Extracting the nucleon axial form factor from Lattice QCD using chiral perturbation theory

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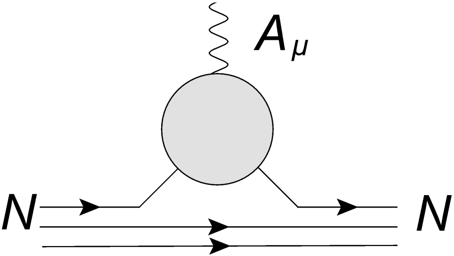
Nucleon Axial Form Factor

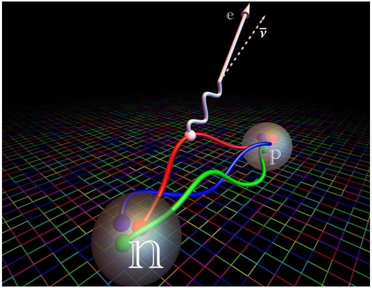
- Nucleon Axial Form Factor, $F_A(q^2)$
  - Electroweak interactions open a doorway to fundamental properties of strong interacting matter: spins distribution
  - $A^i_\mu(x) = \bar{q}(x)\gamma_\mu\gamma_5 \frac{\tau^i}{2} q(x)$
  - $\langle N(p')| A^i_\mu |N(p)\rangle =$
    $\bar{u}\left\{\gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N} G_P(q^2)\right\}\gamma_5 \frac{\tau^i}{2} u(p)$
- $$\boxed{F_A(q^2) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]}$$
- g_A and F_A dependence in q^2 are necessary in ν oscillations experiments
- μ capture, β-decay
- Chiral Perurbation Theory calculation of F_A $\Longrightarrow$ extract $\langle r_A^2\rangle$ from lattice QCD without ad-hoc parametrization



Axial form factor, F_A

- Empirical determinations
  - Rely on neutrino-induced charged-current quasielastic scattering on deuteron targets, muon capture in muonic hydrogen and pion electro-production.
- LQCD
  - Several studies on $F_A(q^2) \longrightarrow$ technical difficulties $\Longrightarrow$ significantly improved control of the systematic error
  - Tension between LQCD and empirical determinations
  - Experimental and lattice q^2 parametrisation:

$$\left.\begin{array}{l}\text{- dipole ansatz}\\ \text{- z-expansion}\end{array}\right\} \Longrightarrow \text{different } \langle r_A^2 \rangle$$



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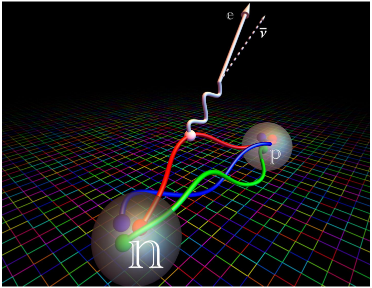


- Chiral Perturbation Theory (χPT)
  - EFT for QCD at low energy
  - QCD based parametrization of q^2 and M_π dependencies $\Longrightarrow$ extrapolate lattice results to the phys. point and extract $\langle r_A^2 \rangle$ from the lattice simulations
  - Account for finite volume, lattice spacing and excited states
  - Determining χPT LECs from the lattice $\Longrightarrow$ predicting other observables
- F_A with quark+diquark Faddeev equation

Chen et al. PLB, 815 (2021), Chen et al. EPJ A, 58 (2022)

F_A Calculation

- NNLO $\mathcal{O}(p^4)$ in relativistic Baryon χPT
- Baryon χPT
  - Problem: $\underbrace{m_B \nrightarrow 0}_{\chi\text{limit}} \Rightarrow$ Power Counting Breaking (PCB)
  - $\Longrightarrow$ additional finite renormalisation: extended on mass-shell (EOMS)
  - PCB terms absorbed by LECs
  - Covariance and analytic properties of loops preserved $\Longrightarrow$ appropriate for chiral extrapolations
- Explicit $\Delta(1232)$
  - SSE: $\delta = m_\Delta - m_N \sim \mathcal{O}(p)$
- $F_A = \mathring{g}_A + 4d_{16}M_\pi^2 + d_{22}t + \text{loops}(M_\pi, t)$
  - $\mathcal{L}_{\pi N}^{(1)} \Longrightarrow \mathring{g}_A, \quad \mathcal{L}_{\pi N}^{(3)} \Longrightarrow d_{16}, d_{22},$ $\mathcal{L}_{\pi N}^{(2)} \Longrightarrow c_1, c_2, c_3, c_4,$
  - $\mathcal{L}_{\pi N\Delta}^{(1)} \Longrightarrow h_A, g_1,$ $\mathcal{L}_{\pi\Delta}^{(2)} \Longrightarrow a_1, \quad \mathcal{L}_{\pi N\Delta}^{(2)} \Longrightarrow b_1, b_2, b_4, b_5$

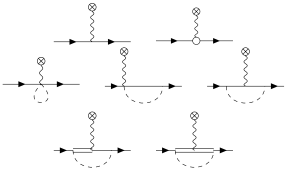


Figure: $\mathcal{O}(p)$ and $\mathcal{O}(p^3)$ (w. f. renormalisation not shown)

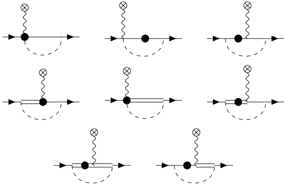
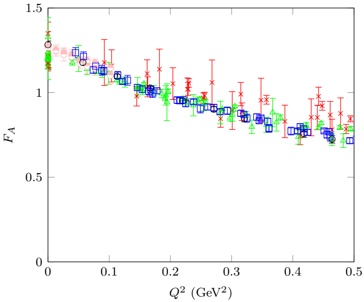
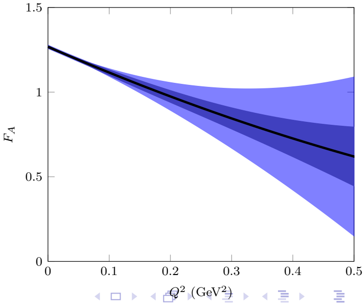


Figure: $\mathcal{O}(p^4)$

Combined fit to lattice data

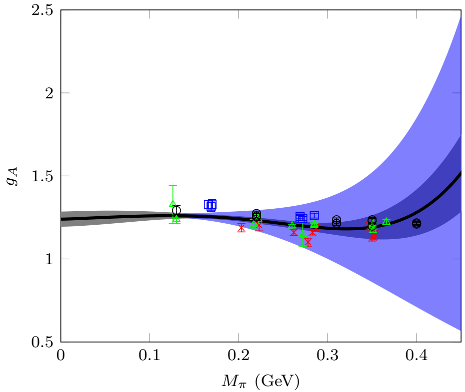
- Lattice data
  - Many recent works ⇒ substantial improvements
  - RQCD[1] + PNDME[2] + "Mainz"[3] + PACS[4] + ETMC[5]
  - data without q^2, finite volume, lattice spacing or M_π extrapolation
  - large vol. only, $M_\pi L \geq 3.5$
  - we correct lattice spacing a: $F_A(a) = F_A + \sum_i (x_i + t y_i) a^{n_i}$

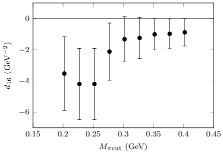


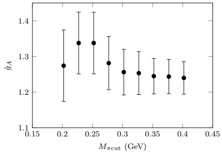


[1] Bali et al. JHEP 05 (2020)
[2] Park et al. 2103.05599
[3] Meyer et al. Modern Phys. A 34 (2019)
[4] Shintani et al. PRD 102 (2020)
[5] Alexandrou et al. PRD 103 (2021)

$F_A(q^2 = 0) = \boxed{g_A}$







- $g_A(M_\pi)$: interesting puzzle in itself
  - we saw that $\not\Delta$ LECs from πN elastic and inelastic scattering fail to describe its M_π dependence
    Alvarado & Alvarez-Ruso PRD 105 (2021)

