Δ Baryon Spectroscopy using Lattice QCD and Hamiltonian Effective Field Theory

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Outline

Motivation - quark model

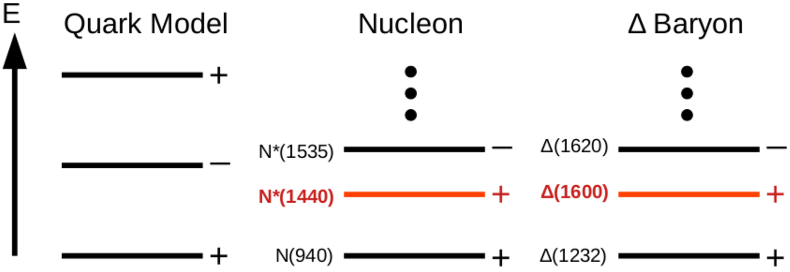
Lattice QCD - Δ spectrum using 3-quark interpolators

Hamiltonian Effective Field Theory

Connecting finite and infinite volume

Conclusions

Quark Model Predictions of Baryon Spectra



***Q*:** Are the Roper $N^*(1440)$ and $\Delta(1600)$ resonances *really* 3-quark states?

Lattice QCD

- Well established method for first principles calculations of hadronic observables in the non-perturbative regime of QCD
- Discretise space-time onto a 4D grid $\implies$ **finite volume**

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Gauge field configurations provided by the PACS-CS collaboration:

- $2+1$ flavour, dynamical-fermion configurations
- $\mathcal{O}(\alpha)$ improved Wilson fermion action + Iwasaki-gauge action
- $32^3 \times 64$ lattice, periodic boundary conditions, $\beta = 1.90$
- 5 ensembles: $m_\pi = 702, 572, 413, 293, 156$ MeV

[arXiv:0807.1661 [hep-lat]]

Hadron Spectroscopy on the Lattice

$$G_{ij}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \Omega | T\{\chi_i(x)\bar{\chi}_j(0)\} | \Omega \rangle = \sum_{B^\pm} \lambda_i^\pm \bar{\lambda}_j^\pm \, e^{-m_{B^\pm} t}$$

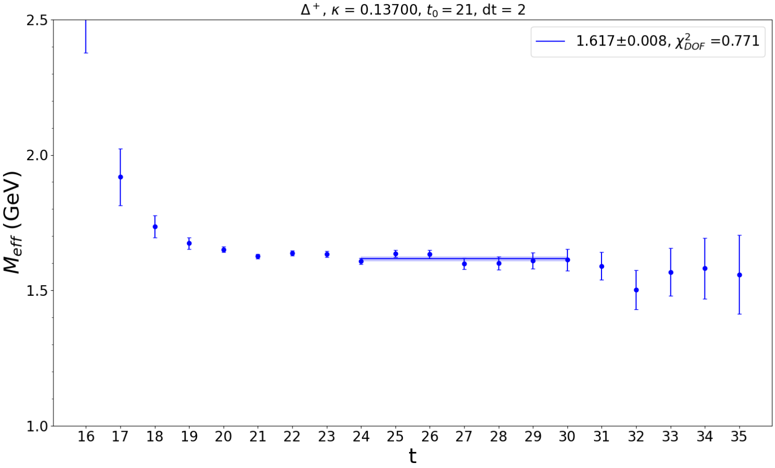
- χ_i are 3-quark interpolating fields coupling to Δ baryons
- Apply positive parity and spin-3/2 projection operators
  Target $\Delta(1232),\ \Delta(1600),\ \Delta(1920)$
- Use a **variational method** to project excited states

$$\left[(G(t_0))^{-1} G(t_0+\Delta t)\right] u^\alpha = e^{-m_\alpha \Delta t} u^\alpha$$

