TMDs in dijet production in SIDIS

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Outline

Kinematic region vs EIC

Factorization formula

- New dijet soft function

Evolution

- ϕ_b-angle and imaginary part
- ζ-prescription
- Scale choice and NP-model

Plots

Check our recent work:
Rafael F. del Castillo, Miguel G. Echevarría, Yiannis Makris, Ignazio Scimemi
https://arxiv.org/abs/2008.07531v4
https://arxiv.org/abs/2111.03703v2

Motivation

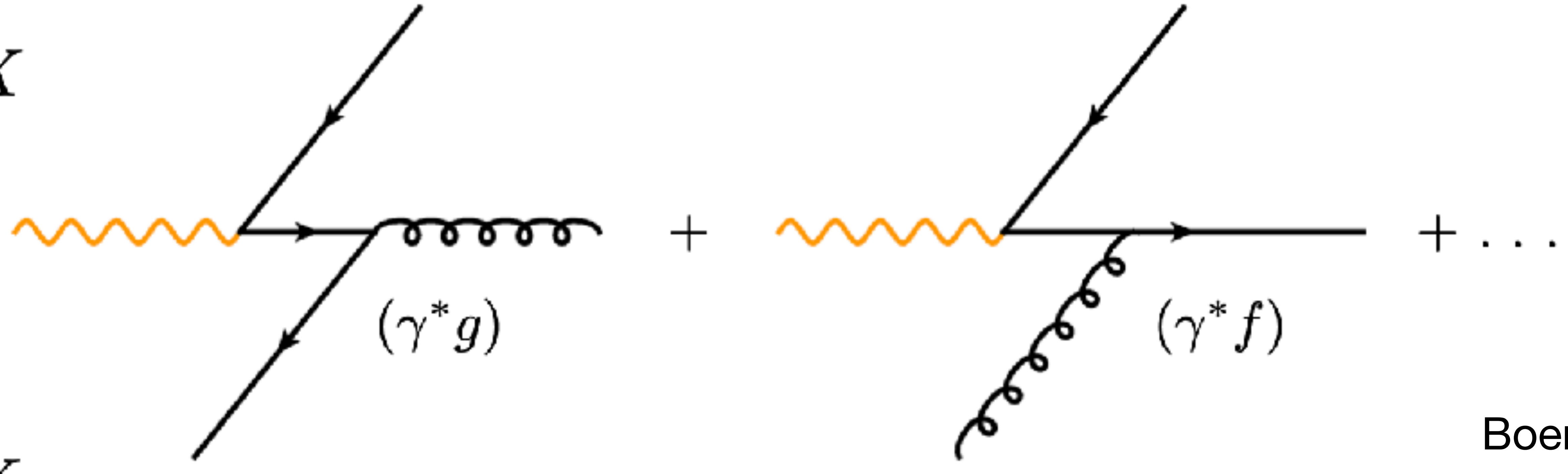
- Gluon transverse momentum dependent distributions (TMDs) are difficult to access due to the lack of clean processes where the factorization of the cross-section holds and incoming gluons constitute the dominant effect. E.g. Higgs production

Gutierrez-Reyez, Leal-Gómez, Scimemi, Vladimirov, 2019

- We consider two processes which are presently attracting increasing attention

$$\ell + h \to \ell' + J_1 + J_2 + X$$

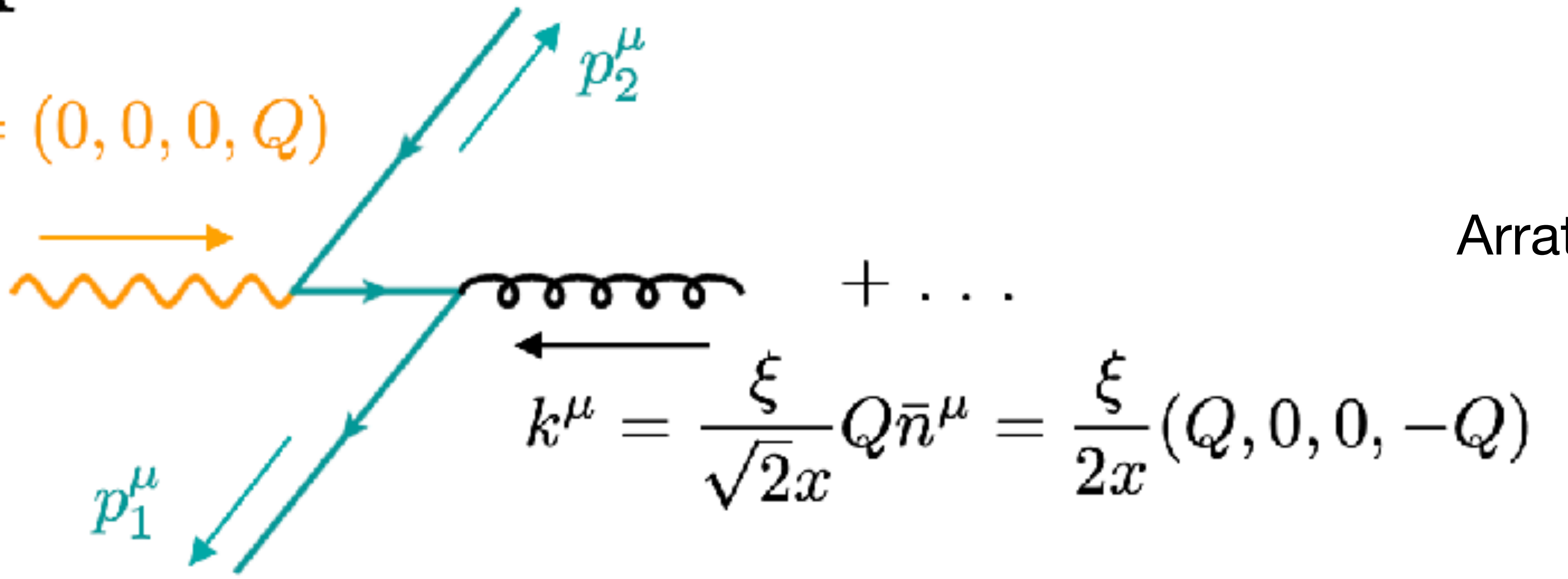
dijet LO process:



$$\ell + h \to \ell' + H + \bar{H} + X$$

$$q^\mu = \frac{Q}{\sqrt{2}}(n^\mu - \bar{n}^\mu) = (0,0,0,Q)$$

heavy meson pair at LO:



