Deuteron VVCS and two-photon exchange effects in (muonic) deuterium in pionless EFT

Vadim Lensky

VL, Hiller Blin, Pascalutsa, PRC 104, 054003 (2021)
VL, Hagelstein, Pascalutsa, PLB 835, 137500 (2022); EPJ A, in press (2022)

Baryons 2022

Seville, November 7-11, 2022

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DFG Deutsche Forschungsgemeinschaft

Muonic Deuterium in Pionless EFT Confirms the Small Proton Radius

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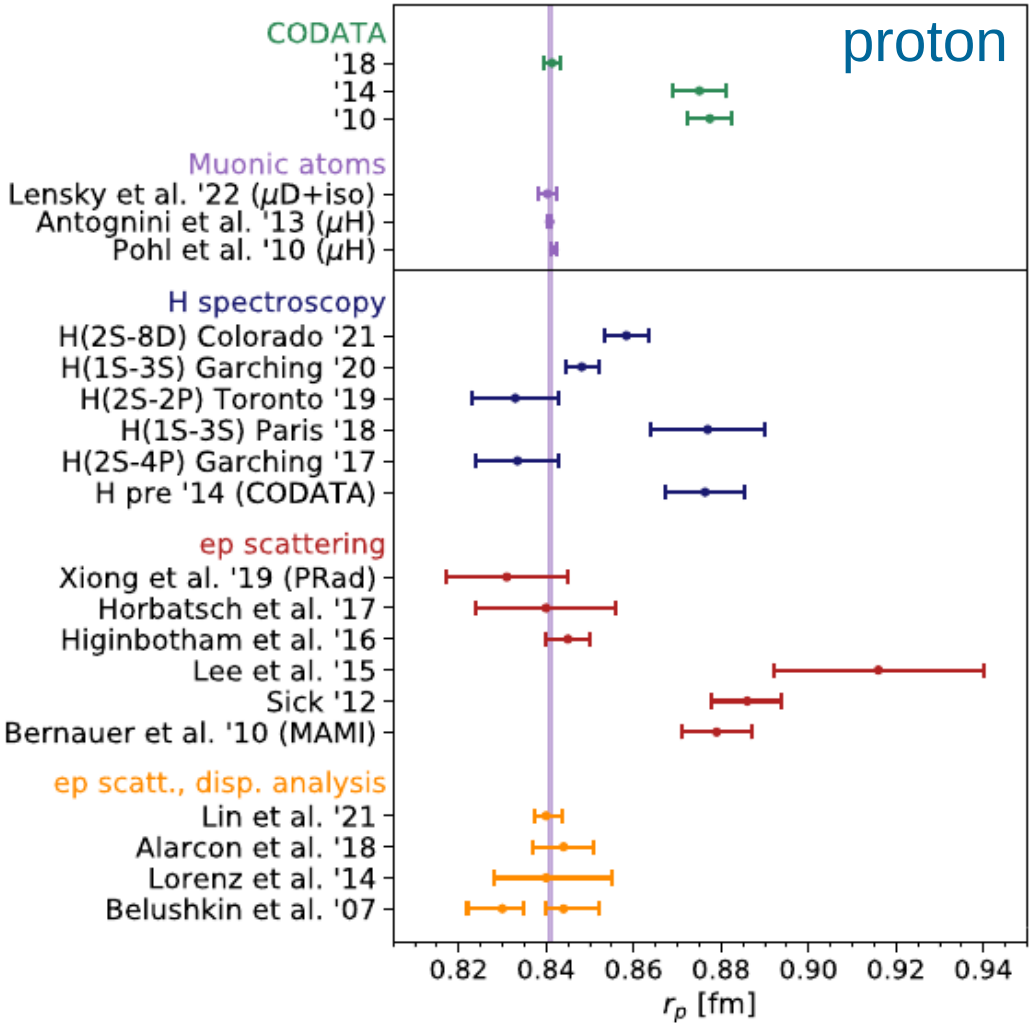
Proton and Deuteron Radii and Isotope Shift

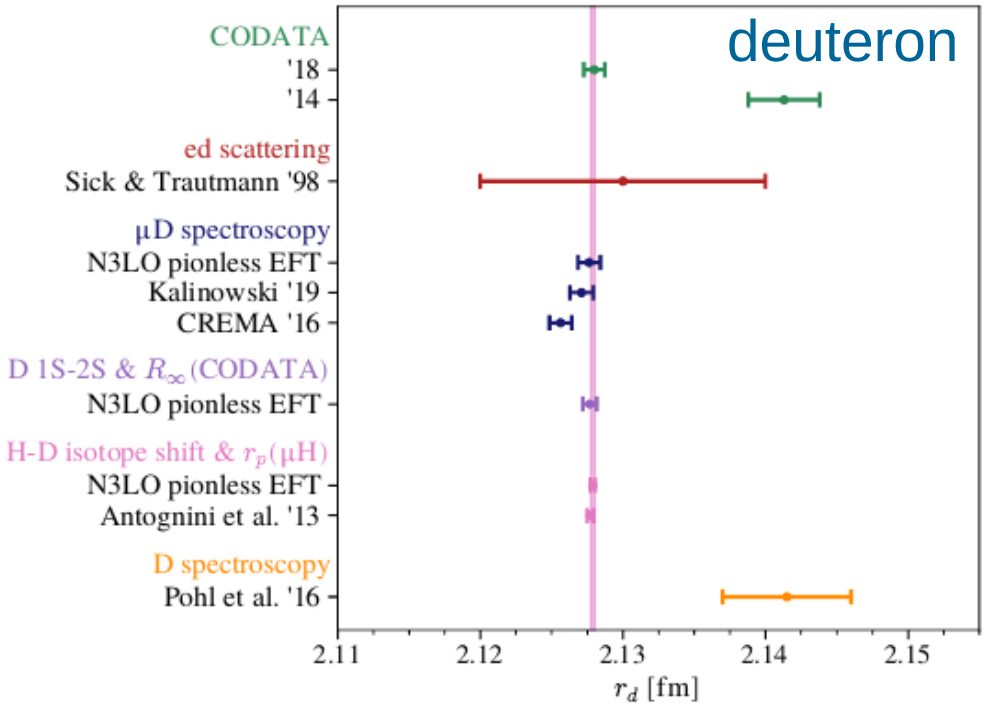
- H-D isotope shift: $E(H, 1S-2S) - E(D, 1S-2S)$

$$r_d^2 - r_p^2 = 3.820\,61(31)\,\mathrm{fm}^2$$

Jentschura et al. (2011)
VL, Hagelstein, Pascalutsa (2022)



Antognini, Hagelstein, Pascalutsa (2022)



