Baryons 2022
Universidad Pablo de Olavide,
Sevilla, Spain

08 November, 2022

Radiative corrections to neutron beta decay from low-energy effective field theory

Los Alamos NATIONAL LABORATORY

Oleksandr Tomalak

LA-UR-22-31609

Outline

1) neutron lifetime: experiment and theory updates

2) effective field theory approach to beta decay: pions

Vincenzo Cirigliano, Jordy de Vries, Leendert Hayen, Emanuele Mereghetti, and Andre Walker-Loud, Phys. Rev. Lett. 129, 12801 (2022)

3) effective field theory approach: four-Fermi theory

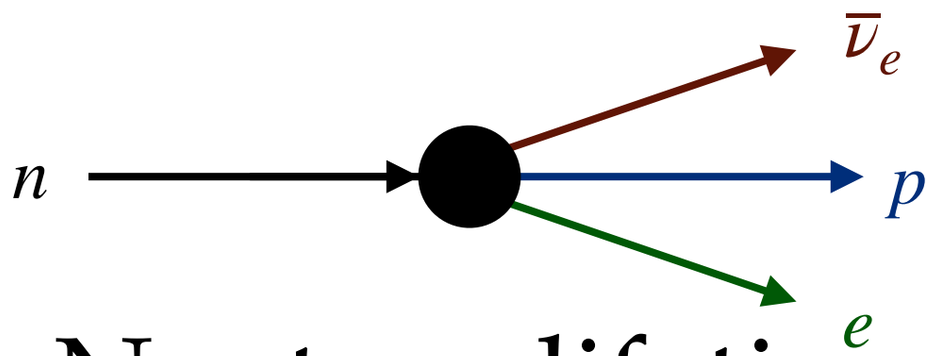
O.T., R. J Hill, Phys. Lett. B 805, 3, 135466 (2020)

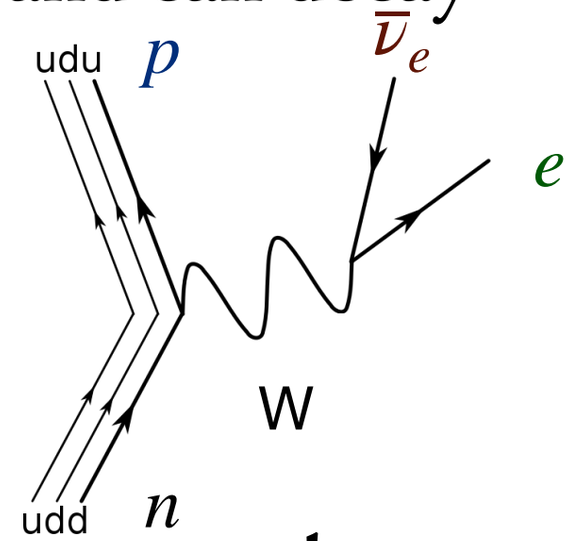
4) low-energy coupling constants in HBχPT

Emanuele Mereghetti, Vincenzo Cirigliano and O. T. (in preparation)

Neutron decay

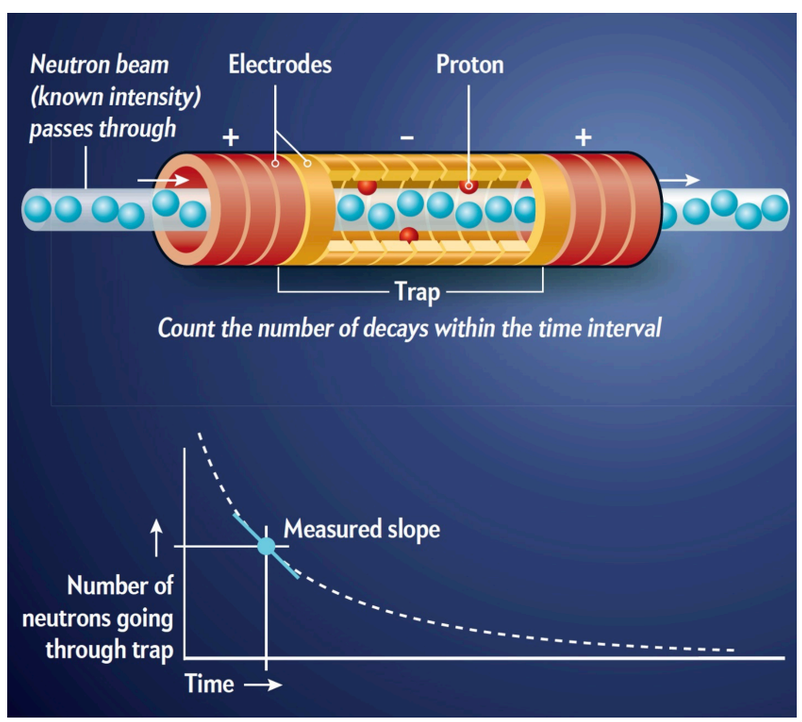
- neutron is heavier than proton by 1.3 MeV and can decay
- neutron lifetime is around 15 mins





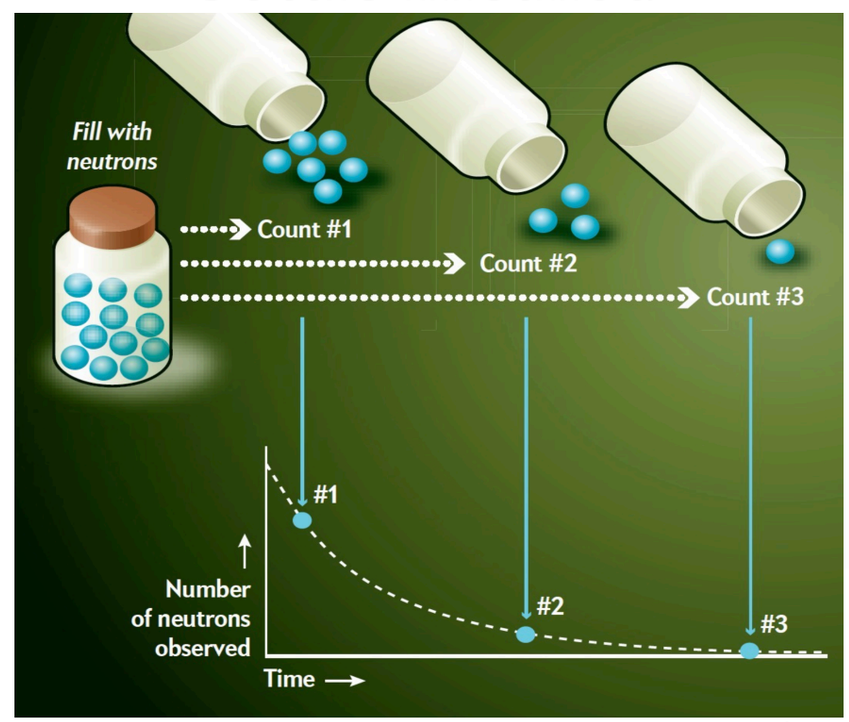
Neutron lifetime measurements

beam method



how many neutrons pass?