- Fit procedure:
  - Differences between $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ are considerable (at larger M_π) and provide a measure of the systematic error [6] arising from the truncation of the perturbative expansion:
    $\Delta g_{A\chi}^{(4)} =$
    $\max\left\{\left(\frac{M_\pi}{\Lambda}\right)^4 |\mathring{g}_A|, \left(\frac{M_\pi}{\Lambda}\right)^2 \left|g_A^{(3)}\right|, \frac{M_\pi}{\Lambda} |g_A^{(4)}|\right\}$
  - $\Delta F_{A\chi}$ is added to LQCD errors in the χ^2
  - LECs have naturalness priors
  - fit range: χ^2 plateau
    $\Longrightarrow M_\pi^{\text{cut}} \simeq 400$ MeV, $Q^2_{\text{cut}} = 0.36$ GeV^2

[6] Epelbaum et al. EPJA 53 (2015)

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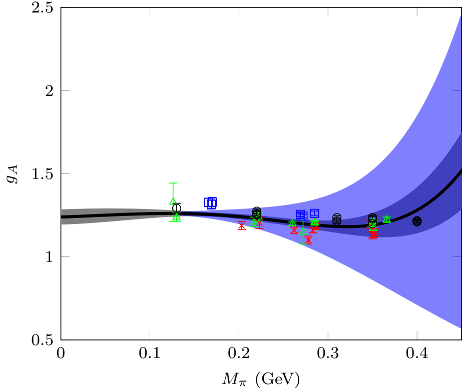

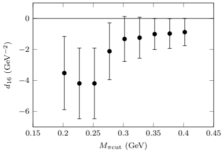

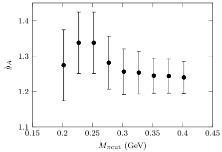

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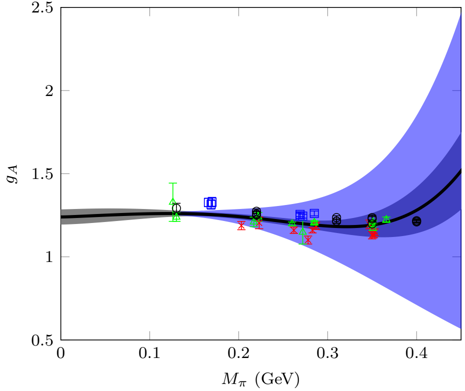
  - $\Delta F_{A\chi}$ is added to LQCD errors in the χ^2
  - LECs have naturalness priors
  - fit range: χ^2 plateau
    $\Longrightarrow M_\pi^{\mathrm{cut}} \simeq 400$ MeV, $Q^2_{\mathrm{cut}} = 0.36$ GeV^2
- Fit results: good description
  - very accurate description at the physical point
  - Δ baryon is a necessary d.o.f.
  - $\mathcal{O}(p^5)$ still needed for full convergence

[6] Epelbaum et al. EPJA 53 (2015)

$F_A(q^2 = 0) = \boxed{g_A}$



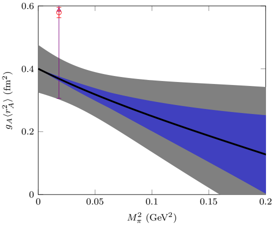
- Axial charge results from $F_A(q^2)$ fit
  - $g_A(M_{\pi\text{phys}}) = 1.273 \pm 0.014$
    vs $g_A^{\text{exp}} = 1.2754(13)_{\text{exp}}(2)_{\text{RC}}$ ⇒excellent agreement with exp.
    vs $g_A^{\text{FLAG}} = 1.246 \pm 0.028$
- $g_A(M_\pi) = \mathring{g}_A + 4d_{16}M_\pi^2 + \text{loop}(M_\pi)$
- $\boxed{d_{16} = -1.46 \pm 1.00\ \text{GeV}^{-2}}$
  $\longrightarrow M_\pi$ dependence of long range nuclear forces
  - Can not be extracted from πN elastic scattering
  - In line with $d_{16} = -1.0 \pm 1.0\ \text{GeV}^{-2}$ from $\pi N \to \pi\pi N$ [6]