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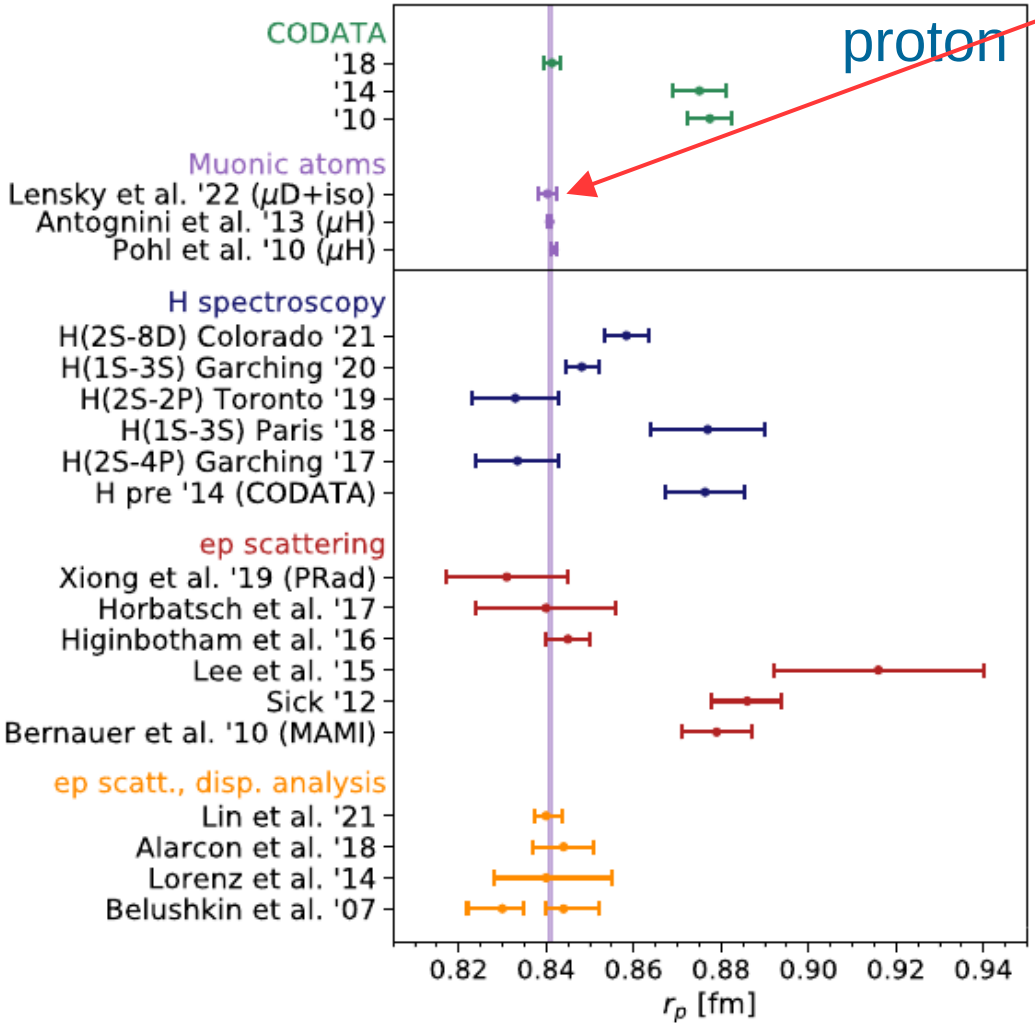

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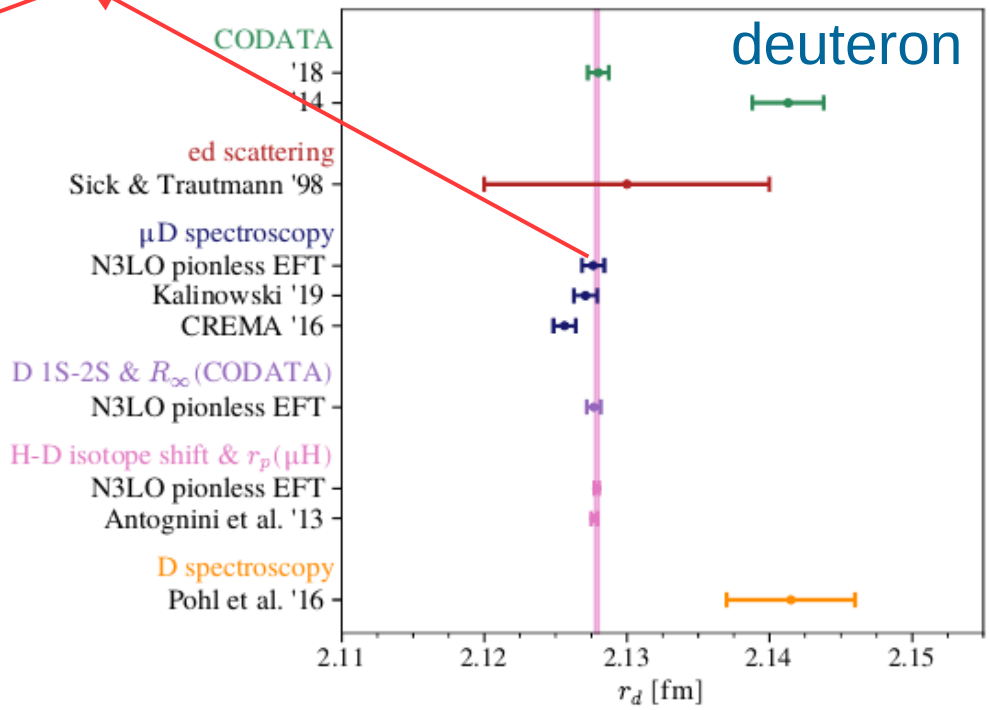


VL, Hagelstein, Pascalutsa (2022)

Two-Photon Exchange (TPE) in (Muonic) Atoms

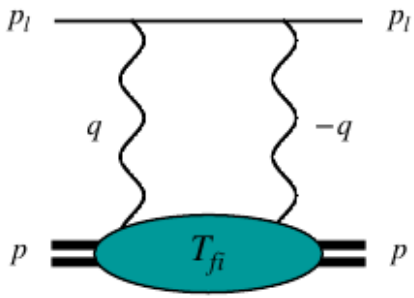
Bohr radius

$$a = (Z\alpha m_r)^{-1}$$

- Muonic atoms: greater sensitivity to charge radii
- But also greater sensitivity to subleading nuclear response

Lamb Shift: $$\Delta E_{nS} = \frac{2\pi Z\alpha}{3}\frac{1}{\pi(an)^3}\left[R_E^2 - \frac{Z\alpha m_r}{2}R_F^3\right] + \ldots$$

