Nucleon Electromagnetic Form Factors at Large Momentum from Lattice QCD

Sergey Syritsyn (Stony Brook University)
for Lattice Hadron Physics collaboration

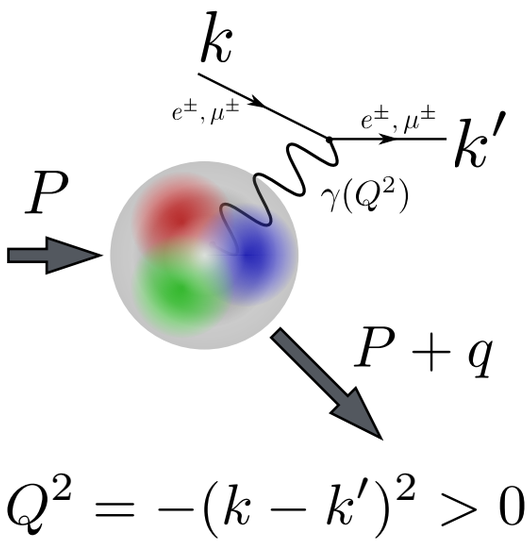
National Energy Research
Scientific Computing Center

BARYONS 2022
Seville, Nov 8

Outline

- Nucleon vector form factors at large momentum
- Challenges for large-Q^2 hadron structure on lattice
- Preliminary results and comparison to experiment & phenomenology
- Examining systematic effects
- Summary and Outlook

Nucleon Elastic E&M Form Factors



$$Q^2 = -(k-k')^2 > 0$$

Elastic e^-p amplitude

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[\overbrace{F_1(Q^2)}^{\text{(Dirac)}}\gamma^\mu + \overbrace{F_2(Q^2)}^{\text{(Pauli)}}\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$

Sachs Electric $\quad G_E(Q^2) = F_1(Q^2) - \dfrac{Q^2}{4M^2}F_2(Q^2)$

Magnetic $\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$

Elastic e^-p cross-section

- $G_{E,M}$ from ϵ-dep. at fixed $\tau(Q^2)$ ("***Rosenbluth separation***")
- dominated by G_M at large Q^2
- 2γ corrections at $Q^2 \gtrsim 1$ GeV^2

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\rm Mott}}{1+\tau}\left[G_E^2 + \frac{\tau}{\epsilon}G_M^2\right]$$

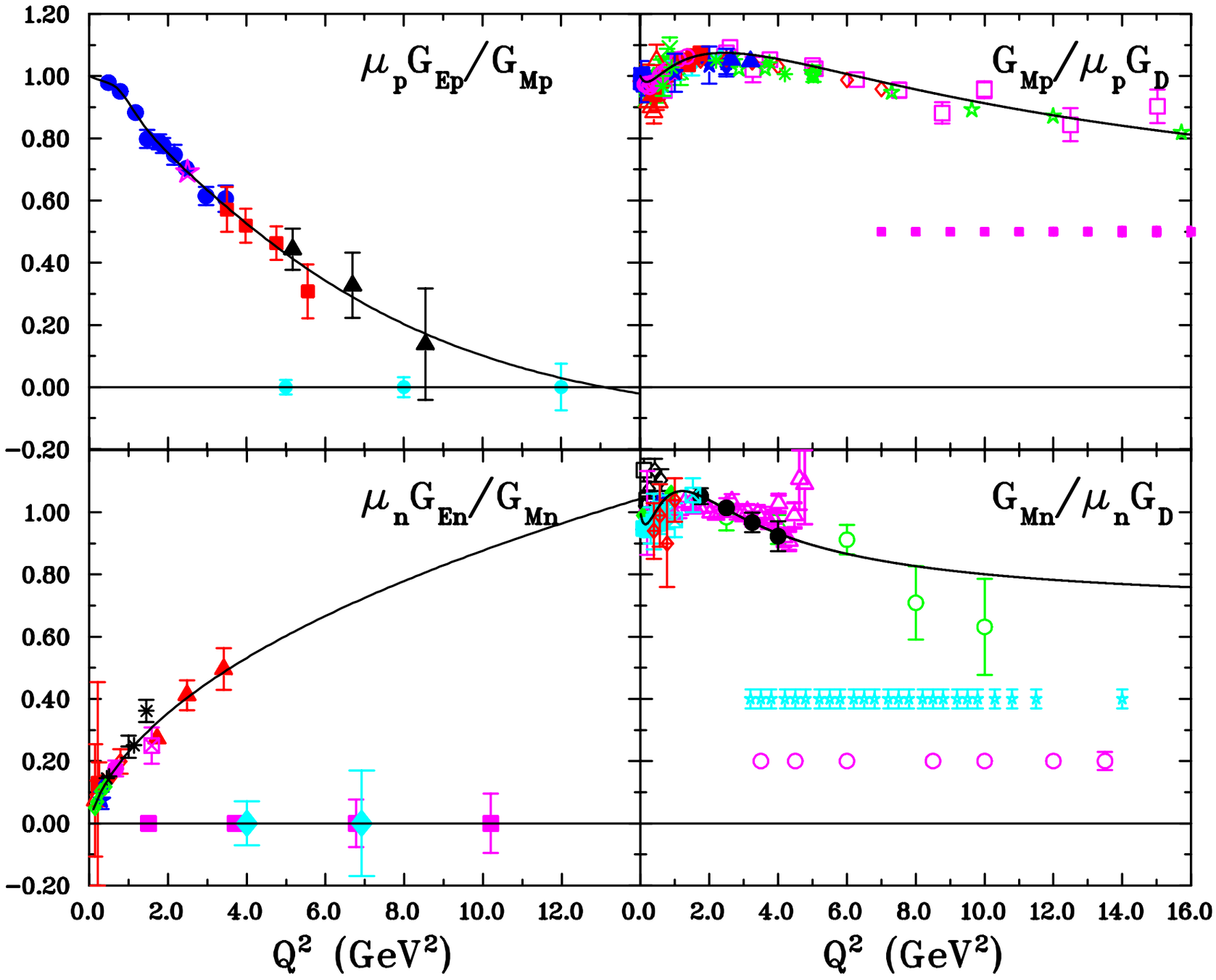
$$\tau = \frac{Q^2}{4M_N^2} \qquad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

Polarization transfer: polarized e^-beam
+ detect polarization of recoil nucleon
(alt.: transverse asymmetry on pol. target)

- G_E/G_M *ratio* (only small radiative corrections)

$$P_t/P_l \propto G_E/G_M$$

Recent/Ongoing Experiments

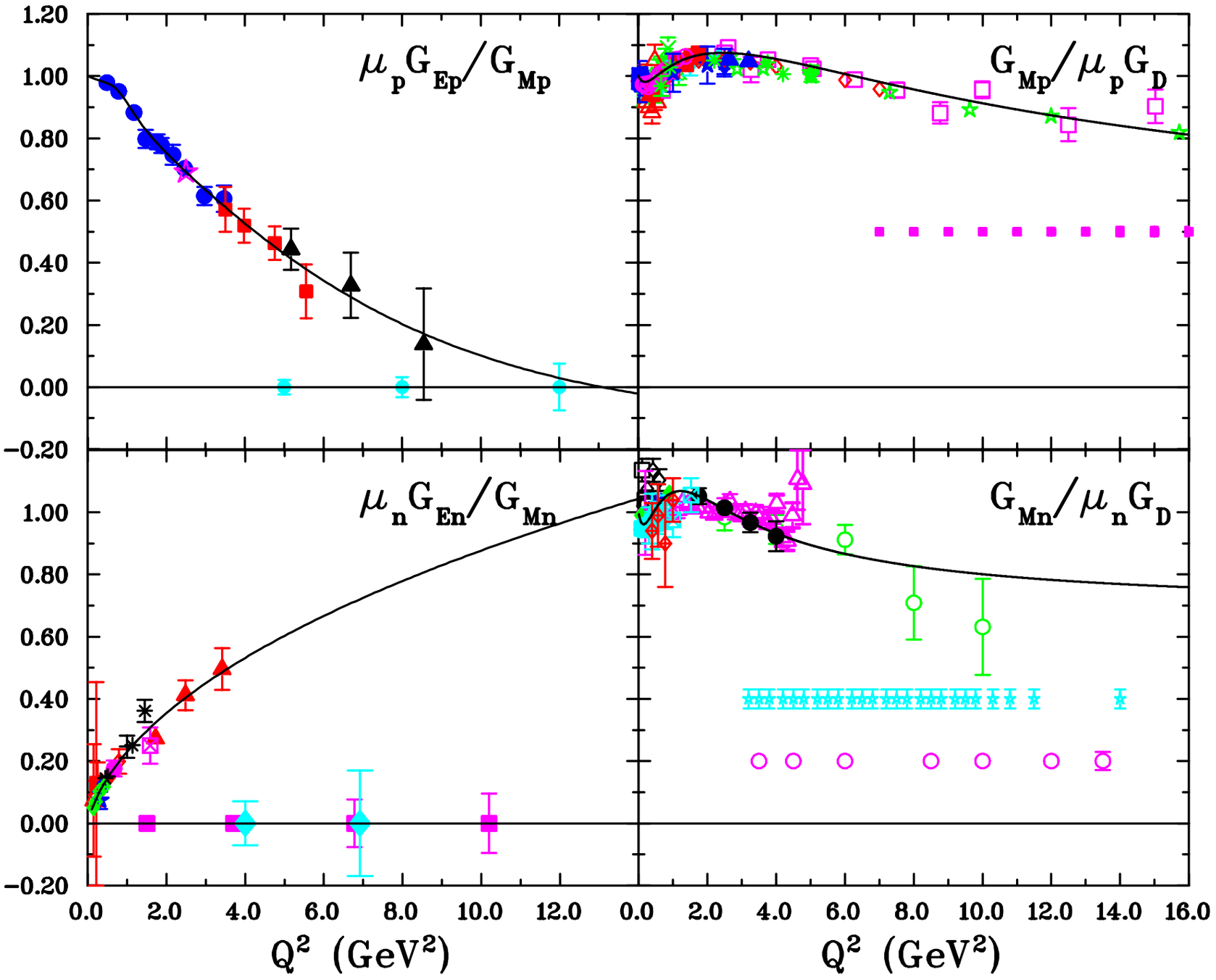


Projected new precision on proton & neutron form factors
[V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]

Experiments at JLab@12GeV

- Hall A (HRS, SBS):
  G_{Mp} @ $Q^2 \lesssim 17.5$ GeV^2
  G_{Ep}/G_{Mp} @ $Q^2 \lesssim 15$ GeV^2 ;
  G_{Mn} @ $Q^2 \lesssim 18$ GeV^2
  G_{En}/G_{Mn} @ $Q^2 \lesssim 10.2$ GeV^2 ;
- Hall B (CLAS12):
  G_{Mn} @ $Q^2 \lesssim 14$ GeV^2
- Hall C :
  G_{En}/G_{Mn} @ $Q^2 \lesssim 6.9$ GeV^2

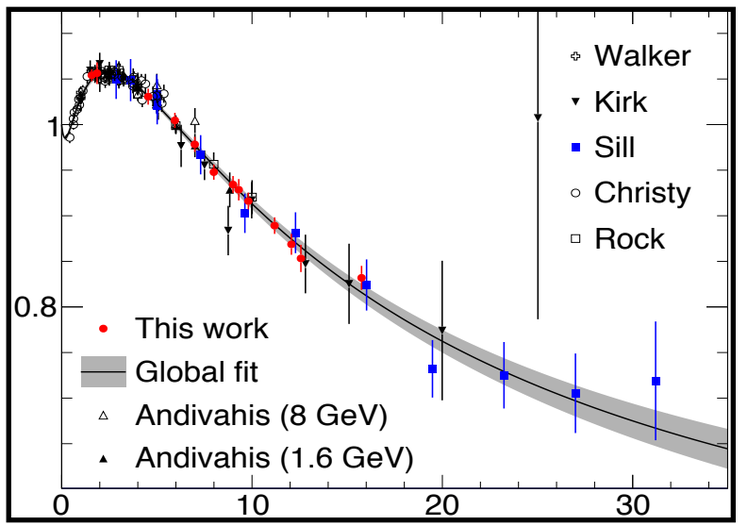
Recent/Ongoing Experiments



Projected new precision on proton & neutron form factors
[V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]

Experiments at JLab@12GeV

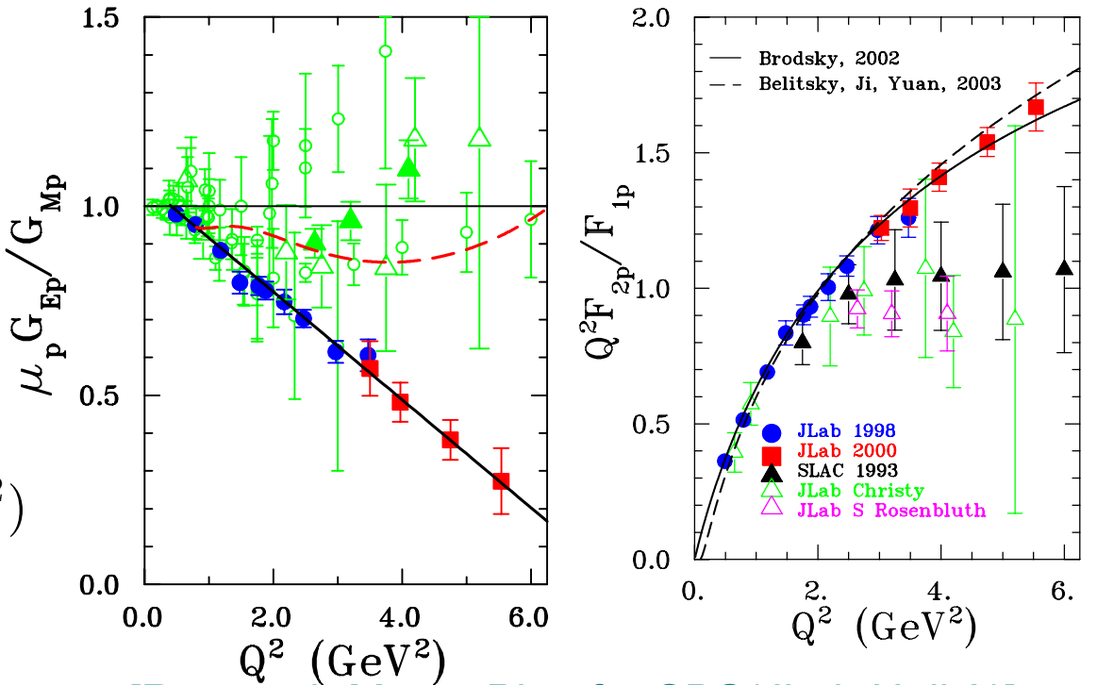
- Hall A (HRS, SBS):
  G_{Mp} @ $Q^2 \lesssim 17.5$ GeV^2
  G_{Ep}/G_{Mp} @ $Q^2 \lesssim 15$ GeV^2 ;
  G_{Mn} @ $Q^2 \lesssim 18$ GeV^2
  G_{En}/G_{Mn} @ $Q^2 \lesssim 10.2$ GeV^2 ;
- Hall B (CLAS12):
  G_{Mn} @ $Q^2 \lesssim 14$ GeV^2
- Hall C :
  G_{En}/G_{Mn} @ $Q^2 \lesssim 6.9$ GeV^2



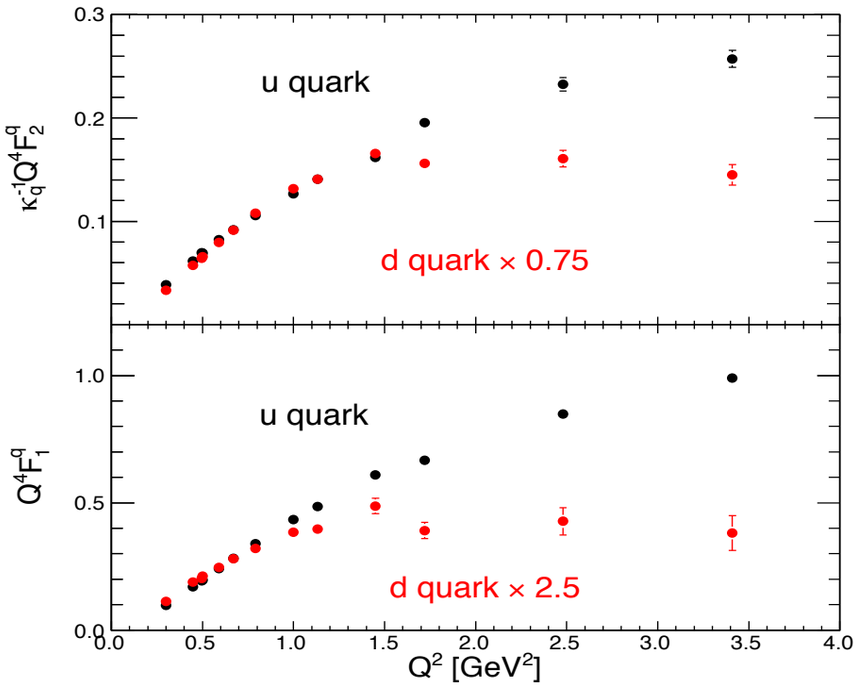
New G_{Mp} data from Hall A
[Christy et al, PRL'22]

Nucleon Form Factors: Open Questions

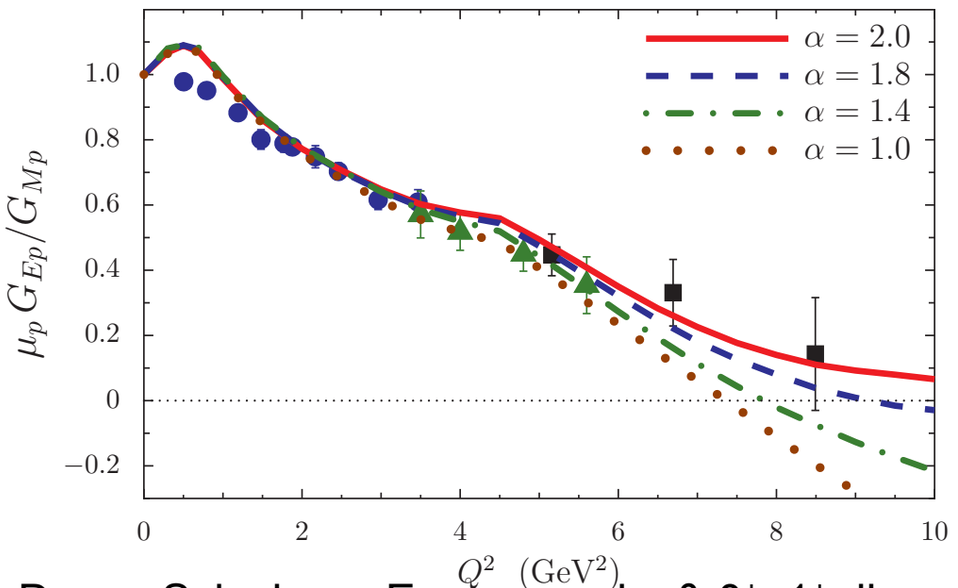
- *Are model descriptions of the nucleon viable ?*
  Nucleon models disagree beyond explored range
- *Role of diquark correlations in elastic scattering ?*
  Neutron & proton G_E/G_M at/above $Q^2 = 8$ GeV^2
- *Scale of transition to perturbative QCD ?*
  (F_2/F_1) scaling at large Q^2 : $Q^2 F_{2p}/F_{1p} \overset{?}{\propto} \log^2(Q^2/\Lambda^2)$
- *What are contributions from u and d flavors?*
  Proton and neutron data needed in wide Q^2 range



[Research Mgmt. Plan for SBS(JLab Hall A)]



[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]



Dyson-Schwinger Eqns : quarks & 0^+, 1^+ diquarks
($\alpha \approx$ rate of transition const.quarks → pQCD with Q^2)
[Cloet, Roberts, Prog.Part.Nucl.Phys 77:1 (2014)]

Nucleon Structure from QCD on a Lattice

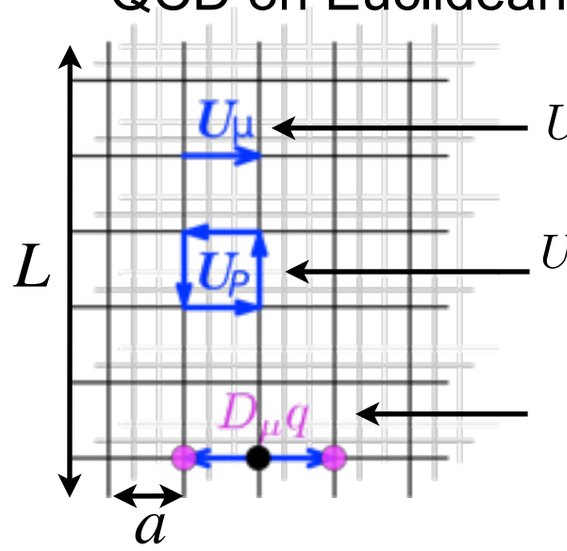
Lattice Field Theory ⇔ Numerical evaluation of the Path Integral

$$\langle q_x \bar{q}_y \ldots \rangle = \int \mathcal{D}\big(Glue\big) \int \mathcal{D}\big(Quarks\big)\; e^{-S_{Glue} - \bar{q}\left(\not{D}+m\right)q}\; \big[q_x \bar{q}_y \ldots\big]$$

Grassmann integration

$$= \int \underbrace{\mathcal{D}\big(Glue\big)\; e^{-S_{Glue}}\; \mathrm{Det}\left(\not{D}+m\right)}_{\text{MC sampling of gluon field}}\; \underbrace{\big[\left(\not{D}+m\right)^{-1}_{x,y} \ldots\big]}_{\text{Quark Propagation}}$$

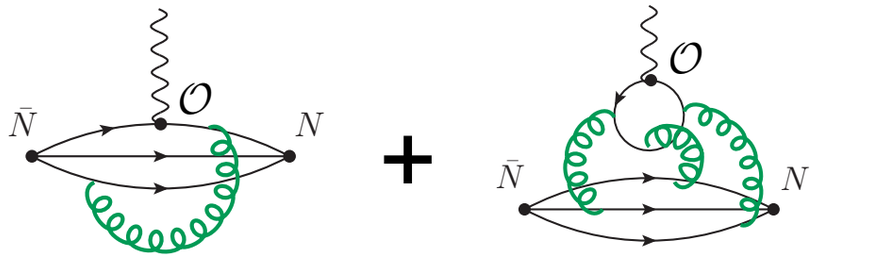
QCD on Euclidean 4D lattice



Hadron Matrix Elements:

$$C^{\mathcal{O}}_{3\mathrm{pt}}(T) = \langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle =$$

$$[N \;\propto\; (uC\gamma_5 d)u]$$



quark-connected **quark-disconnected**

$$\langle N(T)\mathcal{O}(\tau)N(0)\rangle = \sum_{n,m} Z_m e^{-E_n(T-\tau)} \langle n|\mathcal{O}|m\rangle e^{-E_m \tau} Z^*_n$$

$$\xrightarrow[T\to\infty]{} Z_{00} e^{-M_N T}\Big[\langle P'|\mathcal{O}|P\rangle + \mathcal{O}\big(\underbrace{e^{-\Delta E_{10} T}, e^{-\Delta E_{10}\tau}, e^{-\Delta E_{10}(T-\tau)}}_{\text{excited states}}\big)\Big]$$

Ground state form factors

Fit and discard

Systematic uncertainties

- discretization effects
- unphysical-heavy pion mass
- finite volume
- excited state contributions
- (costly) disconnected diagrams

Challenges at Large Q^2

- Multiscale problem: $a \ll p_N^{-1}, Q^{-1}\,; \quad 4m_\pi^{-1} \lesssim L$

- Discretization effects:
  O(a) Correction to current operator

$$(V_\mu)_I = [\bar{q}\gamma_\mu q] + c_V\, a\, \underbrace{\partial_\nu [\bar{q} i\sigma_{\mu\nu} q]}_{\propto Q}$$

- Stochastic noise grows faster with T [Lepage'89]:

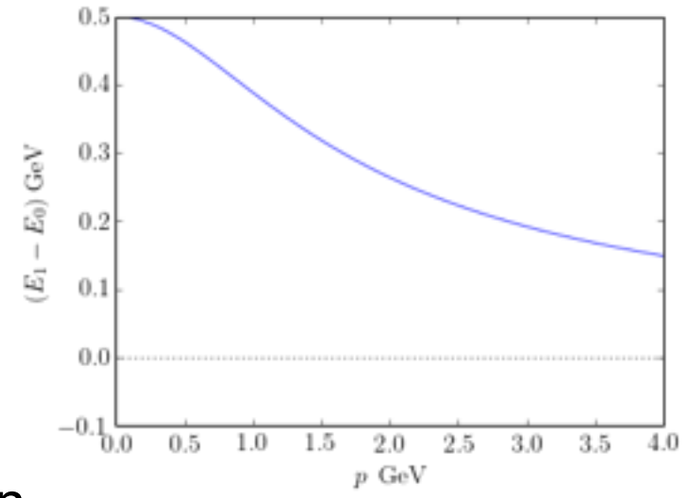
$$\begin{aligned}
&\text{Signal} && \langle N(T)\bar{N}(0)\rangle && \sim e^{-E_N T}\\
&\text{Noise} && \langle |N(T)\bar{N}(0)|^2\rangle - |\langle N(T)\bar{N}(0)\rangle|^2 && \sim e^{-3m_\pi T}\\
&\text{Signal/Noise} && && \sim e^{-(E_N - \frac{3}{2}m_\pi)T}
\end{aligned}$$

SNR reduction
at 1 fm/c ***~ O(10^{-4})***
(phys.quarks, $Q^2 \approx 12$ GeV^2)

- Excited states: boosting "shrinks" the energy gap

$$E_1 - E_0 = \sqrt{M_1^2 + \vec{p}^2} - \sqrt{M_2^2 + \vec{p}^2} < M_1 - M_0$$

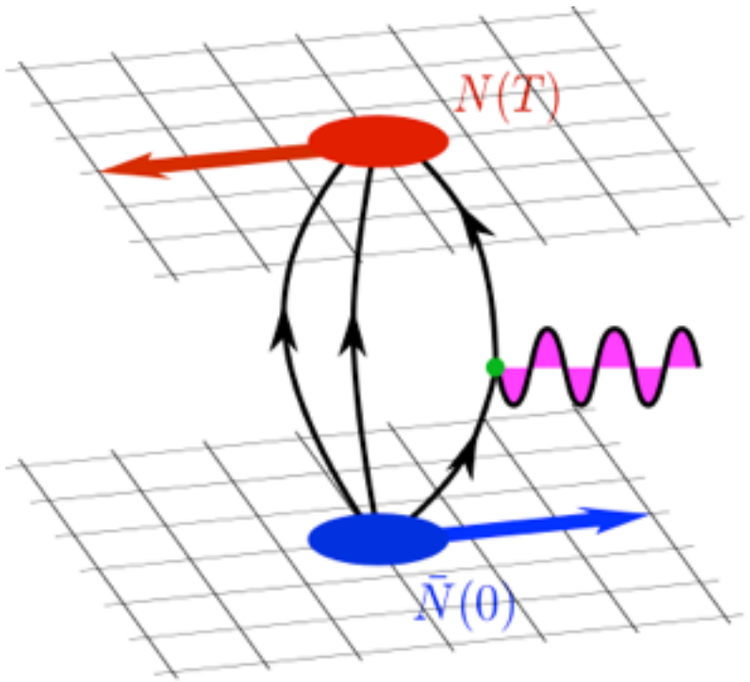
  - N(~1500): pN→1.5 GeV ⇒ ΔE = 500 → 300 MeV



- Large p_N: no reliable EFT/ChPT for mπ-, lattice size-extrapolation

Large statistics required to suppress MC noise in lattice correlators

Accessing Large Q^2 : Breit Frame on a Lattice



Minimize $E_{in,out}$ for target Q^2 :

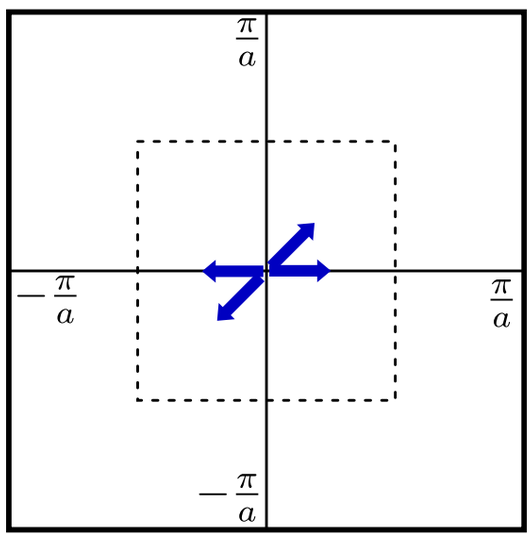
$$Q^2 = (\vec{p}_{in} - \vec{p}_{out})^2 - (E_{in} - E_{out})^2$$

Back-to-back

$$Q^2 = 4\vec{p}^2$$

For $(Q^2)_{max}$ = *10 GeV²*

$$|\vec{p}| = \frac{1}{2}\sqrt{Q^2_{\max}} \approx 1.6 \text{ GeV} \qquad (E_N \approx 1.9 \text{ GeV})$$



lattice kinematics for $Q^2 \approx 10$ GeV²

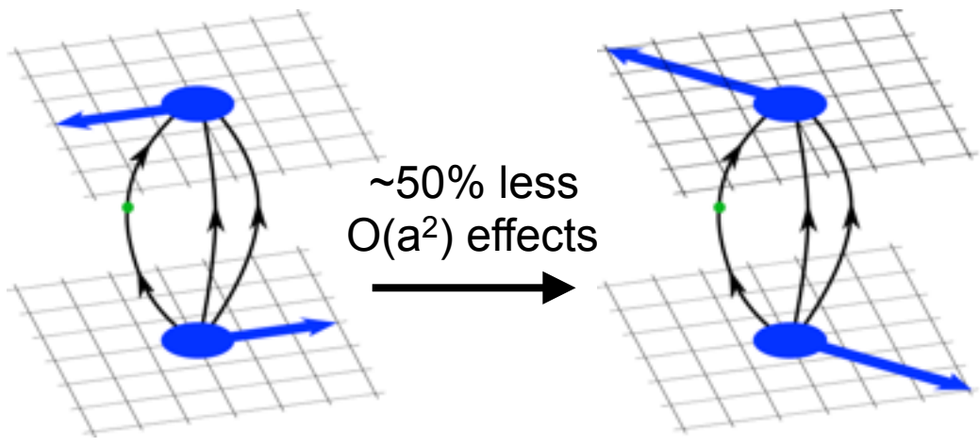
Nucleon momentum ~ Brillouin zone
⇒ unknown distortion of effective nucleon Dirac eqn.

$$\langle N\bar{N}\rangle^{-1}(p) \stackrel{?}{=} -i\not{p}^{\text{lat}} + m_N$$

$$p^{\text{lat}}_\mu = k_\mu + O(k^3)$$

⇒ **addl**. O(a^2) corrections

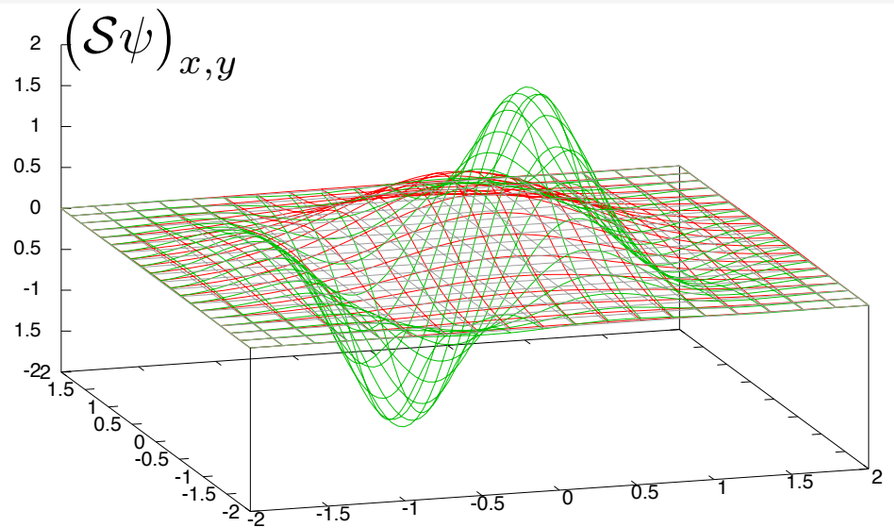
from lattice nucleon polarization



High-momentum Hadron States on a Lattice

Nucleon operator is built from ≈Gaussian smeared quarks

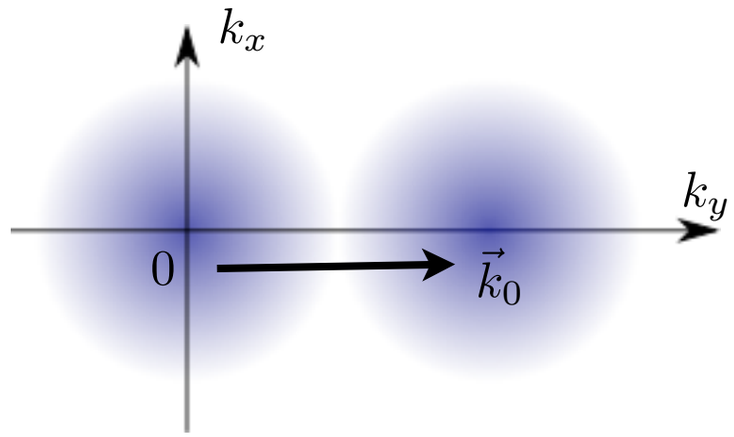
$$N_{\rm lat}(x) = \left(\mathcal{S}\,u\right)^a_x \left[\left(\mathcal{S}\,d\right)^b_x C\gamma_5 \left(\mathcal{S}\,u\right)^c_x\right] \epsilon^{abc}$$



Gaussian shape in momentum space :
reduced overlap with quark WFs in a boosted nucleon

$$\mathcal{S}_{\text{at-rest}} = \exp[-\frac{w^2}{4}(i\vec{\nabla})^2] \;\sim\; exp(-\frac{w^2\vec{k}^2_{\rm lat}}{4})$$

[G.Bali et al, PRD93:094515; arXiv:1602.05525]:
improve the overlap with large-P_N nucleon by shifting the quark trial shape in momentum space (*"momentum smearing"*)



$$\mathcal{S}_{\vec{k}_0} = \exp[-\frac{w^2}{4}(-i\vec{\nabla} - \vec{k}_0)^2] \;\sim\; \exp(-\frac{w^2(\vec{k}_{\rm lat} - \vec{k}_0)^2}{4})$$

Modified smearing operator

$$\left[\mathcal{S}_{\vec{k}_0}(\psi)\right]_x = e^{+\vec{k}_0\vec{x}}\mathcal{S}(e^{-\vec{k}_0\vec{y}}\psi_y) \sim e^{+\vec{k}_0\vec{x}} \cdot \text{smooth fcn.}(x)$$

Modified covariant smearing operator in lattice*color space

$$\left[\mathcal{S}_{\vec{k}_0}\right]_{x,y} = e^{+i\vec{k}_0\vec{x}}\left[\mathcal{S}\right]_{x,y}e^{-i\vec{k}_0\vec{y}}$$

$\Longleftrightarrow$

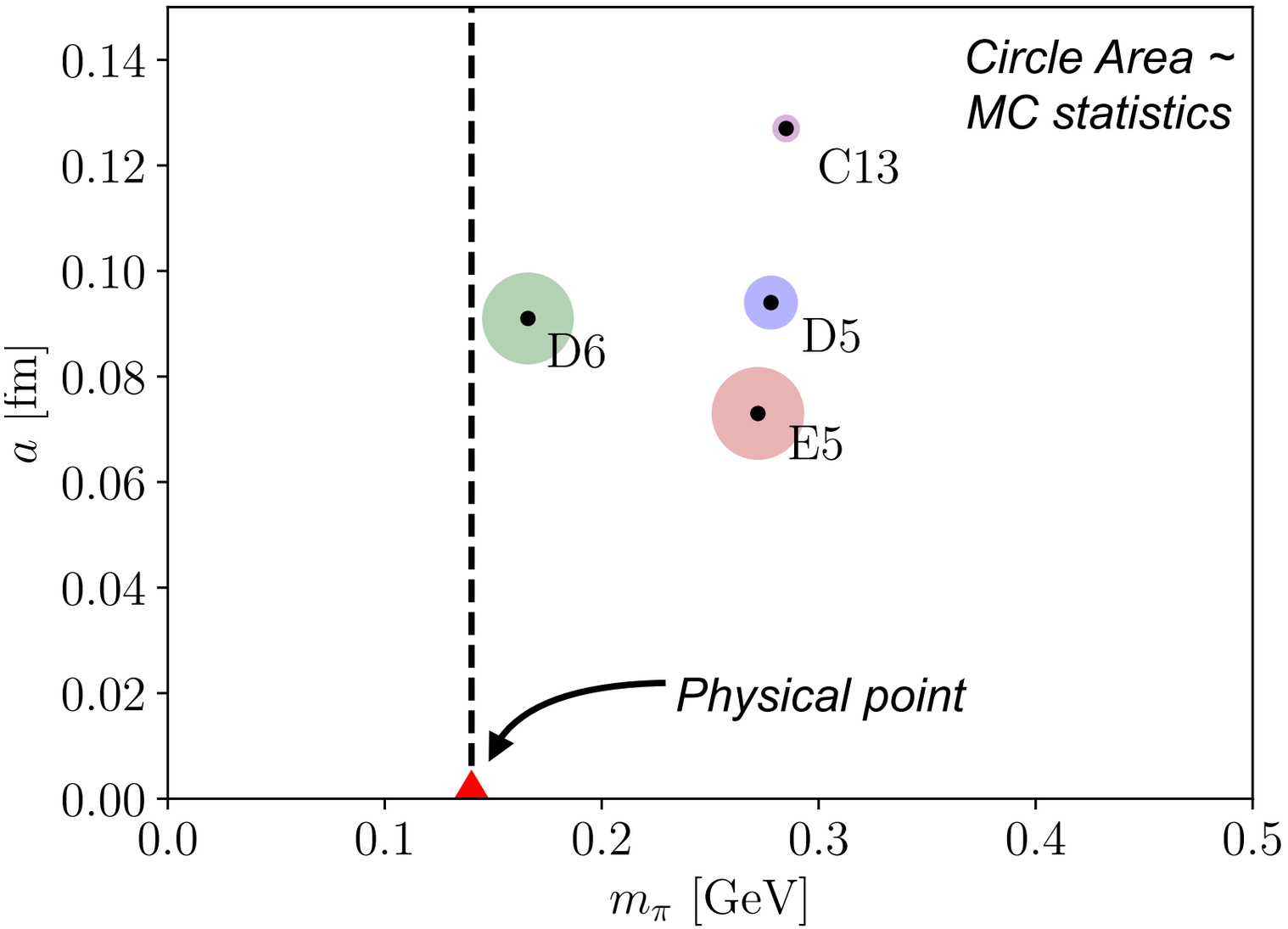
Smearing with twisted gauge links

$$\Delta_{x,y} \longrightarrow e^{+i\vec{k}_0\vec{x}}\Delta_{x,y}e^{-i\vec{k}_0\vec{y}}$$

$$U_{x,\mu} \longrightarrow e^{-ik_\mu}U_{x,\mu}$$

Present QCD Calculation Parameters

- $N_F = 2+1$ clover-improved Wilson fermion ensembles (JLab / W&M / LANL / MIT)
- lattice spacing $a \approx 0.07 \div 0.12$ fm
- unphysical light quark masses : $m\pi = 170 \div 280$ MeV
- large physical volume $L \gtrsim 3.7\ (m\pi)^{-1}$



2022: MC Statistics ~250k on

- D6 ($48^3 \times 96$)
- E5 ($48^3 \times 128$)

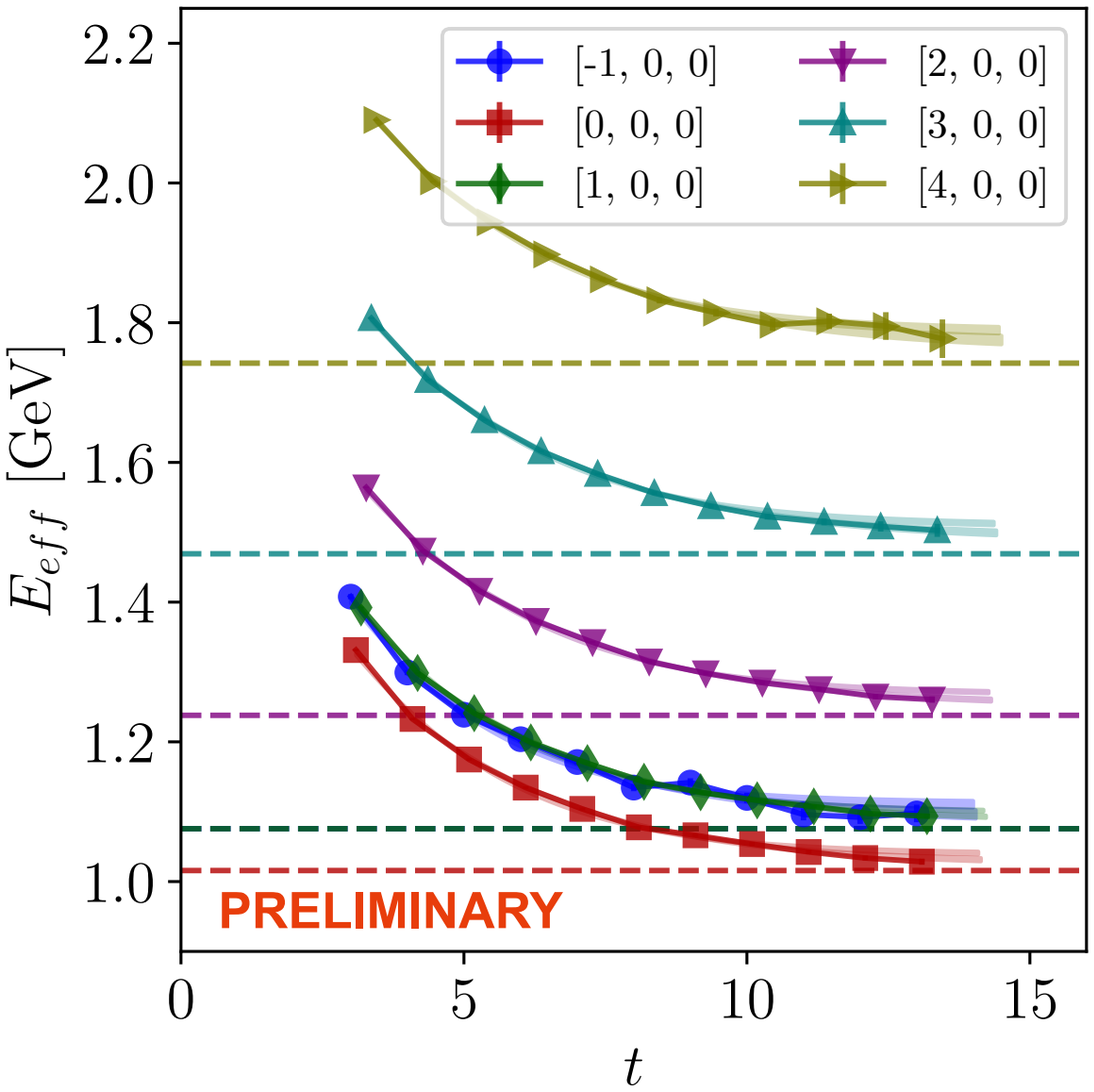
Made possible by new nVidia A100 clusters

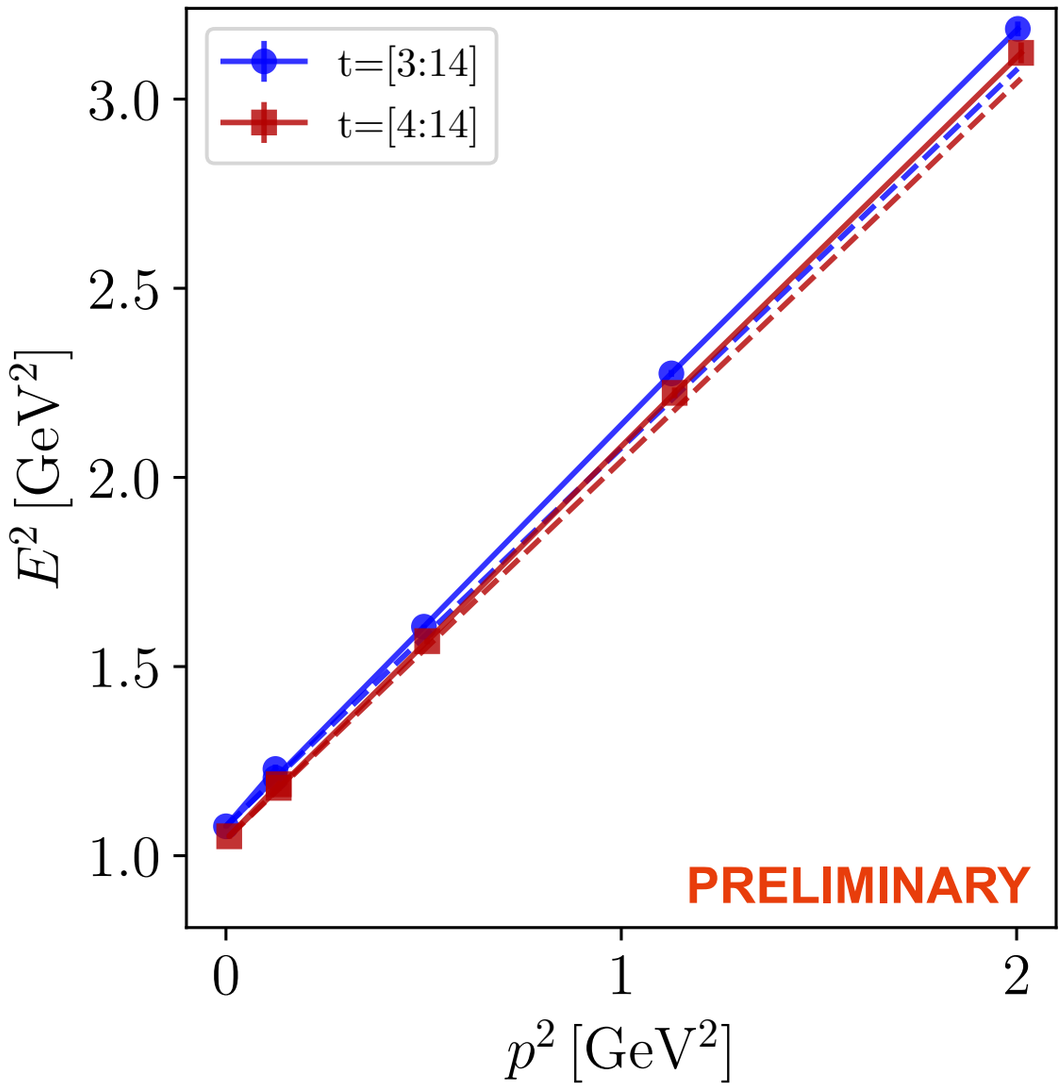
- Perlmutter [NERSC]
- Juwels-Booster [Fz. Juelich]

Special thanks to the QUDA team
[K. Clark, R.Babich, R.Brower, M.Wagner, E.Weinberg, and many others]

Lattice Nucleon Energy & Dispersion Relation (E5)

- E5 : $m\pi = 272$ MeV , spacing $a = 0.073$ fm , 266k MC samples





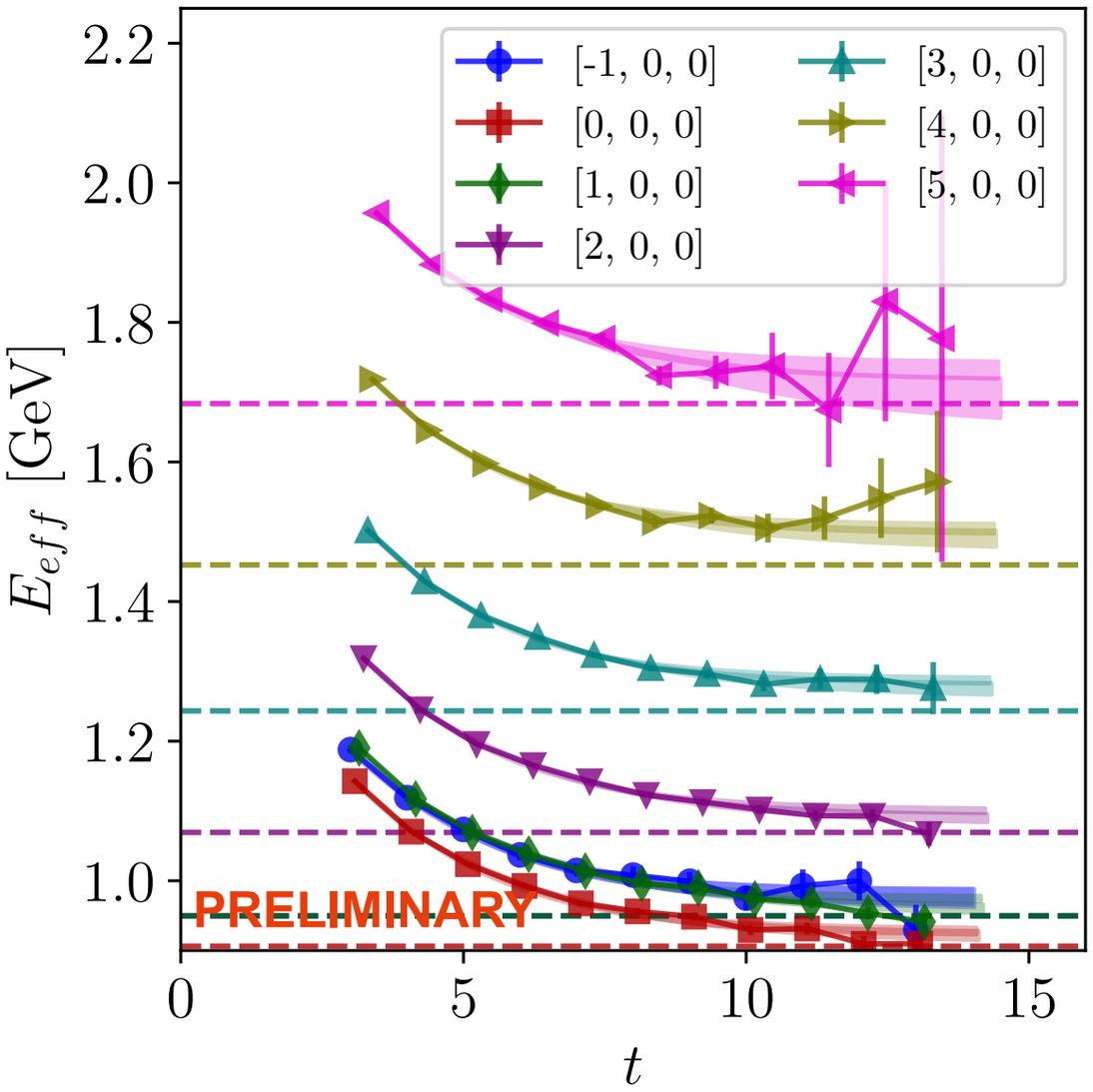
- Effective energy and 2-state fits

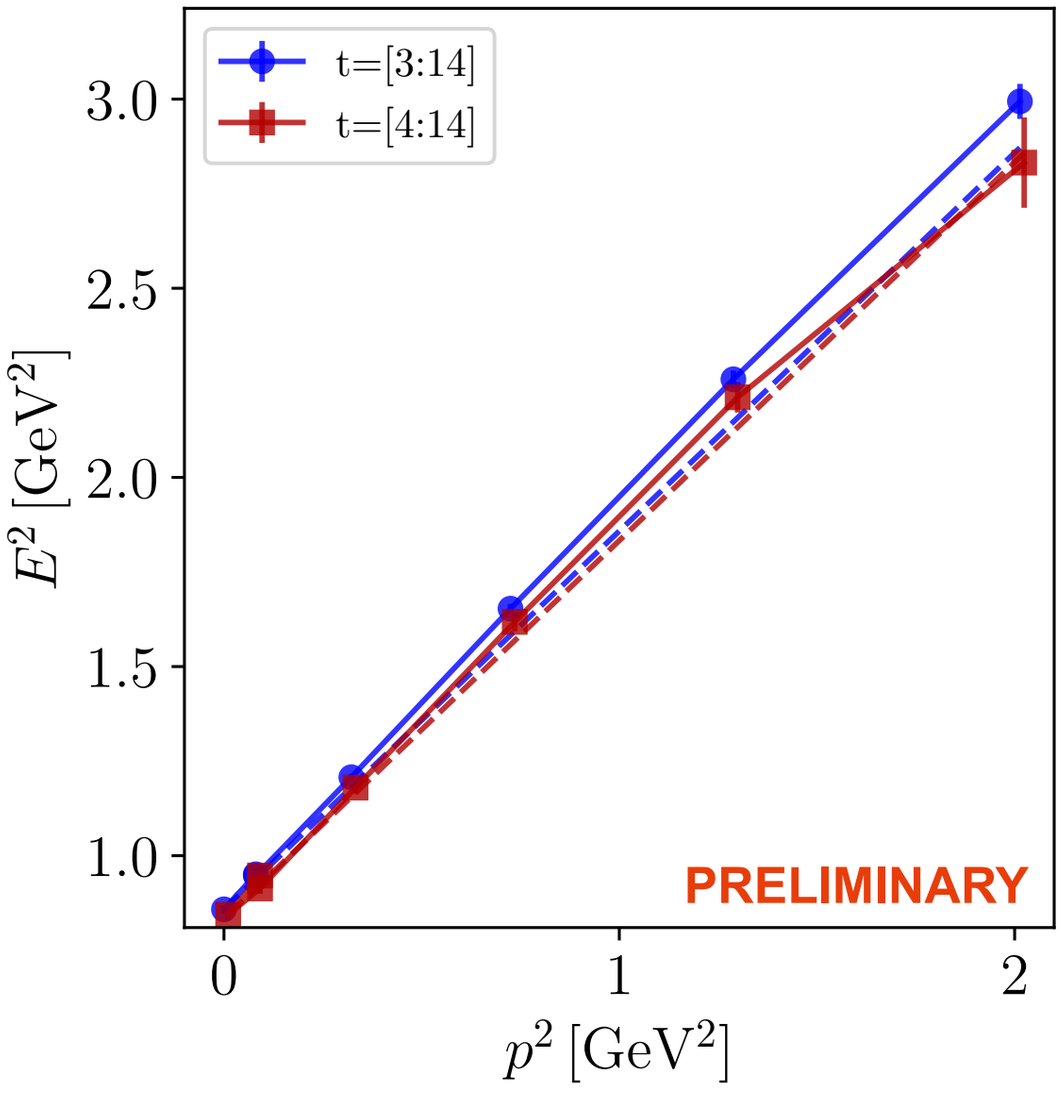
$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$

- Dispersion relation
  Dashed lines: cont. $E^2(p) = E^2(0) + p^2$

Lattice Nucleon Energy & Dispersion Relation (D6)

- D6 : $m\pi =$ 166 MeV , spacing a = 0.091 fm , 261k MC samples



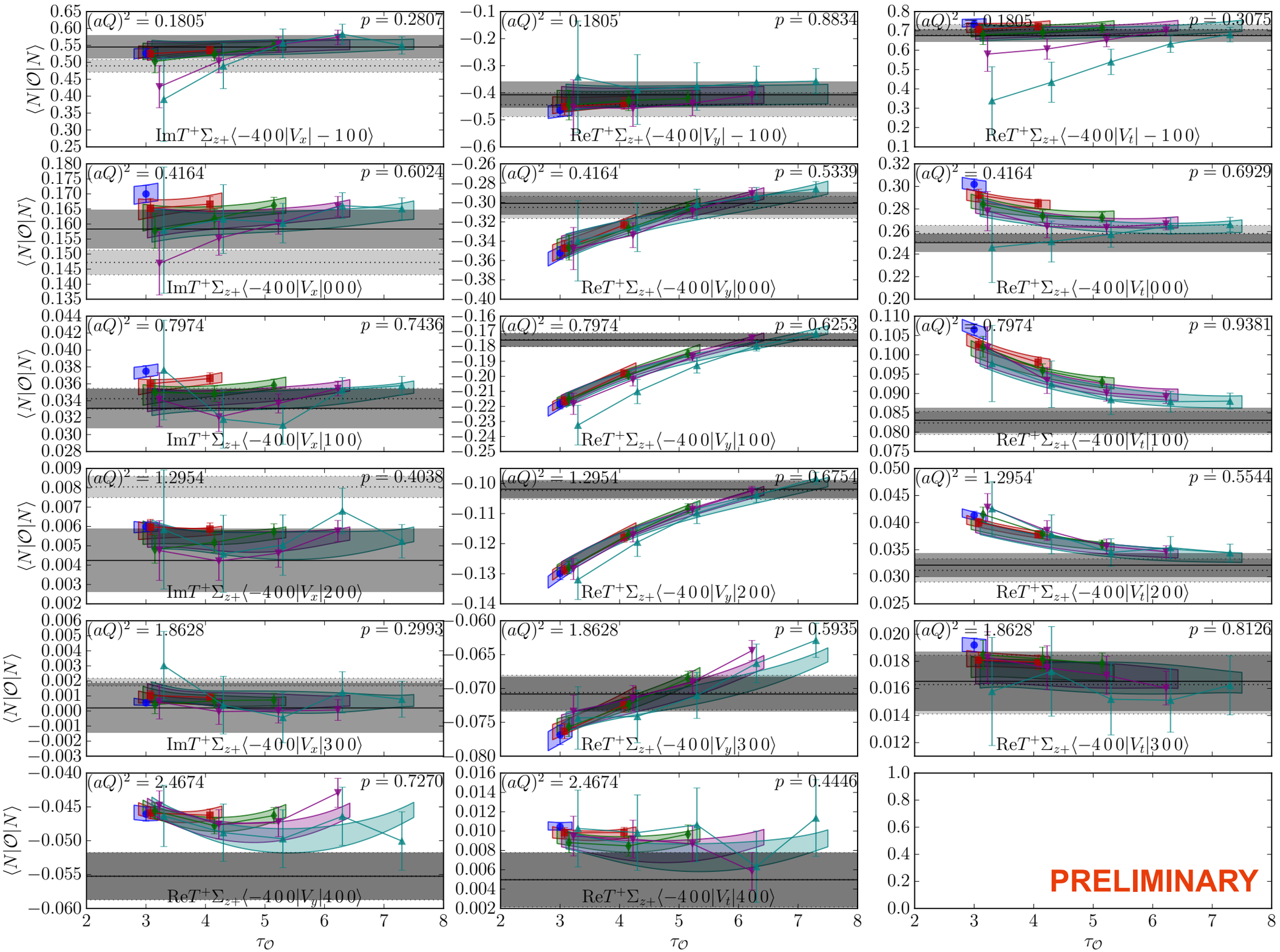


- Effective energy and 2-state fits

$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$

- Dispersion relation
  Dashed lines: cont. $E^2(p) = E^2(0) + p^2$

Nucleon Matrix Element & Form Factor Fits (D5)



$(aQ)^2 = 0.18$ $N_{dof} = 1$

$(aQ)^2 = 0.42$ $N_{dof} = 1$

$(aQ)^2 = 0.80$ $N_{dof} = 1$

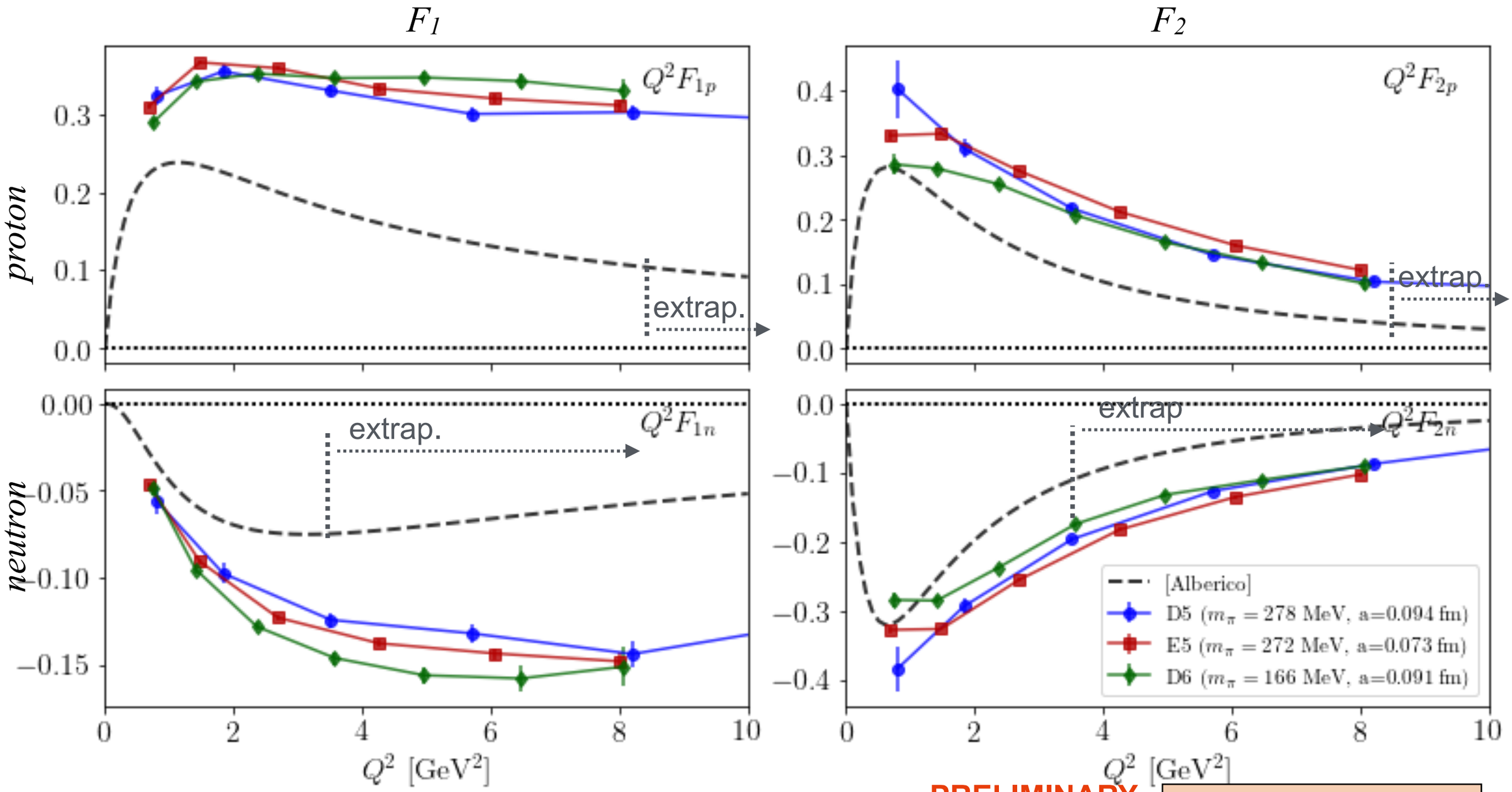
$(aQ)^2 = 1.30$ $N_{dof} = 1$

$(aQ)^2 = 1.86$ $N_{dof} = 1$

$(aQ)^2 = 2.47$ $N_{dof} = 0$

Nucleon Form Factors

- 2-state fits to extract the ground state
- discrepancy x(2..2.5) for Q2>2GeV2: exc.states? discretization? quark mass
- *Phenomenology curves* : [Alberico et al, PRC79:065204 (2008)]

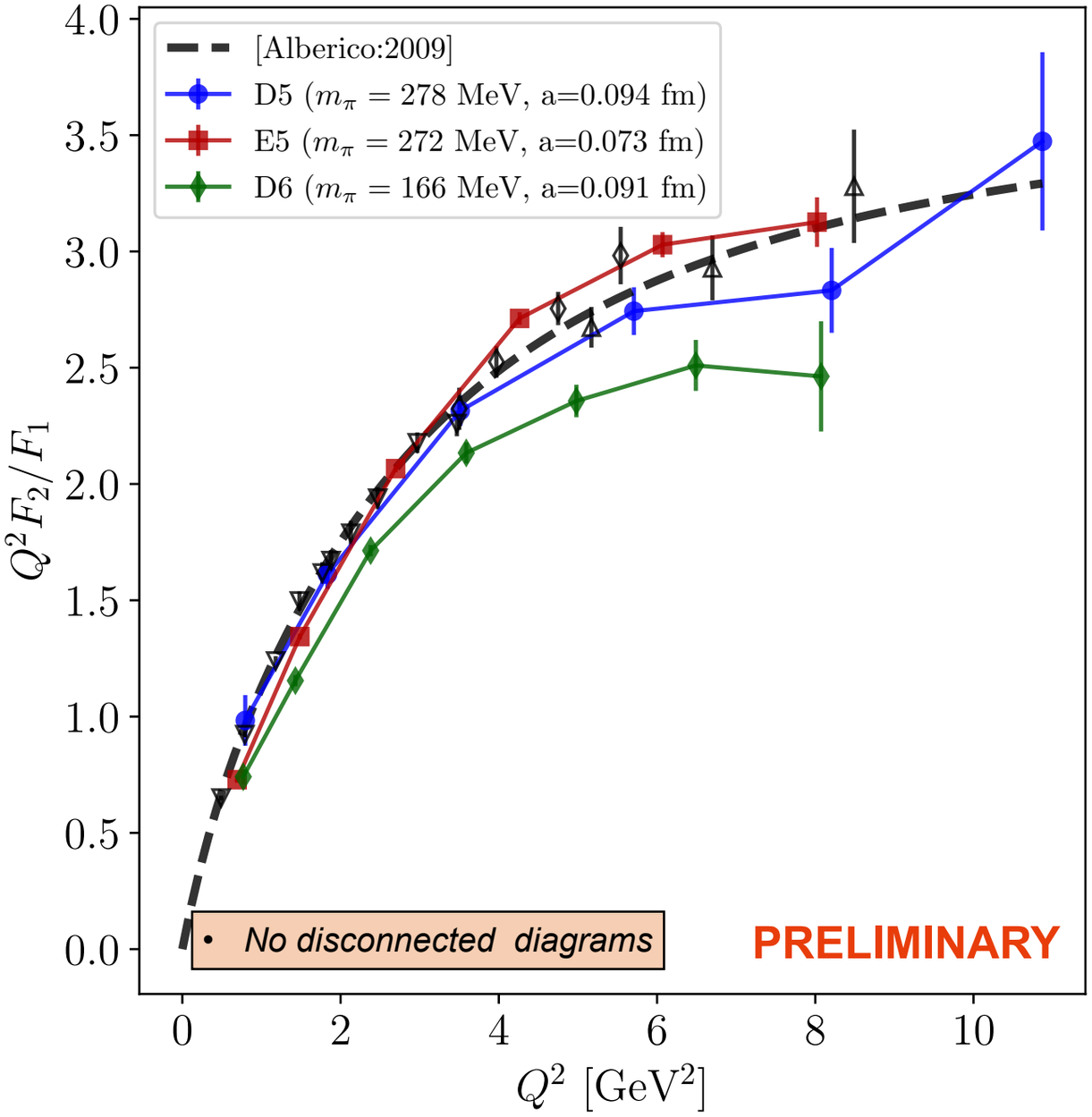


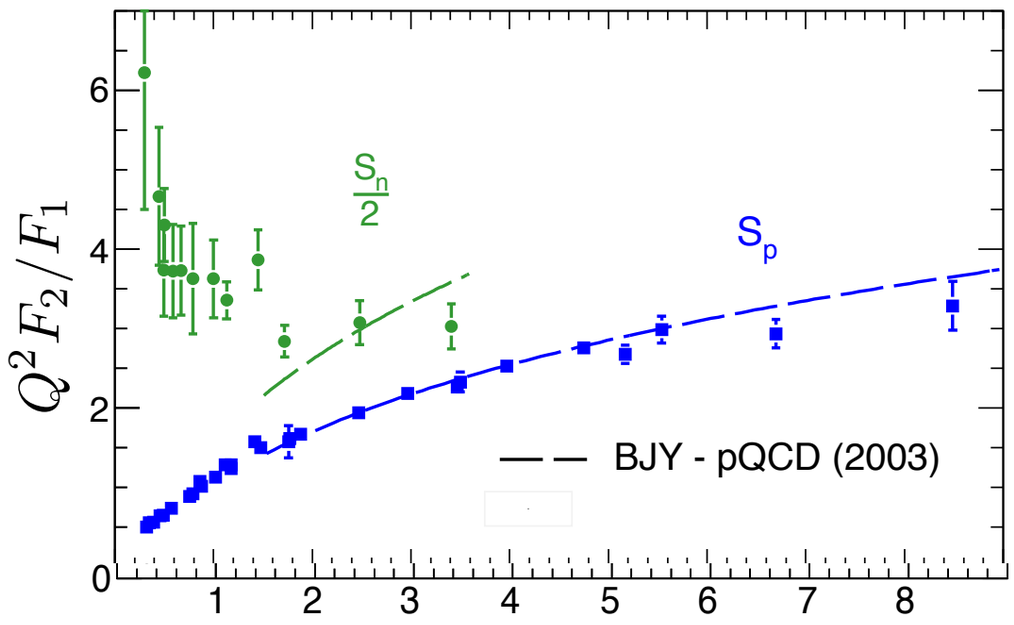
PRELIMINARY

- *No disconnected diagrams*

Proton F_2/F_1 Ratio

- Lattice data: 2-state fits
- Phenomenology curves : [Alberico et al, PRC79:065204 (2008)]
- Comparison to experimental data (black points)

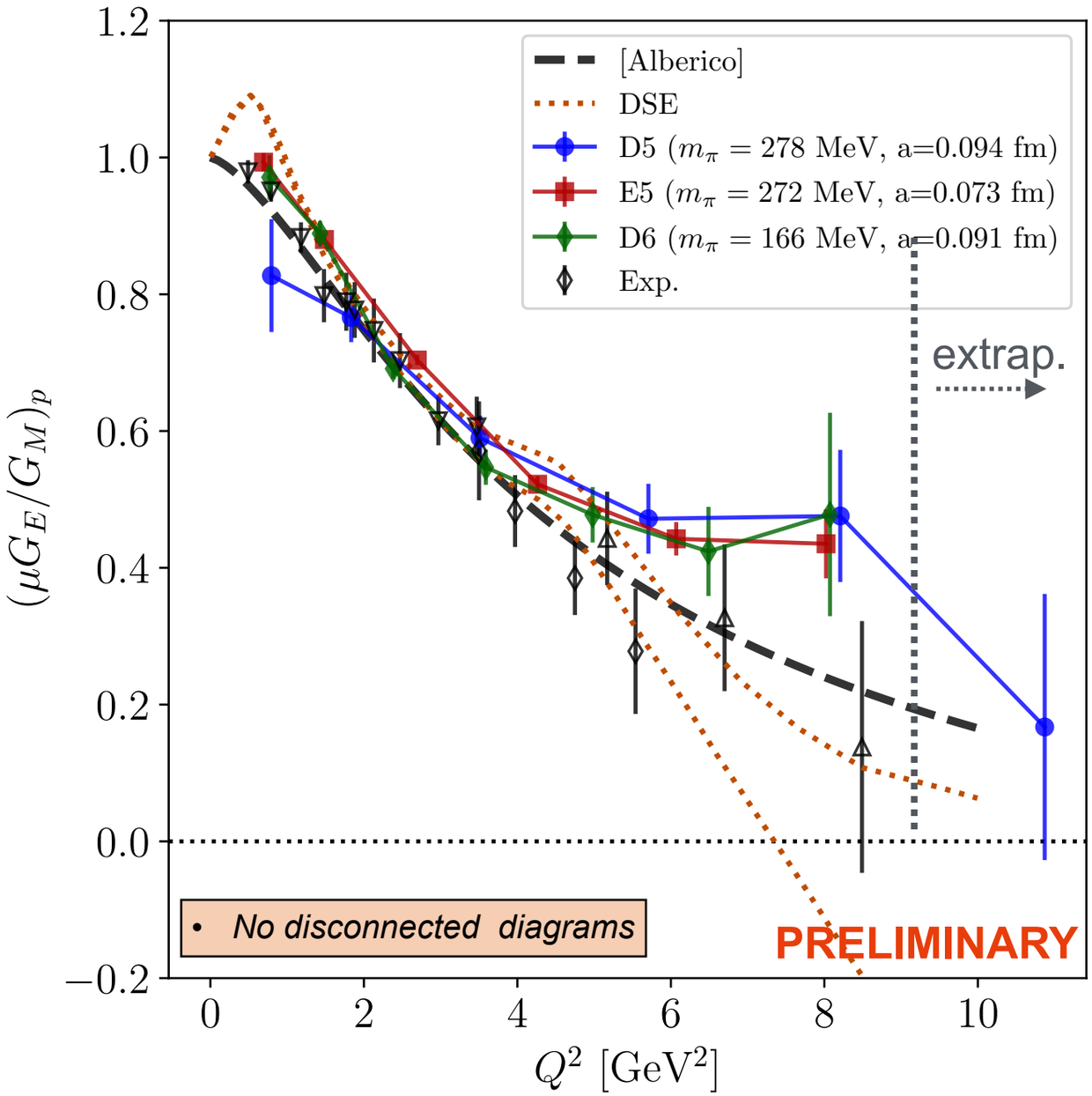


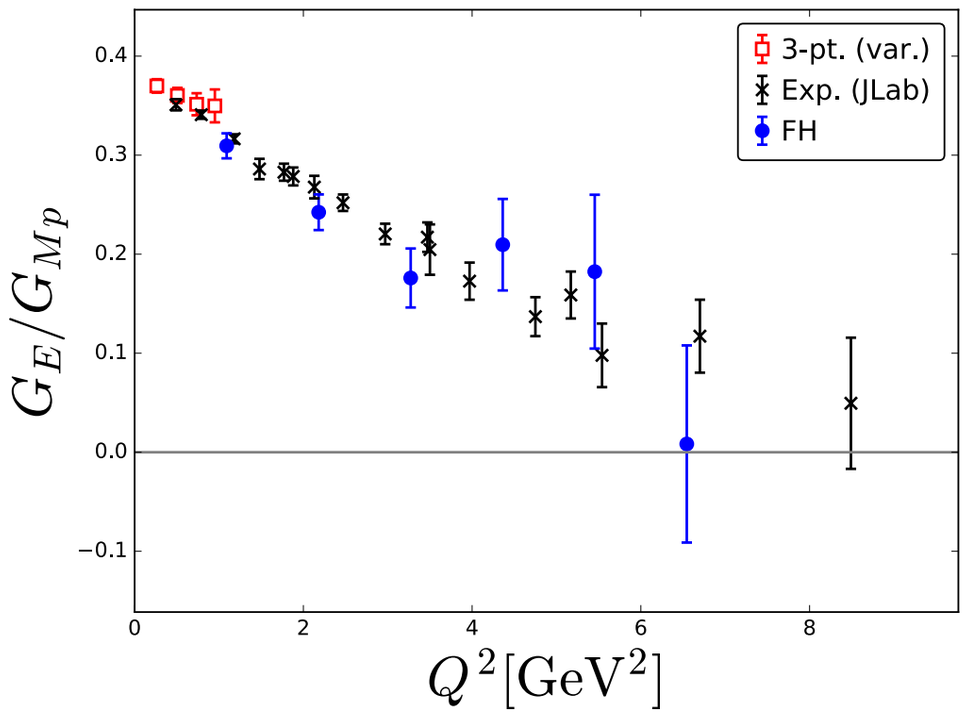


[G.D.Cates, et al, PRL106:252003 (2011)]

Proton G_E/G_M Ratio

- Lattice data: 2-state fits
- Phenomenology curves : [Alberico et al, PRC79:065204 (2008)]
- Comparison to experimental data (black points)





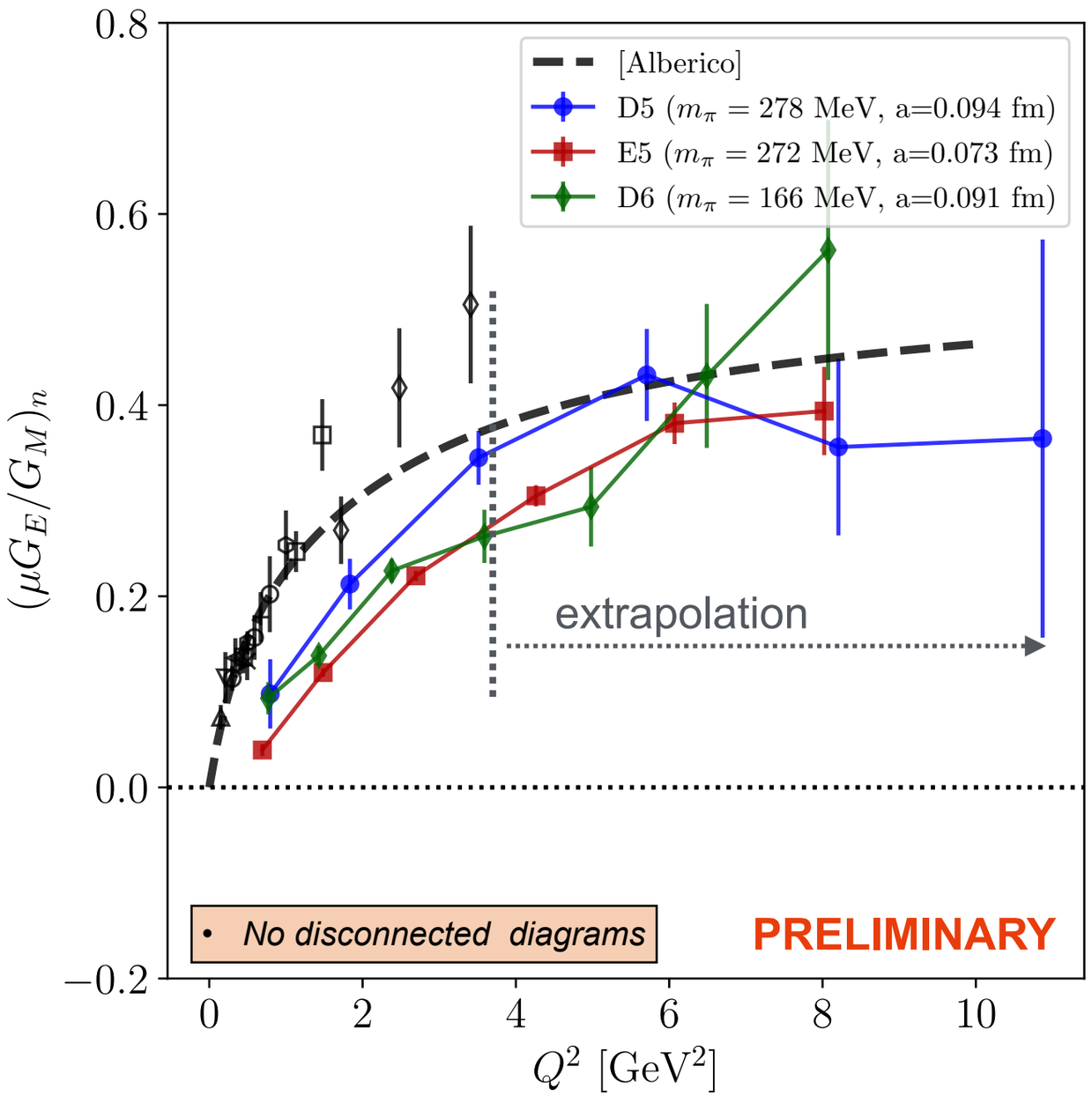
Earler calculation: (a=0.074 fm, mπ=470MeV)

Feynman-Hellman method

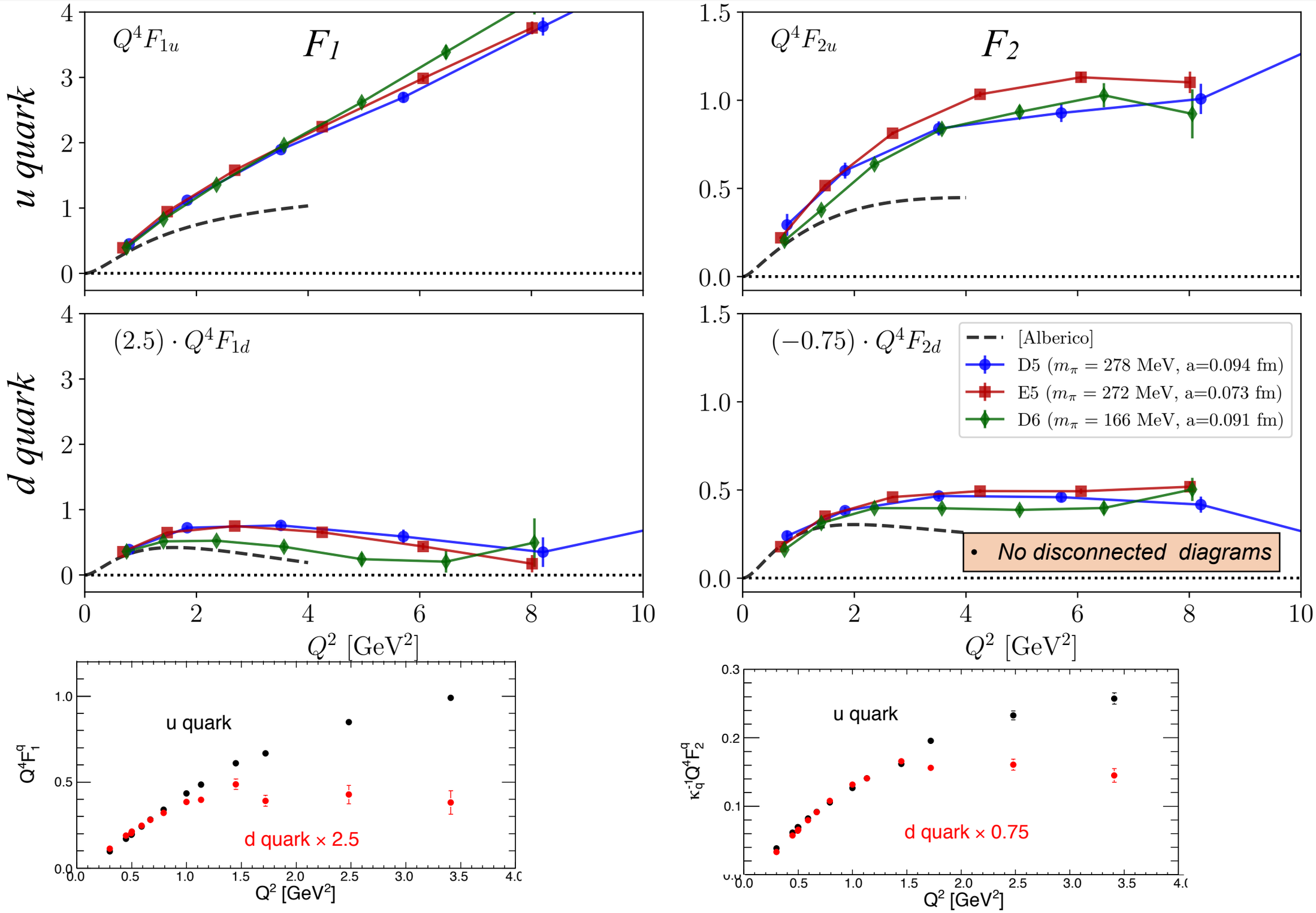
[Chambers et al (CSSM), PRD96: 114509]

G_{En}/G_{Mn} Form Factor Ratio (Neutron)

- Lattice data: 2-state fits
- Phenomenology curves : [Alberico et al, PRC79:065204 (2008)]
- Comparison to experimental data (black points)



Light-Flavor Decomposition



- Similar qual.features of flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]

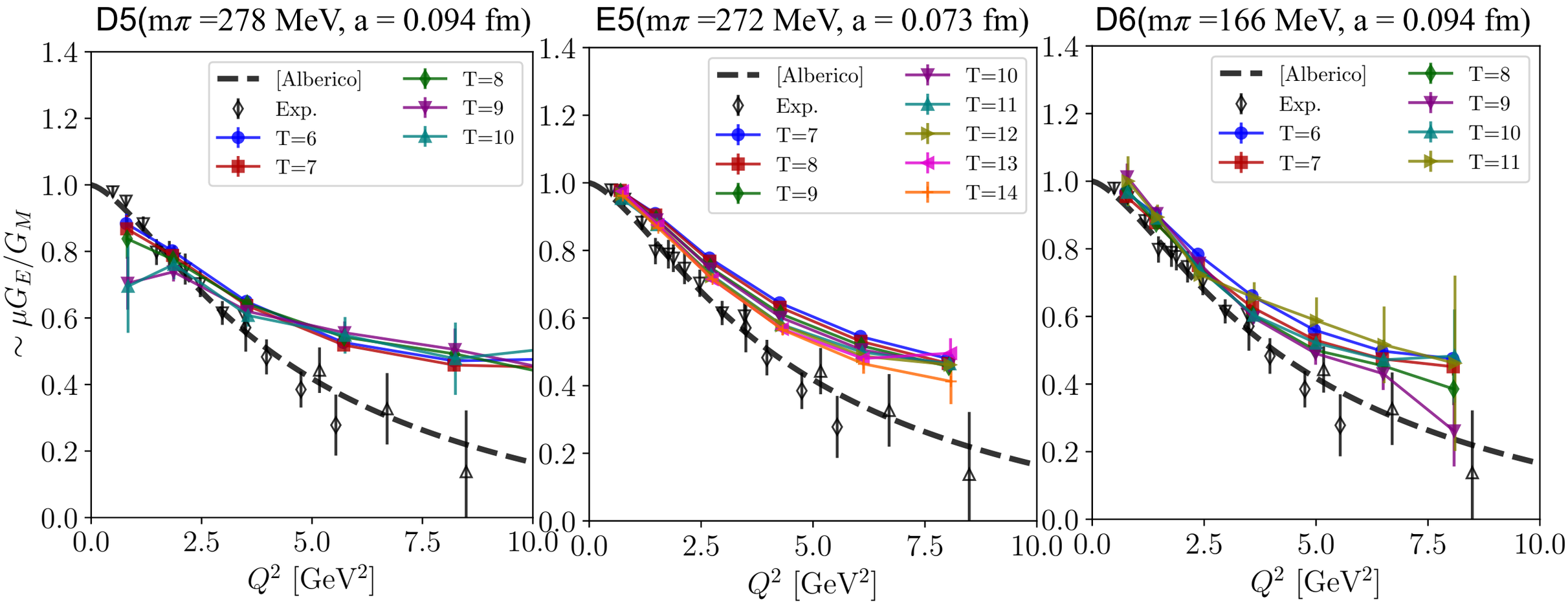
Examine Exc.States : Proton G_E/G_M

- Robust estimator from nucleon-current correlators:
  avoid lattice correlators fits to ~Σ exp(–Et)

$$\mathrm{Re}\,\langle p'\hat{x}|J_t|p\hat{x}\rangle \;\propto\; \cosh\frac{\lambda'+\lambda}{2}G_E$$

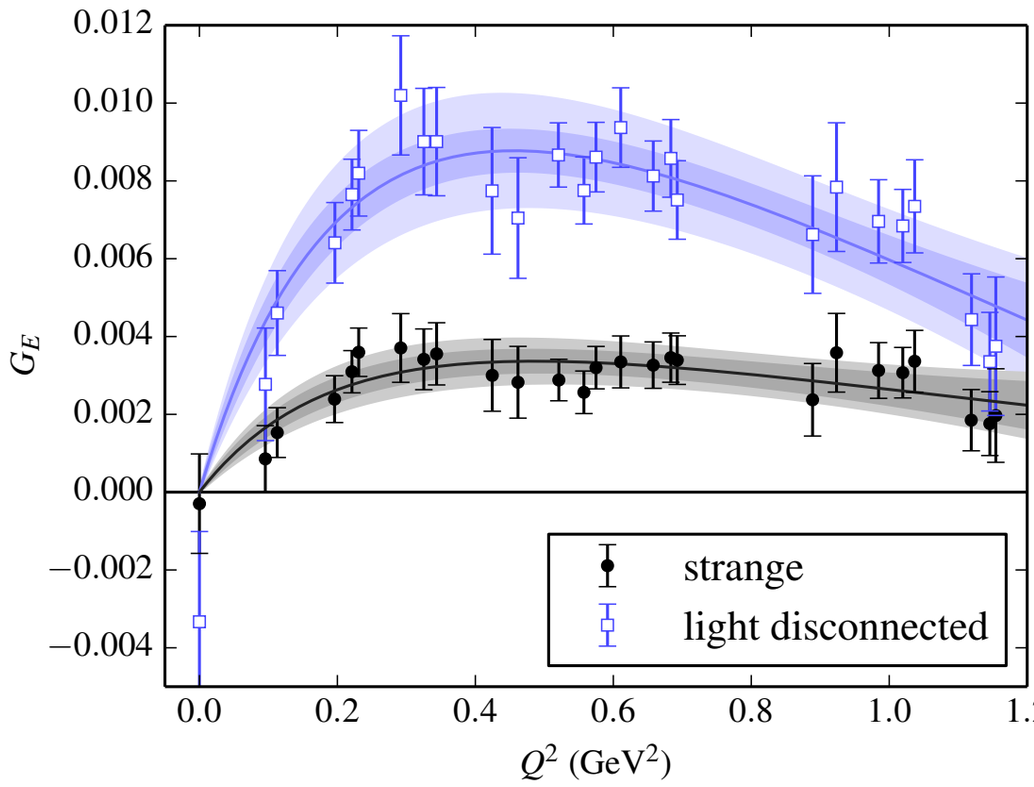
$$\mathrm{Re}\,\langle p'\hat{x}|J_y|p\hat{x}\rangle \;\propto\; \sinh\frac{\lambda'-\lambda}{2}G_M$$

where $\begin{pmatrix} p^{(\prime)} &= m_N \sinh\lambda^{(\prime)} \\ E^{(\prime)} &= m_N \cosh\lambda^{(\prime)} \end{pmatrix}$



$$\left(\frac{\sinh\frac{\lambda'-\lambda}{2}}{\cosh\frac{\lambda'+\lambda}{2}}\right)\frac{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_t(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_y(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}\overset{T\to\infty}{=} G_E/G_M$$

Prior work: Disc.Light & Strange Quark F.F's

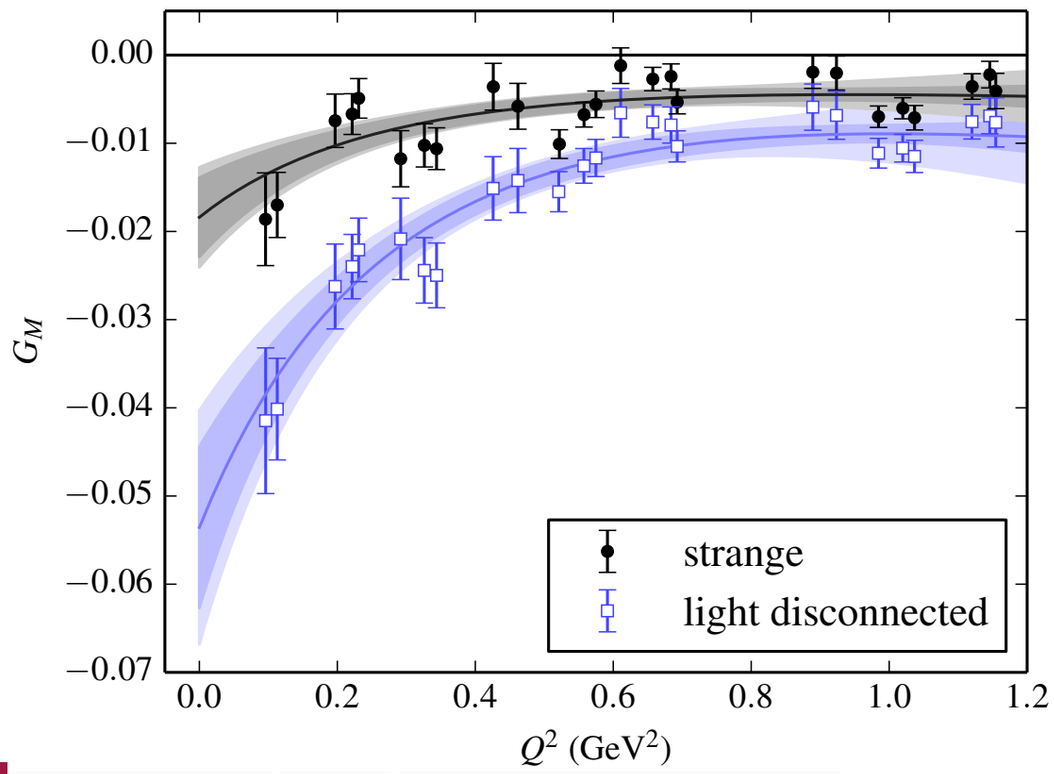


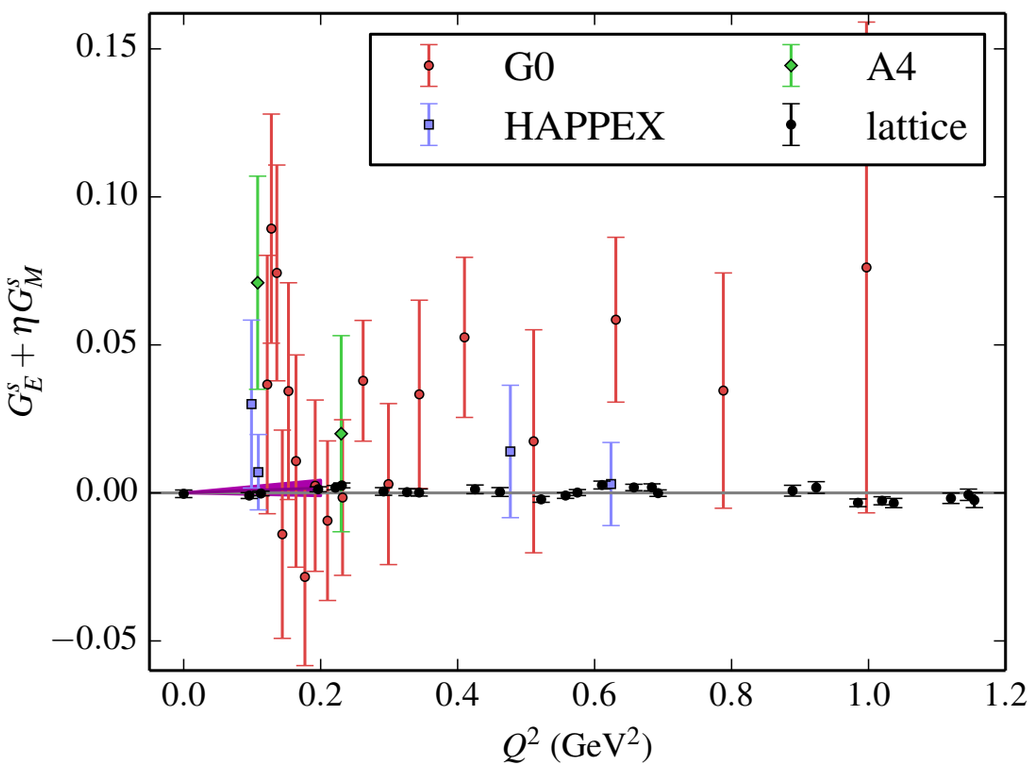
N_f=2+1 dynamical fermions, $m_\pi \approx$ 320 MeV
(C13 ensemble)

$$|(G_E^{u/d})_{\rm disc}| \lesssim 0.010 \text{ of } |(G_E^{u-d})_{\rm conn}|$$
$$|(G_E^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_E^{u-d})_{\rm conn}|$$

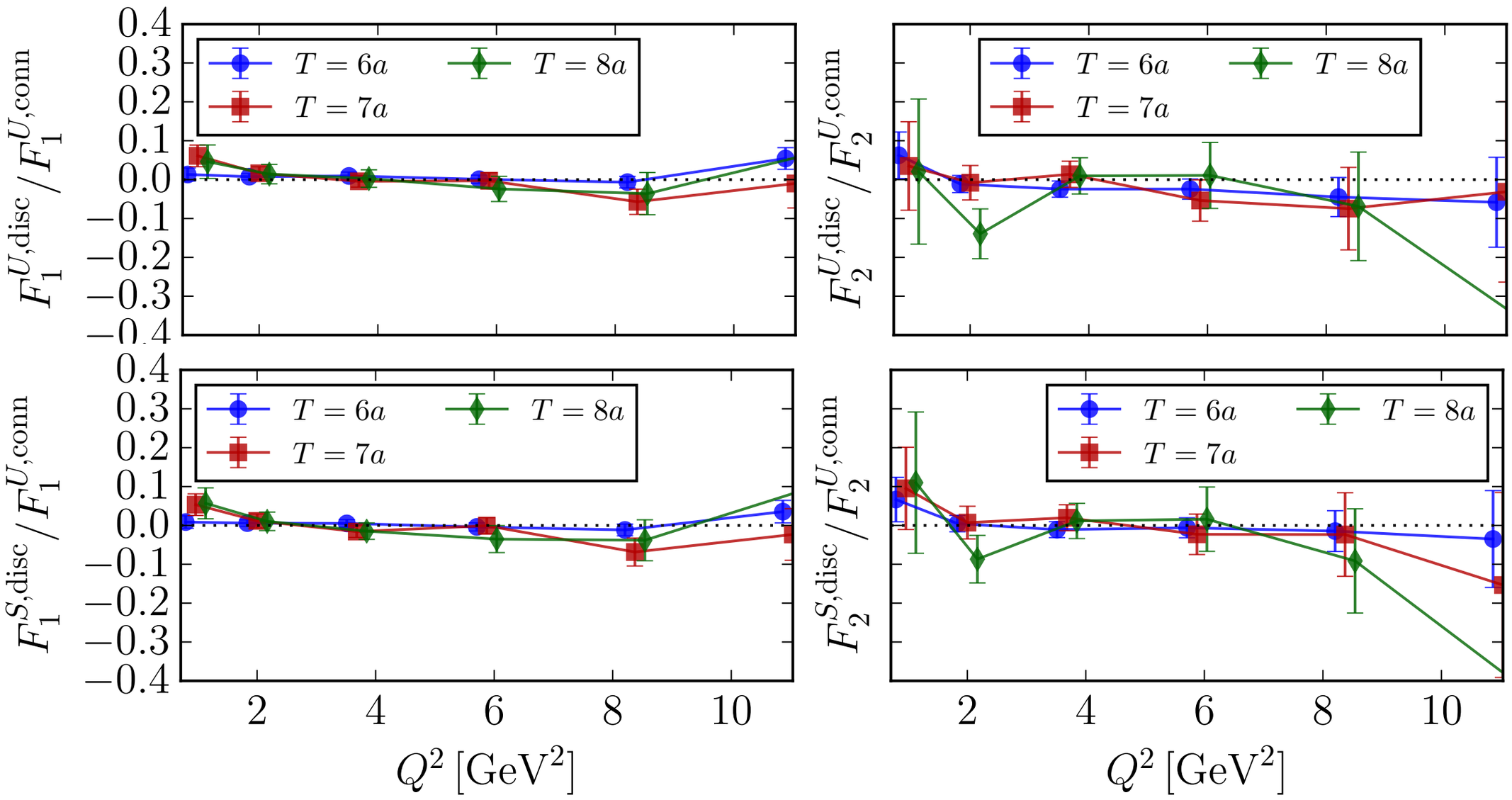
$$|(G_M^{u/d})_{\rm disc}| \lesssim 0.015 \text{ of } |(G_M^{u-d})_{\rm conn}|$$
$$|(G_M^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_M^{u-d})_{\rm conn}|$$

[J. Green, S. Meinel, S.S. et al;
PRD92:031501 (2015)]





Disconnected Light, Strange vs. Connected



- D5 ensemble(mπ=280 MeV, a=0.094 fm)
- s-, disconnected *u,d*- contributions are still small at high Q^2 up to ≲ 10 GeV^2

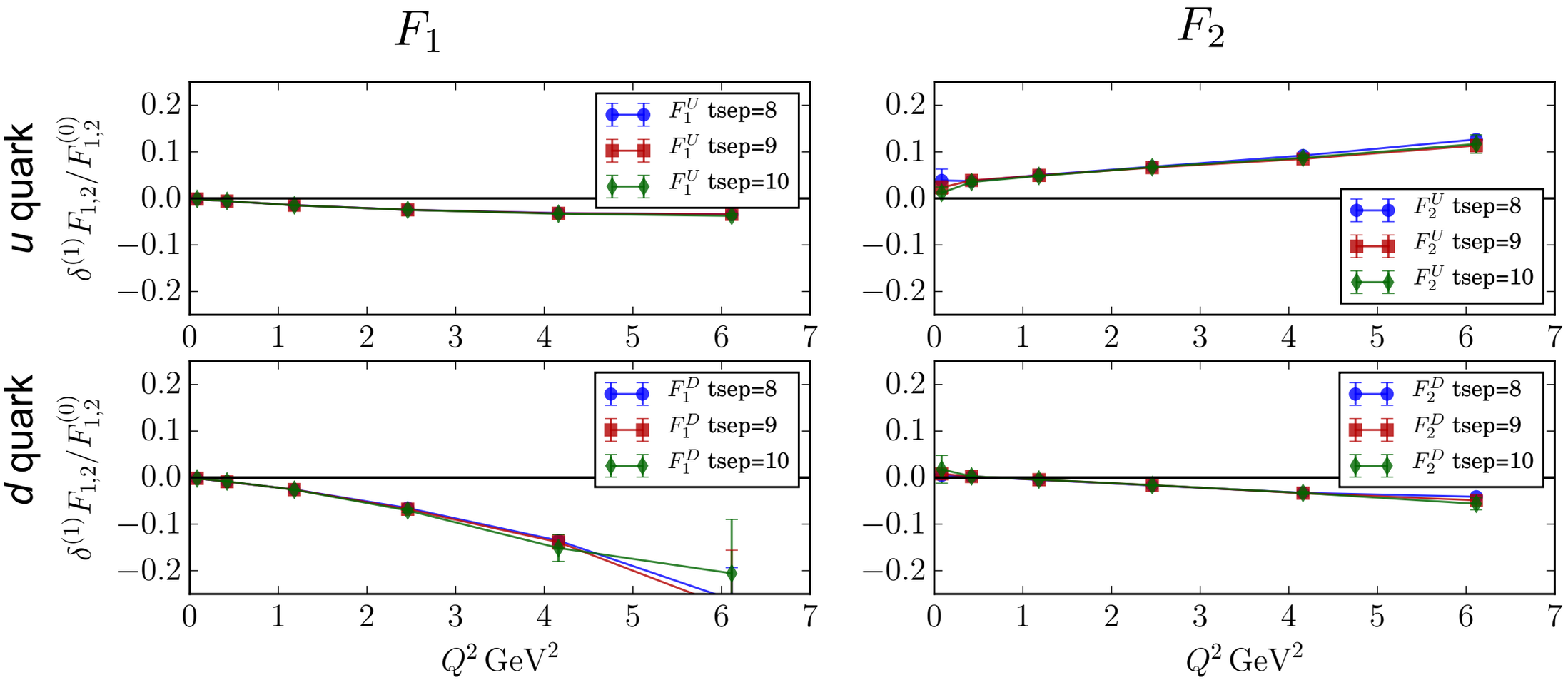
$$|F_1^s| \lesssim |\left(F_1^{u/d}\right)_{\mathrm{disc}}| \lesssim \ 10\% \text{ of } |F_2^{u,d}|$$
$$|F_2^s| \lesssim |\left(F_2^{u/d}\right)_{\mathrm{disc}}| \lesssim \ 20\% \text{ of } |F_2^{u,d}|$$

O(a) Vector Current Correction

- *No disconnected diagrams*

Improved vector current $(V_\mu)_I = \bar{q}\gamma_\mu q + c_V\, a\partial_\nu \bar{q} i\sigma_{\mu\nu} q$

O(a¹) correction : form factors of $a\,\langle N|\partial_\nu(\bar{q} i\sigma^{\mu\nu} q)|N\rangle$



Relative magnitude of *O(a¹)* effects : {*O(a¹)* } / {*O(a⁰)* } form factors
(assuming c_V=0.05)

- improvement coefficient c_V : must be computed on lattice from WI
- perturbation theory: cV ≈ –0.01$C_F(g_0)^2$

Summary

- Preliminary results for high MC-statistics high-momentum form factors
  up to $Q^2 \lesssim 12$ GeV^2, two lattice spacings $a \gtrsim 0.07$ fm, two pion masses $m_\pi \gtrsim 170$ MeV
  (No quark-disconnected contributions yet)

- Form factor results overshoot experimental data x(2 ... 2.5) ;
  G_E/G_M ratios in qualitative agreement
  ~~*Discretization?*~~
  Excited states?
  Non-physical quark masses?
  Quark-Disconnected contributions?

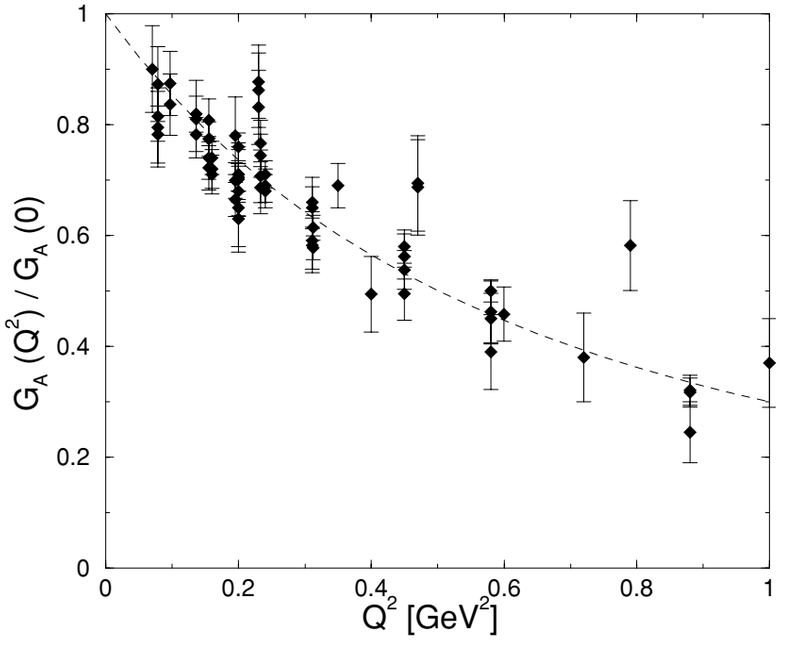
- Comparison to experiment important to validate lattice methods
  for computing relativistic nucleon matrix elements
  Impact on lattice methodology for TMDs, PDFs, DAs calculation

BACKUP

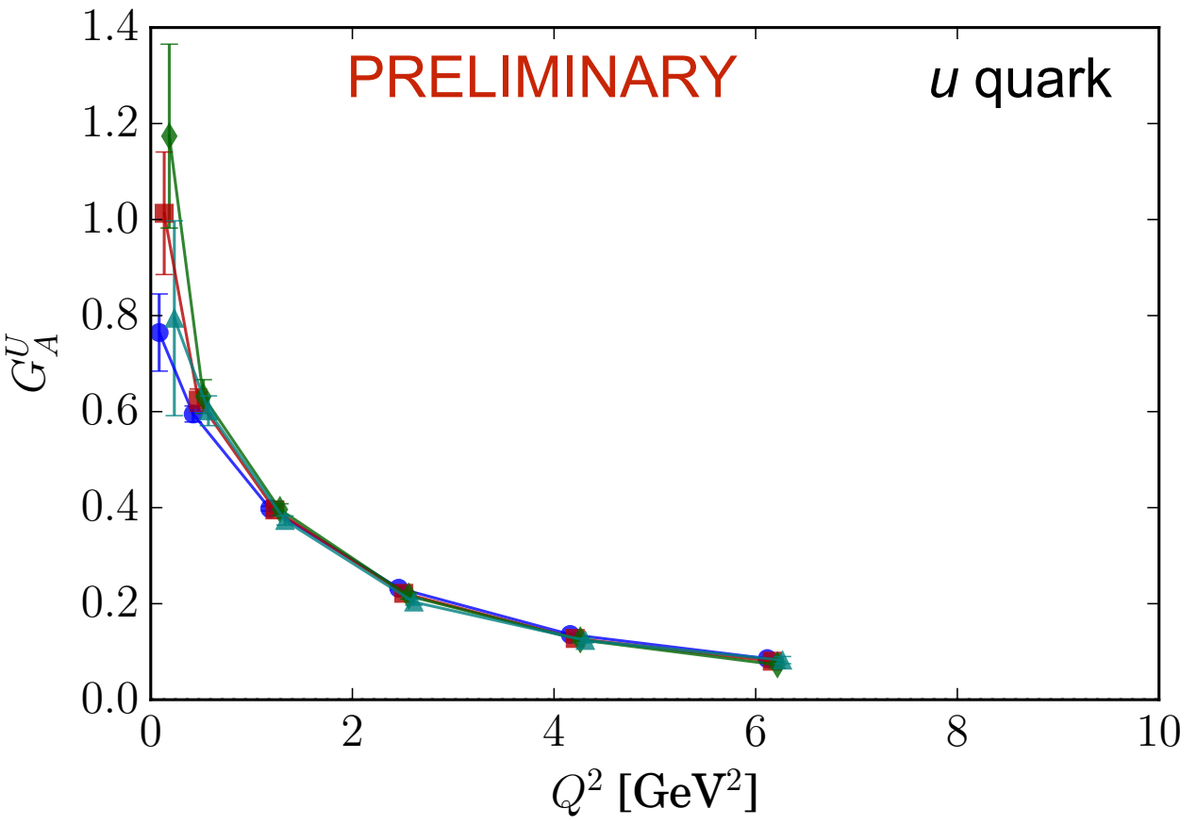
Axial Form Factors

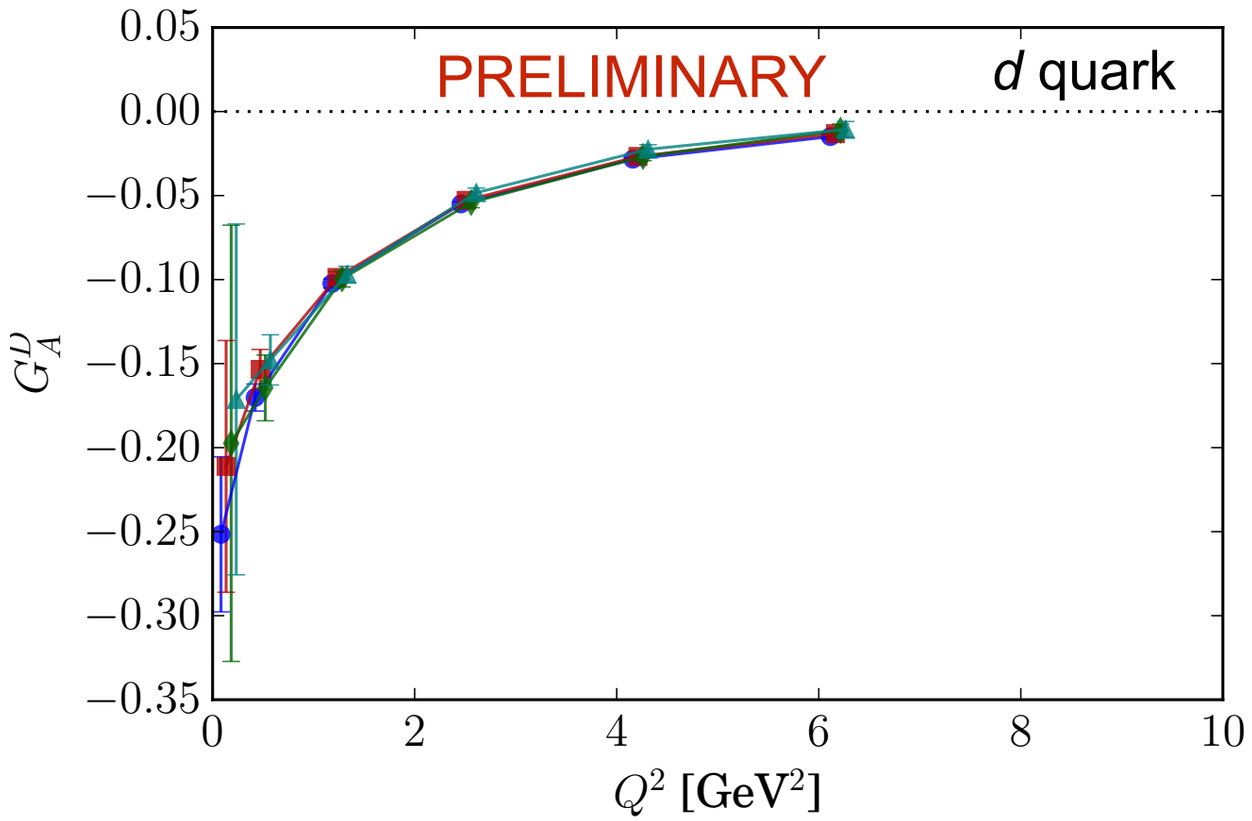
- *No disconnected diagrams*
- *No discretization corrections*

- $G_A(Q^2)$ are measured in ν-scattering, π-production;
  implications for neutrino flux norm. (e.g. in IceCube)
- Large uncertainty in $G_A(Q^2)$ from model dependence
  [B.Bhattacharya,R.Hill,G.Paz, PRD84:073006(2011)]
- Will be accessible in LBNF [see e.g. R.Petti's talk, POETIC'19]
  clean probe (only EW interaction)
  flavor separation in CC interactions



[V.Bernard et at, J.Phys.G28:R1(2002)]





Nucleon Form Factors: Lattice Recipe

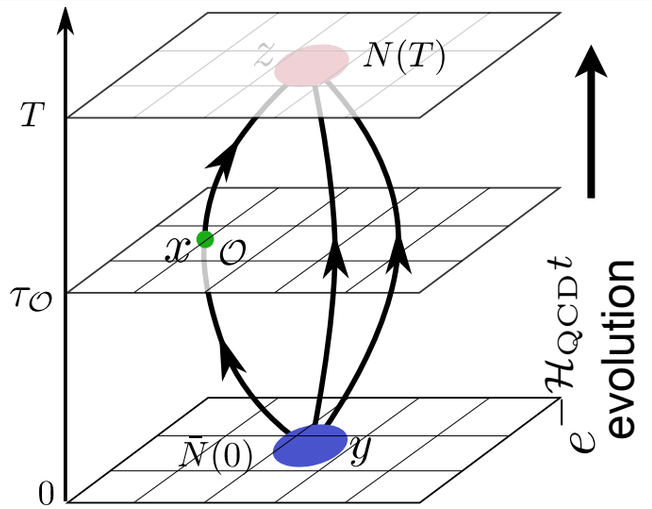
- Generate lattice ensemble

$$P(A_\mu) \propto e^{-S[A_\mu]} \prod_q \det(\not{D} + m_q)$$

- Compute nucleon correlation functions

2-point $$\langle N(\vec{p}', t_{\rm sep})\, \bar{N}(0)\rangle = \sum_y e^{-i\vec{p}'\,\vec{x}} \langle N_{t_{\rm sep},\vec{y}}\, \bar{N}(0,\vec{0})\rangle$$

3-point $$\langle N(\vec{p}', t_{\rm sep})\, \mathcal{O}(\vec{q},\tau)\, \bar{N}(0)\rangle = \sum_{y,z} e^{-i\vec{p}'\,\vec{x}+i\vec{q}\vec{z}} \langle N_{t_{\rm sep},\vec{y}}\, \mathcal{O}_{z,\tau}\, \bar{N}(0,\vec{0})\rangle$$



Quark lines = $(\not{D} + m)^{-1} \cdot \psi$

- Extract ground-state matrix elements (2-state fits typical)

$$\begin{aligned}\langle N(p', t_{\rm sep})\, \mathcal{O}(\tau)\, \bar{N}(p,0)\rangle \sim e^{-E_0'(t_{\rm tsep}-\tau)-E_0\tau} \Big[& \langle 0(p')|\mathcal{O}|0(p)\rangle \\ & + \langle 0(p')|\mathcal{O}|1(p)\rangle e^{-\Delta E_{01}\tau} + \langle 1(p')|\mathcal{O}|0(p)\rangle e^{-\Delta E_{01}'(t_{\rm sep}-\tau)} \\ & + \langle 1(p')|\mathcal{O}|1(p)\rangle e^{-\Delta E_{01}\tau-\Delta E_{01}'(t_{\rm sep}-\tau)} \Big]\end{aligned}$$

- Reduce m.e.'s to form factors: fit over momentum combinations, polarization, etc

$$\langle p', \sigma'|J^\mu|p,\sigma\rangle = [\bar{u}'\gamma^\mu u]F_1 + [\bar{u}'\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}u]F_2$$