The study of *P*-wave strange mesons in coupled channel framework

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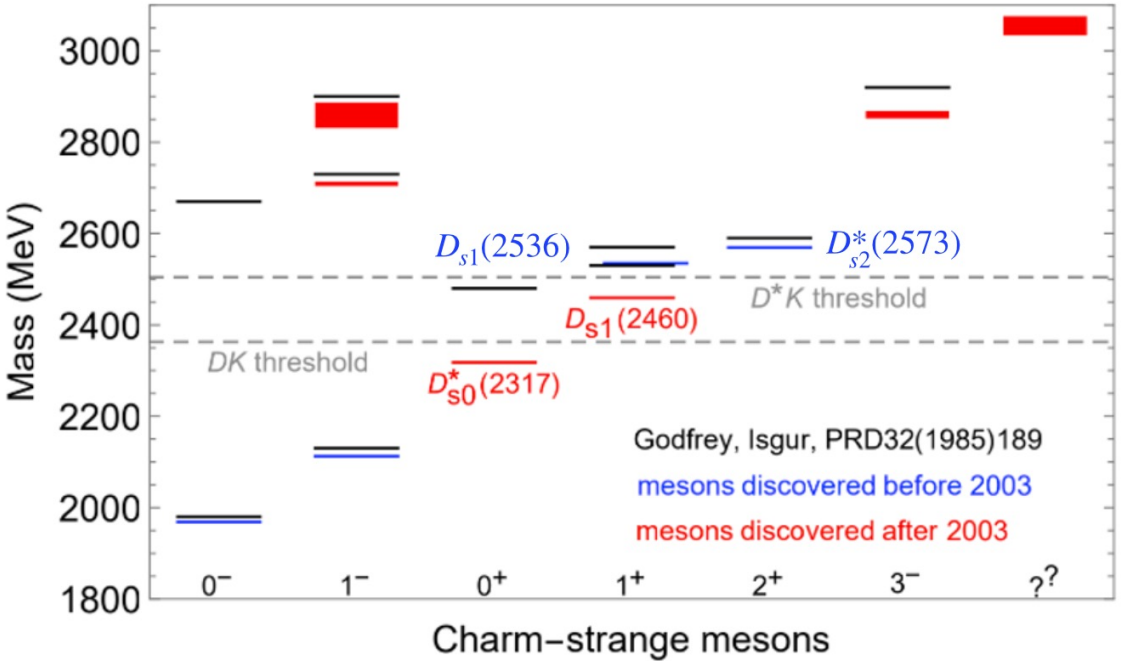
Puzzles of P-wave D_s mesons

- Four P-wave excited $c\bar{s}$ mesons in QM: $S_{c\bar{s}} = 0,\ J^P = 1^+$
  $S_{c\bar{s}} = 1, J^P = 0^+, 1^+, 2^+$



Guo et al., EPJ Web Conf. 202 (2019) 02001

- $D^*_{s0}(2317)$ & $D_{s1}(2460)$: $m_{exp} < m_{c\bar{s}}$?
- $D_{s1}(2536)$ & $D^*_{s2}(2573)$: $m_{exp} \sim m_{c\bar{s}}$.
- Closeness to the $D^{(*)}K$ channels: possible coupled channel effect

$D_{s0}^*(2317)$& $D_{s1}(2460)$

- $D_{s0}^*(2317)$ & $D_{s1}(2460)$: various theoretical explanations.
- $c\bar{s}$ mesons in quenched and unquenched quark model S. Godfrey et al., PRD 32, 189, Y.-B. Dai et al., PRD 68, 114011, D. S. Hwang et al., PLB 601, 137, Y. A. Simonov et al., PRD 70, 114013, H.-Y. Cheng et al., PRD 89, 114017 ⋯
- $D^{(*)}K$ molecule E. E. Kolomeitsev et al., PLB 582, 39, A. P. Szczepaniak, PLB 567, 23. J. Hofmann et al., NPA 733, 142, E. van Beveren et al., PRL 91, 012003,T. Barnes et al., PRD 68, 054006. D. Gamermann et al., PRD 76,074016, F.-K. Guo, et al., PLB 647, 133, J. M. Flynn et al., PRD 75, 074024…
- Tetraquark state H.-Y. Cheng et al., PLB 566, 193,Y.-Q. Chen et al., PRL 93, 232001, V. Dmitrasinovic, PRL 94, 162002, H. Kim et al., PRD 72, 074012, J.-R. Zhang, PLB 789, 432. …
- Inner structures not clear: **lack of experimental data**

$$D^{(*)}K \to D^{(*)}K$$

$D_{s0}^*(2317)$& $D_{s1}(2460)$

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$$D^{(*)}K \rightarrow D^{(*)}K$$

Hamiltonian Effective field theory (HEFT)

J.M.M. Hall, et al., PRD 87, 094510.
J.J. Wu, et al., PRC 90, 055206.
Z. W. Liu, et al., PRL 116,082004…

Lattice energy levels

L. Liu et al., PRD 87, 014508.
D. Mohler et al., PRL 111, 222001.
C. B. Lang et al., PRD 90, 034510.
G. S. Bali, PRD 96, 074501.
C. Alexandrou et al., PRD 101, 034502.

Formalism

- Quark model successfully described other D_s mesons, $D_{s1}(2536)$ & $D_{s2}^*(2573)$: existence of $c\bar{s}$ core

- Coupling with $D^{(*)}K$ - creating $\bar{q}q$ pair from the vacuum: existence of $D^{(*)}K$ and Hadron interaction

- Coupled-channel effect: extremely important for near-threshold states.

- $D_{s0}^*(2317), D_{s1}(2460), D_{s1}(2536)$ & $D_{s2}^*(2573)$: $c\bar{s}$ core+$D^{(*)}K$

Quark model + Coupled-channel effects + Lattice data to study the four P-wave D_s states.

Hamiltonian

- The Hamiltonian reads

$$H = H_0 + H_I,$$

- Non-interacting Hamiltonian

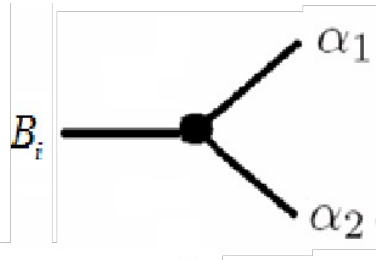
$$H_0 = \sum_B \underline{|B\rangle} m_B \langle B| + \sum_\alpha \int d^3\vec{k} \underline{|\alpha(\vec{k})\rangle} E_\alpha(\vec{k}) \langle \alpha(\vec{k})|.$$