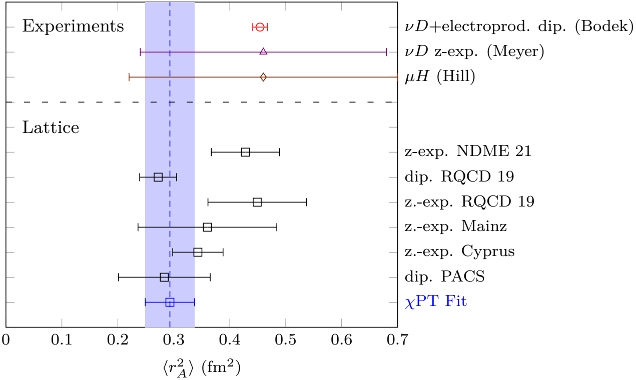
[6] Siemens et al. PRC 96 (2017)
(value converted to standard EOMS)

$F_A = g_A \left(1 + \frac{1}{6} \boxed{\langle r_A^2 \rangle} q^2\right)$ axial radius

(Bodek)=Bodek, Eur. Phys. J. C 53, 349 (2008)
(Meyer)=Meyer, PRD 93, 113015 (2016)

(Hill)=Hill, Rept. Prog. Phys. 81 (2018)





- Our $\mathcal{O}(p^4)$ χPT extraction:
  - M_π slope driven by loops with Δ
  - $d_{22} = 0.29 \pm 1.69$ GeV^{-2} (no assumptions on $\Delta\Delta\pi$ coupling enlarges error)
    - d_{22} compatible with $\mathcal{O}(p^3)$ π electroprod. Guerrero et al. PRD, 102 (2020)
  - $\boxed{\langle r_A^2\rangle(M_{\text{phys}}) = 0.293 \pm 0.044 \text{ fm}^2}$
  - Empirical determinations (model dependent) are in tension with ours and with most of LQCD extractions
  - Tipically the extracted $\langle r_A^2\rangle^{\text{phys}}$ value varies depending on the parametrisation
  - Our QCD based parametrisation leads to a value in line with most of the individual LQCD extractions

Conclusions

- $F_A(q^2)$ essential in ν oscillations
- We extract $F_A(q^2)$ from LQCD using $\mathcal{O}(p^4)$ relativistic χPT
- Our combined fit $\mathcal{O}(p^4)$ with Δ successfully describes the lattice data
  - Δ is a necessary d.o.f.
  - $g_A(M_{\pi\text{phys}}) = 1.273 \pm 0.014$ vs $g_A^{\text{exp}} = 1.2754(13)_{\text{exp}}(2)_{\text{RC}}$ $\Rightarrow$excellent agreement with exp.
  - There is tension between the experimental and lattice extraction of $\langle r_A^2 \rangle$
  - We extract $\boxed{\langle r_A^2 \rangle^{\text{phys}} = 0.291 \pm 0.052 \text{ fm}^2}$ without ad hoc parametrisations
- $d_{16} = -1.46 \pm 1.00 \text{ GeV}^{-2}$, $d_{22} = 0.29 \pm 1.69 \text{ GeV}^{-2}$ and other LECs have been extracted $\Longrightarrow$ agreement with different phenomenological determinations

Thanks!
Any questions?

