Smoking gun signals of the Schwinger mechanism in QCD from lattice simulations

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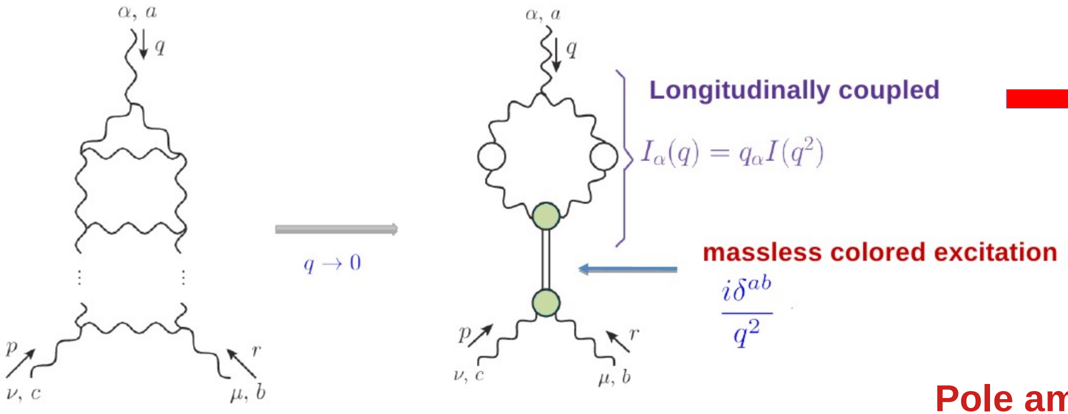
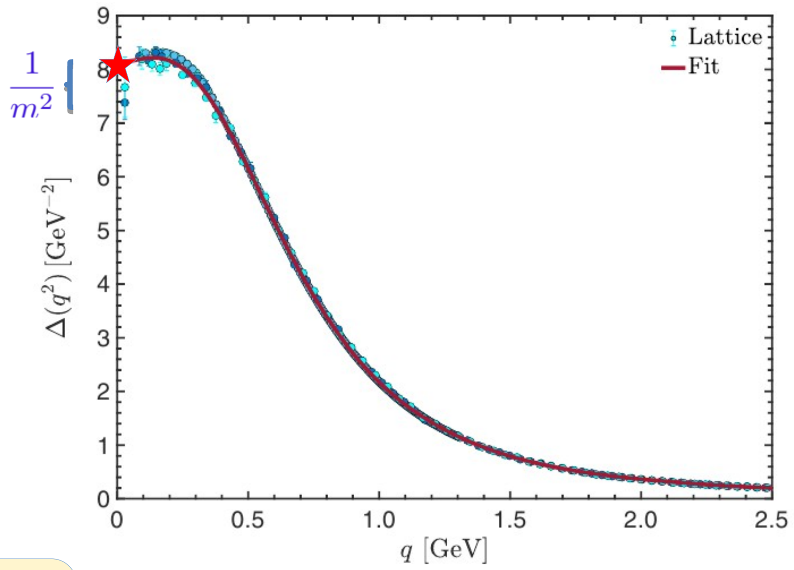
Schwinger mechanism in QCD

The **Schwinger mechanism endows gluons with mass** through the formation of **longitudinally coupled massless poles in the three-gluon vertex**. (See Joannis' talk)





Pole amplitude

$$\mathrm{I}\Gamma^{abc}_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + 2gf^{abc}\frac{q_\alpha}{q^2}g_{\mu\nu}(q\cdot r)\mathbb{C}(r^2) + \ldots$$

- **Lattice simulations only compute transverse projections of the vertex.**
- We can determine $\mathbb{C}(r^2)$ through the **displacement of the Ward Identity.**

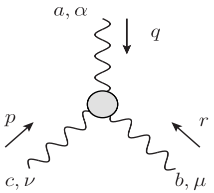
I. L. Bogolubsky, et al, Phys. Lett. B **676**, 69-73 (2009).
A. Cucchieri and T. Mendes, Phys. Rev. D **81**, 016005 (2010).
P. Bicudo, et al, Phys. Rev. D **92**, no.11, 114514 (2015).
A. C. Aguilar, et al, Eur. Phys. J. C **80**, no.2, 154 (2020).0

E. Eichten and F. Feinberg, Phys. Rev. D 10, 3254-3279 (1974).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
D. Binosi, D. Ibanez and J. Papavassiliou, Phys. Rev. D 86, 085033 (2012).
G. Eichmann, J. M. Pawlowski and J. M. Silva, Phys. Rev. D 104, no.11, 114016 (2021).

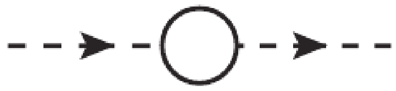
Ward identity displacement in QCD

The **Ward identity** is the *q = 0* limit of the non-Abelian **Slavnov-Taylor identity**:

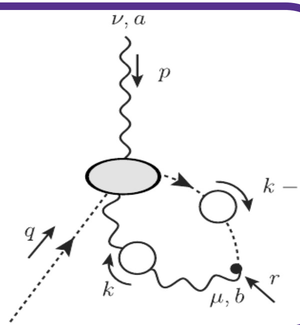
$$q^\alpha \mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$



$$:= \mathrm{I\Gamma}^{abc}_{\alpha\mu\nu}(q,r,p)$$



$$H_{\mu\nu}(r,q,p) :=$$



In deriving the limit, the presence of a longitudinal pole in *q = 0* must be carefully accounted for:

Pole amplitude

$$\mathrm{I\Gamma}^{abc}_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + 2gf^{abc}\frac{q_\alpha}{q^2}g_{\mu\nu}(q\cdot r)\mathbb{C}(r^2) + \dots$$

Ward identity displacement in QCD

$$q^{\alpha}\Gamma_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_{\nu}^{\sigma}(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_{\mu}^{\sigma}(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

Ward identity displacement in QCD

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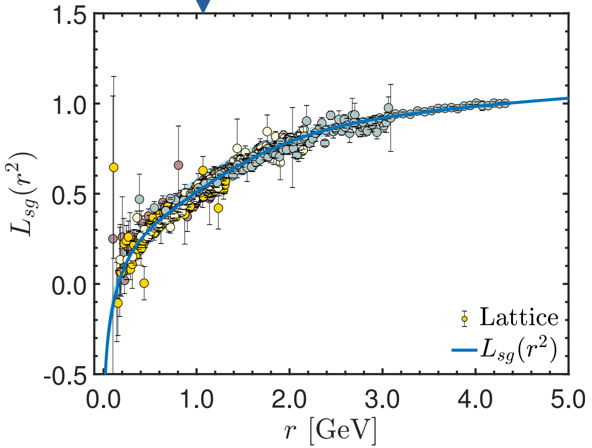

$$P_\mu^{\mu'}(r)P_\nu^{\nu'}(r)\mathrm{I\Gamma}_{\alpha\mu'\nu'}(0,r,-r) = 2L_{\rm sg}(r^2)r_\alpha P_{\mu\nu}(r)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - q_\mu q_\nu/q^2$$

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).

Ward identity displacement in QCD

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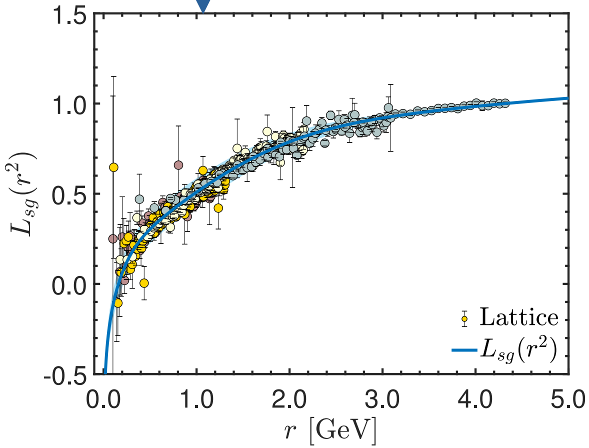

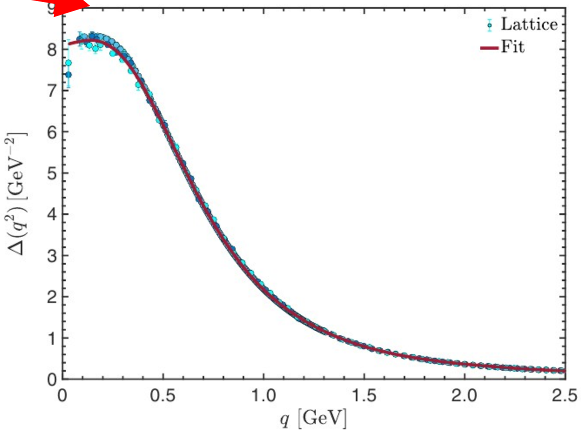

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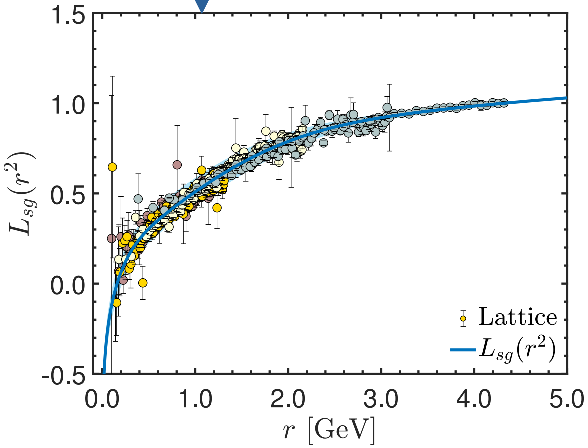

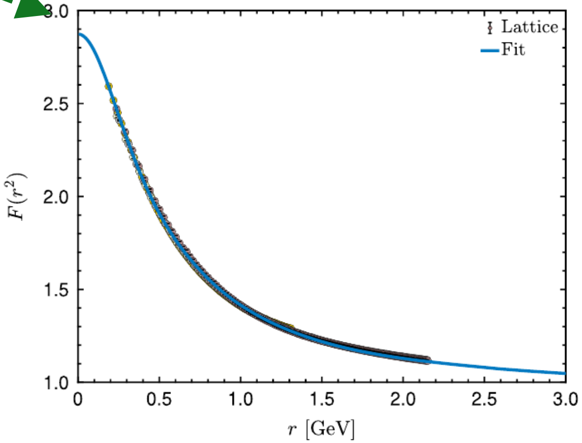

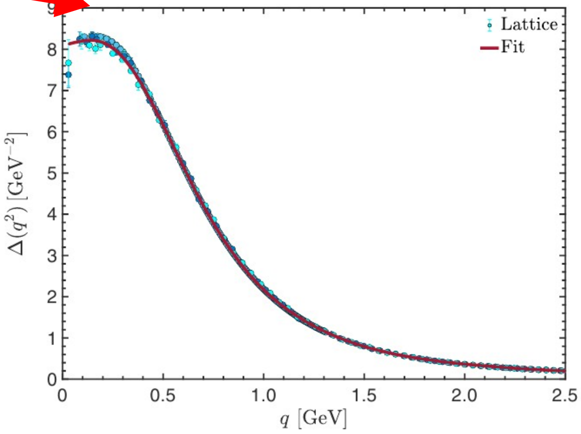

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★ **Only one ingredient not yet determined directly by lattice simulations.**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).