bare $c\bar{s}$ core two-meson state

- Interacting Hamiltonian

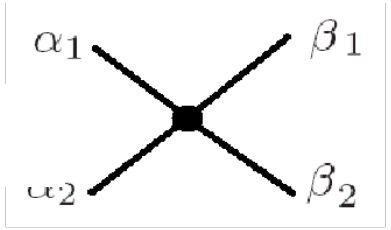
$$H_I = g + v$$

$$c\bar{s} \to D^{(*)}K$$



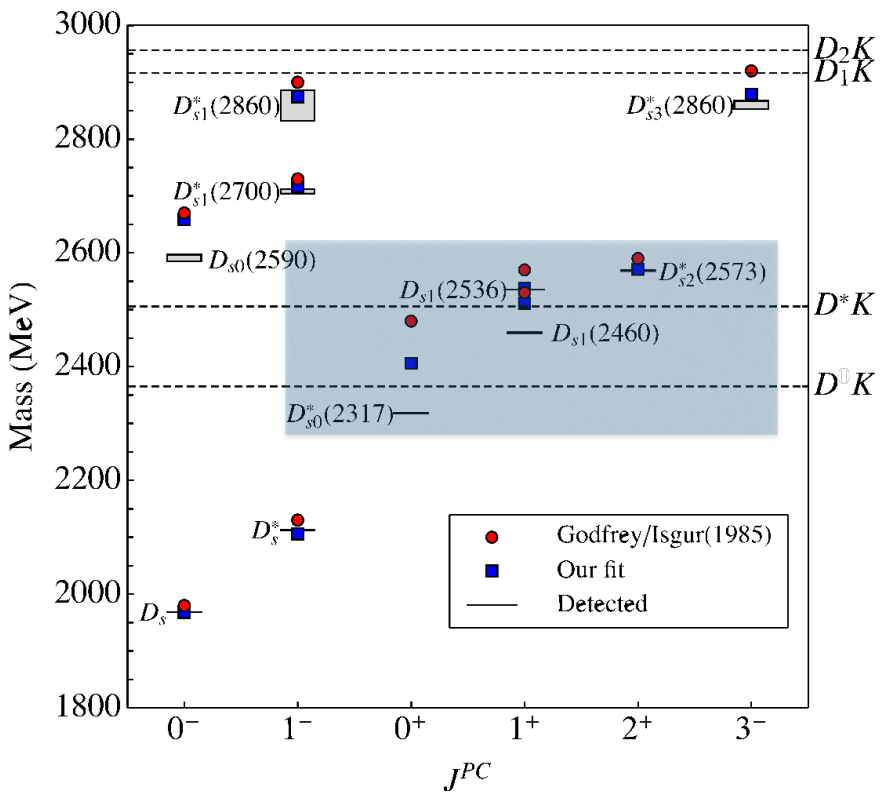
$$g = \sum_{\alpha,B} \int d^3\vec{k} \left\{ |\alpha(\vec{k})\rangle g_{\alpha B}(|\vec{k}|) \langle B| + h.c. \right\}$$

$$D^{(*)}K \to D^{(*)}K$$



$$v = \sum_{\alpha,\beta} \int d^3\vec{k} d^3\vec{k}' |\alpha(\vec{k})\rangle V^L_{\alpha,\beta}(|\vec{k}|, |\vec{k}'|) \langle \beta(\vec{k}')|$$

Quark model: bare $c\bar{s}$ state



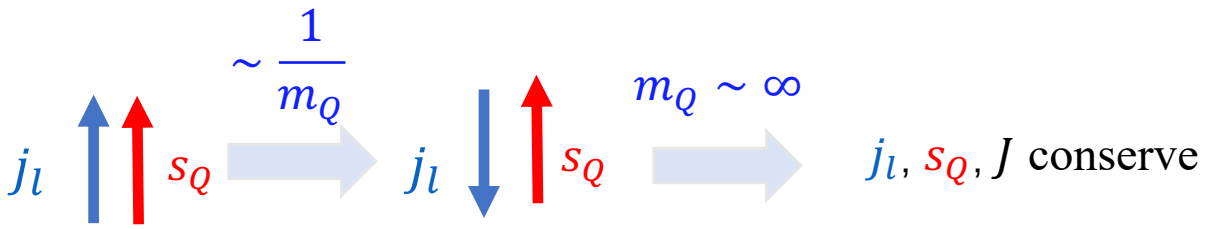
- The predicted lowest $0^+/1^+$ bare $c\bar{s}$ mesons
  -located above the $D^*_{s0}(2317)$&$D_{s1}(2460)$ states.

Godfrey, et al., Phys. Rev. D 32,189 (1985)

- Good heavy quark spin symmetry

- Total Spin

$$J = s_Q \otimes s_q \otimes L = S(s_Q \otimes s_q) \otimes L = s_Q \otimes j_l(s_q \otimes L)$$

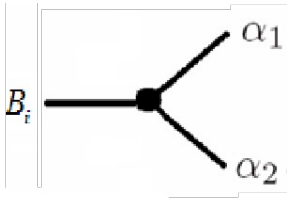


M. Neubert, Phys. Rept. 245 (1994) 259-396

	$c\bar{s}(J^P)$	$c\bar{s}$ (Mass)	$B(\lvert^{2S+1}L_J\rangle)$	HQS	α	L
$D^*_{s0}(2317)$	0^+	2405.9	$\lvert^3P_0\rangle$	$\lvert\frac{1}{2}_l \otimes \frac{1}{2}_H\rangle_0$	DK	S
$D^*_{s1}(2460)$	1^+	2511.5	$0.68\lvert^1P_1\rangle - 0.74\lvert^3P_1\rangle$	$-0.99\lvert\frac{1}{2}_l \otimes \frac{1}{2}_H\rangle_1 + 0.13\lvert\frac{3}{2}_l \otimes \frac{1}{2}_H\rangle_1$	D^*K	S, D
$D^*_{s1}(2536)$	1^+	2537.8	$-0.74\lvert^1P_1\rangle - 0.68\lvert^3P_1\rangle$	$0.13\lvert\frac{1}{2}_l \otimes \frac{1}{2}_H\rangle_1 - 0.99\lvert\frac{3}{2}_l \otimes \frac{1}{2}_H\rangle_1$	D^*K	S, D
$D^*_{s2}(2573)$	2^+	2571.2	$\lvert^3P_2\rangle$	$\lvert\frac{3}{2}_l \otimes \frac{1}{2}_H\rangle_2$	DK, D^*K	S, D

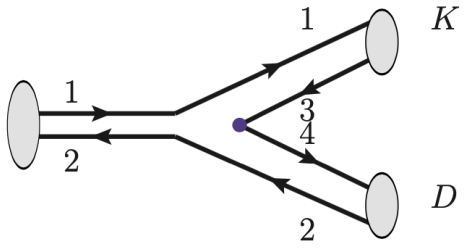
$H_I = g + v$

$\boldsymbol{g}$: $\boldsymbol{c\bar{s} \to D^{(*)}K}$



$^3P_0$ model

at quark level

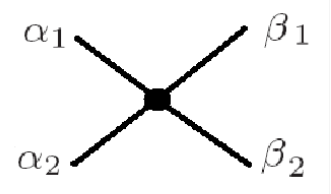


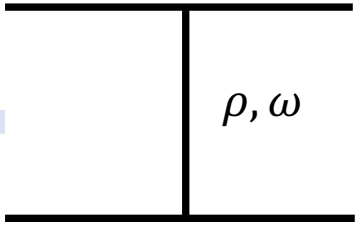
$$g_{\alpha\, B}(|\vec{k}|) = \gamma I_{\alpha\, B}(|\vec{k}|)\boxed{e^{-\frac{\vec{k}^2}{2\Lambda'^2}}}$$

P.G. Ortega, et al., Phys. Rev. D 94, 074037.