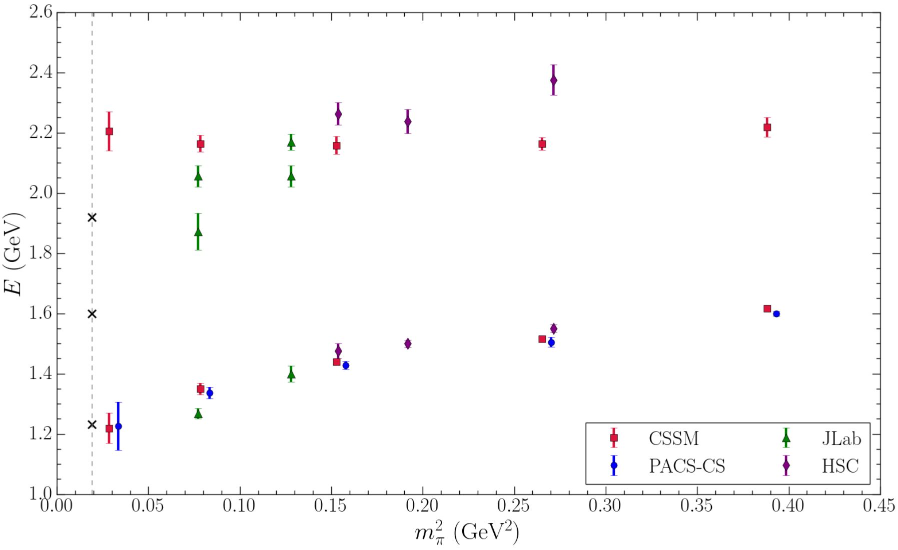
$$v^\alpha \left[G(t_0+\Delta t)(G(t_0))^{-1}\right] = e^{-m_\alpha \Delta t} v^\alpha$$

$$\implies M^\alpha_{\text{eff}}(t) = \ln\left(\frac{v^\alpha G(t,\vec{p}=0) u^\alpha}{v^\alpha G(t+1,\vec{p}=0) u^\alpha}\right)$$

Effective Mass Fits



Lattice Results



- 3-quark interpolators don't produce $\Delta(1600)$
- Use an effective field theory to further examine structure

Hamiltonian Effective Field Theory (HEFT)

$$H_0 = \sum_{B_0} |B_0\rangle m_{B_0} \langle B_0| + \sum_{\alpha, B_0} \int d^3k \; |\alpha(\vec{k})\rangle \omega_\alpha(\vec{k}) \, \langle \alpha(\vec{k})|$$

$|B_0\rangle$: quark model state

$|\alpha(\vec{k})\rangle$: 2-particle state

$$\omega_\alpha(k) \equiv \sqrt{m_{\alpha_B}^2 + k^2} + \sqrt{m_{\alpha_M}^2 + k^2}$$

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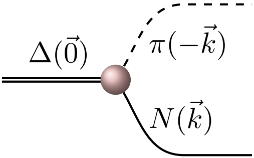
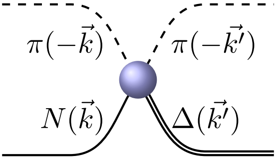
$$H_I = g + v \qquad g = \sum_{\alpha, B_0} \int d^3k \; |B_0\rangle \, G_\alpha^{B_0}(\vec{k}) \, \langle \alpha(\vec{k})| + h.c.$$

$$v = \sum_{\alpha, \beta} \int d^3k \int d^3k' \; |\alpha(\vec{k})\rangle \, V_{\alpha\beta}(\vec{k}, \vec{k}') \, \langle \beta(\vec{k}')|$$





T-matrix

Solve for the t-matrix using the coupled channel integral equations:

$$t_{\alpha\beta}(k,k';E) = \tilde{V}_{\alpha\beta}(k,k';E) + \sum_{\gamma} \int dq\, q^2 \frac{\tilde{V}_{\alpha\gamma}(k,q,E) t_{\gamma\beta}(q,k';E)}{E - \omega_{\gamma}(q) + i\epsilon}$$

where we've defined the coupled channel potential

$$\tilde{V}_{\alpha\beta}(k,k',E) = \frac{G_{\alpha}^{B_0 \dagger}(k) G_{\beta}^{B_0}(k')}{E - m_{B_0}} + V_{\alpha\beta}(k,k')$$