Boer, Brodsky, Mulders, Pisano, 2011

Dominguez, Xiao, Yuan, 2013

Zhang, 2017

Arratia, Furletova, Hobbs, Olness, Nguyen et al. 2020

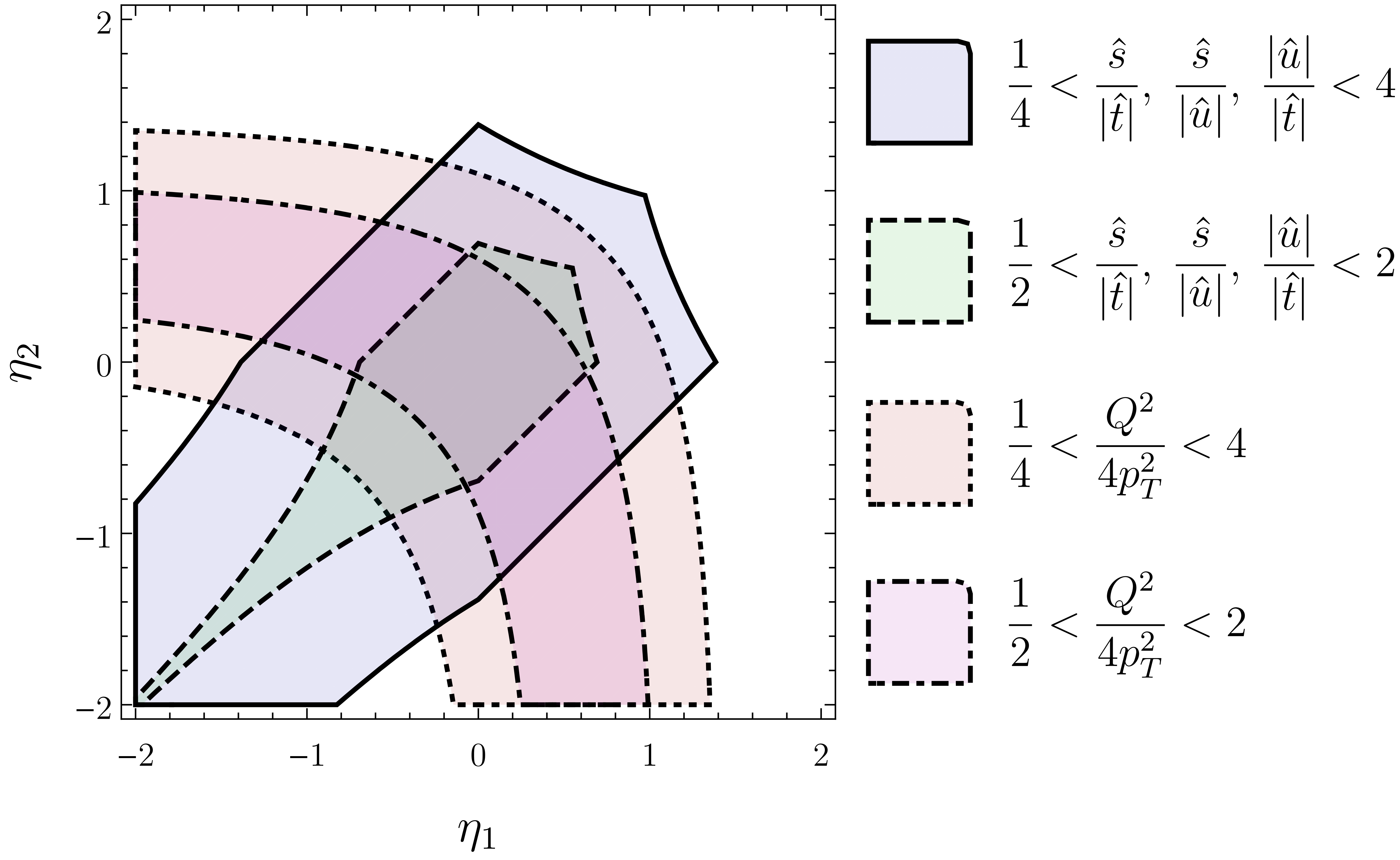
Kinematic region

Dijet production

$$\boldsymbol{r}_T = \boldsymbol{p}_{1T} + \boldsymbol{p}_{2T}$$

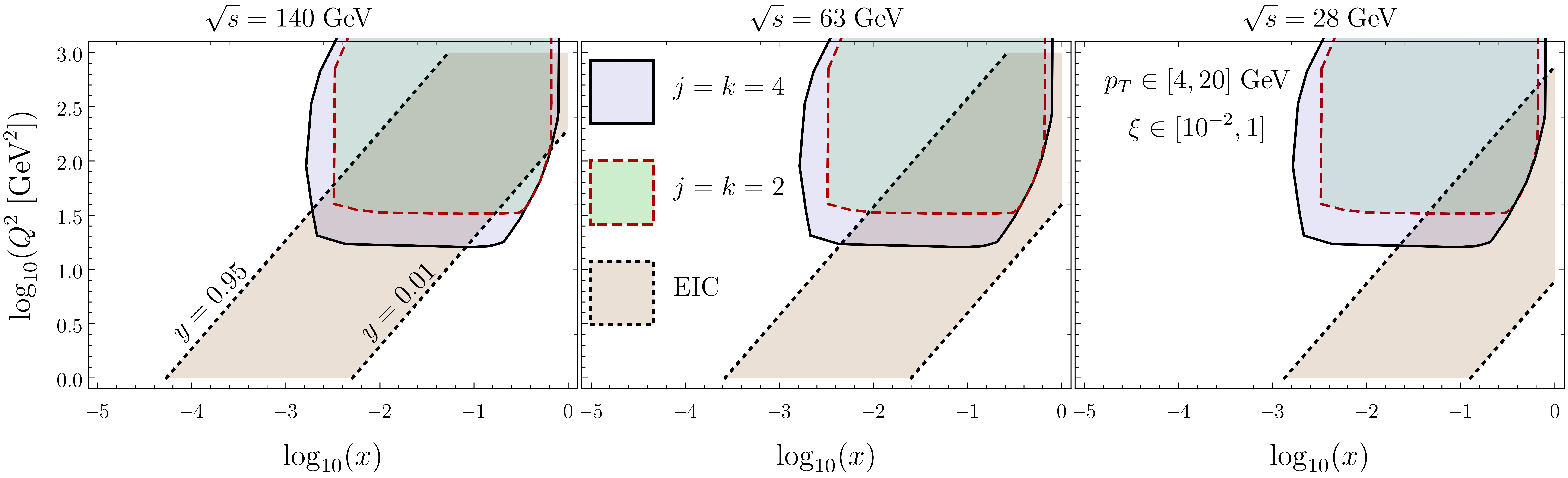
$$p_T = \frac{|\boldsymbol{p}_{1T}| + |\boldsymbol{p}_{2T}|}{2}$$

$$|\boldsymbol{r}_T| \ll p_T$$



Factorization holds for $|\boldsymbol{r}_T| \ll p_T$ and for the central rapidity region

Kinematic region vs EIC coverage



$$\frac{1}{j} < \frac{\hat{s}}{|\hat{t}|},\ \frac{\hat{s}}{|\hat{u}|},\ \frac{|\hat{u}|}{|\hat{t}|} < j \qquad \frac{1}{k} < \frac{Q^2}{4p_T^2} < k$$

Overlapping increases with higher beam energies

Factorization

$$F_g^{\mu\nu}(\xi,\boldsymbol{b}) = f_1(\xi,\boldsymbol{b})\frac{g_T^{\mu\nu}}{d-2} + h_1^{\perp}(\xi,\boldsymbol{b})\left(\frac{g_T^{\mu\nu}}{d-2}+\frac{b^\mu b^\nu}{\boldsymbol{b}^2}\right)$$

Dijet

$$\frac{d\sigma(\gamma^* g)}{dx\,d\eta_1\,d\eta_2\,dp_T\,d\boldsymbol{r}_T} = \sum_f H^{\mu\nu}_{\gamma^* g\to f\bar f}(\hat s,\hat t,\hat u,\mu)\int\frac{d^2\boldsymbol{b}}{(2\pi)^2}\exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)F_{g,\mu\nu}(\xi,\boldsymbol{b},\mu,\zeta_1)$$
$$\times S_{\gamma g}(\boldsymbol{b},\eta_1,\eta_2,\mu,\zeta_2)\left(\mathcal{C}_f(\boldsymbol{b},R,\mu)J_f(p_T,R,\mu)\right)\left(\mathcal{C}_{\bar f}(\boldsymbol{b},R,\mu)J_{\bar f}(p_T,R,\mu)\right)$$

Hornig, Makris, Mehen, 2016

$$\frac{d\sigma^U(\gamma^* f)}{dx\,d\eta_1\,d\eta_2\,dp_T\,d\boldsymbol{r}_T} = \sum_f \sigma_0^{fU} H^U_{\gamma^* f\to g f}(\hat s,\hat t,\hat u,\mu)\int\frac{d^2\boldsymbol{b}}{(2\pi)^2}\exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)F_f(\xi,\boldsymbol{b},\mu,\zeta_1)$$
$$\times S_{\gamma f}(\boldsymbol{b},\zeta_2,\mu)\left(\mathcal{C}_g(\boldsymbol{b},R,\mu)J_g(p_T,R,\mu)\right)\left(\mathcal{C}_f(\boldsymbol{b},R,\mu)J_f(p_T,R,\mu)\right)$$

HHP

$$\frac{d\sigma(\gamma^* g)}{dx\,d\eta_H\,d\eta_{\bar H}\,dp_T\,d\boldsymbol{r}_T} = H^{\mu\nu}_{\gamma^* g\to Q\bar Q}(\hat s,\hat t,\hat u,\mu)\int\frac{d\boldsymbol{b}}{(2\pi)^2}\exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)F_{g,\mu\nu}(\xi,\boldsymbol{b},\mu,\zeta_1)$$
$$\times S_{\gamma g}(\boldsymbol{b},\mu,\zeta_2)H_+(m_Q,\mu)\mathcal{J}_{Q\to H}\left(\boldsymbol{b},\frac{m_Q}{p_T},\mu\right)H_+(m_Q,\mu)\mathcal{J}_{\bar Q\to\bar H}\left(\boldsymbol{b},\frac{m_Q}{p_T},\mu\right)$$

Fickinger, Fleming, Kim, Mereghetti, 2016

New dijet soft function

n - incoming beam direction

v_1 - jet 1 direction

v_2 - jet 2 direction

Soft function

$$\hat{S}_{\gamma g}(\boldsymbol{b}) = \frac{1}{C_F C_A} \langle 0 | \mathcal{S}_n^\dagger(\boldsymbol{b}, -\infty)_{ca'} \mathrm{Tr}\Big[S_{v_2}(+\infty, \boldsymbol{b}) T^{a'} S_{v_1}^\dagger(+\infty, \boldsymbol{b}) \times S_{v_1}(+\infty, 0) T^a S_{v_2}^\dagger(+\infty, 0) \Big] \mathcal{S}_n(0, -\infty)_{ac} | 0 \rangle$$

$$\hat{S}_{\gamma f} = \hat{S}_{\gamma g}(n \leftrightarrow v_2)$$

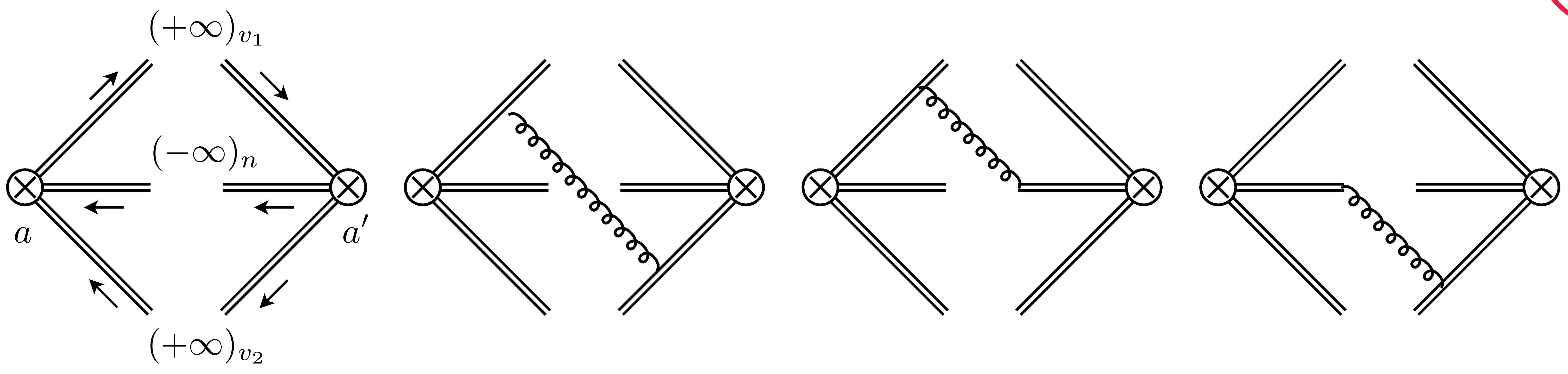
Wilson lines

$$S_v(+\infty, \xi) = P\exp\Big[-ig \int_0^{+\infty} d\lambda \, v \cdot A(\lambda v + \xi) \Big] \qquad S_{\bar{v}}^\dagger(+\infty, \xi) = P\exp\Big[ig \int_0^{+\infty} d\lambda \, \bar{v} \cdot A(\lambda \bar{v} + \xi) \Big]$$

$$S_n(+\infty, \xi) = \lim_{\delta^+ \to 0} P\exp\Big[-ig \int_0^{+\infty} d\lambda \, n \cdot A(\lambda n + \xi) e^{-\delta^+ \lambda} \Big]$$

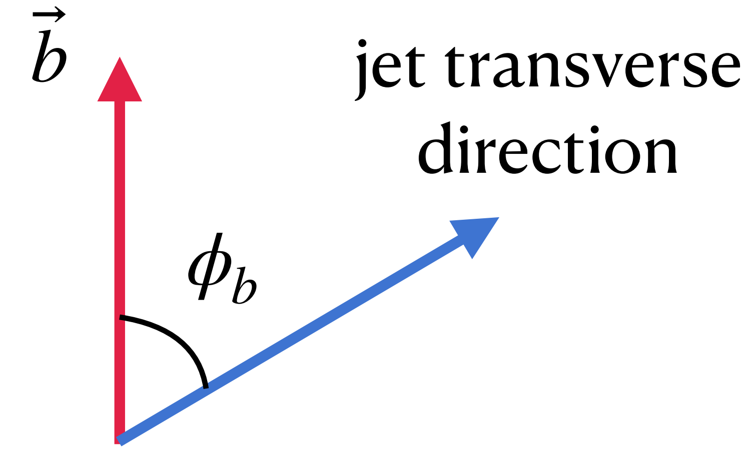
δ - regulator !!!

Echevarría, Scimemi, Vladimirov, 2016



+ virtual diagrams at one-loop order…

Evolution & imaginary part



- We find imaginary parts and ϕ_b-dependent parts in the perturbative result and ADs

$$\gamma_i(\boldsymbol{b},\mu) = \gamma_{\text{cusp}}\left[\alpha_s\right]\left(c_i 2\ln|\cos\phi_b| - c_i' i\pi\Theta\left(\phi_b\right)\right) + \text{ other } \phi_b \text{ independent terms}$$

$$\sum_i c_i = \sum_i c_i' = 0 \qquad \Theta(\phi_b) = \begin{cases} +1 & : -\pi/2 < \phi_b < \pi/2 \\ -1 & : \text{otherwise} \end{cases}$$

- We split the evolution kernels

$$S_{\gamma i}\left(\boldsymbol{b},\mu_f,\zeta_{2,f}\right) = \exp\left[\int_{\mu_0}^{\mu_f}\left(\gamma^{\phi}_{S_{\gamma i}}(\phi)d\ln\mu\right)\right]\exp\left[\int_P\left(\bar{\gamma}_{S_{\gamma i}}\left(b,\mu,\zeta_2\right)d\ln\mu - \mathcal{D}_i(\mu,b)d\ln\zeta_2\right)\right]S_{\gamma i}\left(\boldsymbol{b},\mu_0,\zeta_{2,0}\right)$$

$\mathcal{R}_S^{\phi}$ → Integrate over ϕ_b

$\mathcal{R}_S$ → ζ-prescription

Scimemi, Vladimirov, 2018
Scimemi, Vladimirov, 2020

$$\mathcal{C}_i\left(\boldsymbol{b},R,\mu_f\right) = \exp\left[\int_{\mu_i}^{\mu_f}\gamma^{\phi}_{\mathcal{C}_i}(\phi)d\ln\mu\right]\exp\left[\int_{\mu_i}^{\mu_f}\bar{\gamma}_{\mathcal{C}_i}\left(b,R,\mu\right)d\ln\mu\right]\mathcal{C}_i\left(\boldsymbol{b},R,\mu_i\right)$$

$\mathcal{R}_{\mathcal{C}}^{\phi}$ → Integrate over ϕ_b

$\mathcal{R}_{\mathcal{C}}$ → Single scale evolution

Hornig, Makris, Mehen, 2016

- ϕ_b angle is integrated out with the Fourier transform and imaginary parts cancel

Evolution & imaginary part

- After this manipulation b-space cross-section is proportional to:

ϕ-independent and real kernel

Perturbative result

$$d\sigma(\boldsymbol{b}) \sim |\cos\phi_b|^{2\mathcal{A}} \left(\cos(\mathcal{B}\pi) - i\Theta(\phi_b)\sin(\mathcal{B}\pi)\right) \mathcal{R}\left(\{\mu_k\} \to \mu\right) \left[1 + \sum_{k\in\{H,F,J,S,\mathcal{C}\}} a_s(\mu_k)\, f_k^{[1]}(b, \cos\phi_b)\right]$$

$$\mathcal{A}(\{\mu_i\}) = \sum_{i\in\{S,\mathcal{C}\}} c_i \int_{\mu_i}^{\mu} \gamma_{\text{cusp}}\,[\alpha_s]\, d\ln\mu', \quad \mathcal{B}(\{\mu_i\}) = \sum_{i\in\{S,\mathcal{C}\}} c_i' \int_{\mu_i}^{\mu} \gamma_{\text{cusp}}\,[\alpha_s]\, d\ln\mu'$$

$$\sum_i c_i = \sum_i c_i' = 0$$

- All ϕ_b-integrals can be written in terms of a master integral

Master integral: $$I_n(\mathcal{A}) \equiv \int_{-\pi}^{+\pi} d\phi_b\, |\cos\phi_b|^{2\mathcal{A}} \ln^n|\cos\phi_b|$$

- We need $2\mathscr{A} > -1$ in order for the ϕ_b-integral to be well-defined $\Rightarrow$ restriction over initial scales

- This restriction do not let us completely resum logs in collinear-soft and heavy meson jet function

Evolution & imaginary part

- We need $2\mathscr{A} > -1$ in order for the ϕ_b-integral to be well-defined ⇒ restriction over initial scales

Master integral: $$I_n(\mathcal{A}) \equiv \int_{-\pi}^{+\pi} d\phi_b \, |\cos\phi_b|^{2\mathcal{A}} \, \ln^n |\cos\phi_b|$$

$$I_0(\mathcal{A}) = \frac{2\sqrt{\pi}\,\Gamma(1/2+\mathcal{A})}{\Gamma(1+\mathcal{A})},$$

Not well-defined if $2\mathscr{A} < -1$

$$\mathcal{A}(\{\mu_i\}) = \sum_{i\in\{S,\mathcal{C}\}} c_i \int_{\mu_i}^{\mu} \gamma_{\rm cusp}\,[\alpha_s]\, d\ln\mu'$$

$$I_1(\mathcal{A}) = \frac{\sqrt{\pi}\,\Gamma(1/2+\mathcal{A})}{\Gamma(1+\mathcal{A})}\left(H_{\mathcal{A}-1/2} - H_{\mathcal{A}}\right)$$

$$I_2(\mathcal{A}) = \frac{\sqrt{\pi}\,\Gamma(1/2+\mathcal{A})}{2\Gamma(1+\mathcal{A})}\left[\left(H_{\mathcal{A}-1/2} - H_{\mathcal{A}}\right)^2 + \psi^{(1)}\left(\frac{1}{2}+\mathcal{A}\right) - \psi^{(1)}(1+\mathcal{A})\right]$$

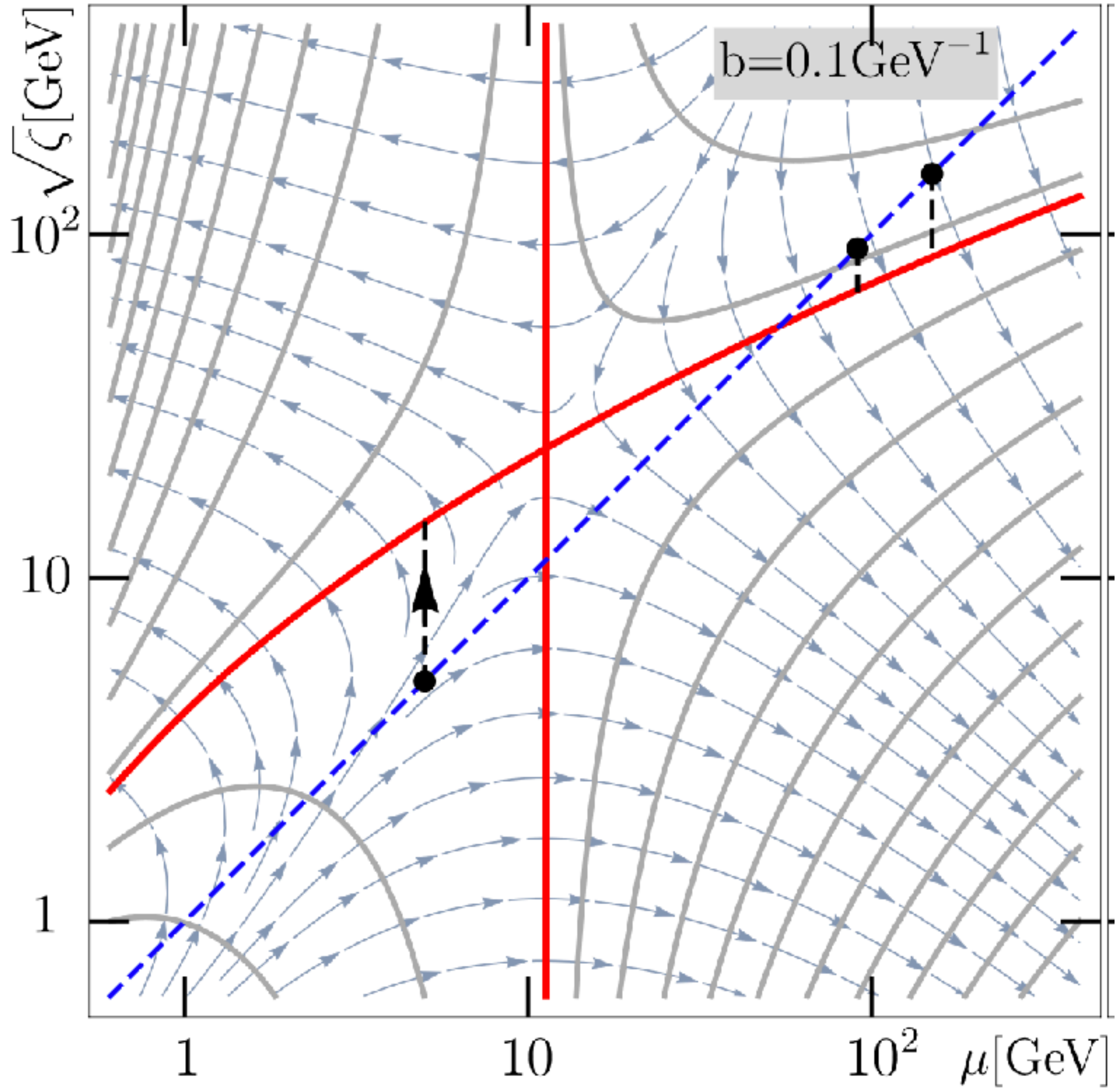
For linearly polarized gluons we have an extra $\cos 2\phi_b$:

$$I_n(\mathcal{A}) \quad \longrightarrow \quad -I_n(\mathcal{A}+1) + \frac{1}{2} I_n(\mathcal{A})$$

Same for angular modulation and Sivers asymmetry…

Evolution, ζ-prescription

Figure: Alexey Vladimirov & Ignazio Scimemi



Scimemi, Vladimirov, 2018
Scimemi, Vladimirov, 2020

fixed μ evolution

Evolution kernel is given by

Saddle point

$$S(\boldsymbol{b};\mu_f,\zeta_{2,f}) = \exp\left[\int_P \left(\gamma_S(\mu,\zeta_2)\, d\ln\mu - \mathcal{D}_S(\mu,b) d\ln\zeta_2\right)\right] S(\boldsymbol{b};\overbrace{\mu_0,\zeta_{2,0}})$$

$$\left.\begin{aligned} \frac{d}{d\ln\mu} S(\boldsymbol{b};\mu,\zeta) &= \gamma_S(b;\mu,\zeta) S(\boldsymbol{b};\mu,\zeta) \\ \frac{d}{d\ln\zeta} S(\boldsymbol{b};\mu,\zeta) &= -\mathcal{D}_S(b,\mu) S(\boldsymbol{b};\mu,\zeta) \end{aligned}\right\} \longrightarrow \boxed{\boldsymbol{\nabla} F = \boldsymbol{E} F}$$

$$\boldsymbol{E} = (\gamma_S(b,\mu,\zeta), -\mathcal{D}_S(b,\mu))$$

Equipotential (null-evolution) line is given by $\gamma_S = 2\mathcal{D}_S \dfrac{d\ln\zeta_\mu}{d\ln\mu^2}$

gluon channel solution $\zeta_{2,\mu}^{\gamma * g}(\boldsymbol{b},\mu) = \left(\dfrac{\mu}{\mu_0}\right)^{\frac{2C_F}{C_A}} \zeta_{2,0} e^{v_S(\boldsymbol{b},\mu)}$

perturbative

$$\boxed{R_S((\mu_0,\zeta_{2,0}) \to (\mu_f,\zeta_f)) = \left(\frac{\zeta_f}{\zeta_{2,\mu}(\boldsymbol{b},\mu_f)}\right)^{-D_S(\boldsymbol{b},\mu_f)}}$$

Scale choices and NP-model

- For the new b-dependent function we consider a gaussian model for NP contribution

$$S_{\gamma i}(b;p_T,1) = \mathcal{R}_S(\{\mu_0,\zeta_0\} \to \{p_T,1\}) S^{pert}_{\gamma i}(b;\mu_0,\zeta_0) f^{\rm NP}_{\mathcal{S}}(b)$$
$$\mathcal{C}(b,R;p_T) = \mathcal{R}_{\mathcal{C}}(b,R;p_T,\mu_{\mathcal{C}}) \mathcal{C}^{pert}(b,R;\mu_{\mathcal{C}}) f^{\rm NP}_{\mathcal{C}}(b,R)$$
$$\mathcal{J}(b,m_Q/p_T;p_T) = \mathcal{R}_{\mathcal{J}}(b,m_Q/p_T;p_T,\mu_{\mathcal{J}}) \mathcal{J}^{pert}(b,m_Q/p_T;\mu_{\mathcal{J}}) f^{\rm NP}_{\mathcal{J}}(b;m_Q)$$

$$f^{\rm NP}_i(b) = \exp\left(-\frac{b^2}{\left(B^i_{\rm NP}\right)^2}\right)$$

Initial scales

$$\mu_{\mathcal{C}} = 2e^{-\gamma_E}\left(\frac{1}{b} + \frac{1}{b_{\max}}\right) \qquad \mu_J = p_T R$$

$$\mu_{\mathcal{J}} = \frac{1}{2}e^{-\gamma_E}\left(\frac{1}{b} + \frac{1}{b_{\max}}\right) \qquad \mu_+ = m_Q$$

$$\mu_S = \frac{2e^{-\gamma_E}}{b^*}, \quad b^* = \frac{b}{\sqrt{1+b^2/b^2_{\max}}} \qquad \zeta^{\gamma g}_{2,0} = \left(\frac{4p_T^2}{\hat{s}}\right)^{\frac{2C_F}{C_A}}$$

→

Final scales

$$\mu_f = p_T$$

$$\zeta_{2,0} = 1$$

	$\mathcal{C}$	$\mathcal{J}$	S
$B^i_{\rm NP}$ (GeV^{-1})	2.5	2.5	2.5

	$\mathcal{C}$	$\mathcal{J}$
$b_{\max}$ (GeV^{-1})	0.5	0.3

Plots for phenomenological analysis

https://teorica.fis.ucm.es/artemide/
https://github.com/vladimirovalexey/artemide-public.”

- We use arTeMiDe to obtain the plots
- TMDPDF and TMDFF structure and evolution is included arTeMiDe
- SF double-scale evolution and jet functions included as new modules

$p_T = 20\,\mathrm{GeV} \quad (p_T \sim Q)$

$\sqrt{s} = 140\,\mathrm{GeV}$

Integrated over x

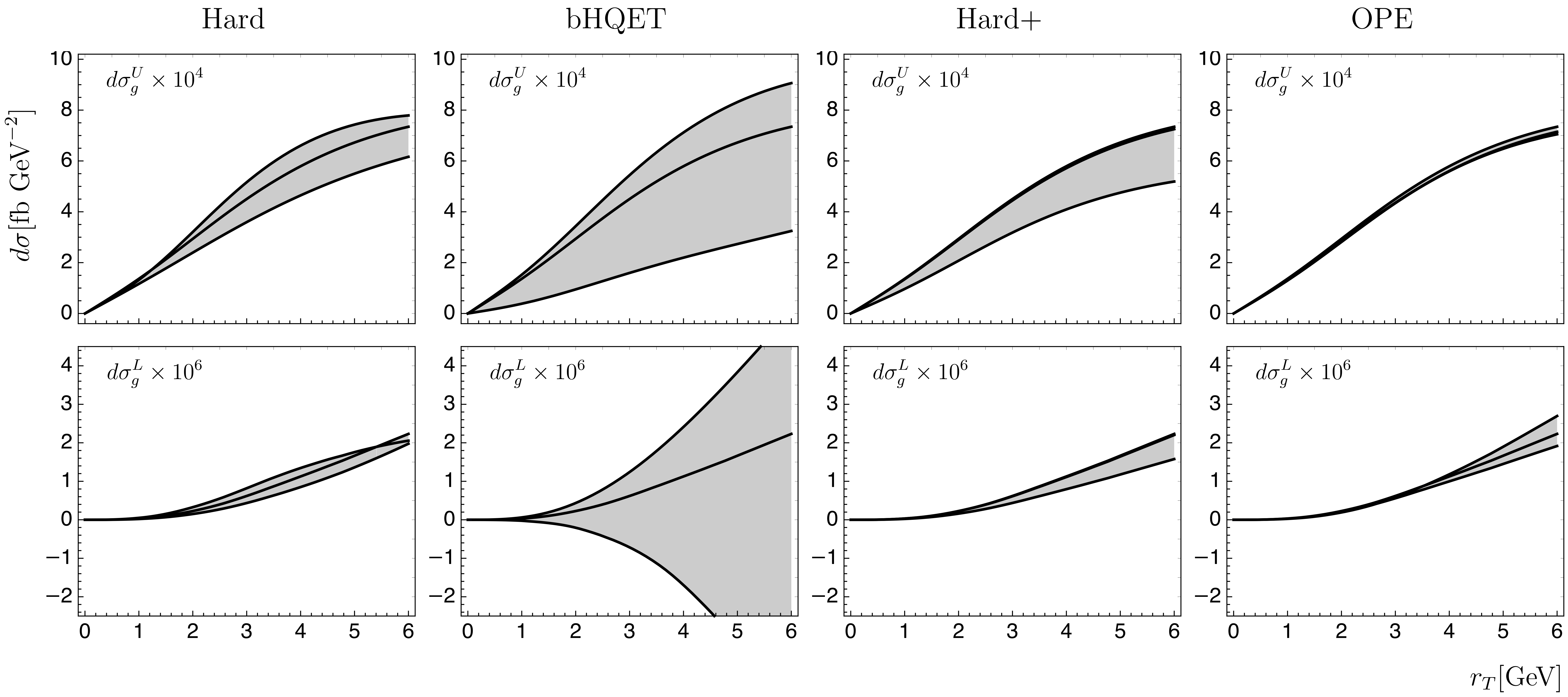
Central rapidity region

Dijet production

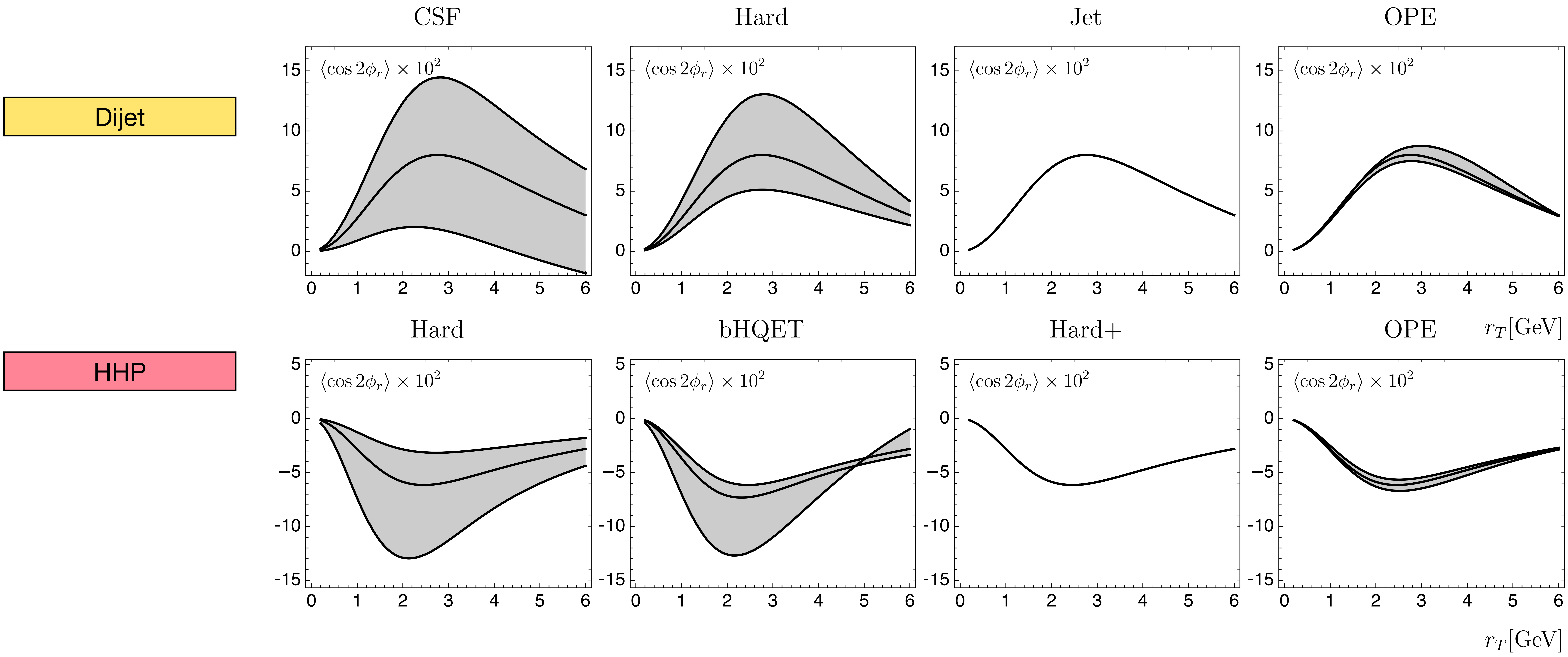
Heavy hadron pair production

Total cross-section

Linearly polarized gluon channel



$\langle \cos 2\phi_r \rangle$ - asymmetry



Conclusion

- We have established factorization for dijet and heavy hadron pair production
- Can be potentially observed in the future EIC
- We have been able to compute the new TMD Soft Function up to NLO and its anomalous dimension up to three-loops
- Rapidity structure of this new SF allows us to use the ζ-prescription
- The presence of the new SF makes the gluon TMDPDF extraction non-trivial
- Analysis of the numerical result for the cross-section shows the effect of linearly polarized gluon TMDs can be neglected compared to unpolarized gluon TMDs
- Future work: Gluon Sivers function, dihadron production,…

Thank you for listening!

Backup

Evolution & imaginary part

Constant terms

$$I_{\text{const.}}(\mathcal{A},\mathcal{B}) \equiv \int_{-\pi}^{+\pi} d\phi_b\,|\cos\phi_b|^{2\mathcal{A}}\Big(\cos(\mathcal{B}\pi) - i\Theta(\phi_b)\sin(\mathcal{B}\pi)\Big) = I_0(\mathcal{A})\cos(\mathcal{B}\pi)$$

Single logarithmic terms

$$I_{\log}(\mathcal{A},\mathcal{B}) \equiv \int_{-\pi}^{+\pi} d\phi_b\,|\cos\phi_b|^{2\mathcal{A}}\Big(\cos(\mathcal{B}\pi) - i\Theta(\phi_b)\sin(\mathcal{B}\pi)\Big)\ln(-i\cos\phi_b)$$

From the perturbative result

We rewrite $\ln(-i\cos\phi_b) = \ln|\cos\phi_b| - \frac{i\pi}{2}\Theta(\phi_b)$

$$I_{\log}(\mathcal{A},\mathcal{B}) = I_1(\mathcal{A})\cos(\mathcal{B}\pi) - \frac{\pi}{2}I_0(\mathcal{A})\sin(\mathcal{B}\pi)$$

Imaginary part cancels in this way for every case

Zero-bin subtraction

Due to the rapidity divergencies structure of the two-direction soft function (zero-bin)

can be split as

ν arbitrary positive number

$$S(\boldsymbol{b},\mu,\sqrt{\delta^+\delta^-}) = S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^+\nu)S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^-/\nu)$$

$$\left(S_i^{\text{bare}}(\boldsymbol{b},\delta)\right)^{\frac{1}{2}} = 1 + a_s C_i\Big\{-\frac{2}{\epsilon^2}+\frac{4}{\epsilon}\ln\Big(\frac{\sqrt{2}\,\delta}{\mu}\Big)+\ln\big(B\mu^2 e^{2\gamma_E}\big)\Big[4\ln\Big(\frac{\sqrt{2}\,\delta}{\mu}\Big)+\ln\big(B\mu^2 e^{2\gamma_E}\big)\Big]+\frac{\pi^2}{6}\Big\}$$

In this way we defined the rapidity divergence-free objects

Universal TMDPDF

Rapidity divergence-free dijet soft function

$$F_i(\xi,\boldsymbol{b},\mu,\zeta_1) = \frac{B_i^{\text{un.}}(\xi,\boldsymbol{b},\mu,k^-/\delta^-)}{S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^-/\nu)}\Bigg|_{\sqrt{2}\,k^-/\nu\to\sqrt{\zeta_1}}$$

$$S_{\gamma i}(\boldsymbol{b},\mu,\zeta_2) = \frac{\hat{S}_{\gamma i}(\boldsymbol{b},\mu,\sqrt{A_n}\,\delta^+)}{S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^+\nu)}\Bigg|_{\nu/\sqrt{2A_n}\to\sqrt{\zeta_2}}$$

ζ scale associated with the δ-regulator and zero-bin split

In the Breit frame $\quad \zeta_1\,\zeta_2 = p_T^2$

Kinematic definitions

We define three light-like vectors for the incoming beam and the outgoing jets

$$n^\mu = \frac{1}{\sqrt{2}}(1,0,0,1), \quad n^2 = \bar{n}^2 = 0, \quad \bar{n}\cdot n = 1$$

$$v_J^2 = \bar{v}_J^2 = 0, \quad v_J \cdot \bar{v}_J = 1, \ \text{with } J = 1,2$$

The standard Lorentz invariants

$$Q^2 = -q^2, \quad x = \frac{Q^2}{2P\cdot q}, \quad \xi = \frac{k^+}{P^+}$$

The transverse momentum imbalance

$$\boldsymbol{r}_T = \boldsymbol{p}_{1T} + \boldsymbol{p}_{2T}, \qquad p_T = \frac{|\boldsymbol{p}_{1T}| + |\boldsymbol{p}_{2T}|}{2}$$

Kinematic definitions

We can rewrite them in terms of the Born level kinematics

$$Q = 2p_T \cosh(\eta_-) \exp(\eta_+), \qquad \xi = 2x \cosh(\eta_+) \exp(-\eta_+)$$

$$\eta_\pm = \frac{\eta_1 \pm \eta_2}{2}$$

The parsonic Mandelstam variables are given by

$$\hat{s} = (q+k)^2 = +4p_T^2 \cosh^2(\eta_-)$$

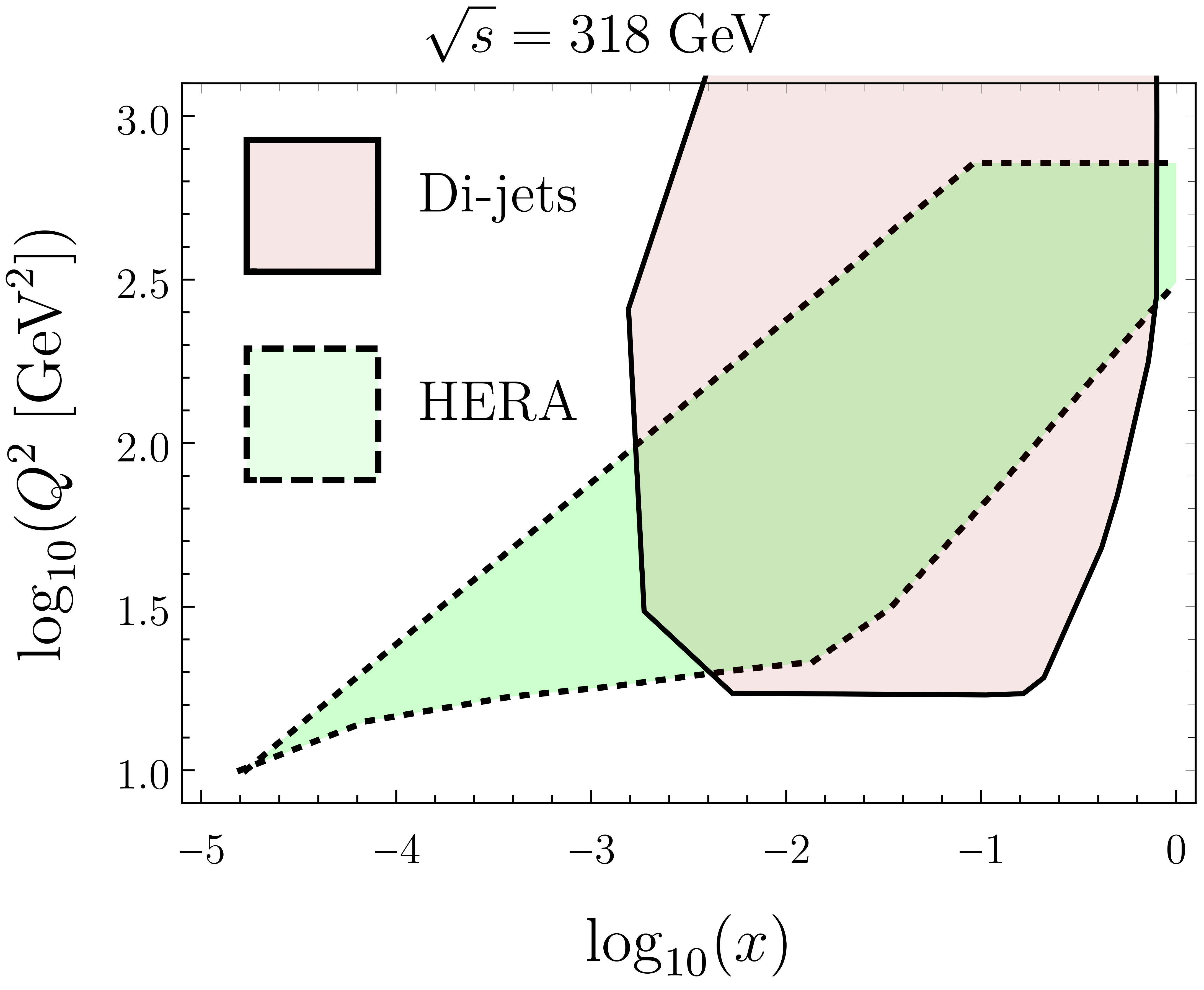
$$\hat{t} = (q-p_2)^2 = -4p_T^2 \cosh(\eta_-) \cosh(\eta_+) \exp(\eta_1)$$

$$\hat{u} = (q-p_1)^2 = -4p_T^2 \cosh(\eta_-) \cosh(\eta_+) \exp(\eta_2)$$

$$\hat{s} + \hat{t} + \hat{u} = -Q^2$$

Kinematic region vs HERA coverage

Dijet production



Cross-section factorization

Dijet production

$$\frac{d\sigma}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T}$$

We measure over

- x Bjorken variable
- η_i jet pseudorapidity
- p_T transverse momentum
- $\boldsymbol{r}_T$ transverse momentum imbalance