Undetermined γ & Λ'

$\boldsymbol{v}$: $\boldsymbol{D^{(*)}K - D^{(*)}K}$





$$\mathcal{L} = \mathcal{L}_{PPV} + \mathcal{L}_{VVV}$$
$$= ig_v \mathrm{Tr}(\partial^\mu P[P, V_\mu]) + ig_v \mathrm{Tr}(\partial^\mu V^\nu [V_\mu, V_\nu]),$$

$$\mathcal{V}(l, l'S, j) = \frac{1}{(2\pi)^3}\sqrt{\frac{1}{2E_D^i 2E_D^f 2E_K^i 2E_K^f}}2\pi \int dcos\theta V^v(\vec{p}_f, \vec{p}_i)\boxed{(\frac{\Lambda^2}{\Lambda^2 + p_f^2})^2(\frac{\Lambda^2}{\Lambda^2 + p_i^2})^2}$$

Form factor

Undetermined $g_{VDD}g_{VKK}(g_{VD^*D^*}g_{VKK})$ & Λ

Z. W. Lin, et al., Phys. Rev. C 69, 034903.
E. Oset, et al., Eur. Phys. J. A 44, 445.

Hamiltonian effective field theory (HEFT)

- In the finite volume, the momentum is discretized as

$$k_n = 2\pi\sqrt{n}/L, \quad n = n_x^2 + n_y^2 + n_z^2, \quad n = 0, 1, 2, \ldots$$

Continuous	$\int d\vec{k}$	and	$\lvert\alpha(\vec{k}_\alpha)\rangle$	and $\langle\beta(\vec{k}_\beta)\lvert\alpha(\vec{k}_\alpha)\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha - \vec{k}_\beta)$
↓	↓		↓	↓
Discrete	$\sum_i \left(2\pi/L\right)^3$	and	$\left(2\pi/L\right)^{-3/2}\lvert\vec{k}_i, -\vec{k}_i\rangle_\alpha$	and ${}_\beta\langle\vec{k}_j, -\vec{k}_j\rvert\vec{k}_i, -\vec{k}_i\rangle_\alpha = \delta_{\alpha\beta}\delta_{ij}$

$$H_0 = \sum_{i=1,n} \lvert B_i\rangle m_i \langle B_i\rvert + \sum_{\alpha,i} \lvert\vec{k}_i, -\vec{k}_i\rangle_\alpha \left[\sqrt{m_{\alpha_B}^2 + k_\alpha^2} + \sqrt{m_{\alpha_M}^2 + k_\alpha^2}\right]_\alpha \langle\vec{k}_i, -\vec{k}_i\rvert$$

$$H_I = \sum_j \left(2\pi/L\right)^{3/2} \sum_\alpha \sum_{i=1,n} \left[\lvert\vec{k}_j, -\vec{k}_j\rangle_\alpha\, g_{i,\alpha}^+ \langle B_i\rvert + \lvert B_i\rangle g_{i,\alpha}\, {}_\alpha\langle\vec{k}_j, -\vec{k}_j\rvert\right]$$

$$+ \sum_{i,j} \left(2\pi/L\right)^3 \sum_{\alpha,\beta} \lvert\vec{k}_i, -\vec{k}_i\rangle_\alpha\, v_{\alpha,\beta}\, {}_\beta\langle\vec{k}_j, -\vec{k}_j\rvert$$

J.M.M. Hall, et al., Phys. Rev. D 87, 094510.
J.J. Wu, et al., Phys. Rev. C 90, 055206.
Z. W. Liu, et al., Phys. Rev. Lett.116,082004

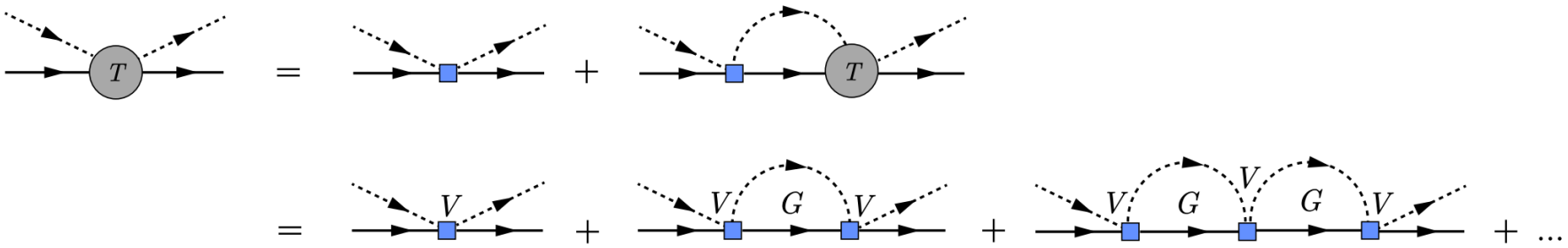
$$(H_0 + H_I)|\Psi\rangle = \underline{E}|\Psi\rangle$$

Energy levels in Lattice QCD

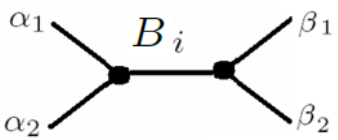
T- matrix

- In the infinite volume, the scattering T-matrix reads

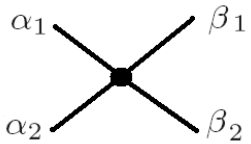
$$T_{\alpha,\beta}(k,k';E) = \mathcal{V}_{\alpha,\beta}(k,k';E) + \sum_{\alpha'} \int q^2 dq \frac{\mathcal{V}_{\alpha,\alpha'}(k,q;E)T_{\alpha,\beta}(q,k';E)}{E-E_{\alpha'}(q)+i\epsilon}$$



$$\mathcal{V}_{\alpha,\beta}(k,k';E) = \sum_B \frac{g_{\alpha B}(k) g^*_{\beta B}(k')}{E - m_B} + V^L_{\alpha,\beta}(k,k').$$



$$\sum_B \frac{g_{\alpha B}(k) g^*_{\beta B}(k')}{E - m_B}$$



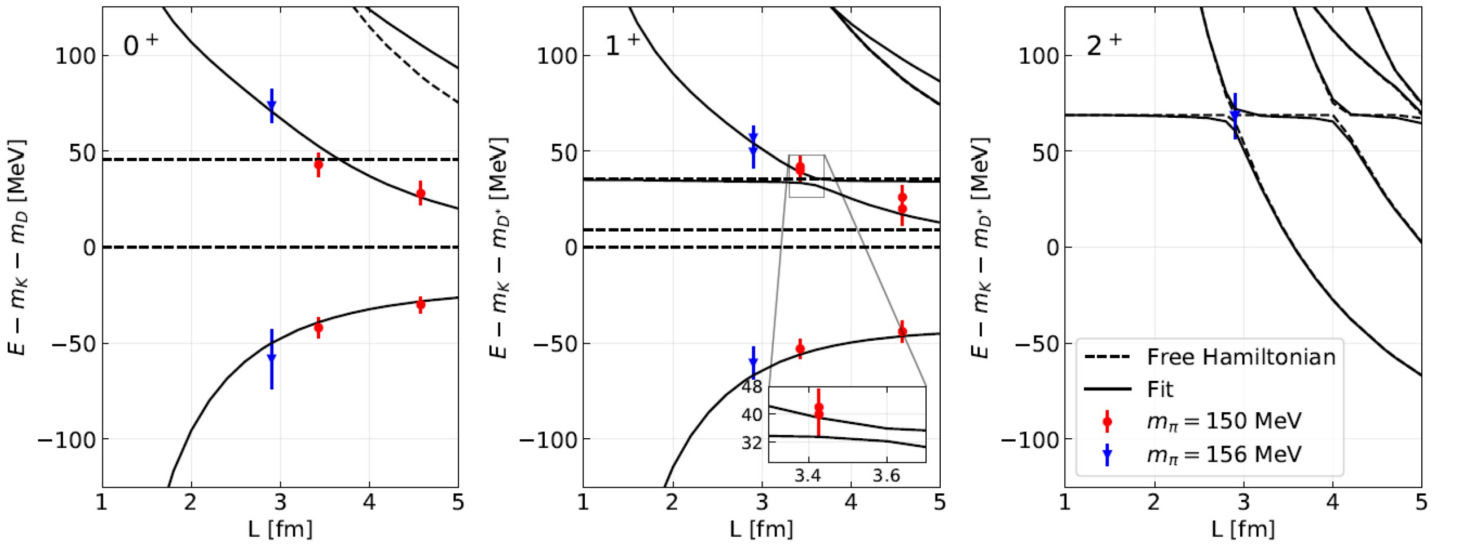
$$V^L_{\alpha,\beta}(k,k')$$

Results

- Probability: wave function with length L=4.57 fm.

	$P(c\bar{s})[\%]$	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

Prediction I: $D_{s2}(2573)$



- Pole position by solving T-matrix at infinite volume.

Phys. Rev. Lett. 128. 112001

- Different mass splitting patterns:

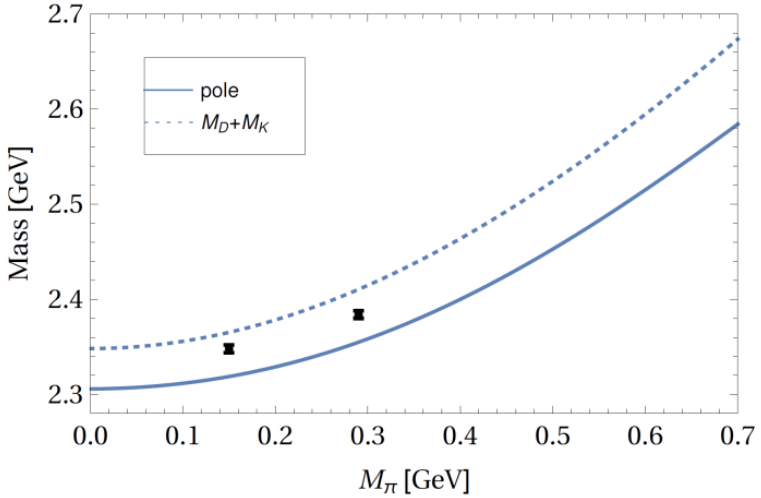
$D_{s0}^*(2317)$ and $D_{s1}(2460)$- coupled with S-wave $D^{(*)}K$ → Sizable mass shift & mixing

$D_{s1}(2536)$ and $D_{s2}(2573)$- coupled with D-wave $D^{(*)}K$ → Small mass shift & tiny mixing

Prediction II: m_π- dependence

- DK molecule: Tends to become larger with larger m_π.

- Latest lattice results in G. Bali et al., PRD96(2017)074501



curves: prediction in Du et al., EPJC77(2017)728

- Bare state ($c\bar{s}$): Tends to become stable with larger m_π.

"…for the lower lying pseudoscalar and vector D_s meson masses which decrease by 3 MeV (from 1980(1) MeV at m_π = 290 MeV to 1977(1) at m_π = 150 MeV) and 7 MeV (from 2101(1) MeV to 2094(1) MeV), respectively, hinting that the 0+ and 1+ states may have a more complicated internal structure."

G. Bali et al., PRD96(2017)074501

Prediction II: m_π- dependence

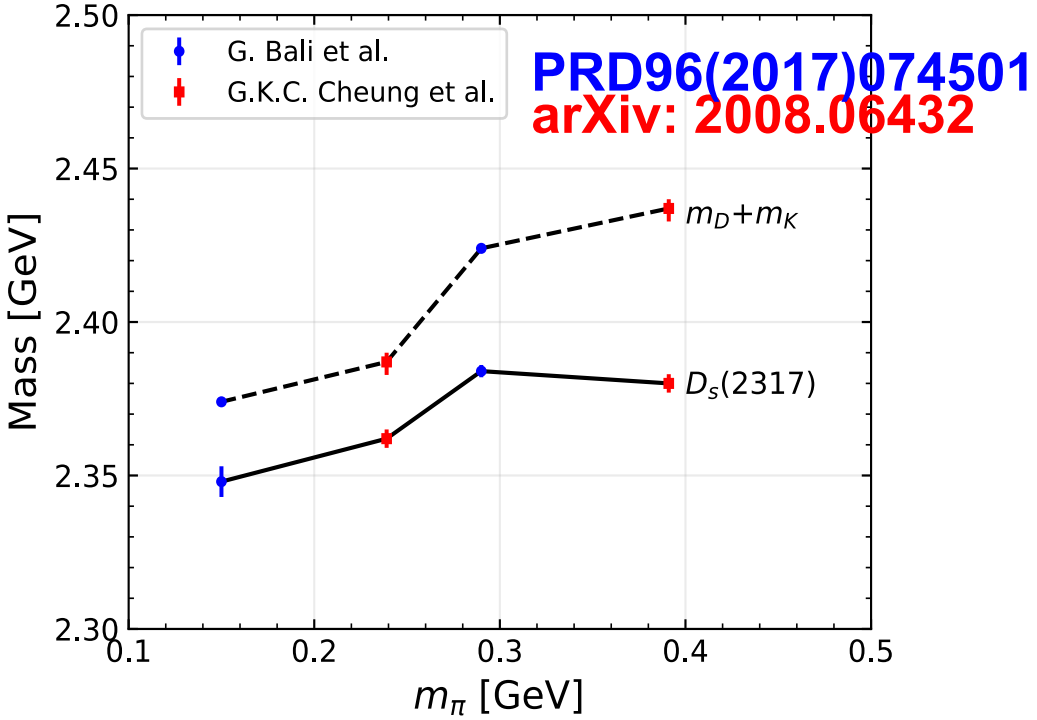
- Our prediction: the mass of $D_{s0}^*(2317)$ finally tends to become stable with increasing m_π.

- m_π ↗ , m_{DK} ↗ , $m_{c\bar{s}}$ → stable
- $m_{DK} < m_{c\bar{s}}$

- $D_{s0}^*(2317)$: dominated by mainly $c\bar{s}$, increasing

- $m_{DK} >> m_{c\bar{s}}$:

- $D_{s0}^*(2317)$ is mainly $c\bar{s}$. $M_{D_{s0}^*(2317)}$ tends to be stable.



G. Bali et al., PRD96(2017)074501

Gavin K. C. Cheung et al., arXiv: 2008.06432

Extension to P-wave B_S mesons

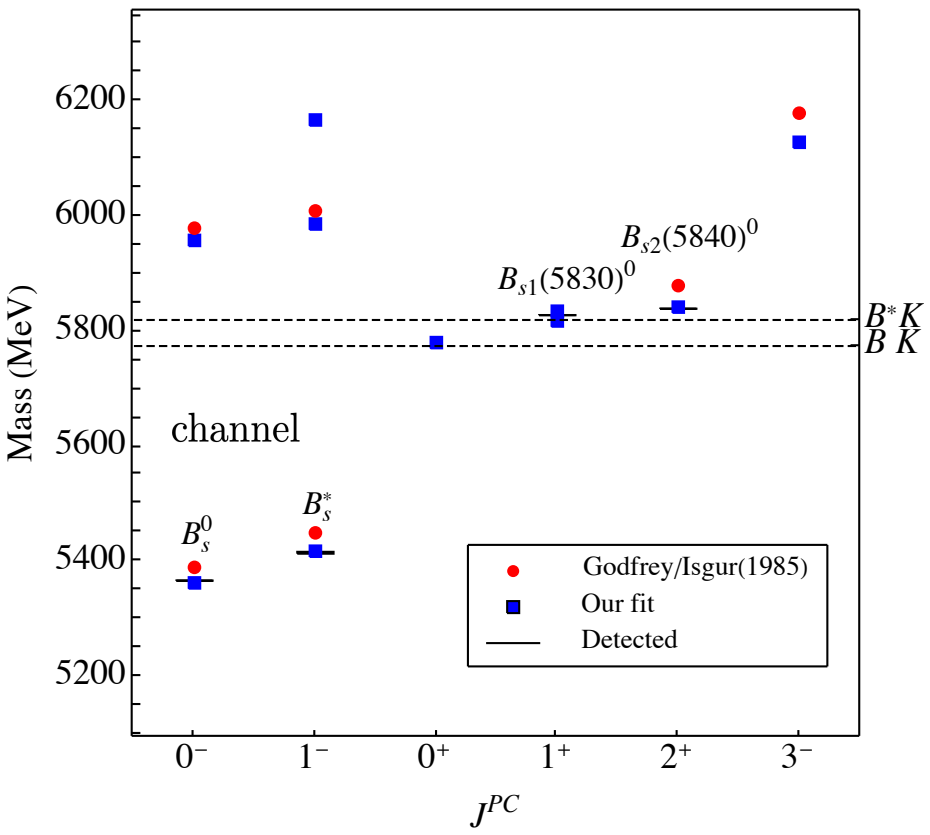
- Quark model

$$S_{\bar{b}s} = 0,\ J^P = 1^+$$
$$S_{\bar{b}s} = 1,\ J^P = 0^+, 1^+, 2^+$$

- Absence of 0^+ and lower 1^+ B_s states

$b\bar{s}$ cores channel

	$b\,(\lvert^{2S+1}L_J\rangle)$	b (mass)	α	L
B^*_{s0}	$\lvert^3P_0\rangle$	5780.9	$B\bar{K}$	S
B^*_{s1}	$-0.74\,\lvert^1P_1\rangle + 0.67\,\lvert^3P_1\rangle$	5818.5	$B^*\bar{K}$	S, D
	$= 0.98\,\phi_s - 0.22\,\phi_d$			
$B^{*\prime}_{s1}$	$0.67\,\lvert^1P_1\rangle + 0.74\,\lvert^3P_1\rangle$	5835.6	$B^*\bar{K}$	S, D
	$= 0.22\,\phi_s + 0.98\,\phi_d$			
$B^{*\prime}_{s2}$	$\lvert^3P_2\rangle$	5842.7	$B\bar{K},\ B^*\bar{K}$	D



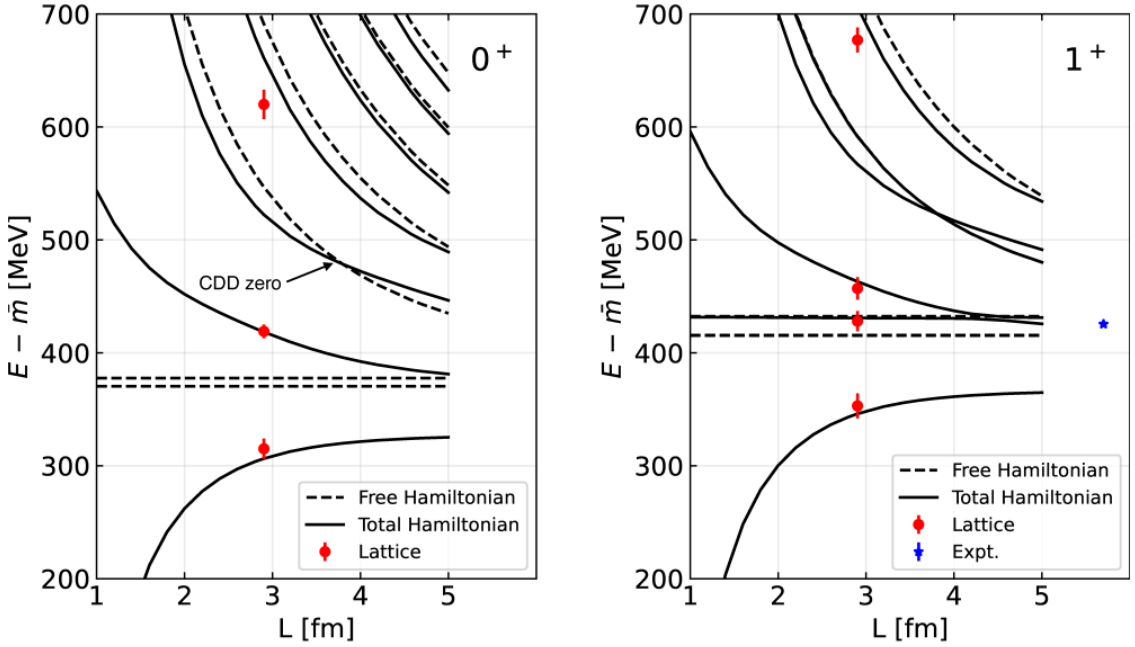
- Similar to D_S mesons: 0^+ and lower 1^+ B_s-S wave $B^{(*)}\,\bar{K}$ channels → Sizable mass shift & mixing

  higher 1^+ and 2^+ B_s-S wave $B^{(*)}\,\bar{K}$ channels → Small mass shift & tiny mixing

Extension to P-wave B_s mesons

- Heavy quark flavor symmetry: Using Previous Parameters

Postprediction, not a fit!



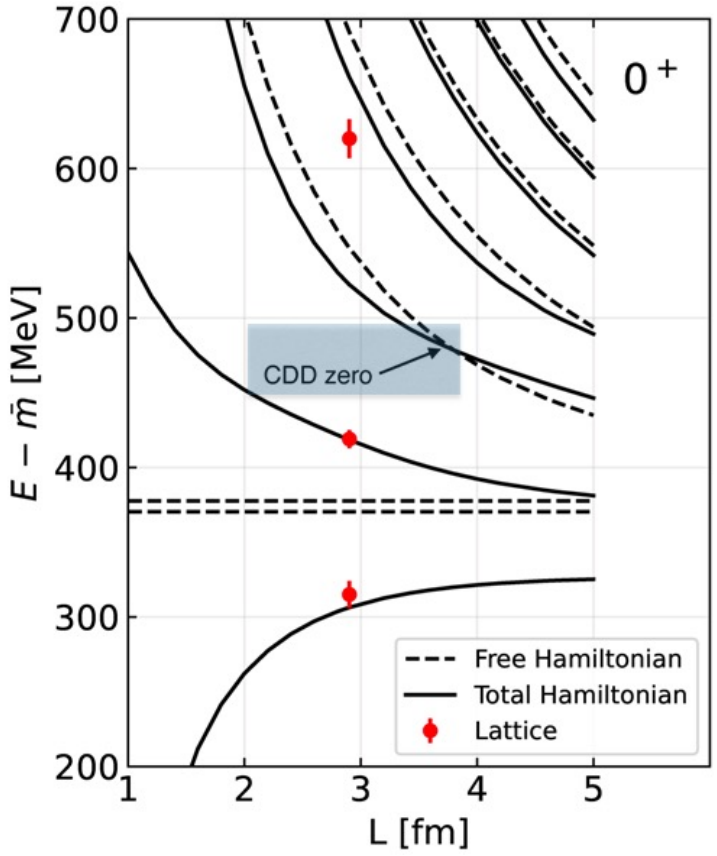
B^*_{s0} (5730) $- B\bar{K}$
B^*_{s1} (5570) - $B^*\bar{K}$