Quark model states:

- $|\Delta^{(1)}\rangle\,,\ |\Delta^{(2)}\rangle$

2-particle states (PDG motivated):

- $|\pi N\rangle$
- $|\pi\Delta\rangle$ in p-wave $\equiv |\pi\Delta_p\rangle$
- $|\pi\Delta\rangle$ in f-wave $\equiv |\pi\Delta_f\rangle$

Fitting to scattering data

The couplings and potentials are derived from chiral EFT i.e.

$$G_{\pi N}^{\Delta}(k) = \frac{\boldsymbol{g_{\pi N}^{\Delta}}}{m_\pi} \frac{k}{\sqrt{\omega_\pi(k)}} u(k, \boldsymbol{\Lambda_{\pi N}})$$

$$V_{\pi N,\pi N}(k, k') = \frac{\boldsymbol{v_{\pi N,\pi N}}}{(m_\pi)^2} \frac{k}{\omega_\pi(k)} \frac{k'}{\omega_\pi(k')} u(k, \boldsymbol{\Lambda_{\pi N}^{v}}) u(k', \boldsymbol{\Lambda_{\pi N}^{v}})$$

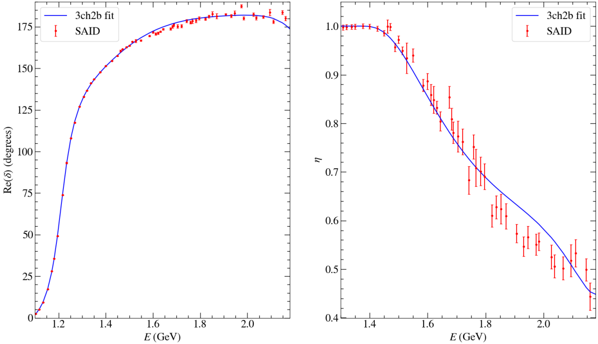
Note we've regularised our integrals by:

$$u(k, \Lambda) = \left(1 + k^2/\Lambda^2\right)^{-2}$$

With the t-matrix calculated, we can extract scattering phase shifts and inelasticities

- constrains the Hamiltonian parameters using infinite volume information

Fit to scattering data



Resonance	PDG Pole	This Work
$\Delta(1232)$	$(1.210 \pm 0.001) - (0.050 \pm 0.001)i$	$1.211 - 0.049i$
$\Delta(1600)$	$(1.510 \pm 0.050) - (0.135 \pm 0.035)i$	$1.444 - 0.219i$
$\Delta(1920)$	$(1.900 \pm 0.050) - (0.150 \pm 0.100)i$	$2.262 - 0.185i$

Finite Volume Factors

- Cube with side length L with periodic b.c. $\implies$ discrete momenta

$$k_n = \frac{2\pi}{L}\sqrt{n}, \qquad n = n_x^2 + n_y^2 + n_z^2$$

$$\int d^3k \to \sum_{\vec{n}\in\mathbb{Z}^3} \to \sum_n C_3(n), \quad C_3(n)\,\text{counts degeneracy in}\,k_n$$

- Finite volume corrections

$$\bar{G}_\alpha^{B_0}(k_n) = \sqrt{\frac{C_3(n)}{4\pi}}\left(\frac{2\pi}{L}\right)^{3/2} G_\alpha^{B_0}(k_n)$$

$$\bar{V}_{\alpha\beta}(k_n, k_m) = \sqrt{\frac{C_3(n)}{4\pi}}\sqrt{\frac{C_3(m)}{4\pi}}\left(\frac{2\pi}{L}\right)^3 V_{\alpha\beta}(k_n, k_m)$$

Finite Volume Hamiltonian Matrix

Construct a finite size Hamiltonian (single channel and single bare state shown)

$$H_0 = \mathsf{diag}(m_\Delta^{(1)},\, \omega_{\pi N}(k_1),\, \omega_{\pi N}(k_2),\, \dots)$$