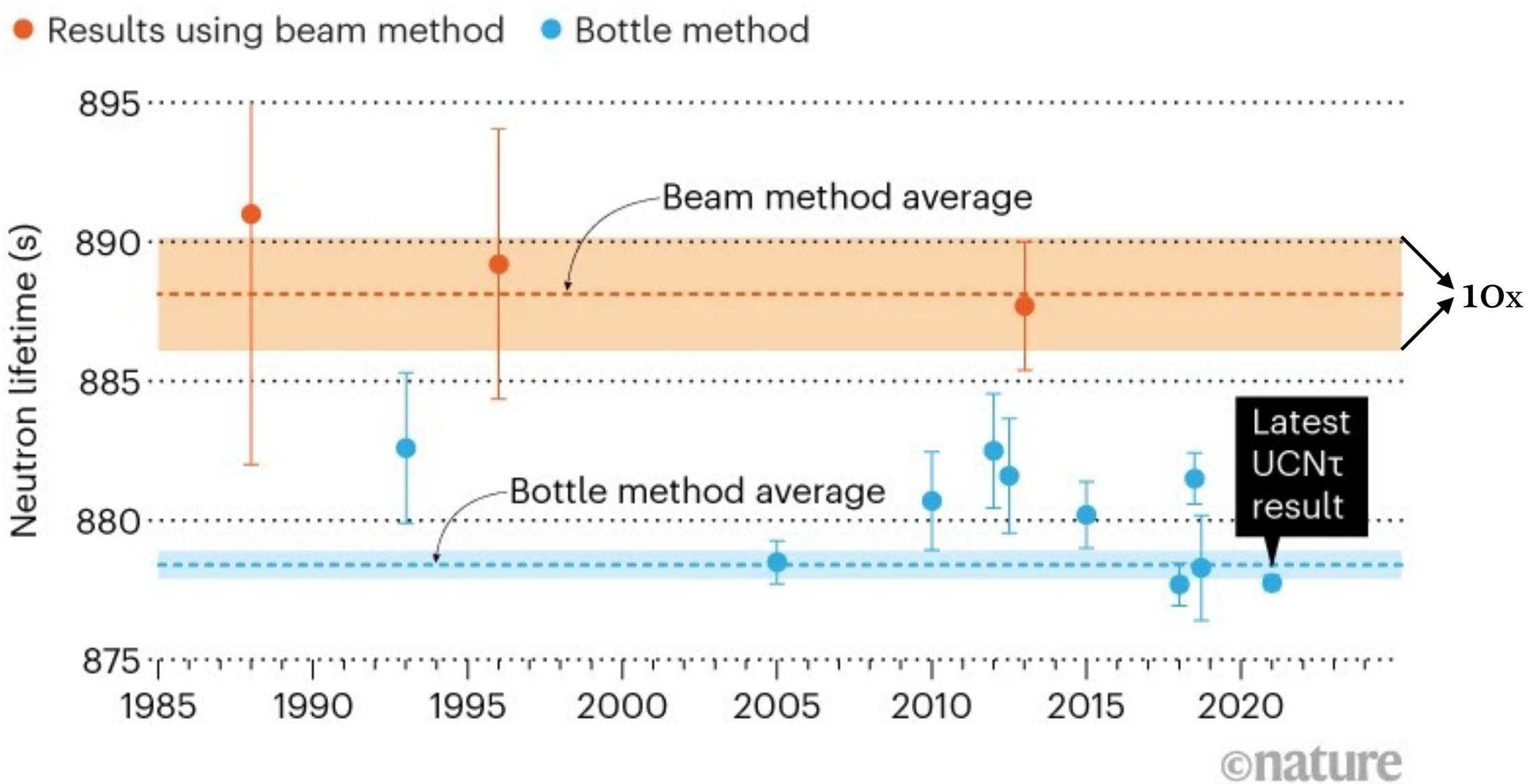
bottle method



how many neutrons survive?

scientificamerican.com

Neutron lifetime

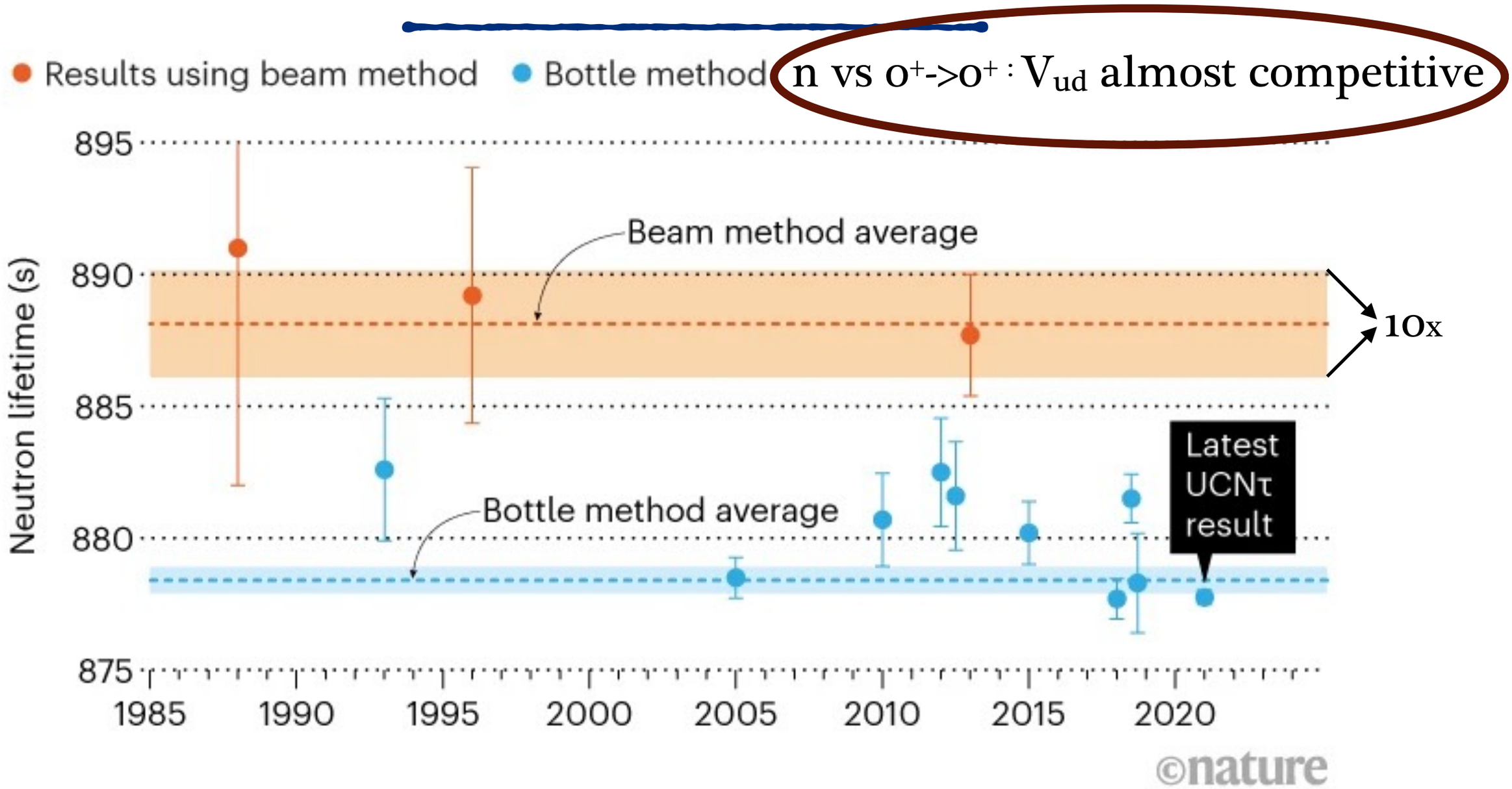


UCNτ: F. Gonzales et al., Phys. Rev. Lett. 127, 162501 (2021)

D. Castelvecchi, Nature 598, 549 (2021)

- 8-9 seconds discrepancy beam vs bottle method : 3-5σ
- 0.3 seconds uncertainty of UCNτ@LANL : $(3-4)\times 10^{-4}$ precision

Neutron lifetime



UCNτ: F. Gonzales et al., Phys. Rev. Lett. 127, 162501 (2021)

D. Castelvecchi, Nature 598, 549 (2021)

- complementary way to determine V_{ud}
- test of CKM unitarity and search for BSM at low energies

Low-energy description

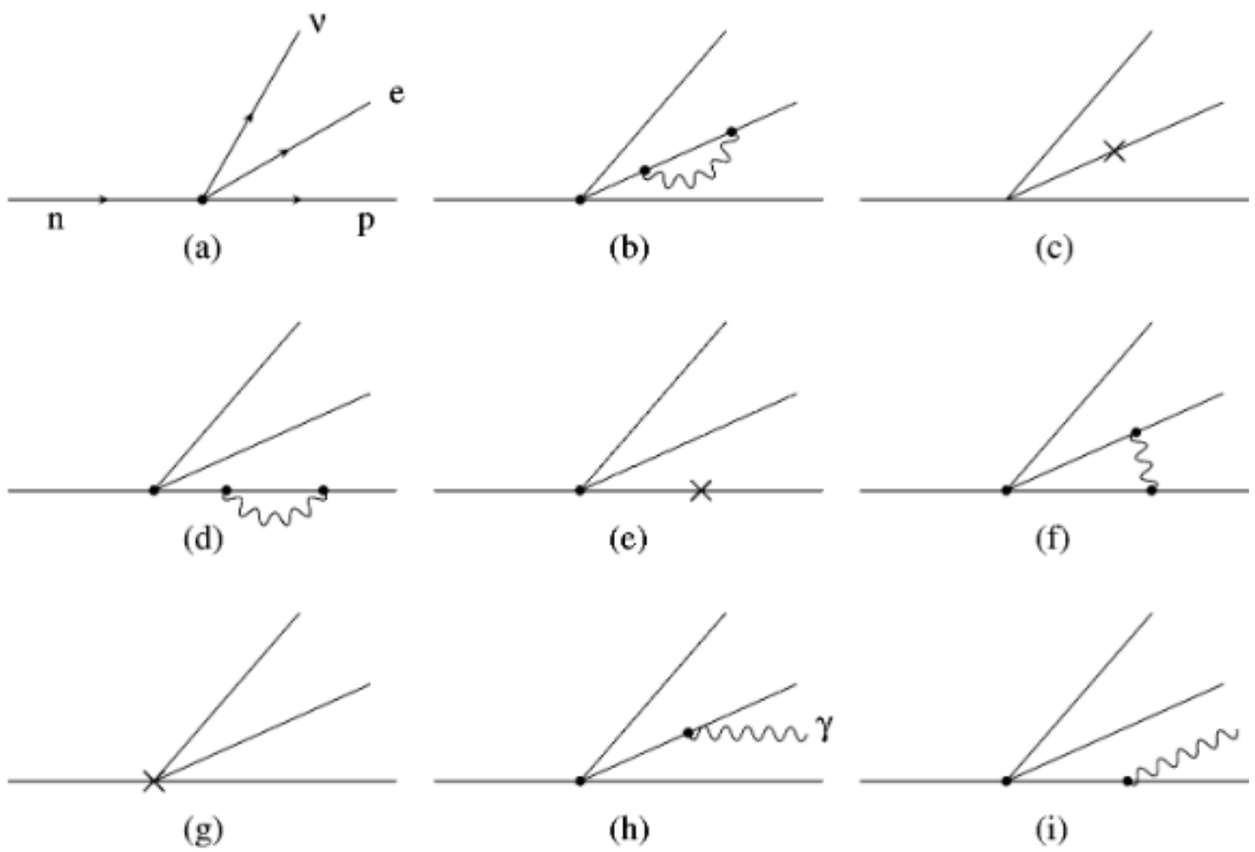
- four-fermion interaction between leptons and heavy nucleons