$(\gamma^* g)$

$$\frac{d\sigma(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sum_f H^{\mu\nu}_{\gamma^* g \to f\bar{f}}(\hat{s},\hat{t},\hat{u},\mu) \int \frac{d^2\mathbf{b}}{(2\pi)^2} \exp(i\mathbf{b}\cdot\boldsymbol{r}_T) F_{g,\mu\nu}(\xi,\mathbf{b},\mu,\zeta_1)$$
$$\times S_{\gamma g}(\mathbf{b},\eta_1,\eta_2,\mu,\zeta_2) \Big(\mathcal{C}_f(\mathbf{b},R,\mu) J_f(p_T,R,\mu)\Big)\Big(\mathcal{C}_{\bar{f}}(\mathbf{b},R,\mu) J_{\bar{f}}(p_T,R,\mu)\Big)$$

$(\gamma^* f)$

$$\frac{d\sigma^U(\gamma^* f)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sigma_0^{fU} \sum_f H^U_{\gamma^* f \to g\bar{f}}(\hat{s},\hat{t},\hat{u},\mu) \int \frac{d^2\mathbf{b}}{(2\pi)^2} \exp(i\mathbf{b}\cdot\boldsymbol{r}_T) F_f(\xi,\mathbf{b},\mu,\zeta_1)$$
$$\times S_{\gamma f}(\mathbf{b},\zeta_2,\mu) \Big(\mathcal{C}_g(\mathbf{b},R,\mu) J_g(p_T,R,\mu)\Big)\Big(\mathcal{C}_{\bar{f}}(\mathbf{b},R,\mu) J_{\bar{f}}(p_T,R,\mu)\Big)$$

Unpolarized & linearly polarized cross-section

Dijet production

$$F_g^{\mu\nu}(\xi, \boldsymbol{b}) = f_1(\xi, \boldsymbol{b}) \frac{g_T^{\mu\nu}}{d-2} + h_1^{\perp}(\xi, \boldsymbol{b}) \left(\frac{g_T^{\mu\nu}}{d-2} + \frac{b^\mu b^\nu}{\boldsymbol{b}^2} \right)$$

$$H^{\mu\nu}_{\gamma^* g \to f\bar{f}} = \sigma_0^{gU}\, H^U_{\gamma^* g \to f\bar{f}} \frac{g_T^{\mu\nu}}{d-2} + \sigma_0^{gL}\, H^L_{\gamma^* g \to f\bar{f}} \left(-\frac{g_T^{\mu\nu}}{d-2} + \frac{v_{1T}^\mu\, v_{2T}^\nu + v_{2T}^\mu\, v_{1T}^\nu}{2\, v_{1T} \cdot v_{2T}} \right)$$

Unpolarized cross-section

$$\frac{d\sigma^U(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sigma_0^{gU} \sum_f H^U_{\gamma^* g \to f\bar{f}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, f_1(\xi, \boldsymbol{b}, \mu, \zeta_1)$$
$$\times S_{\gamma g}(\boldsymbol{b}, \zeta_2, \mu) \Big(\mathcal{C}_f(\boldsymbol{b}, R, \mu) J_f(p_T, R, \mu)\Big)\Big(\mathcal{C}_{\bar{f}}(\boldsymbol{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu)\Big)$$

Linearly polarized cross-section

$$\frac{d\sigma^L(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sigma_0^{gL} \sum_f H^L_{\gamma^* g \to f\bar{f}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, h_1^{\perp}(\xi, \boldsymbol{b}, \mu, \zeta_1)$$
$$\times \frac{s_{\boldsymbol{b}}^2 - c_{\boldsymbol{b}}^2}{2} S_{\gamma g}(\boldsymbol{b}, \zeta_2, \mu) \Big(\mathcal{C}_f(\boldsymbol{b}, R, \mu) J_f(p_T, R, \mu)\Big)\Big(\mathcal{C}_{\bar{f}}(\boldsymbol{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu)\Big)$$

Zero-bin subtraction

- We need to subtract the zero-bin from the TMD beam function
- The zero-bin corresponds to the two-direction back-to-back soft function (the one used in Drell-Yan or SIDIS). Here, we use the subtraction as done in Echevarría, Idilbi, Scimemi, 2013.
- We can reorganize the zero-bin to obtain rapidity divergence-free function as expressed in the cross-section factorization
- This leads to the universal TMDPDF and a rapidity divergence-free new TMD soft function and the introduction of the scale ζ

$$B_i^{\text{un.}}(\xi, \boldsymbol{b}, \mu, k^-/\delta^-) \quad \longrightarrow \quad F_i(\xi, \boldsymbol{b}, \mu, \zeta_1)$$

$$\hat{S}_{\gamma i}(\boldsymbol{b}, \mu, \sqrt{A_n}\,\delta^+) \quad \longrightarrow \quad S_{\gamma i}(\boldsymbol{b}, \mu, \zeta_2)$$

Zero-bin subtraction

ζ scale

The scale can be removed from the final result by introducing the constrain

$$\zeta_1 \, \zeta_2 = \frac{(k^-)^2}{A_n} = \frac{\hat{u} \, \hat{t}}{\hat{s}}$$

In the Breit-frame this leads to

$$\zeta_1 \, \zeta_2 = p_T^2$$

Notice that

ζ_1 has square mass dimension

ζ_2 is dimensionless

→

$\zeta_1 = p_T^2$, $\zeta_2 = 1$ natural way of choosing the scale

Procedure totally analogous to the one used in Drell-Yan or SIDIS

This allows to use ζ-prescription for TMDPDF and SF evolution

Scimemi, Vladimirov, 2020

Zero-bin subtraction

Subtracted soft function, finite result

$(\gamma^* g)$ - channel

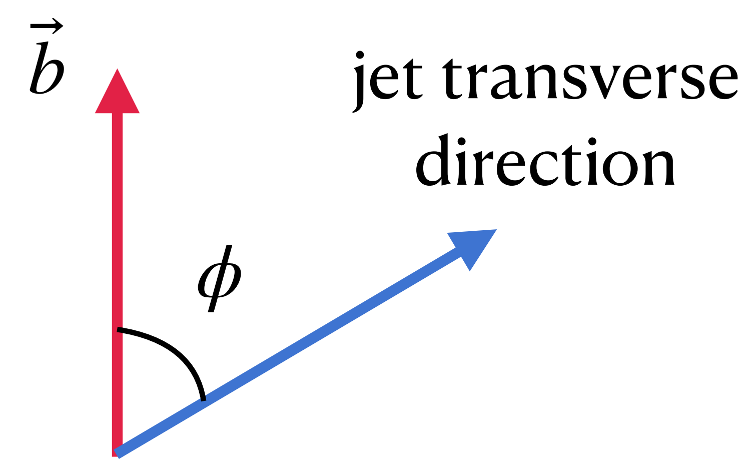
$$S_{\gamma g}(\boldsymbol{b}, \mu, \zeta_2) = 1 + a_s \Big\{ C_F \Big[\frac{\pi^2}{3} + 2\ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 4\mathrm{Li}_2(1 + A_{\boldsymbol{b}}) \Big]$$

$$+ C_A \Big[-2\ln\big(B\mu^2 e^{2\gamma_E}\big) \boxed{\ln\zeta_2} - \ln^2(-A_{\boldsymbol{b}}) - \frac{\pi^2}{3} - 2\mathrm{Li}_2(1 + A_{\boldsymbol{b}}) \Big] \Big\} + \mathcal{O}(a_s^2)$$

with... $$A_{\boldsymbol{b}} = \frac{(v_1 \cdot v_2)}{2\,(v_1 \cdot \hat{b})\,(v_2 \cdot \hat{b})} = -\frac{\hat{s}}{4\,p_T^2\,c_b^2}$$

Consistency check

Dijet-production



$$\frac{d}{d\ln\mu}G(\mu) = \gamma_G(\mu)\,G(\mu)$$

$(\gamma^* g)$-channel $\quad \gamma_{H_{\gamma g}} + \gamma_{S_{\gamma g}} + \gamma_{F_g} + 2\gamma_{J_f} + \gamma_{\mathcal{C}_1} + \gamma_{\mathcal{C}_2} + \gamma_\alpha = 0$

$(\gamma^* f)$-channel $\quad \gamma_{H_{\gamma f}} + \gamma_{S_{\gamma f}} + \gamma_{F_f} + \gamma_{J_f} + \gamma_{J_g} + \gamma_{\mathcal{C}_f} + \gamma_{\mathcal{C}_g} + \gamma_\alpha = 0$

The sum of all anomalous dimensions should cancel for each channel

ζ-logs

ϕ-logs

$$\gamma^{[1]}_{S_{\gamma g}} = 4\Big\{ -C_A \ln\zeta_2 + 2C_F\Big[\ln\big(B\mu^2\, e^{2\gamma_E}\big) - \ln\hat{s} + \ln p_T^2 + \ln\big(4c_{\boldsymbol{b}}^2\big)\Big]\Big\}\,,$$

$$\gamma^{[1]}_{S_{\gamma f}} = 4\Big\{(C_F + C_A)\Big[\ln\big(B\mu^2 e^{2\gamma_E}\big) - \ln\hat{s} + \ln p_T^2 + \ln\big(4c_{\boldsymbol{b}}^2\big)\Big] + (C_F - C_A)\Big[\ln\Big(\frac{\hat{t}}{\hat{u}}\Big) - \kappa(v_f)\Big] - C_F \ln\zeta_2\Big\}$$

$$\gamma^{[1]}_{F_i} = 4C_i\Big[-\ln\Big(\frac{\zeta_1}{\mu^2}\Big) + \gamma_i\Big]\,,$$

$$\gamma^{[1]}_{\mathcal{C}_g} = 4C_A\Big[-\ln\Big(B\mu^2\, e^{2\gamma_E}\Big) + \ln R^2 - \ln\big(4c_{\boldsymbol{b}}^2\big) + \kappa(v_g)\Big]$$

$$\gamma^{[1]}_{\mathcal{C}_i} = 4C_F\Big[-\ln\Big(B\mu^2\, e^{2\gamma_E}\Big) + \ln R^2 - \ln\big(4c_{\boldsymbol{b}}^2\big) + \kappa(v_i)\Big]$$