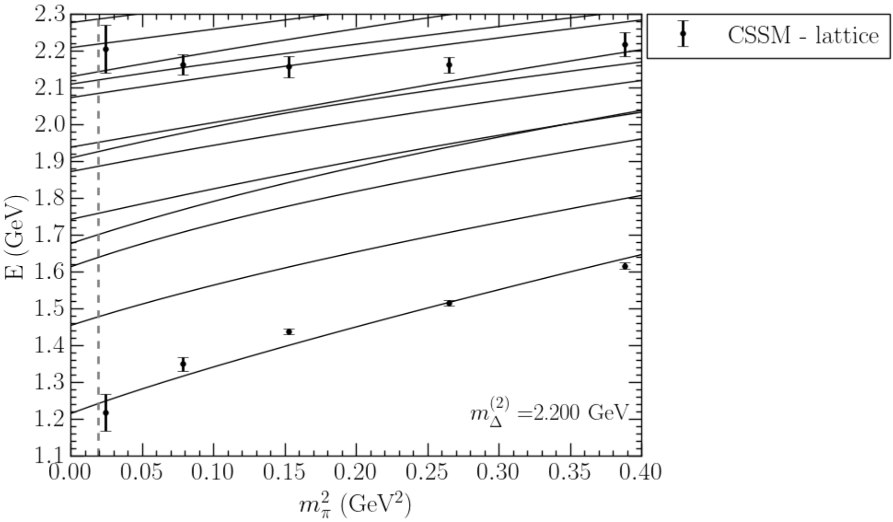
$$H_I = \begin{pmatrix} 0 & \bar{G}^{(1)}_{\pi N}(k_1) & \bar{G}^{(1)}_{\pi N}(k_2) & \dots \\ \bar{G}^{(1)}_{\pi N}(k_1) & \bar{V}_{\pi N,\pi N}(k_1,k_1) & \bar{V}_{\pi N,\pi N}(k_1,k_2) & \dots \\ \bar{G}^{(1)}_{\pi N}(k_2) & \bar{V}_{\pi N,\pi N}(k_2,k_1) & \bar{V}_{\pi N,\pi N}(k_2,k_2) & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Maximum size determined by lower bound on regulator $u(k,\Lambda)$

This symmetric Hamiltonian can be solved for its eigenvalues and eigenvectors at a given lattice size L

Finite Volume Energy Spectrum

Eigenvalues $\implies$ finite volume energy spectrum (extrapolating in m_π^2):



Eigenvector Analysis

Eigenvectors tell us about the composition of energy eigenstates

Consider the states which have largest overlaps with:

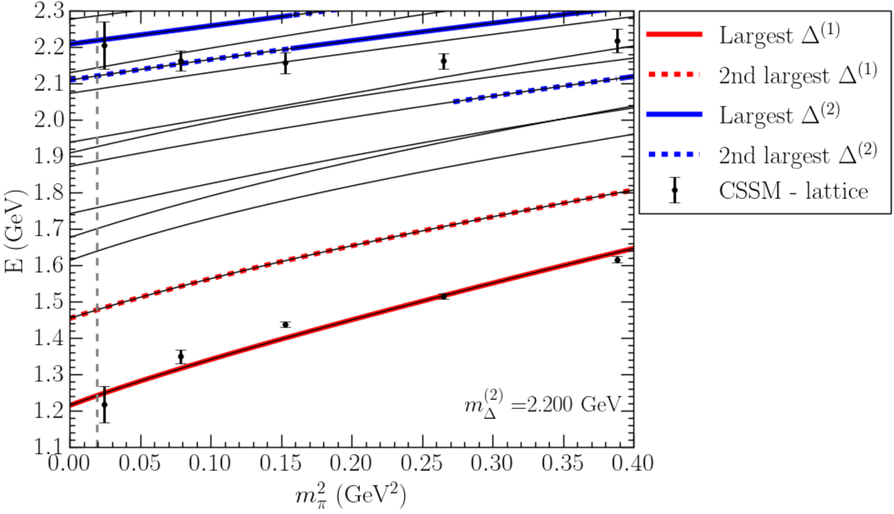
- 3-quark basis states $|\Delta^{(1)}\rangle\,,\ |\Delta^{(2)}\rangle$
- 2-particle states $|\pi N(\vec{k})\rangle\,,\ |\pi\Delta_p(\vec{k})\rangle\,,\ |\pi\Delta_f(\vec{k})\rangle$

i.e.

