1993)]

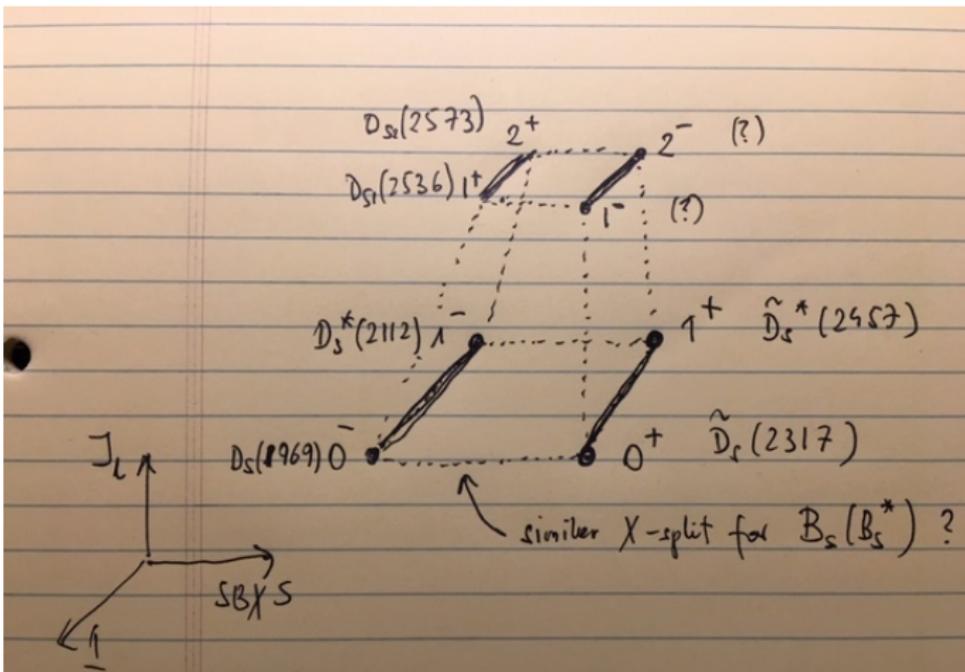
Both symmetries enforce the presence of opposite parity ( $0^+, 1^+$ )

multiplet  $G = \frac{1+\not{v}}{2}(\tilde{D} + \gamma^\mu \gamma_5 \tilde{D}_\mu^*)$

in addition to standard ( $0^-, 1^-$ ) one  $H = \frac{1+\not{v}}{2}(\gamma_5 D + \gamma^\mu D_\mu^*)$

- Consequence of chiral symmetry  $[\not{v}, \gamma_5]_+ = 0$
- Doublers communicate only through axial current
- Physical split in axial couplings and masses,  
 $m_G - m_H \sim O(\Sigma_I) \sim 350 \text{ MeV}$
- Chiral doublers do not double the number of states in quark model, but *reorganize* them in different way.
- Similar construction for excited mesons  $D_1, D_2$  i.e. light spin  $3/2$ , but the effects of chiral split are less pronounced [MAN, Zahed (1993)]

# Visualization of the origin of doubling scenario based on BaBar data



# Several possibilities

- (i) soliton captures  $H$  meson (heavy baryon)
- (ii) soliton captures  $G$  meson (doubler of heavy baryon)
- (iii) soliton captures  $\bar{H}$  (heavy pentaquark)
- (iv) soliton captures  $\bar{G}$  (doubler of heavy pentaquark)
- (v) soliton captures more mesons.....

Model-dependent parameters

# Meson-Baryon HL "supersymmetry"

- Two identical infinitely heavy quarks  $QQ$  "sit on each other", forming diquark configuration with color  $\bar{3}$ , therefore colorwise, nondistinguishable from heavy antiquark  $\bar{Q}$ . This quark-diquark symmetry relates spin-splittings for heavy-light mesons  $\bar{H}L$  to those of doubly heavy baryons  $HHL$
- "Supersymmetric" mass relation  $\Delta M_{HHL} = \frac{3}{4} \Delta M_{\bar{H}L}$   
[Savage Wise (1990)]
- Iterating  $HH \leftrightarrow \bar{H}$  duality leads to further dualities e.g.  $HLL \leftrightarrow \bar{H}\bar{H}LL$  (heavy baryon  $\leftrightarrow$  doubly heavy tetraquark).