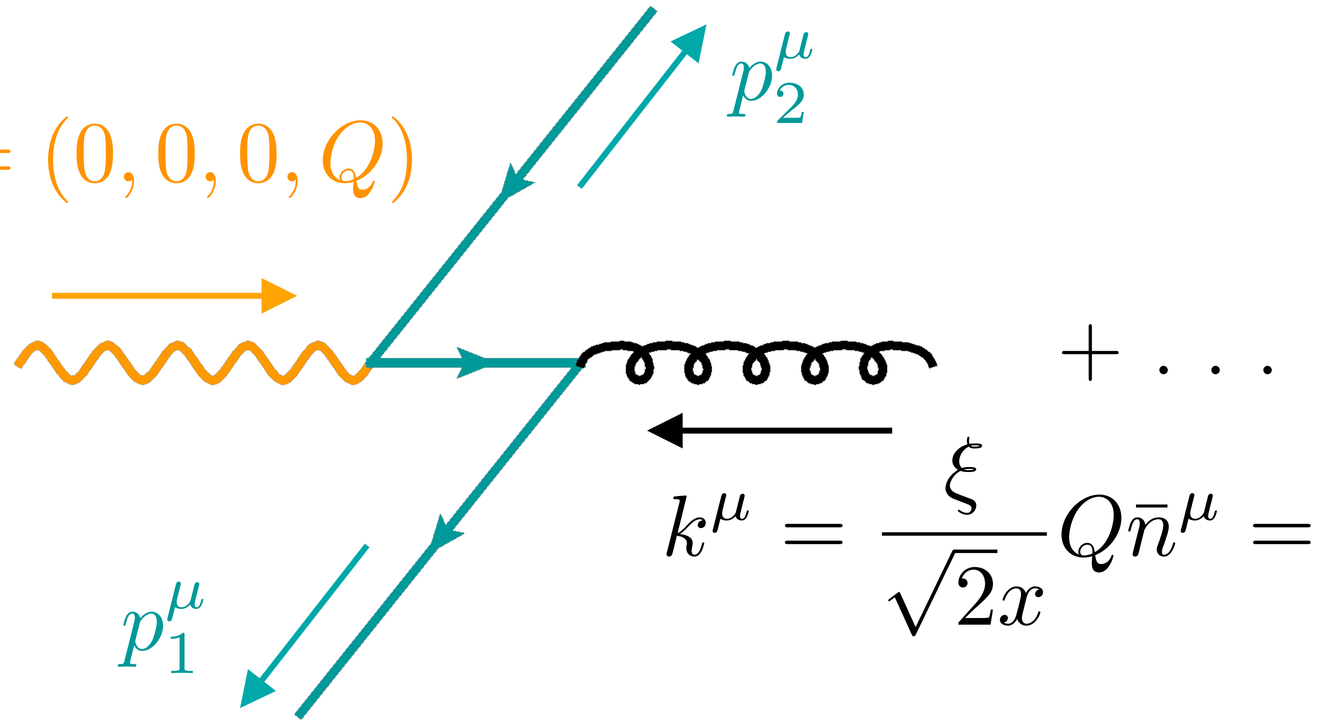
$$\kappa(v_f) = -\kappa(v_{\bar{f}}) = -\kappa(v_g) = i\pi\,\mathrm{sign}(c_{\boldsymbol{b}})$$

They cancel !!!

Heavy-meson pair production

$$\ell + h \to \ell' + H + \bar{H} + X$$

heavy meson pair at LO:



- Experimentally more challenging
- Observation of charmed mesons could be possible

Arratia, Furletova, Hobbs, Olness, Nguyen et al. 2020

Li, Liu, Vitev, 2020

Chudakov, Higinbotham, Hyde, Furletov, Furletova, Nguyen, 2016

Cross-section factorization

Heavy meson pair production

$$\frac{d\sigma(\gamma^* g)}{dx\, d\eta_H\, d\eta_{\bar{H}}\, dp_T\, d\boldsymbol{r}_T}$$

We measure over

- x Bjorken variable
- η_H, $\eta_{\bar{H}}$ heavy meson pseudorapidity
- p_T transverse momentum
- $\boldsymbol{r}_T$ transverse momentum imbalance

$$\frac{d\sigma(\gamma^* g)}{dx\, d\eta_H\, d\eta_{\bar{H}}\, dp_T\, d\boldsymbol{r}_T} = H^{\mu\nu}_{\gamma^* g \to Q\bar{Q}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, F_{g,\mu\nu}(\xi, \boldsymbol{b}, \mu, \zeta_1)$$
$$\times S_{\gamma g}(\boldsymbol{b}, \mu, \zeta_2)\, J_{Q\to H}(\boldsymbol{b}, p_T, m_Q, \mu)\, J_{\bar{Q}\to\bar{H}}(\boldsymbol{b}, p_T, m_Q, \mu)$$

Fickinger, Fleming, Kim, Mereghetti, 2016

Region sensitive to TMD $|\boldsymbol{r}_T| \ll p_T^{H,\bar{H}}$

Factorization for highly boosted heavy mesons $p_T^H \gg m_H$

We have a new scale m_Q

Connection to the fragmentation shape function

Heavy meson pair production

We can see that the shape function is related to the heavy meson jet function

$$\mathcal{J}_{Q\to H}(\boldsymbol{b}) = \frac{m_H}{\sqrt{2}\,p_H^-}\tilde{S}_{Q\to H}\left(\tau \to \frac{\boldsymbol{v}\cdot\boldsymbol{b}}{\sqrt{2}}\right)$$

- We can check our NLO calculation (finite terms)
- We can get the jet function AD up to two loops

Fickinger, Fleming, Kim, Mereghetti, 2016

This sum is known up to three-loops...

$$\gamma_{S_{\gamma g}} = -\left(\gamma_{H_{\gamma g}} + \gamma_{F_g} + \gamma_\alpha + \gamma_{\mathcal{J}}(\boldsymbol{v}_1) + \gamma_{\mathcal{J}}(\boldsymbol{v}_2) + 2\gamma_+\right)$$

Soft function anomalous dimension

$$\gamma_{S_{\gamma g}} = \gamma_{\text{cusp}}\left[2C_F \ln\left(\frac{B\mu^2 e^{2\gamma_E}}{-A_b}\right) - C_A \ln\zeta_2\right] + \delta\gamma_{S_{\gamma g}}$$

$$\delta\gamma^{[1]}_{S_{\gamma g}} = 0$$

$$\delta\gamma^{[2]}_{S_{\gamma g}} = C_F\left[C_A\left(\frac{1616}{27} - \frac{22}{9}\pi^2 - 56\,\zeta_3\right) + n_f T_F\left(-\frac{448}{27} + \frac{8}{9}\pi^2\right)\right]$$

$$\delta\gamma^{[3]}_{S_{\gamma g}} = \dots$$

Connection to the fragmentation shape function

Heavy meson pair production

Shape function is defined as Fickinger, Fleming, Kim, Mereghetti, 2016

$$S_{Q\to H}(\omega) = \frac{1}{2N_c}\sum_X \langle 0|\delta(\omega - i\sqrt{2}\,\bar{v}\cdot\partial)W_v^\dagger h_{v\beta_+}|H_\beta X\rangle\langle H_\beta X|\bar{h}_{v,\beta_+}W_v\frac{\not{\bar{v}}}{\sqrt{2}}|0\rangle$$

and its Fourier transformation…

$$\tilde{S}_{Q\to H}(\tau) = \int d\omega\,\exp(i\omega\tau)S_{Q\to H}(\omega)$$

$$= \frac{1}{2\,m_H N_C}\sum_X \langle 0|\exp\left(-\sqrt{2}\,\tau\,\bar{v}\cdot\partial\right)\,W_v^\dagger h_{v\beta_+}|XH\rangle\langle XH|\bar{h}_{Qv}W_v\frac{\not{\bar{v}}}{\sqrt{2}}|0\rangle$$

We can see that it is related to the bHQET jet function

$$\mathcal{J}_{Q\to H}(\boldsymbol{b}) = \frac{m_H}{\sqrt{2}\,p_H^-}\tilde{S}_{Q\to H}\left(\tau \to \frac{\boldsymbol{v}\cdot\boldsymbol{b}}{\sqrt{2}}\right)$$

- We can check our NLO calculation (finite terms)
- We can get the jet function AD up to two loops

Refactorization of heavy-quark fragmentation

Heavy meson pair production

We use heavy-quark jet function to describe the fragmentation of heavy mesons from heavy quarks. In the limit $r_T \ll p_T$ there are two scales that need to be resummed

$$\mu_+ = m_Q, \qquad \text{and} \qquad \mu_{\mathcal{J}} = m_Q \frac{r_T}{p_T}$$

To do this we use **bHQET (boosted heavy quark effective theory)** to factorize the jet function into a hard matching coefficient and a TMD matrix element

$$J_{Q\to H}(\boldsymbol{b}, p_T, m_Q, \mu) = H_+(m_Q, \mu)\mathcal{J}_{Q\to H}\left(\boldsymbol{b}, \frac{m_Q}{p_T}; \mu\right)$$

Appears for the first time

Known up to two-loops

Refactorization of heavy-quark fragmentation

Heavy meson pair production

Up to one loop order we find

$$H_+(m_Q,\mu) = 1 + \frac{\alpha_s}{4\pi} C_F \left\{ \ln\left(\frac{\mu^2}{m_Q^2}\right) + \ln^2\left(\frac{\mu^2}{m_Q^2}\right) + 8 + \frac{\pi^2}{6} \right\}$$

$$\gamma_+ = \frac{\alpha_s C_F}{\pi} \left\{ \frac{1}{2} - \ln\left(\frac{m_Q^2}{\mu^2}\right) \right\}$$

$$\mathcal{J}^{\text{bare}}_{Q\to Q}\left(\boldsymbol{b}, \frac{m_Q}{p_T}\right) = 1 + \frac{\alpha_s C_F}{\pi} \left\{ -\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\left[1 - 2\ln\mathcal{R}\right] + \ln\mathcal{R} - \ln^2\mathcal{R} - \frac{5\pi^2}{24} \right\}$$

$$\gamma_{\mathcal{J}} = \frac{\alpha_s C_F}{\pi} \{1 - 2\ln\mathcal{R}\}$$

with... $\mathcal{R} = -\frac{i\, p_T \mu\, e^{\gamma_E} (\boldsymbol{v}\cdot\boldsymbol{b})}{m_Q |\boldsymbol{v}|}$

We have separated scales and can now be resummed

Anomalous dimension are consistent $\gamma_{\mathcal{J}} + \gamma_+ = \gamma_J + \gamma_{C_f}$ ⟶ Consistent !!!