Ward identity displacement in QCD

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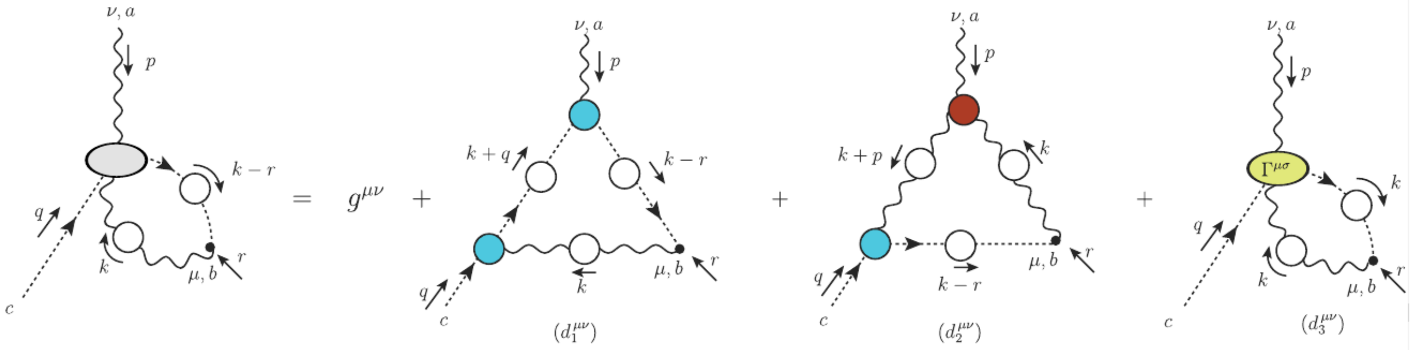
Displacement = pole amplitude

★ **Combine ingredients and determine if there is a nontrivial displacement.**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).

Schwinger-Dyson calculations

The $\mathcal{W}(r^2)$ can be obtained from the Schwinger-Dyson equation for the **ghost-gluon scattering kernel**



A. C. Aguilar, M. N. F. and J. Papavassiliou, Eur. Phys. J. C **81**, no.1, 54 (2021).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Depends on 5 ingredients, three of which are under stringent control already:

1) Ghost propagator;
2) Gluon propagator;

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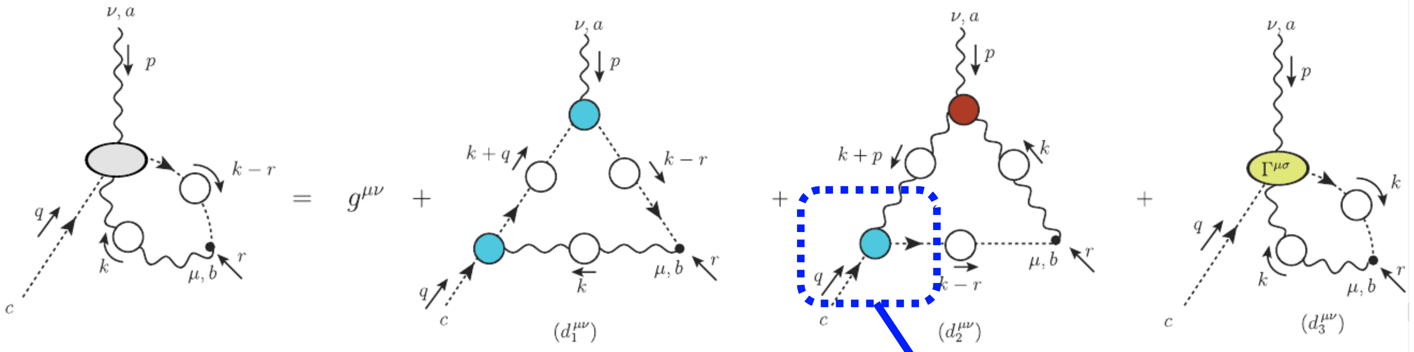

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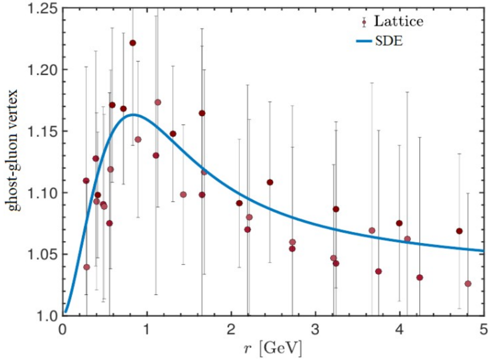

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3) Ghost-gluon vertex;

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M. Q. Huber and L. von Smekal, JHEP 04, 149 (2013).
A. K. Cyrol, L. Fister, M. Mitter, et al. Phys. Rev. D 94, 054005 (2016).
N. Barrios, M. Pelaez, U. Reinosa, et al. Phys. Rev. D102, 11401 (2020).
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., et. al, Phys. Rev. D 104, no.5, 054028 (2021).

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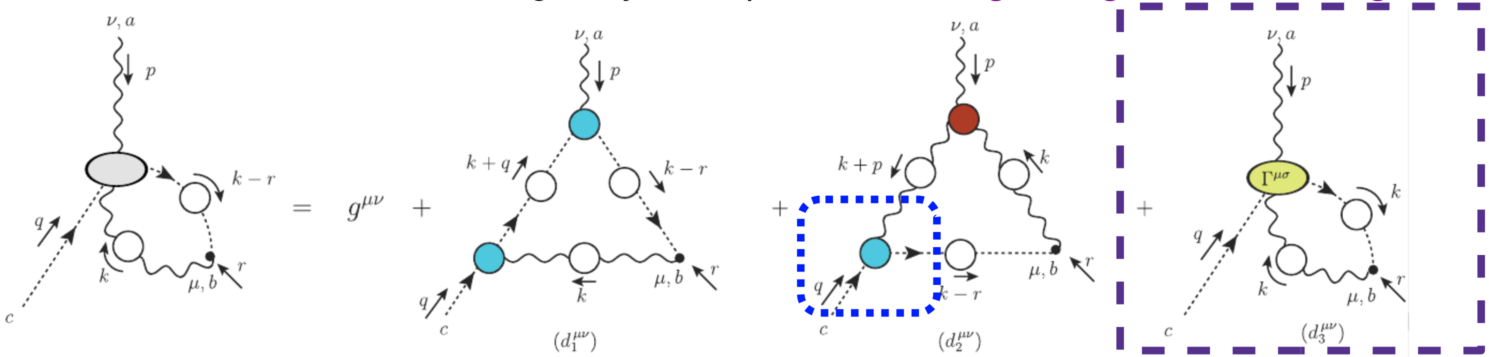

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(2% effect). M. Q. Huber, Eur. Phys. J. C **77**, 733 (2017).

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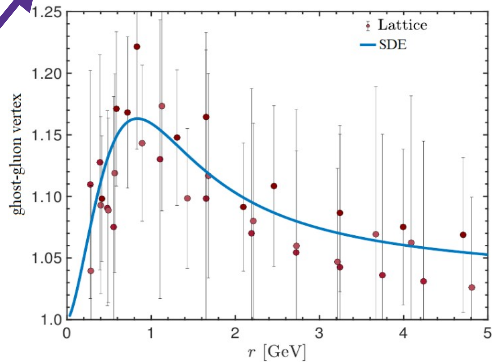
The four-point function is expected to be subleading.



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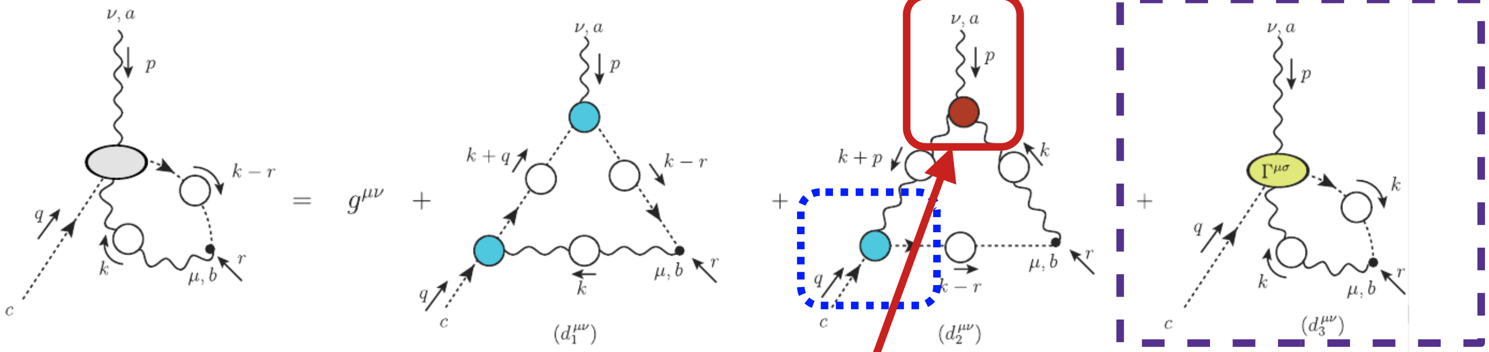

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Depends on 5 ingredients, three of which are under stringent control already:

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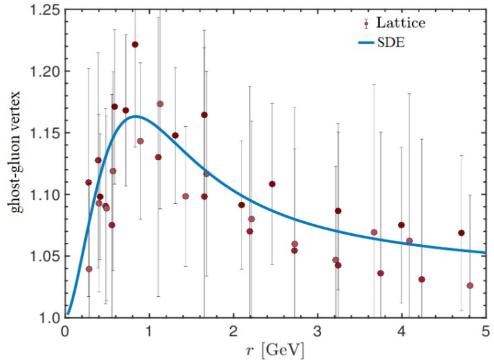
The four-point function is expected to be subleading

The main uncertainty is in the three-gluon vertex



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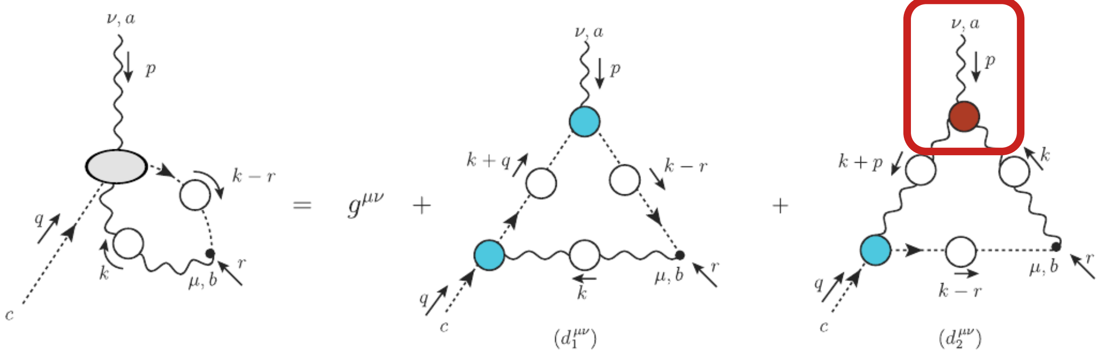
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., et. al, Phys. Rev. D 104, no.5, 054028 (2021).

Schwinger-Dyson calculations

The truncated equation depends on the **three-gluon vertex**:



Complicated object, with 14 independent tensor structures and **rich nonperturbative features**.

A. C. Aguilar, D. Binosi, D. Ibañez, J. Papavassiliou, Phys. Rev. D 89, no. 8, 085008 (2014).
G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D89,105014 (2014).
A. G. Duarte, O. Oliveira and P. J. Silva, Phys. Rev. D 94, no.7, 074502 (2016).

A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016).
R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D 93, no. 3, 034026 (2016).
M. Q. Huber, Phys. Rev. D 101, 114009 (2020).

For $\mathcal{W}(r^2)$, only a particular projection contributes:

$$\mathcal{I}_{\mathcal{W}}(q^2, r^2, p^2) := \frac{1}{2}(q-r)^{\nu}\,\overline{\mathrm{I}\Gamma}^{\mu}_{\mu\nu}(q,r,p)$$

Note that $\mathcal{I}_{\mathcal{W}}$ **projects the vertex transversely**

$$\overline{\mathrm{I}\Gamma}_{\alpha\mu\nu}(q,r,p) := P_{\alpha}^{\alpha'}(q)P_{\mu}^{\mu'}(r)P_{\nu}^{\nu'}(p)\,\mathrm{I}\Gamma_{\alpha'\mu'\nu'}(q,r,p)$$

- **Pole free part of the vertex.**
- **Accessible to lattice simulations.**

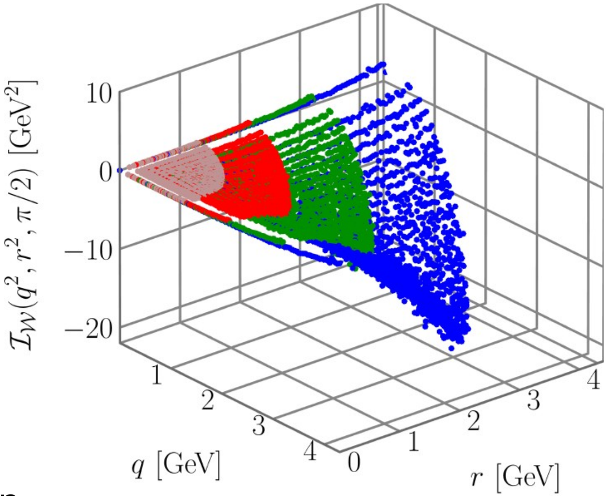
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

Three-gluon vertex from the lattice

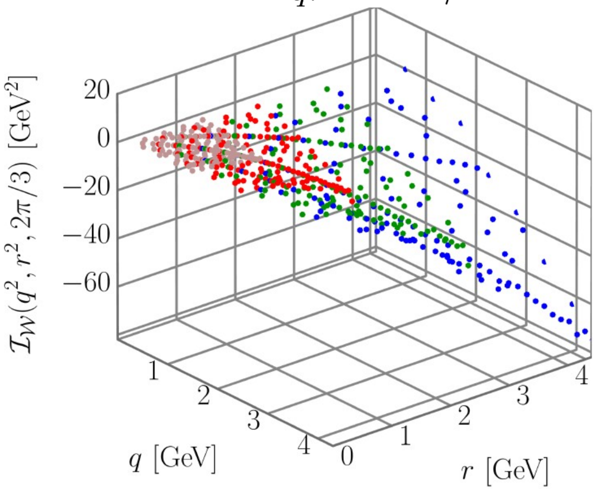
We compute $\mathcal{I}_{\mathcal{W}}(q^2, r^2, p^2)$ in general kinematics on the lattice

$$q \cdot r = |q||r| \cos\theta_{qr}$$

$\theta_{qr} = \pi/2$



$\theta_{qr} = 2\pi/3$



Lattice setups are the same as in F. Pinto-Gómez, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, arXiv:2208.01020. (see Pepe's talk)

To compute $\mathcal{W}(r^2)$ we need to integrate $\mathcal{I}_{\mathcal{W}}(q^2, r^2, p^2)$. This can be achieved through two different methods:

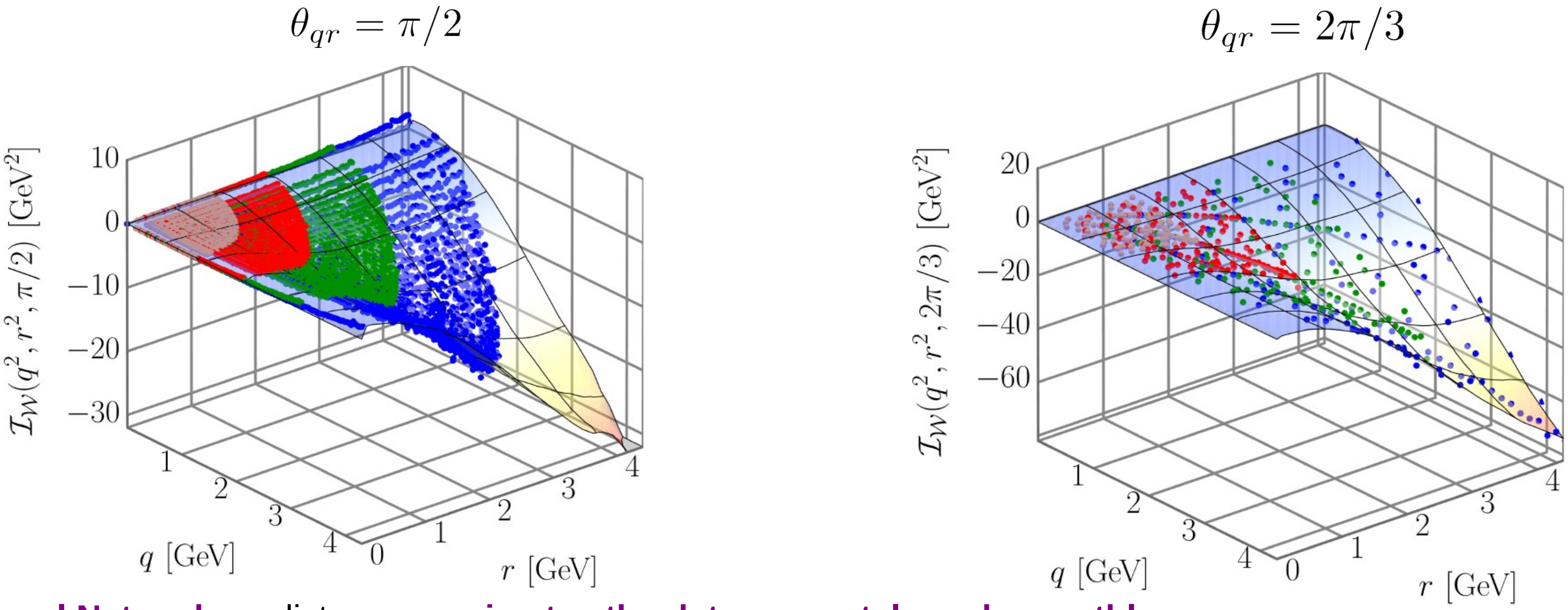
M1) Training a **Neural Network** on the above results to generate a continuous predictor function.

M2) By exploiting **planar degeneracy.**

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

Method 1): Neural Network

To generate a continuous function for integration, we train a **Neural Network** on the lattice points for $\mathcal{I}_{\mathcal{W}}$



- **Neural Network** predictor **approximates the data accurately and smoothly**.
- Data becomes sparse for $\theta_{qr} < \pi/2$ and stops at about q = 4.3 GeV.
- Determination of $\mathcal{W}(r^2)$ requires integration over the whole momentum space.

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

Method 2): Planar degeneracy

In F. Pinto-Gómez, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, arXiv:2208.01020. it was found that the **transversely projected three-gluon vertex can be accurately and compactly approximated by**

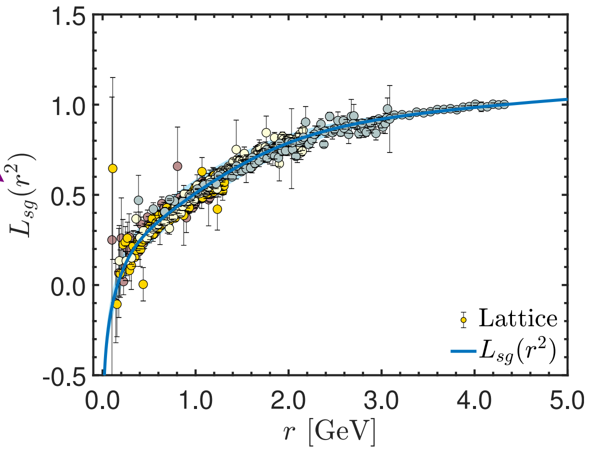
$$\overline{\Gamma}_{\alpha\mu\nu}(q,r,p) \approx \overline{\Gamma}^{0}_{\alpha\mu\nu}(q,r,p) L_{\rm sg}(s^2) \qquad s^2 := (q^2+r^2+p^2)/2$$

where $\overline{\Gamma}^{0}_{\alpha\mu\nu}(q,r,p)$ is the tree-level transversely projected vertex. (See Pepe's talk)

This **planar degeneracy** suggests the following approximation for $\mathcal{I}_{\mathcal{W}}$

$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) \approx \mathcal{I}^{0}_{\mathcal{W}}(q^2,r^2,p^2) L_{\rm sg}(s^2)$$

where $\mathcal{I}^{0}_{\mathcal{W}}$ is the tree level value of $\mathcal{I}_{\mathcal{W}}$.



- Planar degeneracy provides a compact approximation for $\mathcal{I}_{\mathcal{W}}$.
- Allows us to **integrate over entire momentum space**, because UV behavior of $L_{\rm sg}(s^2)$ is known from perturbation theory.

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

Method 2): Planar degeneracy

To quantify the accuracy of the approximation it is convenient to define

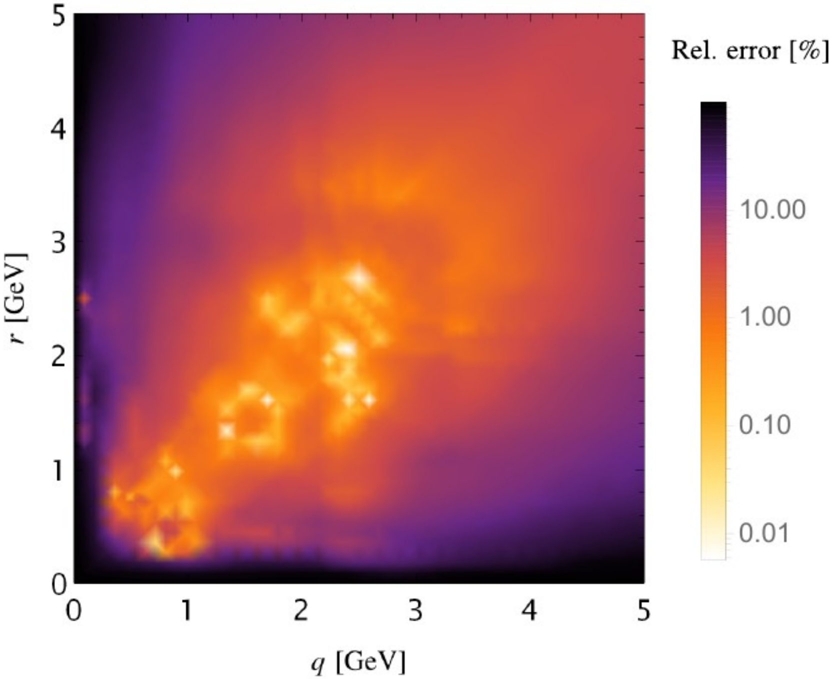
$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) := \frac{\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2)}{\overline{\mathcal{I}}^{0}_{\mathcal{W}}(q^2,r^2,p^2)}$$

Planar degeneracy →

$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) \approx L_{\rm sg}(s^2)$$

Then we can measure the relative difference between $L_{\rm sg}(s^2)$ and $\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2)$

- Approximation is accurate to within 1% near the diagonal.
- And within 10% for most of the kinematics.
- The measured error can then be propagated to the $\mathcal{W}(r^2)$



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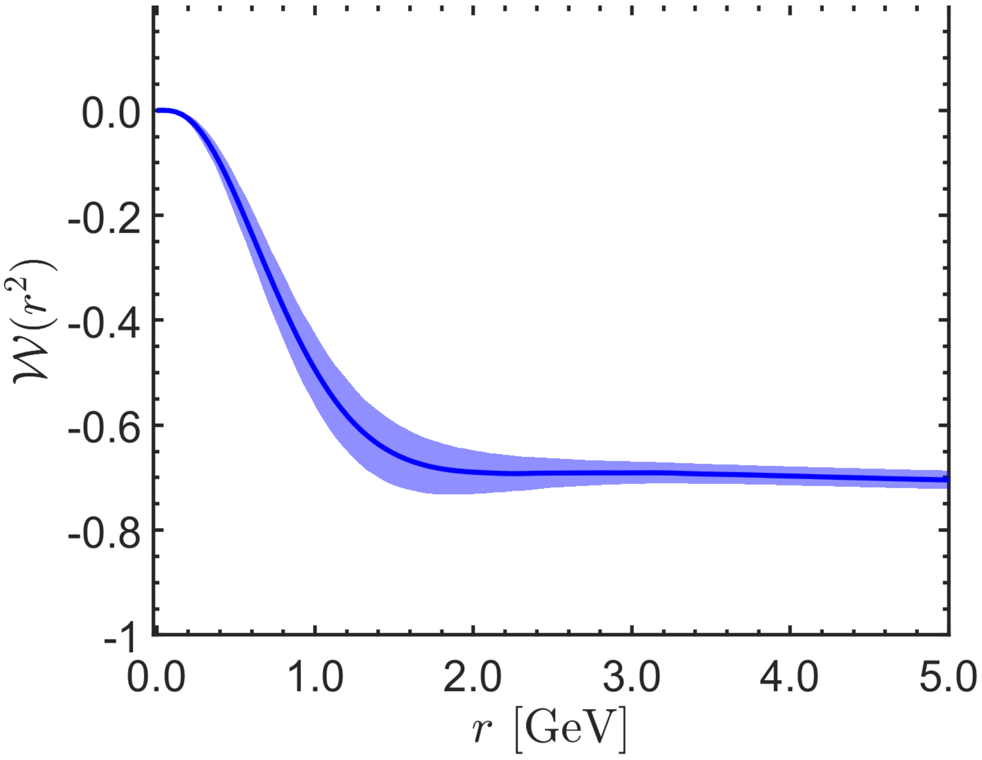
Results for $\mathcal{W}(r^2)$

We use the **planar degeneracy approximation** to obtain the **central curve**.

- **Errors are propagated from known error of the planar degeneracy approximation.**

Impact of three-gluon vertex under control



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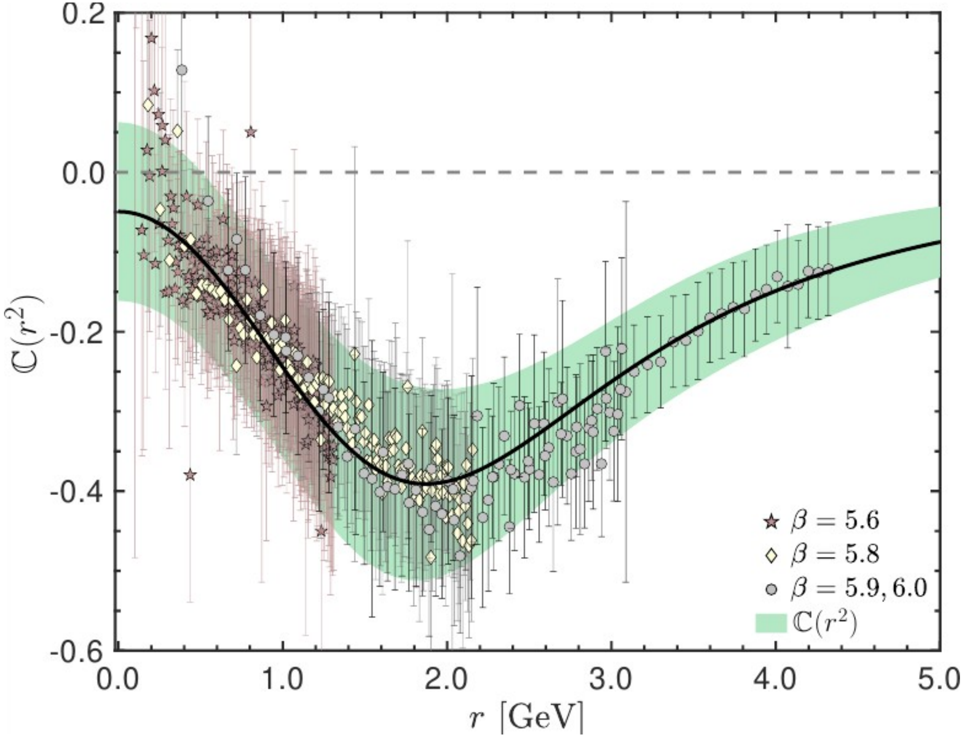
Results for $\mathbb{C}(r^2)$

With $\mathcal{W}(r^2)$ at hand, we can compute $\mathbb{C}(r^2)$ from the **WI displacement**

$$\mathbb{C}(r^2) = L_{\rm sg}(r^2) - F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right]$$

- Clearly nonzero.



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Results for $\mathbb{C}(r^2)$

With $\mathcal{W}(r^2)$ at hand, we can compute $\mathbb{C}(r^2)$ from the **WI displacement**

$$\mathbb{C}(r^2) = L_{\rm sg}(r^2) - F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right]$$

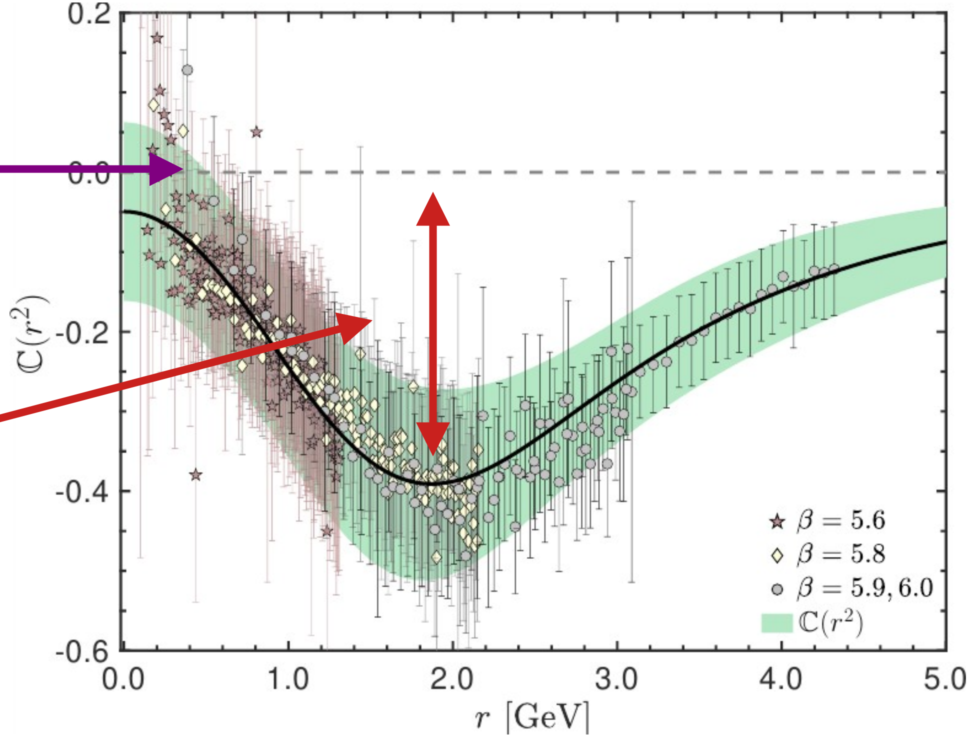
- Clearly nonzero.
- In comparison to **null hypothesis**,

$$\mathbb{C}(r^2) = \mathbb{C}_0 := 0$$

  the χ^2 is huge:

$$\chi^2_{\rm d.o.f.} = 6.36$$

- ***p*-value of null hypothesis is tiny.**
- Even if the errors were 2.2 times as large, the null hypothesis would still be discarded at the 5σ level.



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Conclusions and outlook

- The **WI displacement** allows us to determine the amplitude $\mathbb{C}(r^2)$ of the **poles that trigger the Schwinger mechanism** in **QCD**.
- Only one ingredient, $\mathcal{W}(r^2)$, not directly accessed by lattice simulations.
- We capitalize on **general kinematics lattice simulations of the three-gluon vertex to stringently control** $\mathcal{W}(r^2)$.
- Our results **strongly confirms a nonzero** $\mathbb{C}(r^2)$.
- Additional SDE analysis are been performed to **explore further signals of the Schwinger mechanism**.
- Moreover, it would be instructive to understand the origin of the **planar degeneracy from the vertex SDE**.