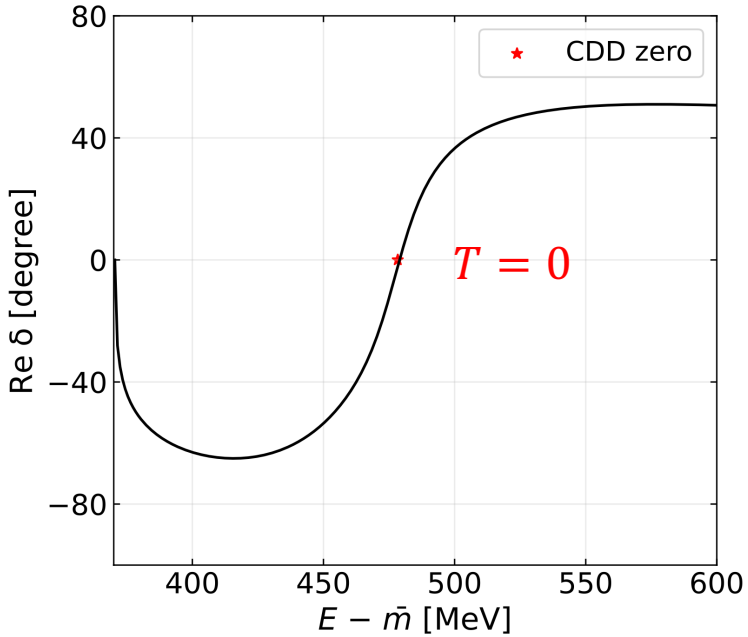
S-wave

Mass moving vs GI Model

J^P		0^+	1^+
mass [MeV]	rel. quark model [63]	5804	5842
	rel. quark model [64]	5833	5865
	rel. quark model [65]	5830	5858
	nonrel. quark model. [66]	5788	5810
	LO $\chi - SU(3)$ [18]	5643	5690
	Bardeen, Eichten, Hill [89]	5718 ± 35	5765 ± 35
	LO UChPT [24, 25]	5725 ± 39	5778 ± 7
	NLO UHMChPT [30]	$5696 \pm 20 \pm 30$	$5742 \pm 20 \pm 30$
	NLO UHMChPT [90]	5720^{+16}_{-23}	5772^{+15}_{-21}
	HQET + ChPT [67]	5706.6 ± 1.2	5765.6 ± 1.2
	Covariant ChPT [68]	5726 ± 28	5778 ± 26
	local hidden gauge [69]	$5475.4 \sim 5457.5$	$5671.2 \sim 5663.6$
	heavy meson chiral unitary [70]	5709 ± 8	5755 ± 8
	lattice QCD [91]	$5752 \pm 16 \pm 5 \pm 25$	$5806 \pm 15 \pm 5 \pm 25$
	lattice QCD [88]	$5713 \pm 11 \pm 19$	$5750 \pm 17 \pm 19$
	this work	$5730.2^{+2.4}_{-1.5}$	$5769.6^{+2.4}_{-1.6}$
$P(b\bar{s})$[%]	heavy meson chiral unitary [70]	$48.2 \pm 1.5/54.2 \pm 1.1$	$50.3 \pm 1.4/51.7 \pm 1.3$
	this work	$54.7^{+5.2}_{-4.1}$	$56.7^{+4.6}_{-3.7}$

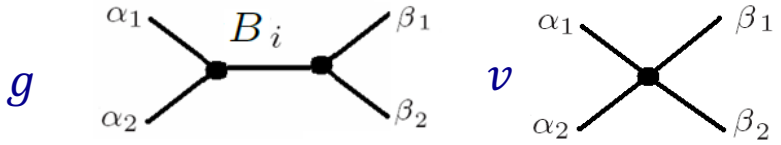
A CDD Zero





$$\bar{m} = \frac{1}{4}\left(m_{B_s} + 3m_{B_s^*}\right) = 5403.3 \text{ MeV}$$

- The CDD zero indicates there are two mechanisms which will cancel at this energy.



- Give a new method to search CDD zero: LQCD.

Summary

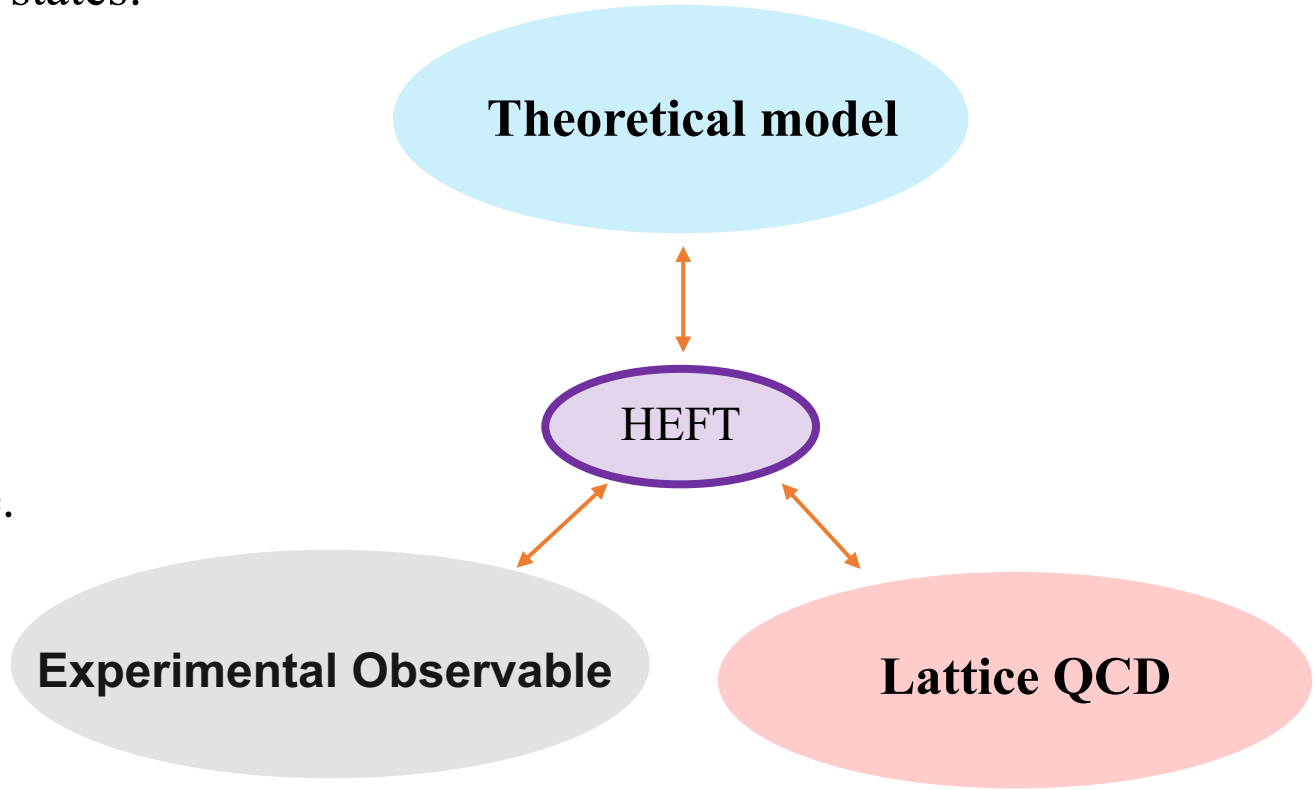
- Quark model + coupled channel effect +HEFT & Lattice QCD: near-threshold states.