New soft function

$$A_{\boldsymbol{b}} = \frac{(v_1 \cdot v_2)}{2\,(v_1 \cdot \hat{b})\,(v_2 \cdot \hat{b})} = -\frac{\hat{s}}{4\,p_T^2\,c_b^2}$$

$$A_n = \frac{(v_1 \cdot v_2)}{2\,(v_1 \cdot n)(v_2 \cdot n)}$$

Bare soft function

$$\hat{S}^{\text{bare}}_{\gamma g}(\boldsymbol{b}) = \hat{S}^{\text{finite}}_{\gamma g}(\boldsymbol{b}) + a_s\Big\{C_A\Big[-\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\Big(2\ln\Big(\frac{\sqrt{2}\,\delta^+}{\mu}\Big) + \ln(2A_n)\Big)\Big] + 2C_F\Big[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\Big(\frac{B\,\mu^2\,e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big)\Big]\Big\}$$

$$\hat{S}^{\text{bare}}_{\gamma f}(\boldsymbol{b}) = \hat{S}^{\text{finite}}_{\gamma f}(\boldsymbol{b}) + a_s\Big\{C_A\Big[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\Big(\ln\frac{(n\cdot v_1)(\boldsymbol{v}_2\cdot\boldsymbol{b})}{(n\cdot v_2)(\boldsymbol{v}_1\cdot\boldsymbol{b})} + \ln\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big)\Big)\Big] + \frac{4}{\epsilon}C_F\ln\Big(-\frac{i\,\boldsymbol{v_1}\cdot\boldsymbol{b}\,\delta^+ e^{\gamma_E}}{n\cdot v_1}\Big)\Big\}$$

Finite soft function

$$\hat{S}^{\text{finite}}_{\gamma g}(\boldsymbol{b}) = 1 + a_s\Big\{C_A\Big[\ln\big(B\,\mu^2 e^{2\gamma_E}\big)\Big(\ln\big(B\mu^2 e^{2\gamma_E}\big) + 4\ln\Big(\frac{\sqrt{2}\,\delta^+}{\mu}\Big) + 2\ln(2A_n)\Big) - \ln^2(-A_{\boldsymbol{b}}) - \frac{\pi^2}{6} - 2\mathrm{Li}_2(1+A_{\boldsymbol{b}})\Big] + C_F\Big[\frac{\pi^2}{3} + 2\ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 4\mathrm{Li}_2(1+A_{\boldsymbol{b}})\Big]\Big\},$$

$$\hat{S}^{\text{finite}}_{\gamma f}(\boldsymbol{b}) = 1 + a_s\Big\{C_A\Big[\frac{\pi^2}{6} + \ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 2\mathrm{Li}_2(1+A_{\boldsymbol{b}}) + 2\ln\big(B\mu^2 e^{2\gamma_E}\big)\ln\frac{(n\cdot v_1)(\boldsymbol{v}_2\cdot\boldsymbol{b})}{(n\cdot v_2)(\boldsymbol{v}_1\cdot\boldsymbol{b})}\Big] + 4C_F\ln\big(B\mu^2 e^{2\gamma_E}\big)\ln\Big(-\frac{i\,\boldsymbol{v_1}\cdot\boldsymbol{b}\,\delta^+ e^{\gamma_E}}{n\cdot v_1}\Big)\Big\}$$

$$A_{\boldsymbol{b}} = \frac{(v_1 \cdot v_2)}{2\,(v_1 \cdot \hat{b})\,(v_2 \cdot \hat{b})} = -\frac{\hat{s}}{4\,p_T^2\,c_b^2}$$

$$A_n = \frac{(v_1 \cdot v_2)}{2\,(v_1 \cdot n)(v_2 \cdot n)}$$

Zero-bin subtraction

Subtracted soft function

Finite soft function

$$S_{\gamma g}(\boldsymbol{b},\mu,\zeta_2) = 1 + a_s\Big\{C_F\Big[\frac{\pi^2}{3} + 2\ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 4\mathrm{Li}_2(1+A_{\boldsymbol{b}})\Big] + C_A\Big[-2\ln\big(B\mu^2 e^{2\gamma_E}\big)\ln\zeta_2 - \ln^2(-A_{\boldsymbol{b}}) - \frac{\pi^2}{3} - 2\mathrm{Li}_2(1+A_{\boldsymbol{b}})\Big]\Big\} + \mathcal{O}(a_s^2)$$

$$S_{\gamma f}(\boldsymbol{b},\mu,\zeta_2) = 1 + a_s\Big\{C_A\Big[\frac{\pi^2}{6} + \ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 2\mathrm{Li}_2(1+A_{\boldsymbol{b}}) + 2\ln\big(B\mu^2 e^{2\gamma_E}\big)\ln\frac{(n\cdot v_1)(\boldsymbol{v}_2\cdot\boldsymbol{b})}{(n\cdot v_2)(\boldsymbol{v}_1\cdot\boldsymbol{b})}\Big] + C_F\ln\big(B\mu^2 e^{2\gamma_E}\big)\Big[\ln\big(B\mu^2 e^{2\gamma_E}\big) - 2\ln\zeta_2 + 2\ln\Big(\frac{2(n\cdot v_2)}{(v_1\cdot v_2)(n\cdot v_2)}\Big) - \frac{\pi^2}{6} + 4\ln(-i\,\boldsymbol{v}_1\cdot\hat{\boldsymbol{b}})\Big]\Big\} + \mathcal{O}(a_s^2),$$

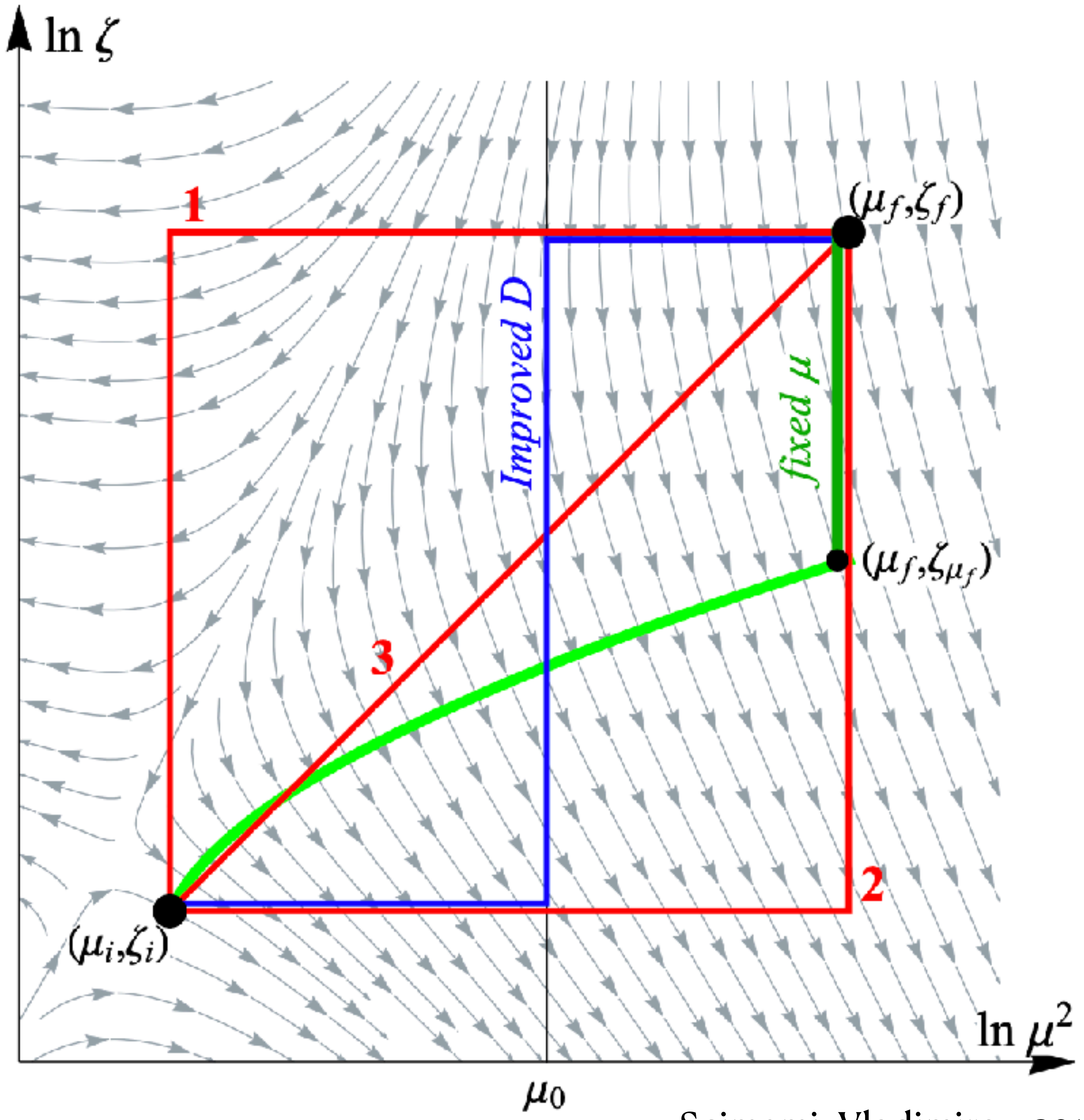
Renormalization function

$$Z^S_{\gamma g}(\boldsymbol{b},\mu,\zeta_2) = 1 + a_s\Big\{C_F\Big[\frac{4}{\epsilon^2} + \frac{4}{\epsilon}\ln\Big(\frac{B\mu^2\,e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big)\Big] - C_A\frac{2}{\epsilon}\ln\zeta_2\Big\} + \mathcal{O}(a_s^2)$$

$$Z^S_{\gamma f}(\boldsymbol{b},\mu,\zeta_2) = 1 + a_s\Big\{C_A\Big[\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\Big(\ln\frac{(n\cdot v_1)(\boldsymbol{v}_2\cdot\boldsymbol{b})}{(n\cdot v_2)(\boldsymbol{v}_1\cdot\boldsymbol{b})}\Big) + \ln\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big)\Big] + \frac{2}{\epsilon}C_F\Big[\ln\big(B\mu^2 e^{2\gamma_E}\big) - \ln\zeta_2 + \ln\Big(\frac{2(n\cdot v_2)}{(v_1\cdot v_2)(n\cdot v_2)}\Big) + 2\ln(-i\,\boldsymbol{v}_1\cdot\hat{\boldsymbol{b}})\Big]\Big\} + \mathcal{O}(a_s^2)$$

Evolution, double-scale evolution

fixed μ evolution



Scimemi, Vladimirov, 2018
Scimemi, Vladimirov, 2020

Describe evolution of functions depending on two scales

TMDPDF | TMDFF | Dijets SF

μ scale	Renormalization scale	ε regulator (DR)
ζ scale	Rapidity scale	δ regulator