# Experimental revolution: Post-Babar-ian era

## Abundance of exotic heavy-light particles

- $\bar{c}du\bar{s}$  :  $X(2866)$ ,  $X_1(2904)$
- $\bar{c}c\bar{q}q$  :  $\chi_{c1}(3872)$
- $\bar{c}c\bar{u}d$ :  $Z_c(3900)$ ,  $Z_c(4020)$ ,  $Z_c(4050)$ ,  $X(4100)$ ,  $Z_c(3985)$ ,  $Z_c(4430)$ ,  $R_{c0}(4240)$
- $\bar{c}c\bar{u}s$ :  $Z_{cs}(3985)$ ,  $Z_{cs}(4000)$ ,  $Z_{cs}(4220)$
- $\bar{b}b\bar{u}d$ :  $Z_b(10610)$ ,  $Z_b(10650)$
- $\bar{c}c\bar{c}c$ :  $X(6900)$
- $\bar{c}c\bar{u}d$ :  $T_{cc}^+(3875)$ , also  $T_{cc}^0$ ,  $T_{cc}^{++}$  (preliminary)
- Pentaquarks  $\bar{c}c\bar{u}ud$ :  $P_c((4380), (4450))$   
 $\rightarrow [(4440), (4457)], (4312)]$ ,  $P_c(4337)$  ( $3\sigma$ ) significance
- $\bar{c}c\bar{u}ds$ :  $P_{cs}(4459)$
- possibilities of further **heavy-light "chemistry"**: **many more expected (?)**

Input from lattice

# Theoretical revolution: Gravity/Gauge duality (holography, AdS/CFT)

- In the 70' QCD became **fundamental theory of quarks and gluons**, strings (flux tubes) appear as **effective**, e.g. Lund model
- Maldacena pointed that gauge theory in 4 dim is equivalent to string theory in higher dimensions: **Two fundamental theories of strong interactions (!)**
- Various versions - conformal window, lower-dimensions (solid state physics)
- Witten (1998) applied duality to QCD: pure YM in 3 + 1 at large N and  $\lambda = g_{YM}^2 N$   
Surprising similarity to spectrum of glueballs at large N lattice

- Adding  $N_f$  massless fermions - geometric  $SB\chi S$
- Low energy limit  $S = S_{YM} + S_{CS}$  where  
$$S_{YM} \sim \int d^4x dz \text{Tr} \left( \frac{1}{2} k(z)^{-1/3} F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right)$$
 where  
$$k(z) = 1 + z^2$$
- Mode expansion:  $A_\mu(x^\mu, z) = \sum_n B_\mu^{(n)}(x_\mu) \Psi_n(z)$ ,  
 $A_5(x^\mu, z) = \sum_n \phi^{(n)}(x^\mu) \Phi_n(z)$
- Keeping only  $\phi^{(0)}$  give  $L = L_\sigma + L_{Skyrme} + L_{WZ}$ . Adding  
 $B_\mu^{(1)} \sim \rho$  and  $B_\mu^{(2)} \sim a_1$  give hidden gauge model
- **Successful phenomenology with very few parameters**

# Baryon in Sakai-Sugimoto scenario

- 4 dim pion - Skyrminion (static solution)
- 5 dim gauge field - **BPST instanton in  $x_1, x_2, x_3, z$  in flavor**
- Topological number  $\equiv$  baryon number
- Direct realization of 1989 Atiyah-Manton idea
- 8 zero modes lead to moduli space quantization

$$U(\vec{x}) = P \exp(i \int dz A_z(\vec{x}, z))$$

$$M = M_0 + \left( \sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N^2} + \sqrt{\frac{2}{3}}(n_\rho + n_z + 1) \right) M_{KK}$$

where  $l = 2I = 2J = 1, 3, 5 \dots$

- CK-like scheme with heavy-spin symmetry
- $M = M_0 + (N_Q + N_{\bar{Q}})m_H +$   
 $(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15}N^2(1 - \frac{15(N_Q - N_{\bar{Q}})}{4N} + \frac{5(N_Q - N_{\bar{Q}})^2}{3N^2})^2})M_{KK} +$   
 $\sqrt{\frac{2}{3}}(n_\rho + n_z + 1))M_{KK}$
- Various combinations of q-numbers give all types of HL hadrons
  - $N_Q = 1, N_{\bar{Q}} = 0$  yield HLL
  - $N_Q = 2, N_{\bar{Q}} = 0$  yield HHL
  - $N_Q = N_{\bar{Q}} = 1$  yield pentaquarks  $H\bar{H}LLL$
  - $n_z \neq 0$  yield excited (Roper-like),  $n_\rho \neq 0$  yield odd parity
- **Three parameters:**  $M_{KK}, M_0, m_H \sim M_D(M_B)$

# Adding spin effect (subleading in $m_H^{-1}$ ) Liu, MAN, Zahed, 2021

- 3 parameters,  $M_0 \rightarrow m_N$ ,  $M_{KK} \rightarrow m_{\Lambda_c}$ ,  $m_H \sim M_D(M_B)$  for  $c(b)$
- 3 pentaquarks  $\frac{1}{2}, \frac{1}{2}^- (S = 1)$ ,  $\frac{1}{2}, \frac{1}{2}^- (S = 0)$ ,  $\frac{1}{2}, \frac{3}{2}^- (S = 1)$ ,  $IJ^\pi$  ( $\frac{1}{2}, \frac{5}{2}^\pm$  ruled out), consistent with  $P_c(4312, 4440, 4457)$  [LHCb]
- Recently reported  $P_c(4337)$  at  $3\sigma$  significance is not supported
- Open and hidden decay widths (Liu, MAN, Zahed, 2021)  
e.g.  $P_c \rightarrow \Lambda_c + \bar{D}$ ,  
 $\Gamma(S = 0, J = \frac{1}{2}) : \Gamma(S = 1, J = \frac{1}{2}) : \Gamma(S = 1, J = \frac{3}{2}) = \frac{1}{2} : \frac{5}{6} : \frac{1}{3}$
- Formfactors (Liu, Mamo, MAN, Zahed; 2021), consistent with recent GLUEX results on  $\gamma p \rightarrow (P_c^+) \rightarrow J/\psi p$

# Charmed baryons and Pentaquarks

TABLE I. Charm baryons and Pentaquarks

| $B$           | $IJ^P$   | $l$ | $n_\rho$ | $n_z$ | $N_Q$ | $N_{\bar{Q}}$ | Mass-MeV         | Exp-MeV          |
|---------------|--|-----|----------|-------|-------|---------------|------------------|------------------|
| $\Lambda_c$   | $0\frac{1}{2}^+$                                     | 0   | 0        | 0     | 1     | 0             | 2286             | 2286             |
| $\Sigma_c$    | $1\frac{1}{2}^+$                                     | 2   | 0        | 0     | 1     | 0             | 2557             | 2453             |
|               | $1\frac{3}{2}^+$                                     | 2   | 0        | 0     | 1     | 0             | 2596             | 2520             |
| $\Lambda_c^*$ | $0\frac{1}{2}^-$                                     | 0   | 0        | 1     | 1     | 0             | 2683             | 2595             |
|               | $0\frac{1}{2}^+$                                     | 0   | 1        | 0     | 1     | 0             | 2726             | 2765             |
| $\Sigma_c^*$  | $1\frac{1}{2}^-, 1\frac{3}{2}^-$                     | 2   | 0        | 1     | 1     | 0             | [2947/2986]      | –                |
|               | $1\frac{1}{2}^+, 1\frac{3}{2}^+$                     | 2   | 1        | 0     | 1     | 0             | [2948/2995]      | –                |
| $P_c$         | $\frac{1}{2}\frac{1}{2}^-, \frac{1}{2}\frac{3}{2}^-$ | 1   | 0        | 0     | 1     | 1             | [4340/4360/4374] | [4312/4440/4457] |
| $P_c^*$       | $1\frac{1}{2}^-, 1\frac{3}{2}^-$                     | 1   | 0        | 1     | 1     | 1             | [4732/4752/4767] | –                |
|               | $1\frac{1}{2}^+, 1\frac{3}{2}^+$                     | 1   | 1        | 0     | 1     | 1             | [4725/4746/4763] | –                |