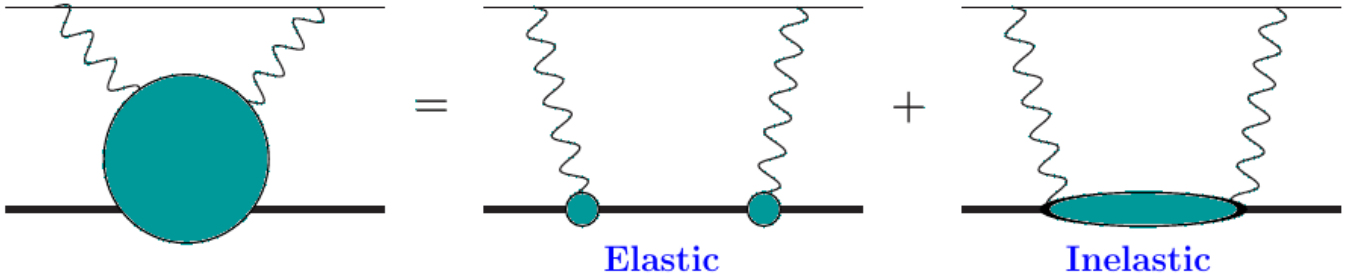
$$\propto \alpha^5$$



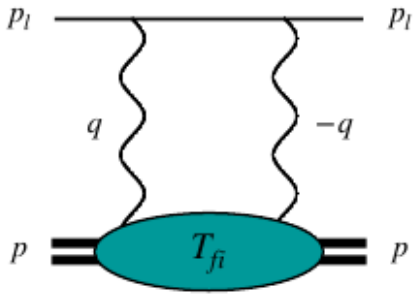
Friar radius: $$R_F^3 = \frac{48}{\pi}\int_0^\infty \frac{dQ}{Q^4}\left[G_C^2(Q^2) - 1 - 2G_C'(0)\,Q^2\right]$$

– (part of the) two-photon response

- Described in terms of (doubly virtual forward) Compton scattering: VVCS
- Elastic ($\nu = \pm Q^2/2M_{\text{target}}$, elastic e.m. form factors) and inelastic (~ nuclear generalised polarisabilities)



VVCS



- Forward unpolarised VVCS amplitude

$$\alpha_{\rm em}\, M^{\mu\nu}(\nu, Q^2) = -\left\{\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\, q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\, q^\nu\right) T_2(\nu, Q^2)\right\}$$

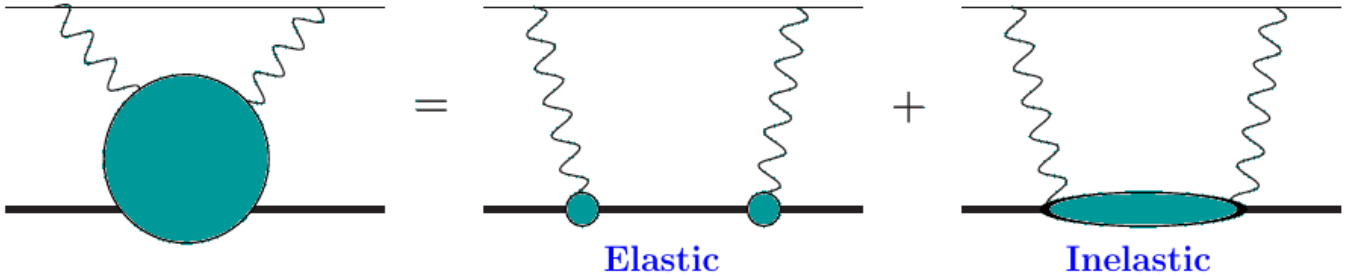
$Q^2 = -q^2,\ \nu = p\cdot q/M_{\rm target}$ photon virtuality and lab frame energy

Lamb Shift:

$$E_{nS}^{2\gamma} = -8i\pi\alpha m\, [\phi_n(0)]^2 \int \frac{{\rm d}^4 q}{(2\pi)^4} \frac{(Q^2 - 2\nu^2)\, T_1(\nu, Q^2) - (Q^2 + \nu^2)\, T_2(\nu, Q^2)}{Q^4(Q^4 - 4m^2\nu^2)}$$

- Need to know both the elastic and inelastic parts of the amplitude

$$T_{1,2}(\nu, Q^2) = T_{1,2}^{\rm elastic}(\nu, Q^2) + T_{1,2}^{\rm inel}(\nu, Q^2)$$



Theory Framework: Pionless EFT

- Typical energies in (muonic) atoms are small: use effective field theories
  - pionless EFT for nuclear effects
  - expansion in powers of a small parameter $p/m_\pi \simeq \gamma/m_\pi \simeq 1/3$
  - order-by-order Bayesian uncertainty estimate
- Why pionless?
  - easier to solve than χEFT (analytic results for *NN*)
  - easier to analyse
  - explicit gauge invariance and renormalisability
  - slower convergence (~larger uncertainty) and (potentially) a narrower range of applicability than χEFT
    - the latter two issues do not seem to affect deuteron VVCS a lot
- We in fact do go beyond strict pionless and use χEFT/data driven DR to estimate higher-order individual nucleon contributions

Counting for VVCS and TPE

- Longitudinal and Transverse amplitudes

$$f_L(\nu, Q^2) = -T_1(\nu, Q^2) + \left(1 + \frac{\nu^2}{Q^2}\right) T_2(\nu, Q^2), \qquad f_T(\nu, Q^2) = T_1(\nu, Q^2)$$

Lamb Shift:

$$\Delta E_{nl} = -8i\pi m \, [\phi_{nl}(0)]^2 \int \frac{d^4q}{(2\pi)^4} \frac{f_L(\nu, Q^2) + 2(\nu^2/Q^2) f_T(\nu, Q^2)}{Q^2(Q^4 - 4m^2\nu^2)}$$

$f_L = O(p^{-2}), \qquad f_T = O(p^0)$ in the VVCS amplitude

$\alpha_{E1} = 0.64 \text{ fm}^3$

$\beta_{M1} = 0.07 \text{ fm}^3$

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- Transverse contribution to TPE starts only at N4LO

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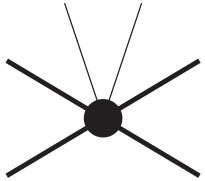
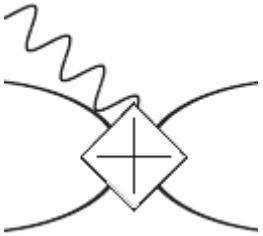
- Transverse contribution to TPE starts only at N4LO
- N4LO: ΔE_{nl} needs to be regularised, an unknown lepton-NN LEC