$$\mathcal{L}_{\text{eff}} = -\sqrt{2}\mathrm{G_F} V_{ud} \bar{e}\gamma_\mu \mathrm{P_L} \nu_e \cdot \overline{N}\left(g_V v^\mu - 2 g_A S^\mu\right)\tau^+ N + \mathrm{O}\left(\frac{m_e}{M_p}, \alpha, \alpha\frac{m_\pi}{M_p}, \alpha\frac{m_e}{m_\pi}\right)$$

for uncertainty $m_e \sim M_p - M_n$

A. Sirlin, Phys. Rev. 164, 50 (1967)

- radiative corrections formulated in modern EFT language



vector and axial-vector counterterms (diagrams c, e, g)

data

Standard Model

S. Ando et al., Phys. Lett. B 595, 250 (2004)

- two coupling constants predict all observables

Radiative corrections to neutron decay

- current-algebra formulation of radiative corrections

A. Sirlin, Rev. Mod. Phys. 50, 573 (1978)

- β decay ($0^+ \to 0^+$ Fermi transition, g_V) corrects by overall factor

Sirlin's function — short-distance EW — short-distance pQCD — long-distance γW

$$\mathrm{RC_{EW}} = \frac{\alpha}{2\pi}\left(g\left(E_m\right) + 3\ln\frac{M_Z}{M_p} + \ln\frac{M_Z}{M_A} + A_g + 2\mathrm{C}\right)$$

W. J. Marciano and A. Sirlin, Phys. Rev. Lett. 56, 22 (1986)

A. Sirlin, Phys. Rev. 50, 164 (1967)

- EW logarithms are resumed by renormalization group analysis

- assuming the same relative change of g_V and g_A, theory uncertainty:

$$8 \times 10^{-4}$$

A. Czarnecki, W. J. Marciano and A. Sirlin, Phys. Rev. D 70, 093006 (2004) and before

- perturbative logarithms separated; ~permille uncertainty

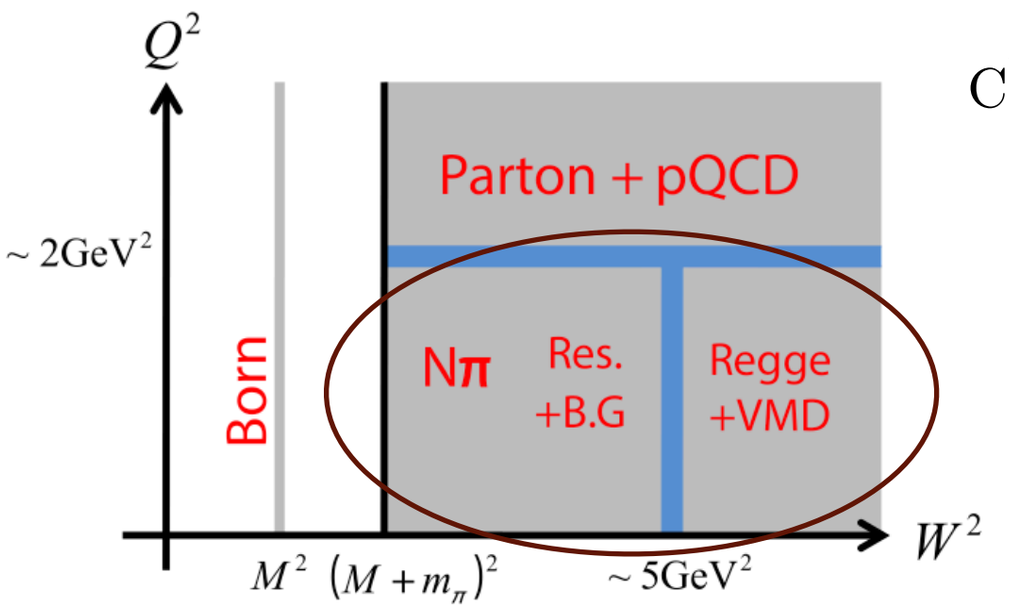
Radiative corrections to neutron decay

- updated calculations of γW contributions

$$\frac{\alpha}{2\pi}\mathrm{C}^V = 3.83(11)\times 10^{-3} \qquad \text{vs} \qquad \frac{\alpha}{2\pi}\mathrm{C}^V = 3.26(19)\times 10^{-3}$$

Ch.-Y. Seng, M. Gorchtein et al., Phys. Rev. Lett. 121, 24 (2018)

- γW box with Born, Nπ, resonance and Regge physics



$$\mathrm{C}^V = \int_0^\infty \frac{\mathrm{d}Q^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\int_0^1 \mathrm{d}x f\left(\frac{x^2}{Q^2}\right) F_3^{(0)}\left(x,Q^2\right)$$

A. Czarnecki, W. J. Marciano and A. Sirlin, Phys. Rev. D 100, 073008 (2019)

K. Shiells, P. G. Blunden and W. Melnitchouk, Phys. Rev. D 104, 033003 (2021)

L. Hayen, Phys. Rev. D 103, 113001 (2021)

- dispersive validation of the same relative change of g_V and g_A:

$$\frac{\alpha}{2\pi}\left(\mathrm{C}^A-\mathrm{C}^V\right) = 0.13(11)_V(6)_A\times 10^{-3} \qquad \frac{\alpha}{2\pi}\left(\mathrm{C}^A-\mathrm{C}^V\right) = 0.6(5)\times 10^{-3}$$

M. Gorchtein and Ch.-Y. Seng, JHEP 10, 053 (2021) L. Hayen, Phys. Rev. D 103, 113001 (2021)

- hadron physics -> precise evaluations of long-distance γW

Effective field theory for β decay

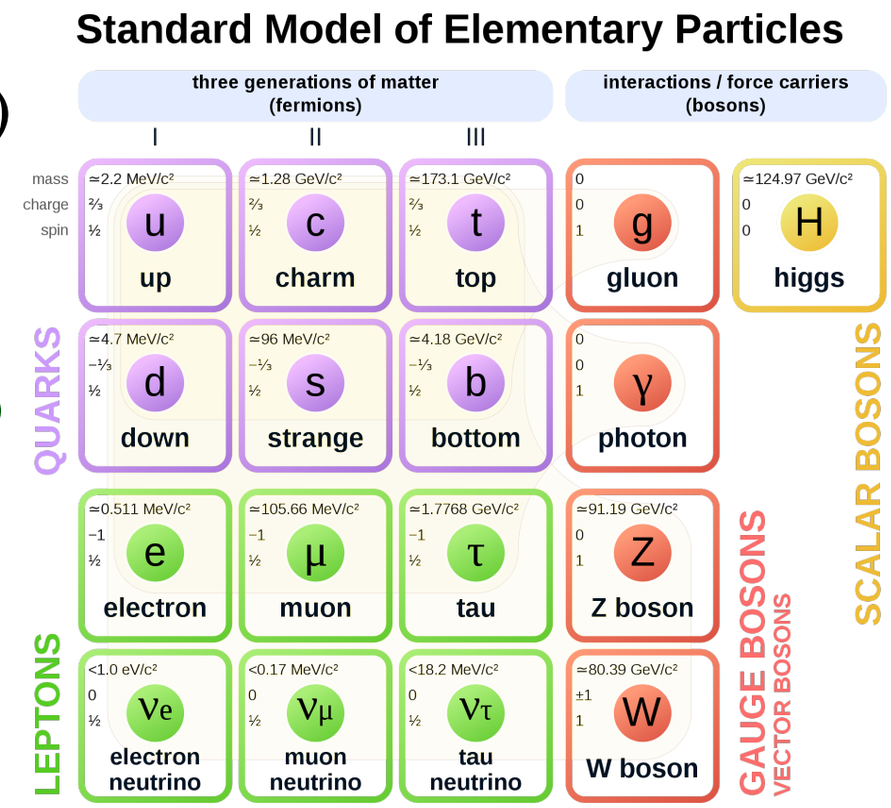
M_Z

full content of Standard Model (SM)

integrate out top, Z, W, h

O.T., R. J Hill, Phys. Lett. B 805, 3, 135466 (2020)

W. Dekens, P. Stoffer, JHEP 07, 107 (2019)



m_b

m_τ

integrate out GeV particles

m_c

α_s becomes too strong going to lower energies

current focus

hadron physics

m_π

dynamical pions

Vincenzo Cirigliano, Jordy de Vries, Leendert Hayen, Emanuele Mereghetti, and Andre Walker-Loud, Phys. Rev. Lett. 129, 12801 (2022)

photons, neutrinos, electrons, external nucleons

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- goal: determine coupling constants starting from SM