To fix the parameters for the charmed heavy baryons, we choose  $M_D = 1.87$  GeV for the D-meson mass in (7) and fix  $M_{KK} = 0.475$  GeV to reproduce the  $M_{\Lambda_c} = 2.286$  GeV. This low value of  $M_{KK}$  is consistent with the value used to reproduce the nucleon

# Bottom baryons and Pentaquarks

TABLE II. Bottom baryons and Pentaquarks

| $B$           | $IJ^P$   | $l$ | $n_\rho$ | $n_z$ | $N_Q$ | $N_{\bar{Q}}$ | Mass-MeV             | Exp-MeV |
|---------------|--|-----|----------|-------|-------|---------------|----------------------|---------|
| $\Lambda_b$   | $0\frac{1}{2}^+$                                     | 0   | 0        | 0     | 1     | 0             | 5608                 | 5620    |
| $\Sigma_b$    | $1\frac{1}{2}^+$                                     | 2   | 0        | 0     | 1     | 0             | 5962                 | 5810    |
|               | $1\frac{3}{2}^+$                                     | 2   | 0        | 0     | 1     | 0             | 5978                 | 5830    |
| $\Lambda_b^*$ | $0\frac{1}{2}^-$                                     | 0   | 0        | 1     | 1     | 0             | 5998                 | 5912    |
|               | $0\frac{1}{2}^+$                                     | 0   | 1        | 0     | 1     | 0             | 6029                 | (6072)  |
| $\Sigma_b^*$  | $1\frac{1}{2}^-, 1\frac{3}{2}^-$                     | 2   | 0        | 1     | 1     | 0             | [6351/6367]          | –       |
|               | $1\frac{1}{2}^+, 1\frac{3}{2}^+$                     | 2   | 1        | 0     | 1     | 0             | [6344/6367]          | –       |
| $P_b$         | $\frac{1}{2}\frac{1}{2}^-, \frac{1}{2}\frac{3}{2}^-$ | 1   | 0        | 0     | 1     | 1             | [11155/11163/11167]  | –       |
| $P_b^*$       | $1\frac{1}{2}^-, 1\frac{3}{2}^-$                     | 1   | 0        | 1     | 1     | 1             | [11544/11553/11556]  | –       |
|               | $1\frac{1}{2}^+, 1\frac{3}{2}^+$                     | 1   | 1        | 0     | 1     | 1             | [11532 /11543/11579] | –       |

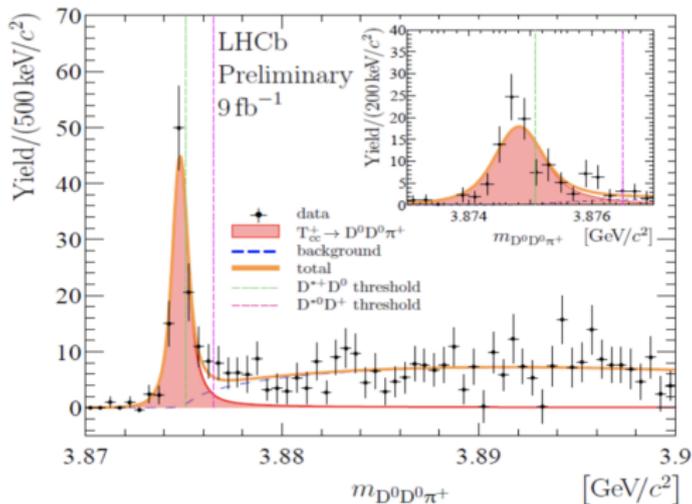


FIG. 1. Newly measured isoscalar charmed  $T_{cc}^+$  tetraquark by LHCb [48].