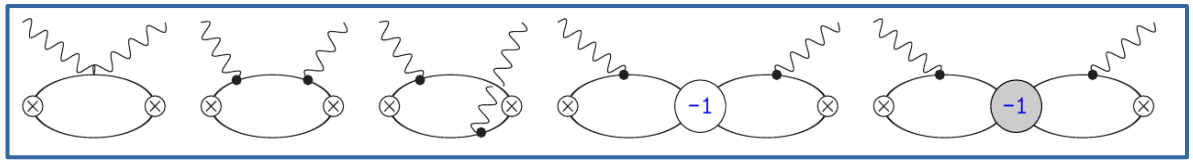


- We go up to N3LO in f_L, and up to (relative) NLO in f_T [cross check]
- One unknown LEC at N3LO in f_L
  - important for the charge form factor
  - extracted from the H-D isotope shift and proton R_E

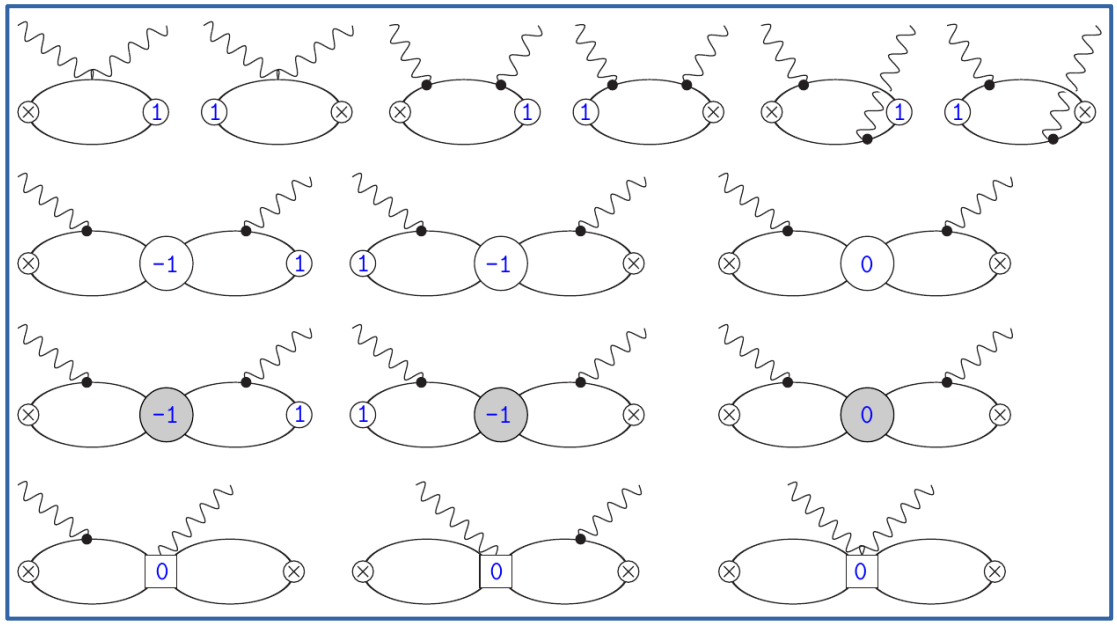


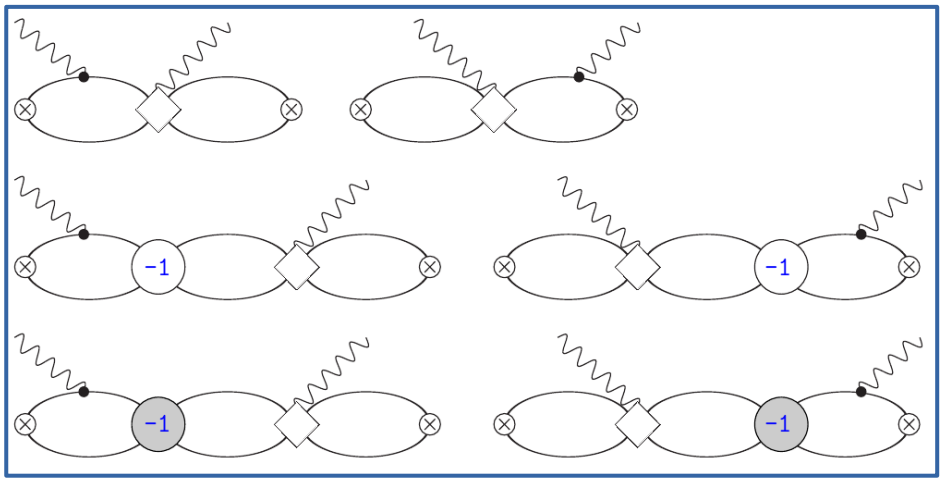
Deuteron VVCS: Feynman Graphs

LO



NLO

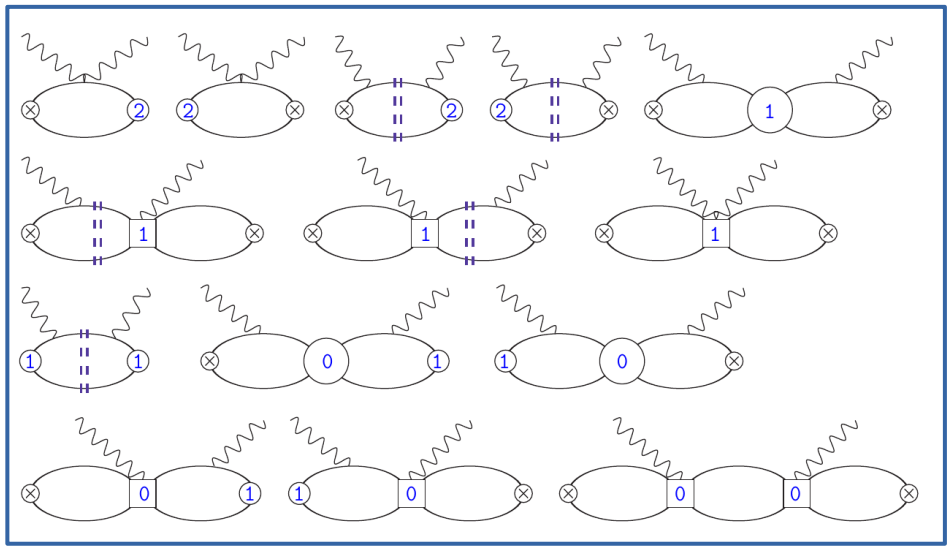


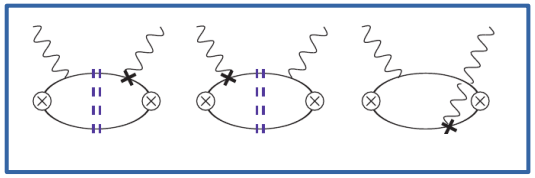


- Amplitudes are calculated analytically
- Checks:
  - the sum of each subgroup (+ respective crossed graphs) is gauge invariant
  - regularisation scale dependence has to vanish

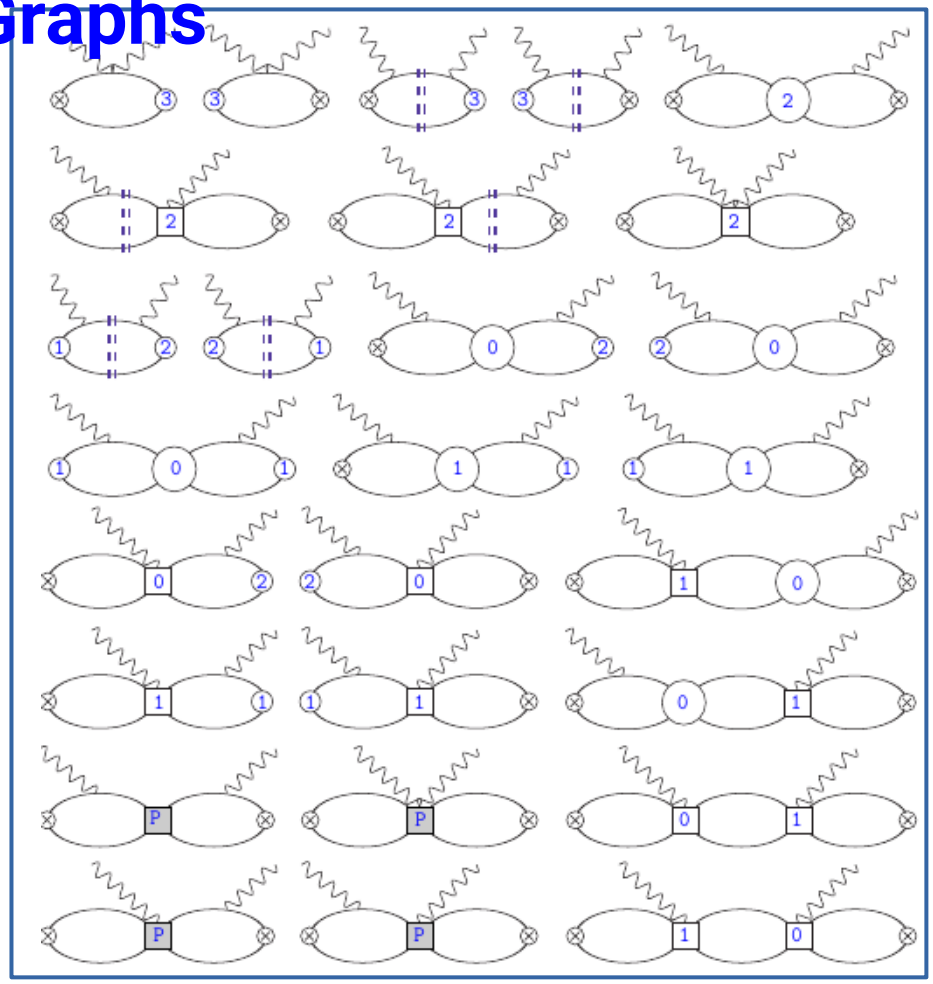
Deuteron VVCS: Feynman Graphs

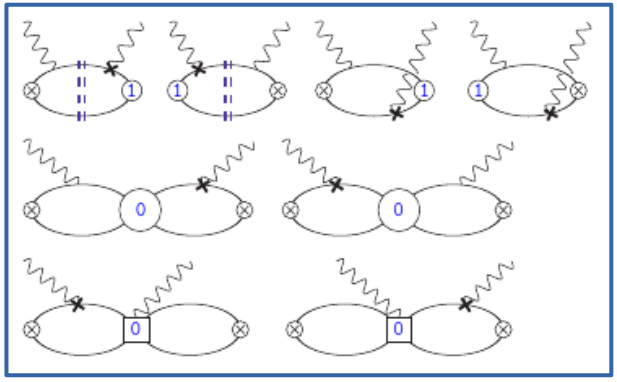
NNLO

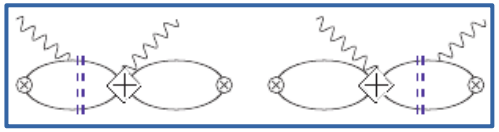




N3LO

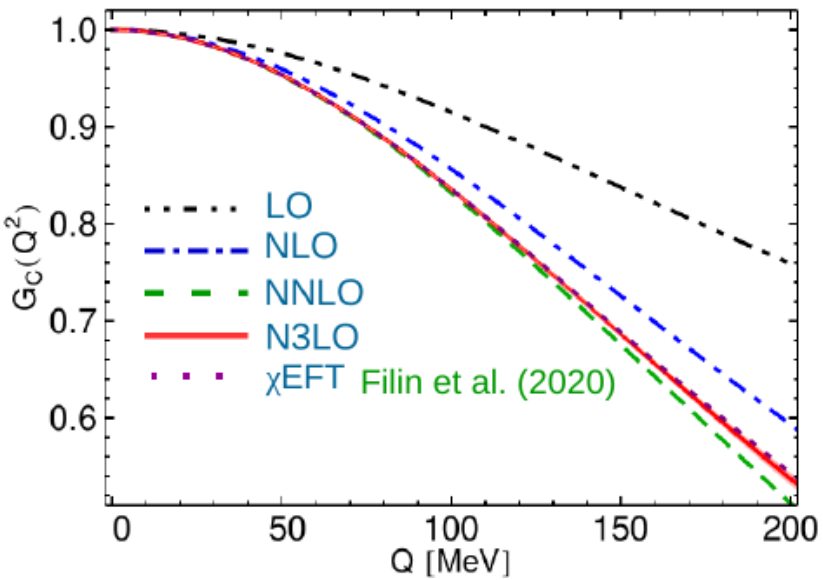






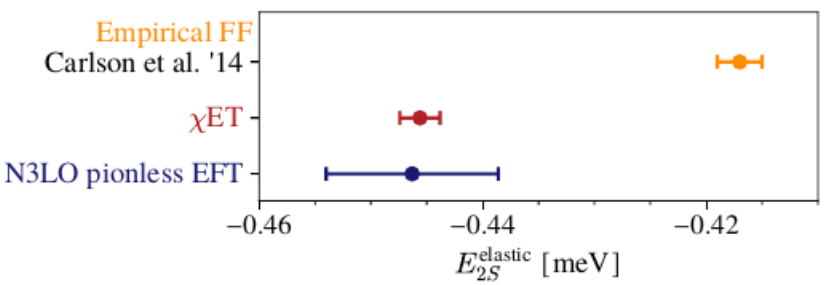
Deuteron Charge Form Factor and TPE in μD