Low-energy description

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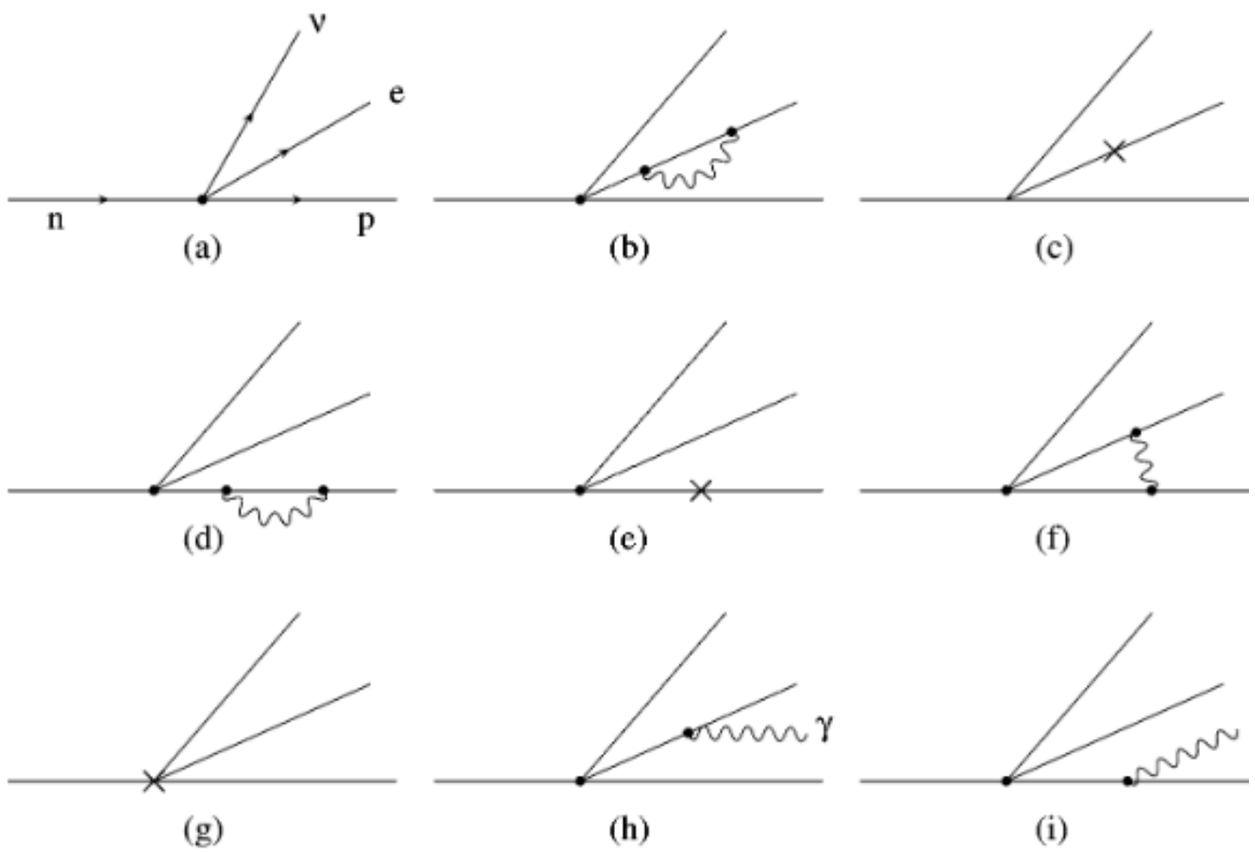

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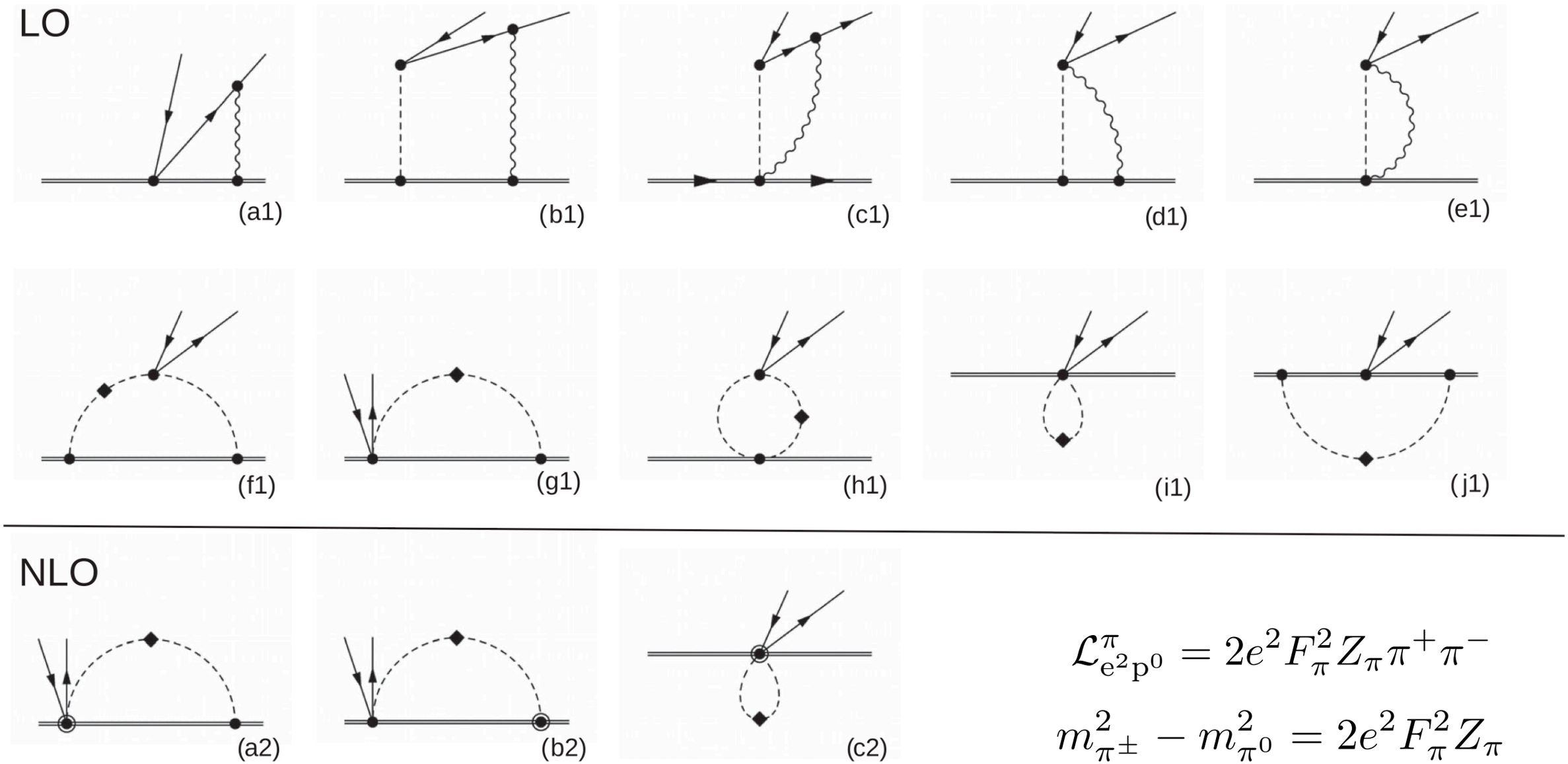
$\not{\pi}$EFT and role of pions



$$\mathcal{L}^{\pi}_{e^2p^0} = 2e^2 F_\pi^2 Z_\pi \pi^+\pi^-$$

$$m^2_{\pi^\pm} - m^2_{\pi^0} = 2e^2 F_\pi^2 Z_\pi$$

Vincenzo Cirigliano, Jordy de Vries, Leendert Hayen, Emanuele Mereghetti, and Andre Walker-Loud, Phys. Rev. Lett. 129, 12801 (2022)

- pion-mediated correction to g_A: for data vs SM comparison
- first steps in matching to χPT with baryons

Effective field theory for β decay

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full content of Standard Model (SM)

integrate out top, Z, W, h

O.T., R. J Hill, Phys. Lett. B 805, 3, 135466 (2020)