- Investigation of inner structures of P-wave D_s states:
  $D_{s0}(2317)$ [$c\bar{s}$-DK(S-wave)],
  $D_{s1}(2460)$ [$c\bar{s}$-DK*(S-wave)],
  $D_{s1}(2536)$ [$c\bar{s}$](DK*(D-wave)),
  $D_{s2}(2573)$ [$c\bar{s}$] DK*(D-wave).

- Prediction of the B_{s0}^* (5730) and B_{s1}^* (5570).

- $\boldsymbol{q\bar{q}}$ and **hadron interactions are** always there.

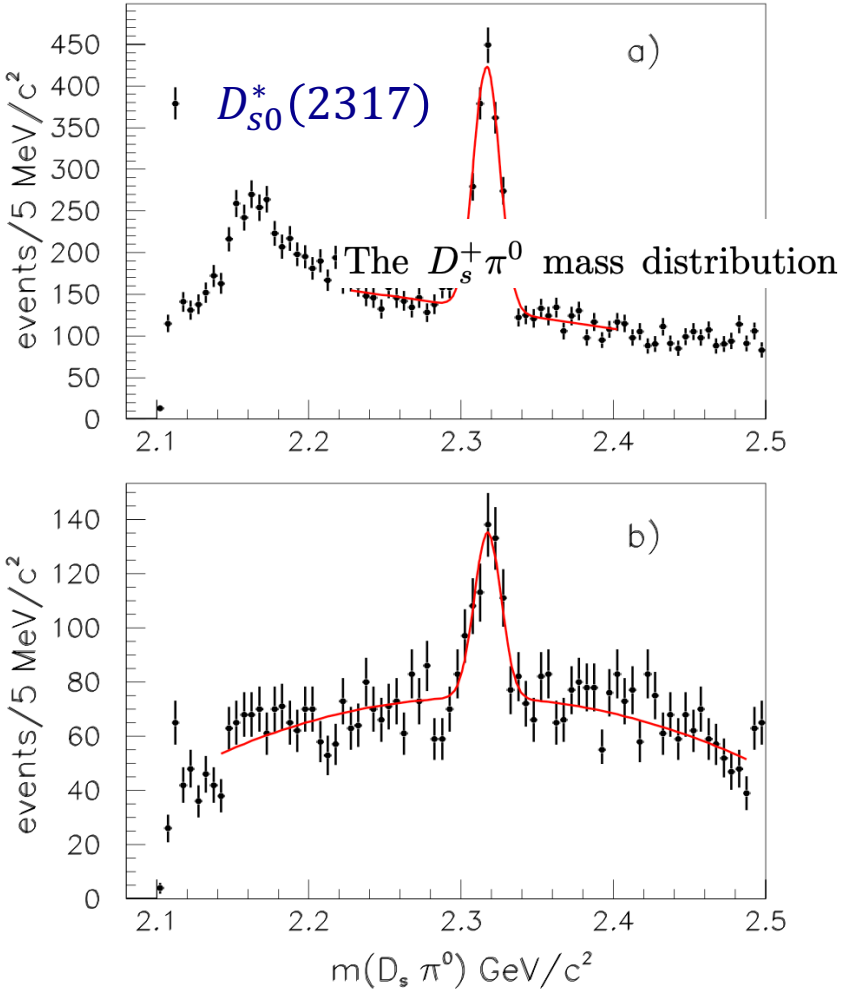
- Extension to other near-threshold states.



Thank you for your attention!

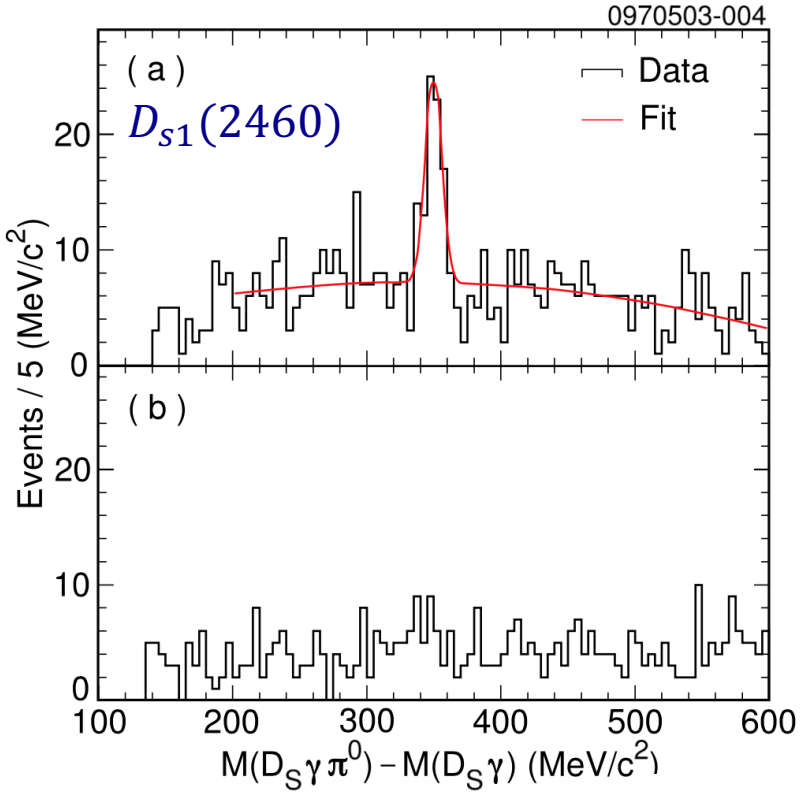
Backup slide

Observation of $D_{s0}^*(2317)$ & $D_{s1}(2460)$



The $D_s\pi^0$ mass distribution

BaBar, PRL90 (2003) 242001



CLEO, PRD68 (2003) 032002

$D_{s0}^*(2317)$: M= 2317.8 ± 0.5 MeV, $\Gamma < 3.8$ MeV.
$I(J^P) = 0(0^+)$.

$D_{s1}(2460)$: M=2459.5 ± 0.6 MeV, $\Gamma < 3.5$ MeV.
$I(J^P) = 0(1^+)$.

PDG20

Quark model: bare $\bar{c}s$ meson

The relativized quark model:

$$V = G_{\text{eff}}(r) + S_{\text{eff}}(r)$$

1. $G(r) = -\frac{4\alpha_s(r)}{3r}$
   $S(r) = br + c$

smearing →

$$\widetilde{G}(r) = -\sum_k \frac{4\alpha_k}{3r}\left[\frac{2}{\sqrt{\pi}}\int_0^{\tau_{kij'}} e^{-x^2}dx\right]$$

$$\widetilde{\boldsymbol{S}}(r) = br\left[\frac{e^{-\sigma_{ij}^2 r^2}}{\sqrt{\pi}\sigma_{ij} r} + \left[1 + \frac{1}{2\sigma_{ij}^2 r^2}\right]\frac{2}{\sqrt{\pi}}\int_0^{\sigma_{ij} r} e^{-x^2}dx + c\right.$$