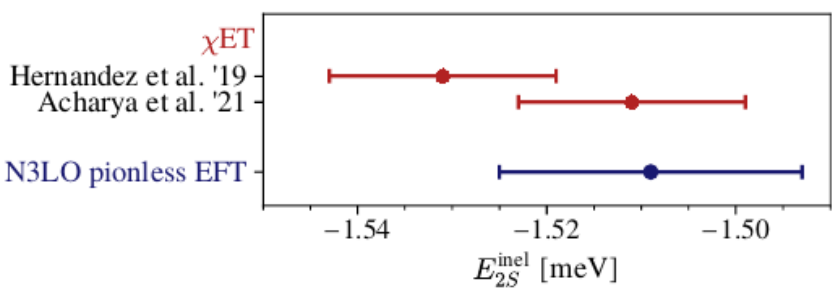
- The deuteron charge form factor obtained from the residue of the VVCS amplitude
- The result is consistent with χEFT
- Elastic TPE is several std. deviations larger than with the empirical form factor of Abbott et al (JLab t_{20})
- Inelastic TPE agrees with other calculations



VL, Hiller Blin, Pascalutsa (2021)

$$\Delta E_{2S} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} = -1.955(19)\text{meV}$$





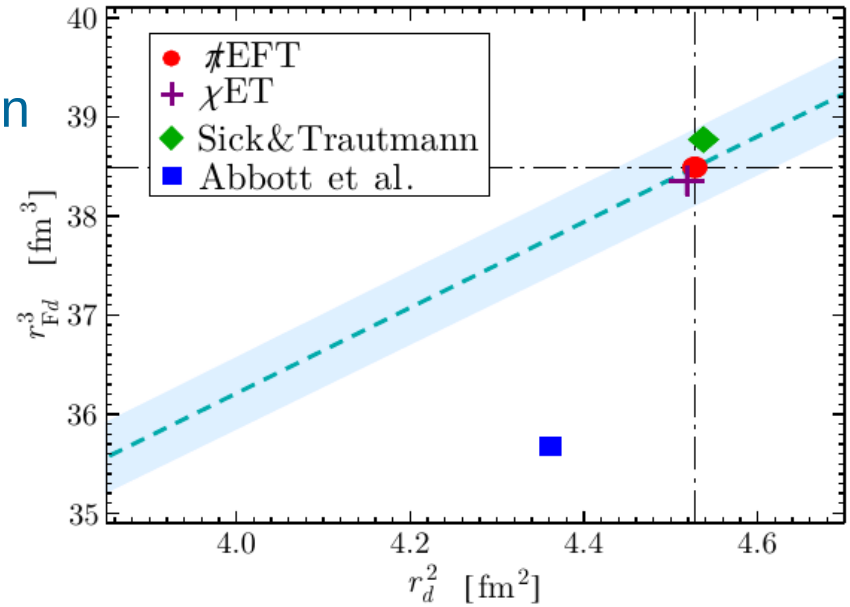
VL, Hagelstein, Pascalutsa (2022)

- Uncertainty is quantified using Bayesian inference
- Disagreement in the elastic TPE?

Deuteron Charge Form Factor and TPE in μD

- Correlation between the charge and Friar radii; can be used to test FF parametrisation

$$R_F^3 = \frac{48}{\pi} \int_0^\infty \frac{dQ}{Q^4} \left[G_C^2(Q^2) - 1 - 2G_C'(0)\, Q^2 \right]$$



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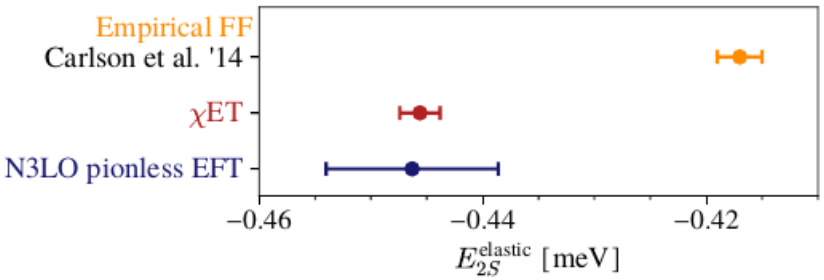

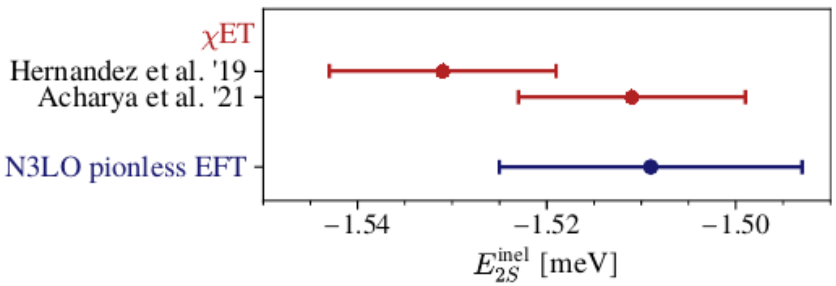

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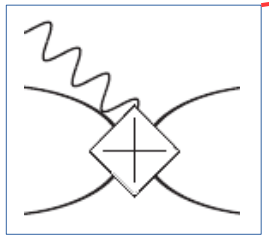
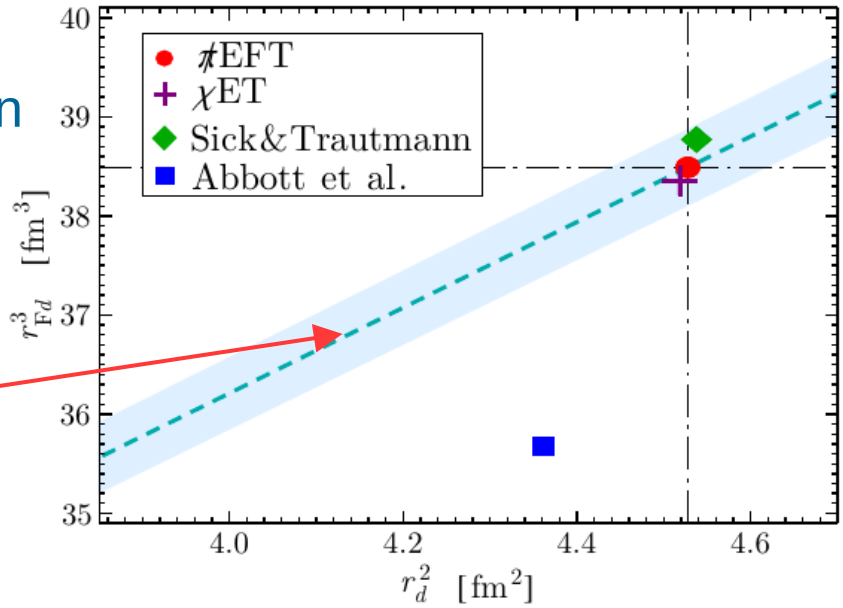
- The correlation is generated by the N3LO LEC