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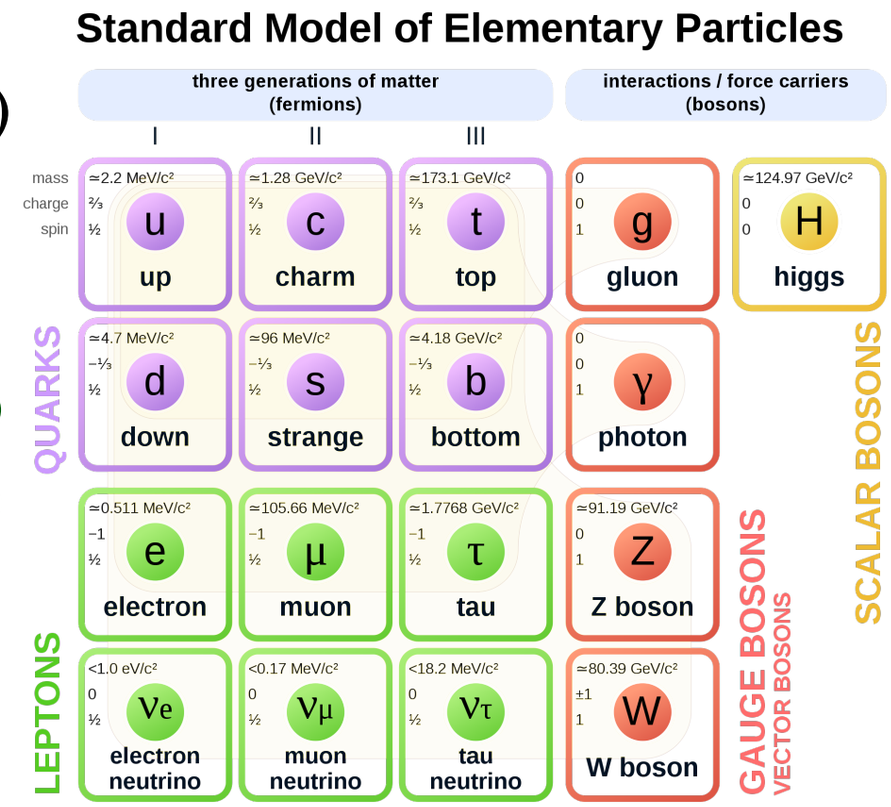


- goal: determine coupling constants starting from SM

Semileptonic operators and muon decay

M_Z

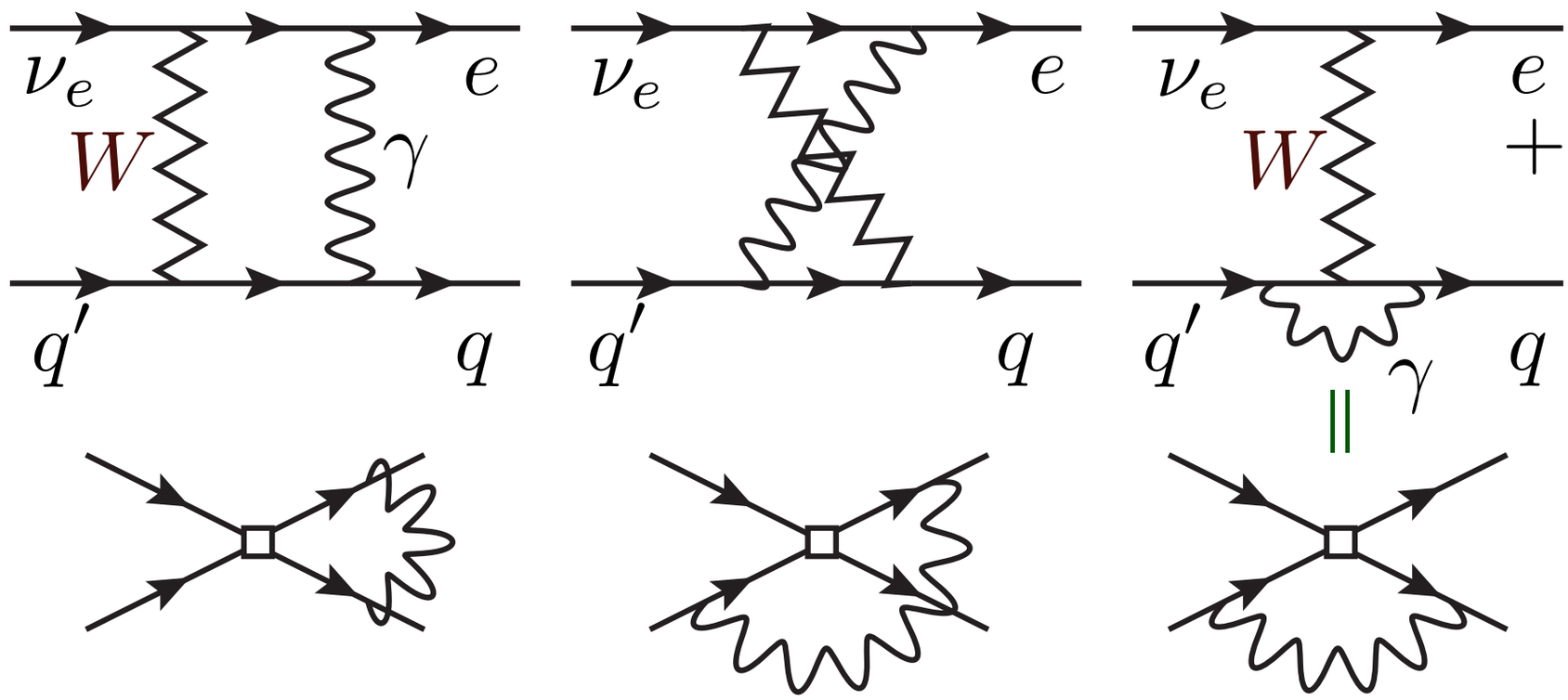
$$\mathcal{L}_{\text{eff}} = -2\sqrt{2}\mathrm{G_F} \sum_{\ell \neq \ell'} \bar{\nu}_{\ell'} \gamma^\mu \mathrm{P_L} \nu_\ell \, \bar{\ell} \gamma_\mu \mathrm{P_L} \ell' - c^{qq'} \sum_{q \neq q'} \bar{\ell} \gamma^\mu \mathrm{P_L} \nu_\ell \, \bar{q} \gamma_\mu \mathrm{P_L} q'$$

O.T., R. J Hill, Phys. Lett. B 805, 3, 135466 (2020)

- Fermi coupling is scale independent

m_b

m_τ



$+\cdots$ scheme independent

scheme dependent for quarks

m_c

m_π

- NDR scheme for γ_5 with a=-1 for evanescent operators E

$$\gamma^\alpha \gamma^\beta \gamma^\mu \mathrm{P_L} \otimes \gamma_\mu \gamma_\beta \gamma_\alpha \mathrm{P_L} = 4\left(1 + a\left(4 - d\right)\right) \gamma^\mu \mathrm{P_L} \otimes \gamma_\mu \mathrm{P_L} + \mathrm{E}\left(a\right)$$

Buras and Weisz (1990)

- Wilson coefficient of semileptonic operator depends on scale

- in progress: compare matching to work on BSM operators

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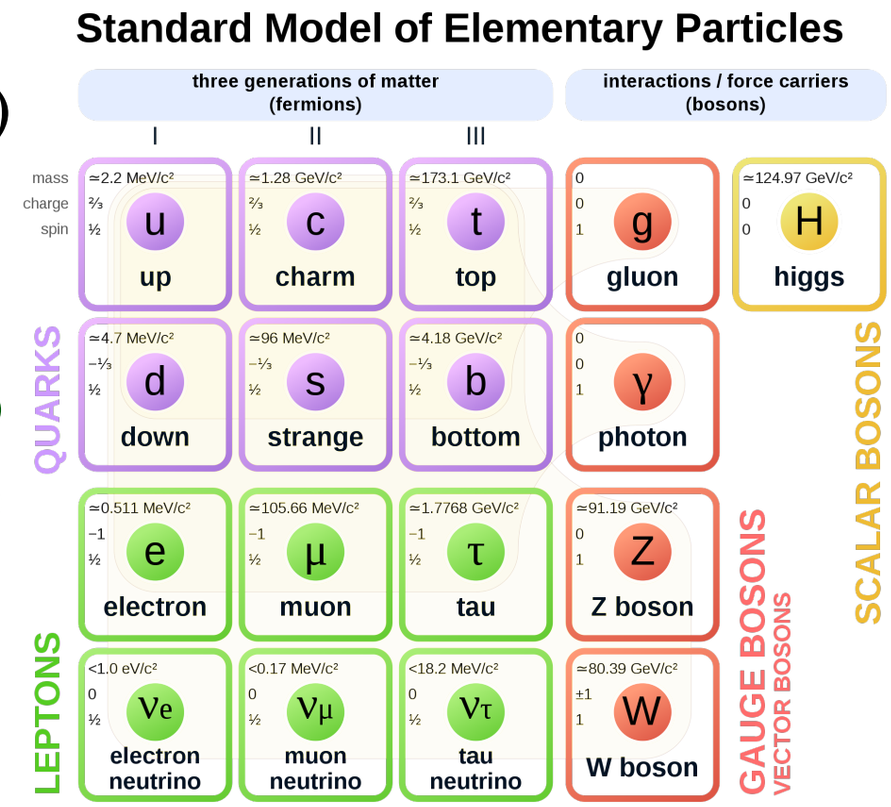