$$\langle E_i|B_0\rangle\ , \qquad \text{and} \qquad \langle E_i|\alpha(\vec{k})\rangle$$

Hamiltonian eigenstates with the largest overlaps with the 3-quark states should match the lattice QCD results

Comparison with Lattice Data



Conclusions

$\Delta(1232)$:

- HEFT suggests a predominantly quark model-like state

$\Delta(1600)$:

- 3-quark interpolators reveal a first excitation ~ 2.15 GeV
- HEFT predicts a 3-quark dominated state close to these lattice points
- HEFT suggests $\Delta(1600)$ is predominantly a dynamical resonance generated through πN and $\pi\Delta$ scattering

First excitation results on the lattice are quite preliminary, expect these to move (but only a little bit)

References

Lattice data from others:

- **PACS-CS:** S. Aoki et al. [PACS-CS], Phys. Rev. D 79 (2009) 034503 [arXiv:0807.1661 [hep-lat]]
- **JLab:** T. Khan, D. Richards and F. Winter, Phys. Rev. D 104 (2021) 034503 [arXiv:2010.03052 [hep-lat]]
- **Hadron Spectrum Collaboration:** J. Bulava, et al., Phys. Rev. D 82 (2010) 014507 [arXiv:1004.5072 [hep-lat]]

$\pi N \to \pi N$ scattering data used in fits:

- George Washington University's SAID partial wave analysis program

Backup Slides

Lattice Ensembles

κ	m_π (MeV)	a (fm)	Number configs
0.13700	702	0.1023(15)	399
0.13727	572	0.1009(15)	397
0.13754	413	0.0961(13)	449
0.13770	293	0.0951(13)	400
0.13781	156	0.0933(13)	198*

*Lightest ensemble needs additional statistics

Spin Projection Operators

$$P^{3/2}_{mn}(\vec{p}) = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}(\gamma \cdot p\gamma_\mu p_\nu + p_\mu\gamma_\nu\gamma \cdot p)$$

At rest $\vec{p} = 0$:

$$P^{3/2}_{mn}(0) = \delta_{mn}\mathbb{I} - \frac{1}{3}\gamma^S_m\gamma^S_n$$

where S denotes the Sakurai representation:

$$\gamma^4_S = \gamma^0_{\text{Dirac}},\ \gamma^i_S = -i\gamma^i_{\text{Dirac}}$$

Spin projected correlator:

$$G^{3/2}_{\mu\nu}(\vec{p} = 0) = \sum_{\sigma,\rho=1}^{4} G_{\mu\sigma}g^{\sigma\rho}P^{3/2}_{\rho\nu}(0)$$

Chiral Extrapolation

Expand the bare Δ masses about the physical pion mass:

$$m_{\Delta}^{(i)} = m_{\Delta}^{(i)}\big|_{\text{phys}} + \alpha_i \left(m_\pi^2 - m_\pi^2\big|_{\text{phys}} \right) \qquad i = 1, 2$$

Likewise

$$m_B = m_B\big|_{\text{phys}} + \alpha_B \left(m_\pi^2 - m_\pi^2\big|_{\text{phys}} \right) \qquad B = N, \Delta$$

Fix the lattice size at $L = 3$ fm and solve the matrix eigenvalue problem for unphysical values of m_π^2

- α_N and α_Δ have known values from previous lattice studies
- Perform fits to our lattice data to obtain α_1 and α_2

2-particle dominated states

Consider overlaps with 2-particle basis states with e.g. $n = 1$: