Definite Orbital Angular Momentum Nucleon GPD Contributions via Light Front Wave Function Overlap

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October 28 2022

Working on the Lightfront with Lightcone Coordinates

$$v^{\pm} \equiv \frac{v^0 \pm v^3}{\sqrt{2}}, \tag{1}$$

$$\vec{v}_{\perp} = (v^1, v^2) \tag{2}$$

such that Minkowski 4-vectors become

$$v = (v^+, \vec{v}_{\perp}, v^-) \tag{3}$$

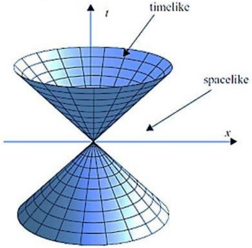
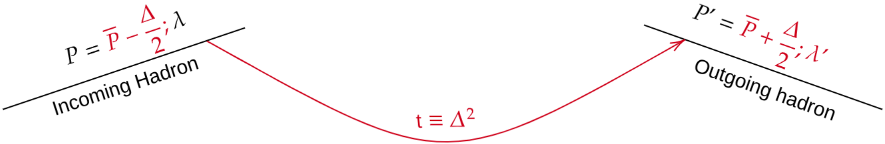


Figure: Light Cone

Defining GPDs

[Ji, 1997a] [D. Müller et al., 1994] [Radyushkin, 1997]

$$\overline{P} \equiv \frac{P' + P}{2}, \quad \Delta = P' - P \to t = \Delta^2$$



$$\equiv \mathcal{H}^q_{\lambda'\lambda} = \langle P'; \lambda' || P; \lambda \rangle \tag{4}$$

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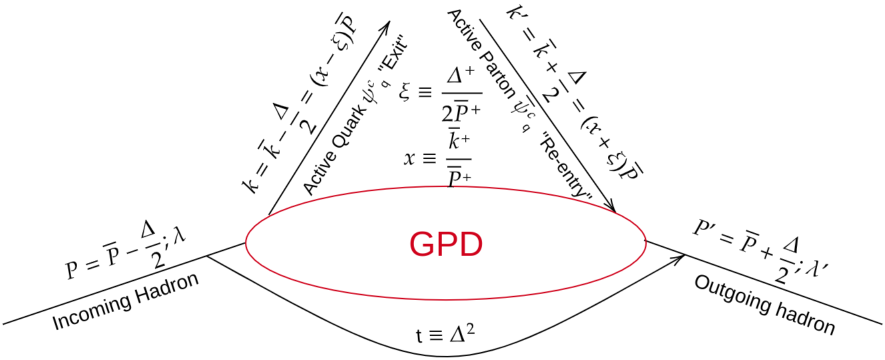

$$\equiv \mathcal{H}^q_{\lambda'\lambda} = \frac{\sum_c \int \frac{dz^-}{2\pi} e^{i\bar{k}^+ z^-}}{2\sqrt{1-\xi^2}} \langle P'; \lambda' | \bar{\psi}^c_q(-\frac{\bar{z}}{2}) \gamma^+ \psi^c_q(\frac{\bar{z}}{2}) | P; \lambda \rangle \tag{4}$$

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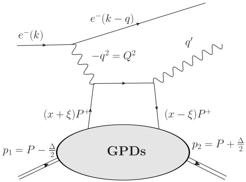

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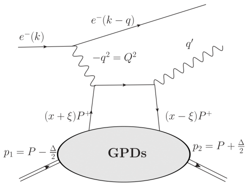

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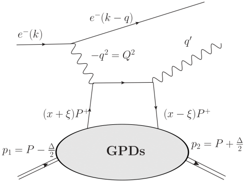

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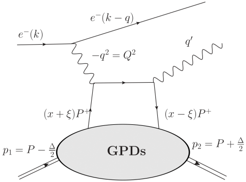

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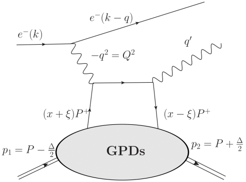

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- Access quark and gluon contributions to the total angular momentum of the nucleon [Ji, 1997b]

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The literature now includes simple algebraic models [Mezrag et al., 2015] and advanced computations [Raya et al., 2022]

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- Polynomality: By Radon Transform [Chouika et al., 2017]

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DVCS Meson Sector Prediction: Publication for phenomenological studies [Chávez et al., 2022]