- goal: determine coupling constants starting from SM

Electromagnetic coupling constants

- SU(2) strong interaction Lagrangian with $< \mathcal{Q}_L - \mathcal{Q}_R >= 0$

$$\mathcal{L}_{\pi\mathrm{N}}^{\mathrm{e^2p}} = e^2 \sum_{i=1}^{14} g_i \bar{N} \mathrm{O}_i N$$

extension of S. Steininger's Ph.D. thesis

$$u = e^{i\frac{\pi^a \mathrm{T}^a}{2}} \qquad \mathbf{q} = \mathbf{q}^a \mathrm{T}^a$$

$$u_\mu = i\left(u^\dagger\left(\partial_\mu - i r_\mu + i\mathbf{q}_R A_\mu\right)u - u\left(\partial_\mu - i l_\mu + i\mathbf{q}_L A_\mu\right)u^\dagger\right)$$

$$\tilde{D}_\mu = \partial_\mu + \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] - \frac{i}{2}u^\dagger r_\mu u - \frac{i}{2}u l_\mu u^\dagger$$

$$c_\mu^\pm = -\frac{i}{2}\left(u\left(i\partial_\mu \mathbf{q}_L + [l_\mu, \mathbf{q}_L]\right)u^\dagger \pm u^\dagger\left(i\partial_\mu \mathbf{q}_R + [r_\mu, \mathbf{q}_R]\right)u\right)$$

$$\mathcal{Q}_L = u\mathbf{q}_L u^\dagger, \qquad \mathcal{Q}_R = u^\dagger \mathbf{q}_R u, \qquad \mathcal{Q}_\pm = \frac{\mathcal{Q}_L \pm \mathcal{Q}_R}{2}$$

$$\mathbf{q}_V = \mathbf{q}_L + \mathbf{q}_R \qquad v = l + r$$

$$\mathbf{q}_A = \mathbf{q}_L - \mathbf{q}_R \qquad a = l - r$$

- notations for next slide

Electromagnetic coupling constants

- SU(2) strong interaction Lagrangian with $< \mathcal{Q}_L - \mathcal{Q}_R >= 0$

$$\mathcal{L}_{\pi\mathrm{N}}^{\mathrm{e}^2\mathrm{p}} = e^2 \sum_{i=1}^{14} g_i \bar{N} \mathrm{O}_i N$$

extension of S. Steininger's Ph.D. thesis

- spurion-dependent operators

$$O_{13} =< \mathcal{Q}_+^2 + \mathcal{Q}_-^2 > u \cdot S$$

$$O_1 =< \mathcal{Q}_+^2 - \mathcal{Q}_-^2 > u \cdot S$$

$$O_2 =< \mathcal{Q}_+ >^2 u \cdot S$$

$$O_6 = \frac{i}{2M} < \mathcal{Q}_+ >< \mathcal{Q}_- u \cdot \tilde{D} > + \text{h.c.}$$

$$O_7 = \frac{i}{2M} \mathcal{Q}_- < \mathcal{Q}_+ u \cdot \tilde{D} > + \text{h.c.}$$

$$O_8 = \frac{i}{2M} \mathcal{Q}_+ < \mathcal{Q}_- u \cdot \tilde{D} > + \text{h.c.}$$

$$O_{14} = \frac{i}{2M} < \mathcal{Q}_+ \mathcal{Q}_- > u \cdot \tilde{D} + \text{ h.c.}$$

$$O_3 =< \mathcal{Q}_+ >< \mathcal{Q}_+ u \cdot S >,$$

$$O_4 = \mathcal{Q}_+ < \mathcal{Q}_+ u \cdot S >,$$

$$O_5 = \mathcal{Q}_- < \mathcal{Q}_- u \cdot S >$$

$$O_9 = -\frac{i}{2M} [\mathcal{Q}_+, c^+ \cdot \tilde{D}] + \text{h.c.}$$

$$O_{10} = -\frac{i}{2M} [\mathcal{Q}_-, c^- \cdot \tilde{D}] + \text{h.c.}$$

$$O_{11} = i[\mathcal{Q}_+, c^- \cdot S]$$

$$O_{12} = i[\mathcal{Q}_-, c^+ \cdot S]$$

- 5 LECs enter β decay in 2 combinations

Electromagnetic coupling constants

- functional derivatives of generating functional w.r.t. EM isoscalar and isovector, axial and vector spurions

$$g_9 \bar{N}' \mathrm{T}^c N = \frac{i\varepsilon_{abc}}{F_0^2} v_\mu \frac{\partial}{\partial r_\mu} \left(\int \mathrm{d}^d x e^{ir\cdot x} < N' | \frac{\delta^2 W(\mathbf{q}_V, \mathbf{q}_A)}{\delta \mathbf{q}_{V^b}(x)\, \delta \mathbf{q}_{V^a}(0)} \bigg|_{\mathbf{q}=0} | N > \right) \bigg|_{r_\mu = 0}$$

$$g_{11} \bar{N}' \mathrm{T}^c \gamma_5 N = \frac{\varepsilon_{abc}}{F_0^2 M} \int \mathrm{d}^d x < N' | \frac{\delta^2 W(\mathbf{q}_V, \mathbf{q}_A)}{\delta \mathbf{q}_{V^b}(x)\, \delta \mathbf{q}_{A^a}(0)} \bigg|_{\mathbf{q}=0} | N >$$

+ χPT loops
+ 4-Fermi counterterms

$$g_{11} \bar{N}' \mathrm{T}^c N = \frac{S_\mu}{F_0^2} i \int \mathrm{d}^d y \int \mathrm{d}^d x < N' | \delta^{ab} \frac{\delta^2 W(v, \mathbf{q}_V, \mathbf{q}_A)}{\delta v_\mu^b(y)\, \delta \mathbf{q}_{A^c}(x)\, \delta \mathbf{q}_{V^a}(0)} \bigg|_{\mathbf{q}=0} | N >$$

$$(g_1 + 2g_2 + g_{13}) p \cdot S \bar{N}' \mathrm{T}^c N + \ldots = -\frac{1}{2F_0} \int \mathrm{d}^d x < N' | \frac{\delta^2 W(\mathbf{q}_V, \mathbf{q}_A)}{\delta \mathbf{q}_{V^0}(x)\, \delta \mathbf{q}_{V^0}(0)} \bigg|_{\mathbf{q}=0} | N \pi^c(p) >$$

$$\left(g_1 + g_2 - \frac{g_{11}}{2} + g_{13} \right) \bar{N}' \mathrm{T}^c N =$$

$$\frac{S_\mu}{F_0^2} \int \mathrm{d}^d y \int \mathrm{d}^d x < N' | \frac{i}{5} \left(\frac{\delta^{ad}\delta^{bc}}{2} - \delta^{ab}\delta^{cd} \right) \frac{\delta^2 W(a, \mathbf{q}_V, \mathbf{q}_A)}{\delta a_\mu^d(y)\, \delta \mathbf{q}_{V^b}(x)\, \delta \mathbf{q}_{V^a}(0)} - \frac{i}{2} \frac{\delta^2 W(a, \mathbf{q}_V, \mathbf{q}_A)}{\delta a_\mu^c(y)\, \delta \mathbf{q}_{V^0}(x)\, \delta \mathbf{q}_{V^0}(0)} \bigg|_{\mathbf{q}=0} | N >$$

- 2-, 3-point functions: product of quark currents in 4-Fermi theory

- a few ways to determine LECs for β decay

Electromagnetic coupling constants

- relevant Lagrangian in 4-Fermi theory

example

$$\mathcal{L}_{4-\mathrm{Fermi}}^{\mathrm{spurions}} = -eA_\mu \bar{q}\mathbf{q}_V \gamma^\mu q + \ldots$$

$$g_9 \bar{N}\mathrm{T}^c N = \frac{i\varepsilon_{abc}e^2}{F_0^2} v_\mu \frac{\partial}{\partial r_\mu}\left(\int \mathrm{d}^d x e^{ir\cdot x} < N|\frac{\delta^2 W\left(\mathbf{q}_V, \mathbf{q}_A\right)}{\delta \mathbf{q}_{V^b}\left(x\right)\delta \mathbf{q}_{V^a}\left(0\right)}\bigg|_{\mathbf{q}=0}|N>\right)\bigg|_{r_\mu=0}$$

$$g_9 \bar{N}\mathrm{T}^c N = -\frac{2\varepsilon_{abc}e^2}{F_0^2}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{g_{\lambda\rho}k\cdot v}{(k^2-\lambda^2+i\varepsilon)^2}\int \mathrm{d}^d x e^{ik\cdot x} < N|\bar{q}\mathrm{T}^b\gamma^\lambda q\left(x\right)\bar{q}\mathrm{T}^a\gamma^\rho q(0)|N>$$

forward Compton tensor

$$\mathrm{T}^{\lambda\rho}_{V^a V^b}\left(v,k\right)$$

- 2-, 3-point functions: product of quark currents in 4-Fermi theory

- a few ways to determine LECs for β decay

Electroweak coupling constants

- SU(2) weak interaction Lagrangian with $< \mathcal{Q}_L^W >= 0$

$$\mathcal{L}_{\pi N \ell}^{e^2 p} = e^2 \sum_{i=1}^{6} \sum_{\ell} \left(\tilde{X}_i v^\mu - \tilde{Y}_i S^\mu \right) \bar{\ell}_L \gamma_\mu \nu_L \bar{N}_v O_i N_v + \text{h.c.}$$