Smearing : $\tilde{f}_{ij}(r) \equiv \int d^3r' \rho_{ij}(\mathbf{r}-\mathbf{r}') f(r')$ with $\rho_{ij}(\mathbf{r}-\mathbf{r}') = \frac{\sigma_{ij}^3}{\pi^{3/2}} e^{-\sigma_{ij}^2(\mathbf{r}-\mathbf{r}')^2}$

$$\widetilde{G}(r) \to \left(1 + \frac{p^2}{E\bar{E}}\right)^{1/2} \widetilde{G}(r) \left(1 + \frac{p^2}{E\bar{E}}\right)^{1/2}$$

$$\frac{\widetilde{V}_i(r)}{m_1 m_2} \to \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon_i} \frac{\widetilde{V}_i(r)}{m_1 m_2} \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon_i}$$

Godfrey, et al, Phys. Rev. D 32,189 (1985)

Quark model: bare $\bar{c}s$ state

The relativized quark model:

$$H = H_0 + V$$

$$H_0 = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$

$$V = G_{\text{eff}}(r) + S_{\text{eff}}(r)$$

$$G_{\text{eff}}(r) = \left[1 + \frac{p^2}{E_1 E_2}\right]^{1/2} \widetilde{G}(r) \left[1 + \frac{p^2}{E_1 E_2}\right]^{1/2}$$
$$+ \left[\frac{\mathbf{S}_1 \cdot \mathbf{L}}{2m_1{}^2} \frac{1}{r} \frac{\partial \widetilde{G}^{\,\text{so}(v)}_{11}}{\partial r} + \frac{\mathbf{S}_2 \cdot \mathbf{L}}{2m_2{}^2} \frac{1}{r} \frac{\partial \widetilde{G}^{\,\text{so}(v)}_{22}}{\partial r} + \frac{(\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L}}{m_1 m_2} \frac{1}{r} \frac{\partial \widetilde{G}^{\,\text{so}(v)}_{12}}{\partial r}\right]$$
$$+ \frac{2\mathbf{S}_1 \cdot \mathbf{S}_2}{3m_1 m_2} \nabla^2 \widetilde{G}^{\,c}_{12} - \left[\frac{\mathbf{S}_1 \cdot \hat{r} \mathbf{S}_2 \cdot \hat{r} - \frac{1}{3} \mathbf{S}_1 \cdot \mathbf{S}_2}{m_1 m_2}\right] \left[\frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r}\right] \widetilde{G}^{\,t}_{12}$$

$$S_{\text{eff}}(r) = \widetilde{S}(r) - \frac{\mathbf{S}_1 \cdot \mathbf{L}}{2m_1{}^2} \frac{1}{r} \frac{\partial \widetilde{S}^{\,\text{so}(s)}_{11}}{\partial r} - \frac{\mathbf{S}_2 \cdot \mathbf{L}}{2m_2{}^2} \frac{1}{r} \frac{\partial \mathbf{S}^{\text{so}(s)}_{22}}{\partial r}$$

Godfrey, Isgur, Phys. Rev. D 32,189 (1985)

- Relativized Modification:

- Relativistic kinematic energy

- Energy dependence

$$\omega_{ij} = 1 + \frac{p_i p_j}{E_i E_j} \qquad \rho_{ij} = \frac{m_i m_j}{E_i E_j}$$

- Mass & wavefunction:

$$H|B\rangle = m_B |B\rangle$$

- Input of fit:

All the well-established mesons far away from two-meson thresholds as input (from π to Υ).

Our fit VS GI model

GI-model		Our fit	
Mass (MeV)		Mass (MeV)	
$m_q = 220$		$m_q = 294$	
$m_s = 419$		$m_s = 497$	
$m_c = 1628$		$m_c = 1720$	
$m_b = 4977$		$m_b = 5065$	
Potential		Potential	
$b = 0.18$ GeV^2		$b = 0.18$ GeV^2	
$\alpha_s^{\text{critical}} = 0.6$		$\alpha_s^{\text{critical}} = 0.6$	
$\Lambda = 200$ MeV		$\Lambda = 200$ MeV	
$c = -253$ MeV		$c = -426$ MeV	
Relativistic effects		Relativistic effects	
smearing	$\sigma_0 = 1.80$ GeV	smearing	$\sigma_0 = 1.45$ GeV
	$s = 1.55$		$s = 1.55$
$m \leftrightarrow E$ ambiguity	$\epsilon_c = -0.168$	$m \leftrightarrow E$ ambiguity	$\epsilon_c = -0.194$
	$\epsilon_t = +0.025$		$\epsilon_t = -0.016$
	$\epsilon_{so(V)} = -0.035$		$\epsilon_{so(V)} = -0.277$
	$\epsilon_{so(s)} = +0.055$		$\epsilon_{so(s)} = -0.289$

Our fit VS GI quark model

	$\bar{c}s$ cores		Coupled channels	
	$B(\|^{2S+1}L_J\rangle)$	B(mass)	α	L
$D_{s0}^*(2317)$	$\|^3P_0\rangle$	2405.9	DK	S
$J^P = 1^+$ $D_{s1}^*(2460)$	$0.68\|^1P_1\rangle - 0.74\|^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D_{s1}^*(2536)$	$-0.74\|^1P_1\rangle - 0.68\|^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D_{s2}^*(2573)$	$\|^3P_2\rangle$	2571.2	DK, D^*K	D

	$\bar{c}s$ cores		Coupled channels	
$J^P = 1^+$	$B(\|^{2S+1}L_J\rangle)$	B(mass)	α	L
$D_{s1}^*(2460)$	$-0.97\|^1P_1\rangle + 0.24\|^3P_1\rangle$ $0.76\phi_s - 0.65\phi_d$	2549.7	D^*K	S, D
$D_{s1}^*(2536)$	$-0.24\|^1P_1\rangle - 0.97\|^3P_1\rangle$ $= -0.65\phi_s - 0.76\phi_d$	2559.46	D^*K	S, D

- GI model: Two 1^+ $\bar{c}s$ meson almost on the basis $^{2S+1}L_J$.
- Our fit: good HQS. Two 1^+ $\bar{c}s$ meson are almost on the heavy quark spin basis.

$$\phi_s = \left|\frac{1}{2}_l \otimes \frac{1}{2}_h\right\rangle \longrightarrow \text{S-wave } D^{(*)}K$$

$$\phi_d = \left|\frac{3}{2}_l \otimes \frac{1}{2}_h\right\rangle \longrightarrow \text{D-wave } D^{(*)}K$$

Parameters

- Parameters

Parameters	g_c	Λ' [GeV]	γ
Best fit	$4.2^{+2.2}_{-3.1}$	$0.323^{+0.033}_{-0.031}$	$10.3^{+1.1}_{-1.0}$
Ref. [1]		0.84	6.5
Ref. [2]	-	-	6.9
Ref. [3]	18.2/9.8	-	-
Ref. [4]	8.4	-	-

[1] P. Ortega, et al. Phys. Rev. D 94 , 074037 (2016).
[2] S. Godfrey, et al. Phys .Rev. D 93 (2016) 3, 034035.
[3] C. W. Shen, et al. Phys. Rev. D 100, 056006 (2019).
[4] Z.W. Lin, et al. Phys. Rev. C 61, 024904 (2000).

- Pole mass: solving the scattering T-matrix in infinite limit,

$$T_{\alpha,\beta}(k,k';E) = \mathcal{V}_{\alpha,\beta}(k,k';E) + \sum_{\alpha'} \int q^2 dq \frac{\mathcal{V}_{\alpha,\alpha'}(k,q;E)T_{\alpha,\beta}(q,k';E)}{E-E_{\alpha'}(q)+i\epsilon}$$