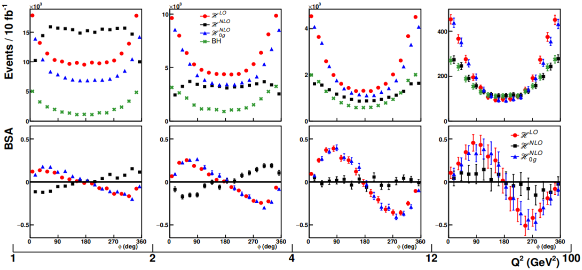

FIG. 3. Number of DVCS events (upper charts) and expected beam-spin asymmetries (lower chart) as a function of Q^2 for $x_B^\pi \in [10^{-3}; 10^{-2}]$. Red circles: LO evaluation of the CFF; blue triangles: NLO evalution but without taking gluon GPDs into account; black circles: full NLO results. The BH event rates is as well displayed by the green crosses.

Light Front Wave Functions (LFWFs) as Fock Coefficients

Matrix element of ultimate interest:

$$\langle P'; \lambda' | \bar{\psi}_q^c(-\bar{z}/2) \gamma^+ \psi_q^c(\bar{z}/2) | P; \lambda \rangle \tag{5}$$

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For further investigation a basis for the incoming and outgoing nucleon states is required:

$$| P; \lambda_{\mathrm{N}} \rangle = \sum_{\mathrm{Fock}} \Psi^{\mathrm{Fock}}_{\lambda_{\mathrm{N}}} | \mathrm{Fock} \rangle \tag{6}$$

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- $\rightarrow$ Each valence LFWF corresponds to a particular set of quark helicities (with sum λ_q)
  - $\rightarrow$ Due to conservation of angular momentum each LFWF corresponds to a definite quark orbital angular momentum (qOAM $= \lambda_{\rm N} - \lambda_q$)

λ_N	$1/2$	$1/2$	$1/2$	$1/2$	$-1/2$	$-1/2$	$-1/2$	$-1/2$
λ_q	$3/2$	$1/2$	$-1/2$	$-3/2$	$3/2$	$1/2$	$-1/2$	$-3/2$
qOAM	-1	0	1	2	-2	-1	0	1

Accessing LFWFs from Hadronic Matrix Elements

How can we actually access them?

We define $$N_\sigma^+ \Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0 | (\prod_{i=1}^{3} q^+_{i,f_i,\alpha_i}(\kappa_i)) | P; \lambda_{\mathrm{N}} \rangle \tag{8}$$

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The $\phi_j(\kappa)$ exhibit various symmetry properties due to u-quark symmetry and futher due to imposition of isospin symmetry

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    - ★ There are 6 independent functions $\Psi_{\lambda_\mathrm{N},\lambda_q}$
    - ★ By projecting our general matrix element onto carefully selected Dirac structures we may isolate the $\Psi_{\lambda_\mathrm{N},\lambda_q}$ directly

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    - ⋆ → After using your favorite way of calculating $\Omega_{f,\alpha,\sigma}$ you may directly calculate the LFWFs
- Let's calculate GPDs using our convenient new basis!

Overlap Representation of GPDs

According to [Diehl et al., 2001] one may calculate GPDs as sums of overlaps of LFWFs.

Original Matrix Element

$$\mathcal{H}^{q}_{\lambda'_N \lambda_N} \equiv \frac{1}{2\sqrt{1-\xi^2}} \sum_c \int \frac{dz^- e^{ik^+ z^-}}{2\pi} \langle P'; \lambda' | \bar{\psi}^c_q(-\bar{z}/2) \gamma^+ \psi^c_q(\bar{z}/2) | P; \lambda \rangle$$

Leading Fock $\rightarrow$

$$= \sqrt{1-\xi}\sqrt{1+\xi} \sum_{\lambda_q = \lambda'_q} \sum_j \delta_{s_j q} \int [d\bar{x}]_N [d^2 \bar{\mathbf{k}}_\perp]_N \delta(\bar{x} - \bar{x}_j)$$

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$$\equiv \int \mathcal{D}\sum_{\sigma\in S_3}\delta^{c,f,h}_{\sigma(c'),\sigma(f'),\sigma(h')}\sum_{l=1}^{3}\delta_{f_l,q}\Psi^*_{\lambda'_N,\lambda'_q}(\sigma(\kappa))|_l\Psi_{\lambda_N,\lambda_q}$$