- spurion-dependent operators

$$O_1 = [\mathcal{Q}_L, \mathcal{Q}_L^W] \qquad O_3 = \{\mathcal{Q}_L, \mathcal{Q}_L^W\} \qquad O_5 =< \mathcal{Q}_L \mathcal{Q}_L^W >$$

$$O_2 = [\mathcal{Q}_R, \mathcal{Q}_L^W] \qquad O_4 = \{\mathcal{Q}_R, \mathcal{Q}_L^W\} \qquad O_6 =< \mathcal{Q}_R \mathcal{Q}_L^W >$$

- scattering amplitude determined by spurion's physical values

$$T = e^2 \left(\left(\tilde{X}_1 + \tilde{X}_2 + \tilde{X}_3 + \tilde{X}_4 \right) v^\mu - \left(\tilde{Y}_1 + \tilde{Y}_2 + \tilde{Y}_3 + \tilde{Y}_4 \right) S^\mu \right) \bar{\ell}_L \gamma_\mu \nu_L \bar{N}' T^a N$$

- 8 LECs enter physical observables in 2 combinations

Electroweak coupling constants

- functional derivatives of generating functional w.r.t. isoscalar and isovector spurious

$$\mathrm{T} =$$

$$e^2 \left(\left(\tilde{X}_1 + \tilde{X}_2 + \tilde{X}_3 + \tilde{X}_4 \right) v^\mu - \left(\tilde{Y}_1 + \tilde{Y}_2 + \tilde{Y}_3 + \tilde{Y}_4 \right) S^\mu \right) \bar{\ell}_{\mathrm{L}} \gamma_\mu \nu_{\mathrm{L}} \bar{N}' \mathrm{T}^a N =$$

$$\int \mathrm{d}^d x < \ell \bar{\nu} N' | \left(\frac{\varepsilon_{abc}}{2} \frac{\delta^2 W\left(\mathbf{q}_V, \mathbf{q}_A, \mathbf{q}_W\right)}{\underline{\delta \mathbf{q}_{V^b}(x)}\, \delta \mathbf{q}_{W^c}(0)} + i \frac{\delta^2 W\left(\mathbf{q}_V, \mathbf{q}_A, \mathbf{q}_W\right)}{\underline{\delta \mathbf{q}_{V^0}(x)}\, \delta \mathbf{q}_{W^a}(0)} \right) \Bigg|_{\mathbf{q}=0} | N >$$

- vector-vector and vector-axial products of quark currents

- 2 correlation functions determine all electroweak LECs

Matching $\not{\pi}$EFT to χPT

- g_V and g_A in theory with pions renormalized as

$$C_V = \hat{C}_V$$

$$C_A = Z_\pi \left[\frac{1 + 3g_A^{(0)2}}{2} \left(\ln \frac{\mu^2}{m_\pi^2} - 1 \right) - g_A^{(0)2} \right] + \hat{C}_A(\mu)$$

- purely leptonic counterterm

$$\mathcal{L}_{\text{lept}}^{\text{CT}} = e^2 X_6 \bar{e} \left(i\partial_\mu + eA_\mu \right) \gamma^\mu e$$

- $\not{\pi}$EFT counterterms from χPT coupling constants in baryon sector

$$\hat{C}_A = 8\pi^2 \left[-\frac{X_6}{2} + \frac{1}{g_A^{(0)}} \left(\tilde{Y}_1 + \tilde{Y}_2 + \tilde{Y}_3 + \tilde{Y}_4 + g_1 + g_2 + \frac{g_{11}}{2} + g_{13} \right) \right]$$

$$\hat{C}_V = 8\pi^2 \left[-\frac{X_6}{2} + 2 \left(\tilde{X}_1 + \tilde{X}_2 + \tilde{X}_3 + \tilde{X}_4 \right) + g_9 \right]$$

- vector and axial products of quark currents

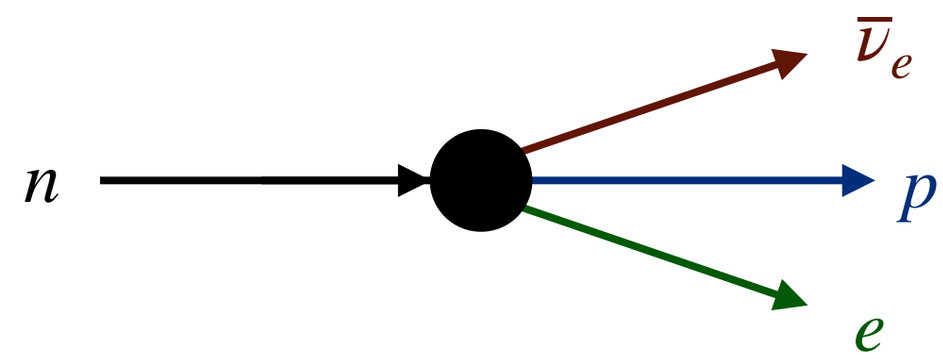
- LECs in terms of 2- or 3-point correlation functions

Conclusions

- radiative corrections evaluated in effective field theory approach
- missing and uncertain contributions are identified
- contribution of correlation functions to EFT couplings clarified

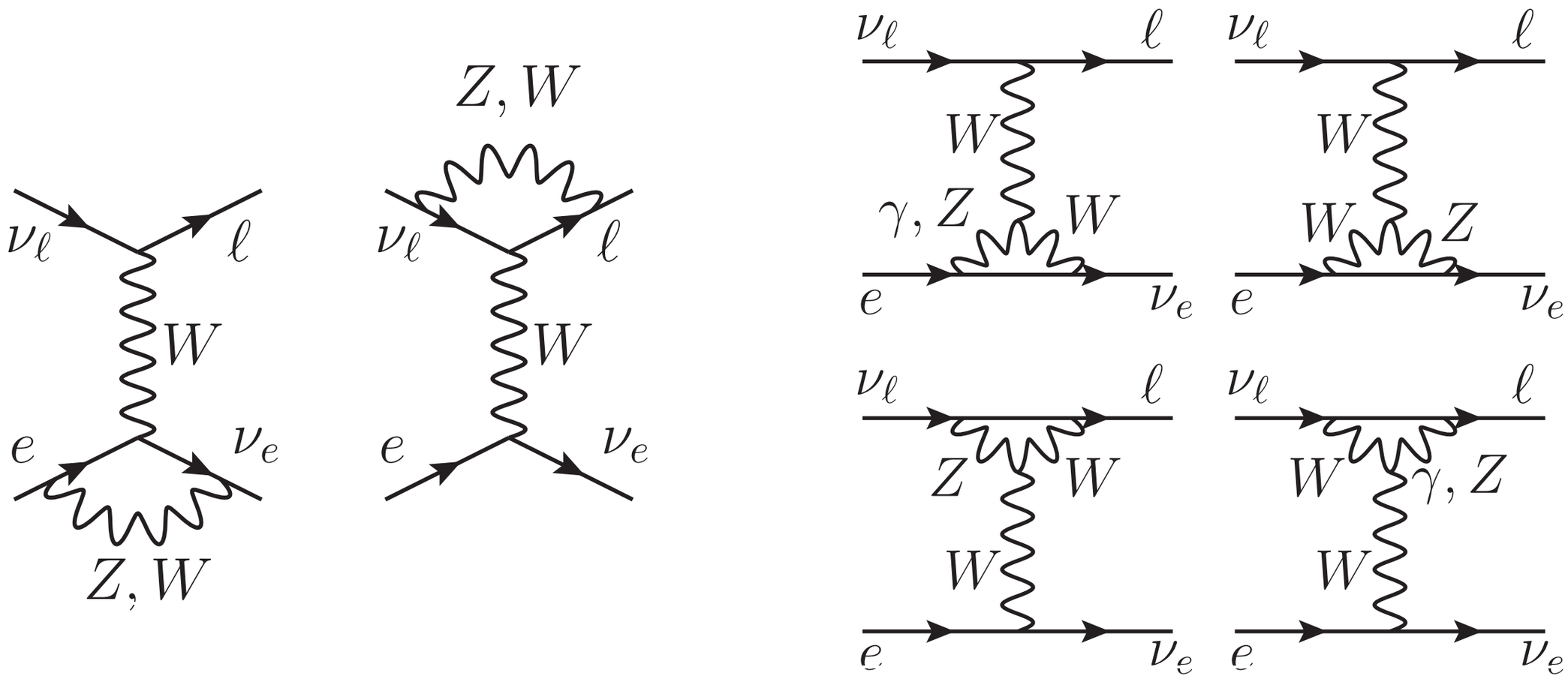
Outlook

- matching to four-Fermi theory and relations to experimental data
- relations to lattice QCD