# Tetraquark puzzle

- $HH\bar{L}\bar{L}$  - several predictions (positive/negative  $\pm 200 \text{ MeV}$ )
- Measurement of  $\Xi_{cc}^{++}$  (3621) fixed the normalization for  $\bar{b}\bar{b}ud$  tetraquark (Karliner, Rosner (2017); Eichten, Quigg (2017)), bound up to  $200 \text{ MeV}$  (!)
- $T_{cc}^+$  ( $01^+$ ) (LHCb), narrow, bounded at  $-360 \text{ keV}$ ,  $\Gamma \sim 50 \text{ keV}$
- Holographic picture: instanton-antiinstanton "molecule" binds two mesons (Liu, MAN, Zahed (2019)),  $b(c)$  tetraquarks bounded by  $80(40) \text{ MeV}$
- Normalizing mass to  $T_{cc}^+$ , (Liu, MAN, Zahed (2022)) predict mass of  $T_{bc}$  and  $T_{bb}$ , and calculate very narrow width.

# Binding energies for tetraquarks

TABLE I. Binding energies for tetraquarks versus the 't Hooft coupling  $\lambda = g_{\text{YM}}^2 N_c$  with  $M_{\text{KK}} = 1$  GeV

| $\lambda$ | $QQ\bar{q}\bar{q}$ GeV | $bb\bar{q}\bar{q}$ GeV | $bc\bar{q}\bar{q}$ GeV | $cc\bar{q}\bar{q}$ GeV |
|-----------|------------------------|------------------------|------------------------|------------------------|
| 10        | -0.097                 | -0.088                 | -0.080                 | -0.072                 |
| 15        | -0.107                 | -0.091                 | -0.077                 | -0.062                 |
| 20        | -0.108                 | -0.085                 | -0.064                 | -0.041                 |
| 25        | -0.103                 | -0.073                 | -0.045                 | -0.018                 |
| 30        | -0.093                 | -0.056                 | -0.024                 | -0.0016                |
| 32        | -0.089                 | -0.048                 | -0.015                 | 0.00073                |

TABLE II. Binding energies for tetraquarks versus the 't Hooft coupling  $\lambda = g_{\text{YM}}^2 N_c$  with  $M_{\text{KK}} = 0.475$  GeV.

| $\lambda$ | $QQ\bar{q}\bar{q}$ GeV | $bb\bar{q}\bar{q}$ GeV | $bc\bar{q}\bar{q}$ GeV | $cc\bar{q}\bar{q}$ GeV |
|-----------|------------------------|------------------------|------------------------|------------------------|
| 10        | -0.046                 | -0.044                 | -0.042                 | -0.040                 |
| 15        | -0.051                 | -0.047                 | -0.044                 | -0.040                 |
| 20        | -0.051                 | -0.046                 | -0.040                 | -0.035                 |
| 25        | -0.049                 | -0.042                 | -0.035                 | -0.028                 |
| 30        | -0.045                 | -0.035                 | -0.027                 | -0.019                 |
| 40        | -0.031                 | -0.018                 | -0.0076                | 0.0011                 |

- **Strongly coupled QCD could be approached via duality from string theory in large  $N$  and large  $\lambda$  limit, including spectra of heavy-light hadrons**
- **Few parameters and very restrictive predictions, so models are confutable**
- Approach based on confinement,  $SB\chi S$  and heavy spin symmetry, in the limit of large  $N$  and  $\lambda$ .
- "High brow" theory boils down to relatively simple QM in moduli space (top-down approach)
- **Astonishing and deep analogies to "old" physics**