Expressing GPDs

Quark orbital angular momentum: 0, 1, 2.

$$\mathcal{H}^q_{++} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^q(\hat{\Psi}_{\frac{1}{2},\frac{1}{2}},\hat{\Psi}_{\frac{1}{2},\frac{1}{2}}) + \mathcal{O}^q(\hat{\Psi}_{\frac{1}{2},\frac{-1}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-1}{2}}) \tag{10}$$

$$+ \mathcal{O}^q(\hat{\Psi}_{\frac{1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^q(\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big)$$

$$\mathcal{H}^q_{-+} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big) \tag{11}$$

$$H^q = \mathcal{H}^q_{++} + \frac{\xi^2 2m|\vec{\Delta}_\perp|}{(\Delta_1 + i\Delta_2)\sqrt{1-\xi^2}\sqrt{\frac{4\xi^2 m^2}{\xi^2-1} - t}}\mathcal{H}^q_{-+} \tag{12}$$

$$E^q = \frac{2m|\vec{\Delta}_\perp|\sqrt{1-\xi^2}}{(\Delta_1 + i\Delta_2)\sqrt{\frac{4\xi^2 m^2}{\xi^2-1} - t}}\mathcal{H}^q_{-+} \tag{13}$$

(14)

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$\rightarrow$ Non-zero quark OAM states are therefore expected to contribute to the energy momentum tensor through the GPD E.

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- Subgoal: Express individual qOAM contributions to GPDs
- Hurdle: Decomposition of the nucleon states $|P;\lambda_N\rangle$ is necessary
    - Leap: We choose to parametrize the matrix element characterizing the contribution of various Fock states, $\Omega_{f,\alpha,\sigma}$, to the state $|P;\lambda_N\rangle$, in terms of the $\Psi_{\lambda_N,\lambda_q}$, a Fock basis which makes manifest contributions of distinct qOAM
        - ⋆ $\rightarrow$ After using your favorite way of calculating $\Omega_{f,\alpha,\sigma}$ you may directly calculate the LFWFs
- Distinct qOAM contributions to GPDs H, E, $\tilde{\text{H}}$, $\tilde{\text{E}}$ and are calculable from the LFWF basis

Conclusions and Future Perspectives

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- Future work will include calculating valence nucleon LFWFs from projected nucleon Fadeev amplitudes in a quark-diquark framework
  - Relative contributions of distinct values of qOAM to GPDs, PDFs, FFs, and the electric radius of the nucleus will be assessed

Thank you

Thank you!

Projection of Ω

$$
\begin{aligned}
& \epsilon^{c_1,c_2,c_3} \langle 0 | q^{+,c_1}_{\alpha_1,f_1}(z_1^-, z_{\perp 1}) q^{+,c_2}_{\alpha_2,f_2}(z_2^-, z_{\perp 2}) q^{+,c_3}_{\alpha_3,f_3}(z_3^-, z_{\perp 3}) | P, \lambda \rangle |_{z^+=0} \\
= \quad & \frac{1}{4} f_N N_\sigma(P,\lambda) \int \left[\prod_{j=1}^{3} \mathrm{d}k_j^+ \mathrm{d}^{(2)} k_{\perp j} \right] e^{-i(k_j^+ z_j^- - k_{\perp j} z_{\perp j})} \delta(P^+ - \sum_j k_j^+) \\
\times \quad & \delta^{(2)}(P_\perp - \sum_j k_{\perp j}) \Omega_{\alpha_1 \alpha_2 \alpha_3;\sigma} \qquad (18)
\end{aligned}
$$

Framed Coordinates

For the incoming frame:

$$x'_i \equiv \frac{\bar{x}_i}{1+\xi}, \qquad \vec{k}'_{iT} \equiv \vec{\bar{k}}_{iT} + \frac{\bar{x}_i}{1+\xi}\frac{\vec{\Delta}_T}{2}$$
$$x'_j \equiv \frac{\bar{x}_j+\xi}{1+\xi}, \qquad \vec{k}'_{jT} \equiv \vec{\bar{k}}_{jT} - \frac{1-\bar{x}_j}{1+\xi}\frac{\vec{\Delta}_T}{2} \tag{19}$$

and for the outgoing frame

$$x_i \equiv \frac{\bar{x}_i}{1-\xi}, \qquad \vec{k}_{iT} \equiv \vec{\bar{k}}_{iT} - \frac{\bar{x}_i}{1-\xi}\frac{\vec{\Delta}_T}{2}$$
$$x_j \equiv \frac{\bar{x}_j-\xi}{1-\xi}, \qquad \vec{k}_{jT} \equiv \vec{\bar{k}}_{jT} + \frac{1-\bar{x}_j}{1-\xi}\frac{\vec{\Delta}_T}{2} \tag{20}$$

Proton PDFs and FFs

$$
\begin{aligned}
f^{p,q} &= H^{p,q}|_{t=\xi=0} = \Big(\mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{1}{2}}, \hat{\Psi}_{1,\frac{1}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-1}{2}}, \hat{\Psi}_{1,\frac{-1}{2}}) \quad (21) \\
&+ \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{3}{2}}, \hat{\Psi}_{1,\frac{3}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-3}{2}}, \hat{\Psi}_{1,\frac{-3}{2}})\Big)|_{t=\xi=0}
\end{aligned}
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\end{aligned}
$$

$$
F_i^{p;P}(t) = \frac{4}{3} F_i^{p,u}(t) - \frac{1}{3} F_i^{p,d}(t) \qquad (22)
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$$+ \quad \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{3}{2}}, \hat{\Psi}_{1,\frac{3}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-3}{2}}, \hat{\Psi}_{1,\frac{-3}{2}})\Big)|_{t=\xi=0}$$

$$F_i^{p;P}(t) = \frac{4}{3}F_i^{p,u}(t) - \frac{1}{3}F_i^{p,d}(t) \quad (22)$$

$$F_1^{p,q}(t) \equiv \int_{-1}^{1} dx H^{p,q}(x,0,t) = \int_{-1}^{1} dx \Big(\mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{1}{2}}, \hat{\Psi}_{1,\frac{1}{2}})$$
$$+ \quad \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-1}{2}}, \hat{\Psi}_{1,\frac{-1}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{3}{2}}, \hat{\Psi}_{1,\frac{3}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-3}{2}}, \hat{\Psi}_{1,\frac{-3}{2}})\Big)|_{\xi=0}$$

$$F_2^{p,q}(t) \equiv \int_{-1}^{1} dx E^{p,q}(x,0,t) = \int_{-1}^{1} dx \frac{(-1)^p 2M_N|\vec{\Delta}_\perp|}{(\Delta_1 + i\Delta_2)\sqrt{-t}}$$
$$\times \quad \Big(\mathcal{O}^{p,q}(\hat{\Psi}_{-1,\frac{-3}{2}}, \hat{\Psi}_{1,\frac{3}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{-1,\frac{3}{2}}, \hat{\Psi}_{1,\frac{-3}{2}})\Big)|_{\xi=0}$$

Proton Electric Radius

$$\langle (r_E^P)^2 \rangle = 6\hbar^2 \partial_t \left(F_1^P(t) - \frac{t}{4M_N^2} F_2^P(t) \right)\Big|_{t=0} \qquad (23)$$

M_N represents the nucleon mass.

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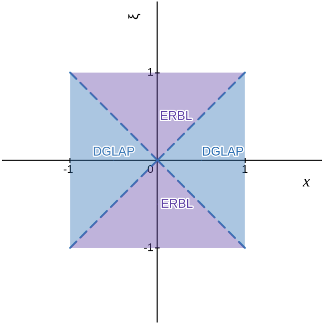
M_N represents the nucleon mass.
This object relies on the $t = 0$ behaviour of the LFWFs, which will be used in future work to constrain modeling assumptions, with specific regard given to Nakanishi weight function based models.