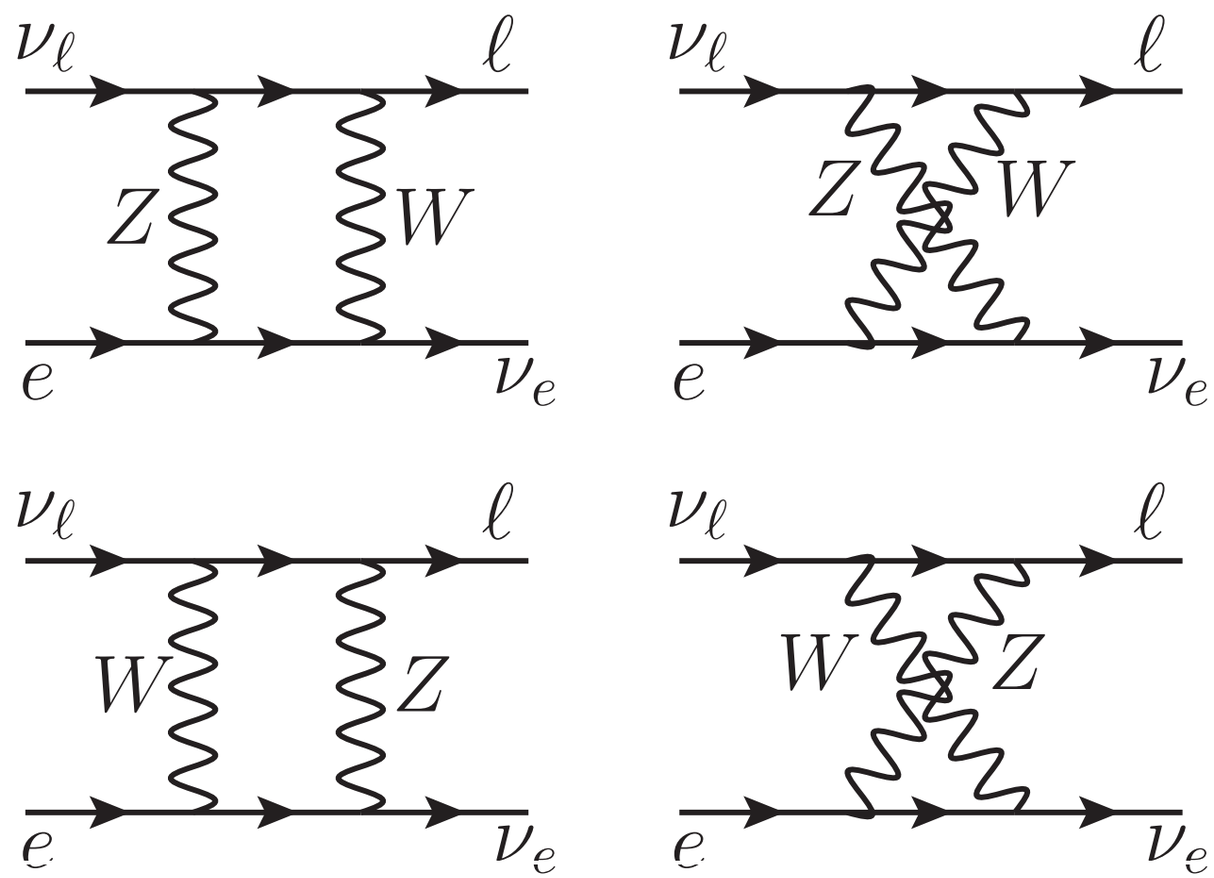
Thanks for your attention !!!

Charged current in SM. Vertexes



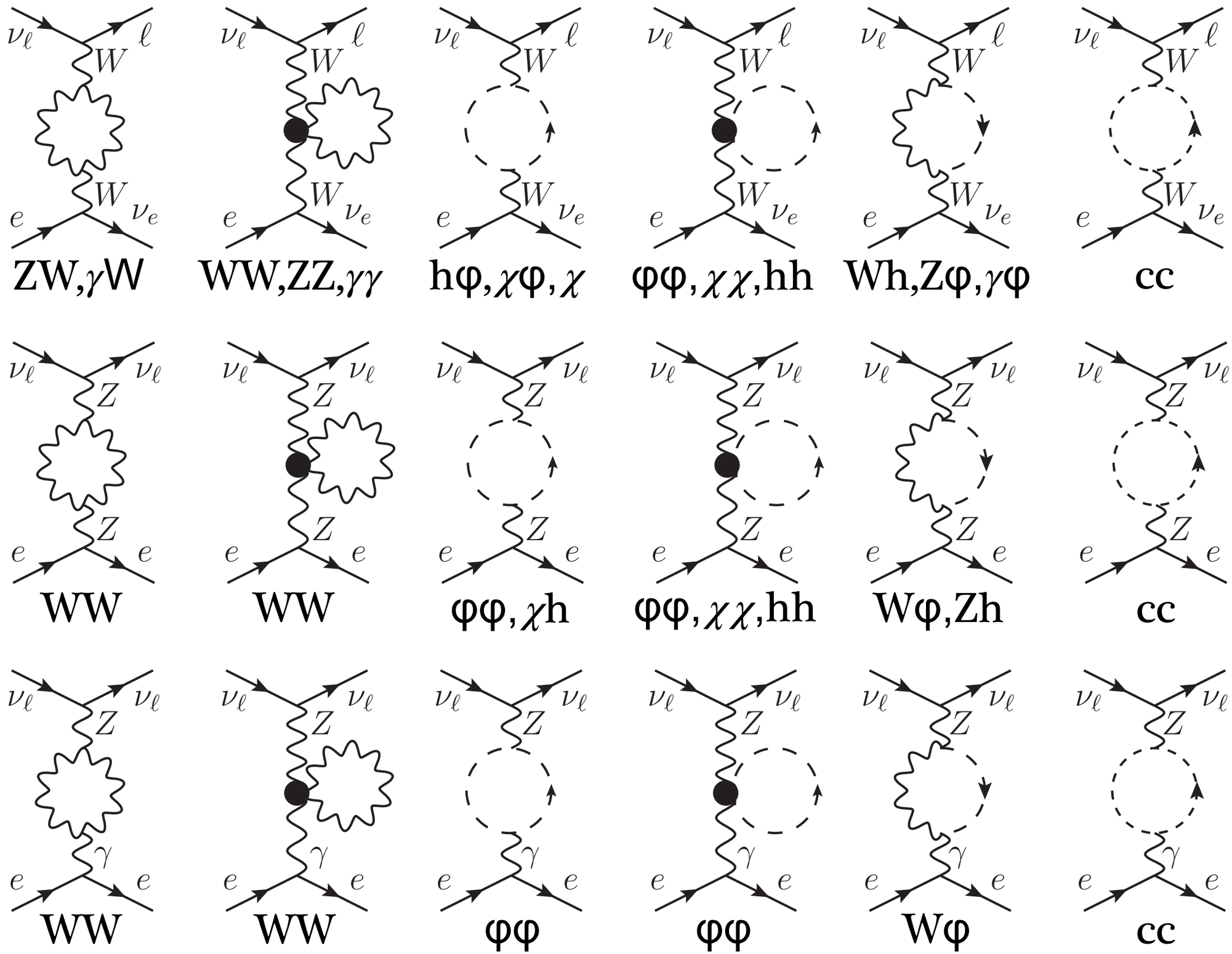
- contribution to effective couplings

Charged current in SM. Boxes

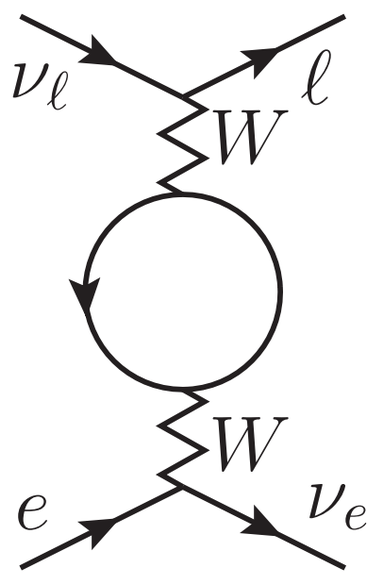


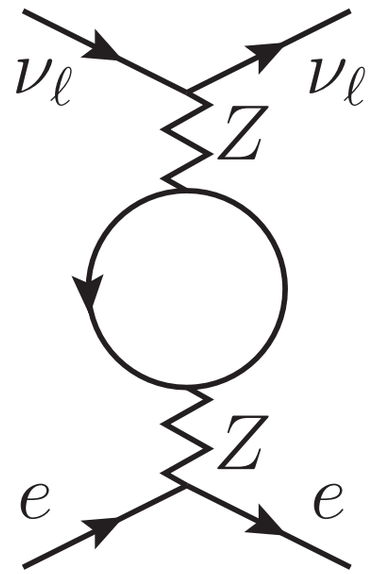
- contribution to effective couplings

Self energy and γZ mixing. Boson loops

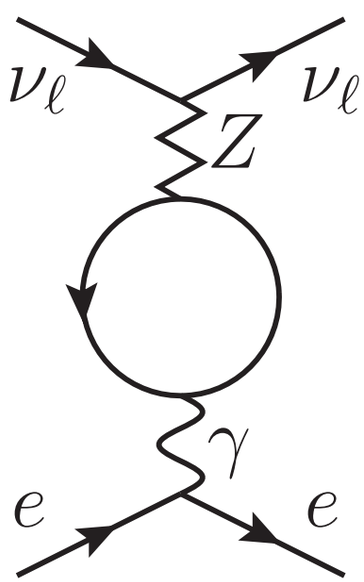


Self energy and γZ mixing. Fermion loops





- vanishing contribution to matching besides loops with t quark
- finite contribution to self energy pole vs $\overline{\text{MS}}$ masses
- do not consider Higgs tadpoles (hVV): matching vs self energy cancellation



- gauge-independent contribution