Nucleon Axial Form Factor: Extra

- Dipole ansatz: $F_A(q^2) = g_A(1 - \frac{q^2}{M_A^2})^{-2}$
- z-exp.: $F_A(q^2) = \sum_k a_k z^k(q^2)$, with $z(q^2, t_{\mathrm{cut}}, t_0)$

Extra

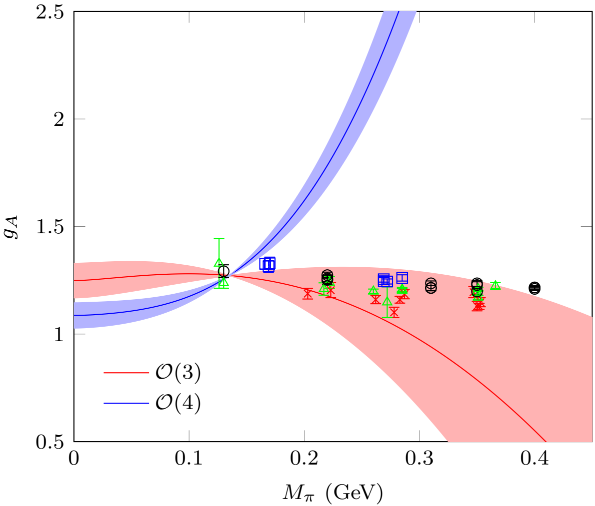


Figure: Pion-mass dependence of g_A at $\mathscr{O}(p^3)$ (red) and $\mathscr{O}(p^4)$ (blue) using phenomenological input from Ref. **?** and 1σ error bands.

Extra

	$\mathcal{O}(p^3)$ $\not\Delta$	$\mathcal{O}(p^4)$ $\not\Delta$	$\mathcal{O}(p^3)$ Δ	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free)	1.1782 ± 0.0073		1.2041 ± 0.0074	1.274 ± 0.041
d_{16} (GeV^{-2}) (free)	-1.021 ± 0.048		0.983 ± 0.062	-1.46 ± 1.00
d_{22} (GeV^{-2}) (free)	1.275 ± 0.086		3.77 ± 1.96	0.29 ± 1.69 large error (free g_1)
h_A	-	-	1.35	1.35
g_1 (free)	-	-	-0.69 ± 0.69	0.66 ± 0.56
c_1 (GeV^{-1})	-	-0.89 ± 0.06	-	-1.15 ± 0.05
c_2 (GeV^{-1})	-	3.38 ± 0.15	-	1.57 ± 0.10
c_3 (GeV^{-1})	-	-4.59 ± 0.09	-	-2.54 ± 0.05
c_4 (GeV^{-1})	-	3.31 ± 0.13	-	2.61 ± 0.10
a_1 (GeV^{-1})	-	-	-	0.90
b_1 (GeV^{-2}) (free)	-	-	-	-0.27 ± 4.96
b_2 (GeV^{-2}) (free)	-	-	-	2.27 ± 2.28
$\tilde{b}_4$ (GeV^{-2}) (free)	-	-	-	-12.48 ± 1.28
x_1 (fm^{-2}) (free)	-8.4 ± 5.8	-	-5.6 ± 5.9	-0.25 ± 16.5 (consistent)
x_2 (fm^{-2}) (free)	-8.6 ± 2.6	-	-7.1 ± 2.6	-6.36 ± 4.20
x_3 (fm^{-1}) (free)	-0.25 ± 0.21	-	-0.08 ± 0.22	0.36 ± 0.47
y_1 (fm^{-2} GeV^{-2}) (free)	-100 ± 40	-	-76 ± 44	-64 ± 121
y_2 (fm^{-2} GeV^{-2}) (free)	-31 ± 21	-	-21 ± 22	-15 ± 46
y_3 (fm^{-1} GeV^{-2}) (free)	-0.63 ± 1.49	-	0.36 ± 1.63	2.54 ± 3.98
$\mathring{m}$ (GeV)	0.874	0.874	0.855	0.855
$\mathring{m}_\Delta$ (GeV)	-	-	1.166	1.166
χ^2/dof	$46.13/(127-9) = 0.391$		$39.17/(127-10) = 0.326$	$14.64/(127-13) = 0.129$
χ^2_0/dof	$857.31/(127-9) = 7.27$		$533.87/(127-10) = 4.45$	$196.58/(127-13) = 1.724$