DGLAP & ERBL



- LFWFs calculated in the DGLAP region feature incoming and outgoing states with identical numbers of partons, whereas the ERBL region requires an unequal number of partons in each state.
- By Radon transforming and subsequently inverse Radon transforming expressions for GPDs in the DGLAP region, one finds ERBL GPDs satisfying important modeling assumptions (i.e. polynomality is conserved)

Expressing Polarized GPDs

Quark orbital angular momentum: 0, 1, 2.
($p = 0 \leftrightarrow$ unpolarized; $p = 1 \leftrightarrow$ polarized)

$$\mathcal{H}^{q,p}_{++} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^{q,p}(\hat{\Psi}_{\frac{1}{2},\frac{1}{2}},\hat{\Psi}_{\frac{1}{2},\frac{1}{2}}) + \mathcal{O}^{q,p}(\hat{\Psi}_{\frac{1}{2},\frac{-1}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-1}{2}}) + \mathcal{O}^{q,p}(\hat{\Psi}_{\frac{1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^{q,p}(\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big) \tag{24}$$

$$\mathcal{H}^{q,p}_{-+} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^{q,p}(\hat{\Psi}_{\frac{-1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^{q,p}(\hat{\Psi}_{\frac{-1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big) \tag{25}$$

$$H^{q,p} = \mathcal{H}^{q,p}_{++} + \frac{\xi^2 2m|\vec{\Delta}_\perp|}{(\Delta_1 + i\Delta_2)\sqrt{1-\xi^2}\sqrt{\frac{4\xi^2 m^2}{\xi^2-1} - t}}\mathcal{H}^{q,p}_{-+} \tag{26}$$

$$E^{q,p} = \frac{2m|\vec{\Delta}_\perp|\sqrt{1-\xi^2}}{(\Delta_1 + i\Delta_2)\sqrt{\frac{4\xi^2 m^2}{\xi^2-1} - t}}\mathcal{H}^{q,p}_{-+} \tag{27}$$

$$\mathcal{O}^{q,p}(\hat{\Psi}'_{\lambda'_N,\lambda_q},\hat{\Psi}_{\lambda_N,\lambda_q})$$

$$\equiv \int \mathcal{D} \sum_{\sigma \in S_3} \delta^{c,f,h}_{\sigma(c'),\sigma(f'),\sigma(h')} \sum_{l=1}^{3} \delta_{f_l,q} \Psi^*_{\lambda'_N,\lambda'_q}(\sigma(\kappa))|_l \Psi_{\lambda_N,\lambda_q} \mathrm{sign}^p(\lambda_{\mathrm{active}})$$

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^q(\Psi^*_{\lambda'_N,\lambda'_q}, \Psi_{\lambda_N,\lambda_q})$$

$$\equiv$$

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^q(\Psi^*_{\lambda'_N,\lambda'_q}, \Psi_{\lambda_N,\lambda_q}) \equiv \int \mathcal{D}$$

$$\mathcal{D} \equiv \frac{1}{2}\prod_{l=1}^{3}(\frac{dx_l d^2\vec{k}_{lT}}{(2\pi)^2\sqrt{x_l}})\delta(1-\sum_{l=1}^{3}x_l)\delta^{(2)}(\sum_{l=1}^{3}\vec{k}_{lT})$$

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Sum over all possible permutations of:

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Sum over all possible permutations of:
-Color

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Sum over all active quarks

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Sum over all active quarks (of the correct flavor)
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$$\Psi'^*|_l\Psi \equiv \Psi'^*|^{\text{Outgoing variables}}_{l\text{th quark active}} \Psi|^{\text{Incoming variables}}_{l\text{th quark active}} \delta(\bar{x} - x_l)$$

The lth quark is active.

Accessing LFWFs from Hadronic Matrix Elements

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We define $$N_{\sigma}^{+}\Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0|(\prod_{i=1}^{3} q_{i,f_i,\alpha_i}^{+}(\kappa_i))|P;\lambda_{\mathrm{N}}\rangle \tag{28}$$

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where the quark annhilation operator q_i

- has been projected onto its '+' lightcone component $q_i^+ \equiv \frac{1}{2}\gamma^-\gamma^+ q_i$ to restrict the analysis to leading twist

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and where N^+_σ is the nucleon spinor with Dirac index σ

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$$N^+_\sigma \Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0|(\prod_{i=1}^{3} q^+_{i,f_i,\alpha_i}(\kappa_i))|P;\lambda_\mathrm{N}\rangle \tag{28}$$

We parametrize at leading twist
$$N^+_\sigma \Omega_{f,\alpha,\sigma} = (\sum_j T_{j,\beta}\phi_j(\kappa))N^+_\beta \tag{29}$$

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by using Lorentz covariance as a constraint on the tensorial structures $T_{j,\beta}$ [Ji et al., 2003][Braun et al., 2000]

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The $\phi_j(\kappa)$ exhibit various symmetry properties due to u-quark symmetry and futher due to imposition of isospin symmetry