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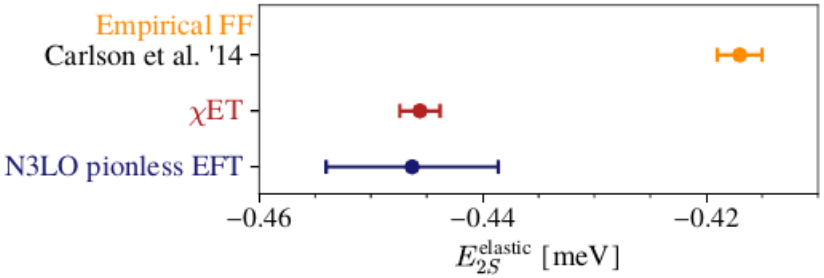

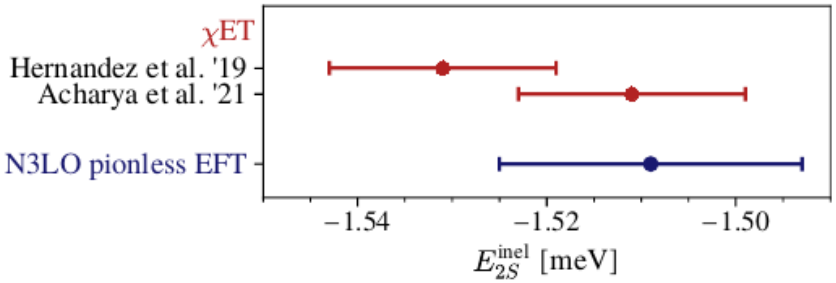

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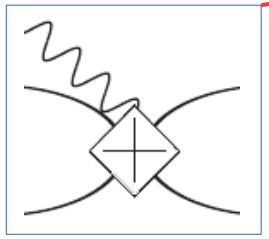

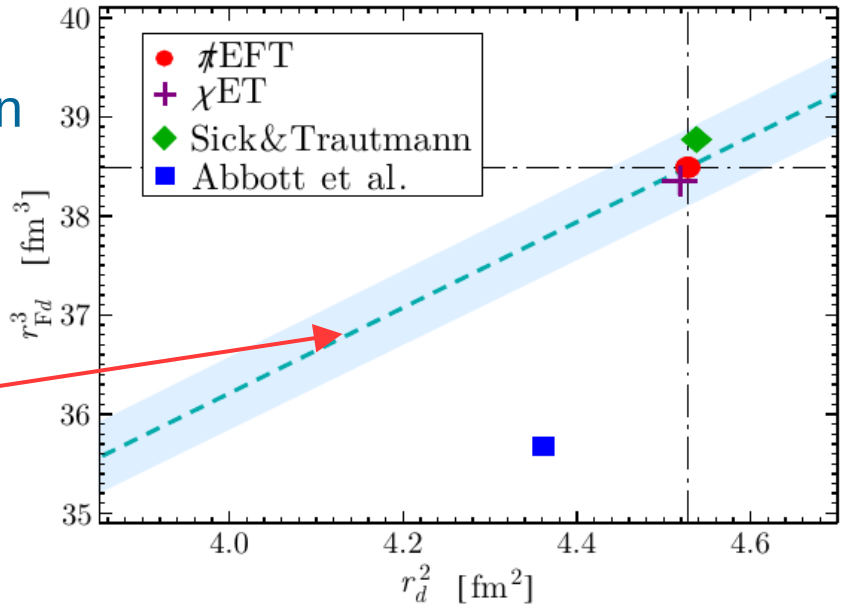

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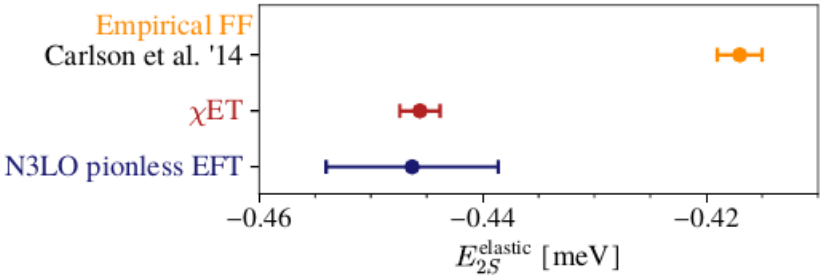
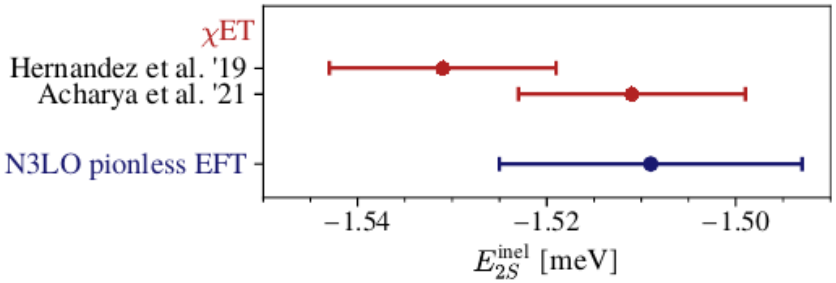




- Abbott et al. charge FF is not suitable for studying the low-Q properties
- Agreement with χEFT vindicates both EFTs

TPE in μD: Higher-Order Corrections

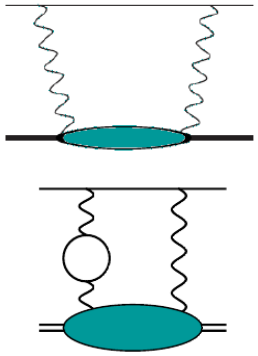
$$\Delta E_{2S} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} = -1.955(19)\text{meV}$$

- Higher-order in α terms are important in D
  - Coulomb $\left[\mathcal{O}(\alpha^6 \log \alpha)\right]$

    taken from elsewhere $\Delta E_{2S}^{\text{Coulomb}} = 0.2625(15)\,\text{meV}$
  - eVP $\left[\mathcal{O}(\alpha^6)\right]$ Kalinowski (2019)

    reproduced in pionless EFT $\Delta E_{2S}^{\text{eVP}} = -0.027\ \text{meV}$

non-forward



- Single-nucleon terms at N4LO in pionless EFT and higher
  - insert empirical FFs in the LO+NLO VVCS amplitude
  - polarisability contribution (inelastic+subtraction)
    - inelastic: *ed* scattering data Carlson, Gorchtein, Vanderhaeghen (2013)
    - subtraction: nucleon subtraction function from χEFT

      VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)
  - in total: small but sizeable: $\Delta E_{2S}^{\text{hadr}} = -0.032(6)\ \text{meV}$

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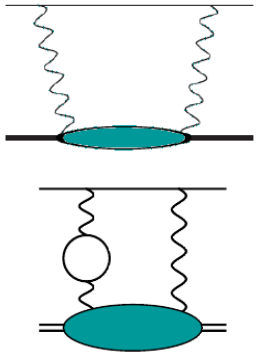

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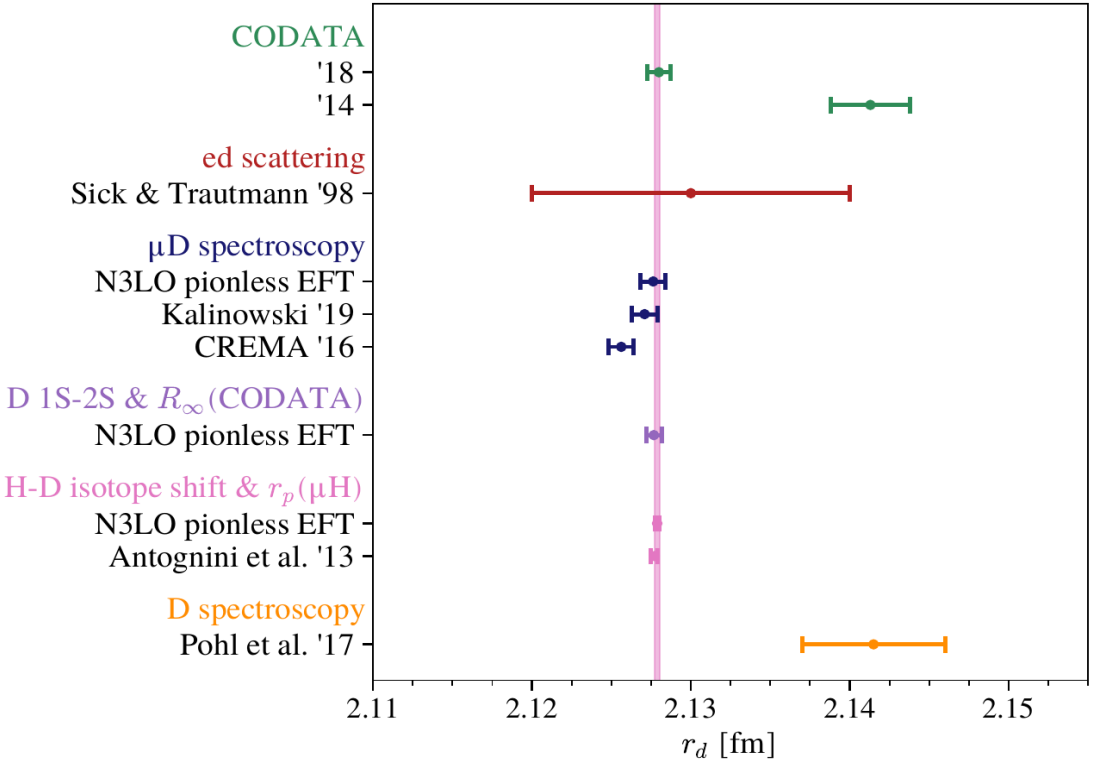
      VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)
  - in total: small but sizeable: $\Delta E_{2S}^{\text{hadr}} = -0.032(6)$ meV

$$\boxed{\Delta E_{2S}^{2\gamma} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} + \Delta E_{2S}^{\text{hadr}} + \Delta E_{2S}^{\text{eVP}} + \Delta E_{2S}^{\text{Coulomb}} = -1.752(20)\ \text{meV}}$$

Deuteron Charge Radius and TPE in μD

- Reassessed with pionless EFT
- μD, D, and H-D isotope shift all consistent with one another
- Agreement with the very precise empirical value of 2γ exchange

	$E_{2S}^{2\gamma}$ [meV]
Theory prediction	
Krauth et al. '16 [5]	−1.7096(200)
Kalinowski '19 [6, Eq. (6) + (19)]	−1.740(21)
$\not{\pi}$EFT (this work)	−1.752(20)
Empirical (μH + iso)	
Pohl et al. '16 [3]	−1.7638(68)
This work	−1.7585(56)



VL, Hagelstein, Pascalutsa (2022)

- Nuclear-level response well under control (in heavier nuclei: use χEFT)
- Single-nucleon structure starts to be important at this level of precision
  - even more important in heavier nuclei (He, Li, …)
- Experimental precision presents a challenge for theory

Thank You for Your Attention!

Slighly More Details on Pionless EFT

- Nucleons are non-relativistic $\rightarrow$ $E \simeq p^2/M = O(p^2)$
- Loop integrals $dE\, d^3p = O(p^5)$
- Nucleon propagators $(E - p^2/2M)^{-1} = O(p^{-2})$
- Typical momenta $p \sim \gamma = \sqrt{ME_d} \simeq 45$ MeV
- Expansion parameter $p/m_\pi \simeq \gamma/m_\pi \simeq 1/3$
- *NN* system has a low-lying bound/virtual state $\rightarrow$ enhance S-wave coupling constants, resum the LO *NN* S-wave scattering amplitude

- Easier to solve than χEFT (analytic results for *NN*)
- Easier to analyse (e.g., discover correlations between various quantities)
- Explicit gauge invariance and renormalisability
- Slower convergence (~larger uncertainty) and (potentially) a narrower range of applicability than χEFT

More Details on the Counting for VVCS and TPE

$$\Delta E_{nl} = -8i\pi m\,[\phi_{nl}(0)]^2 \int \frac{d^4q}{(2\pi)^4} \frac{f_L(\nu, Q^2) + 2(\nu^2/Q^2) f_T(\nu, Q^2)}{Q^2(Q^4 - 4m^2\nu^2)}$$

- Transverse contribution starts at N4LO in TPE

$\alpha_{E1} = 0.64\ \mathrm{fm}^3$

$\beta_{M1} = 0.07\ \mathrm{fm}^3$

$$f_L(\nu, Q^2) = 4\pi\alpha_{E1} Q^2 + \ldots$$

$$f_T(\nu, Q^2) = -\frac{e^2}{M_d} + 4\pi\beta_{M1} Q^2 + 4\pi(\alpha_{E1} + \beta_{M1})\nu^2 + \ldots$$

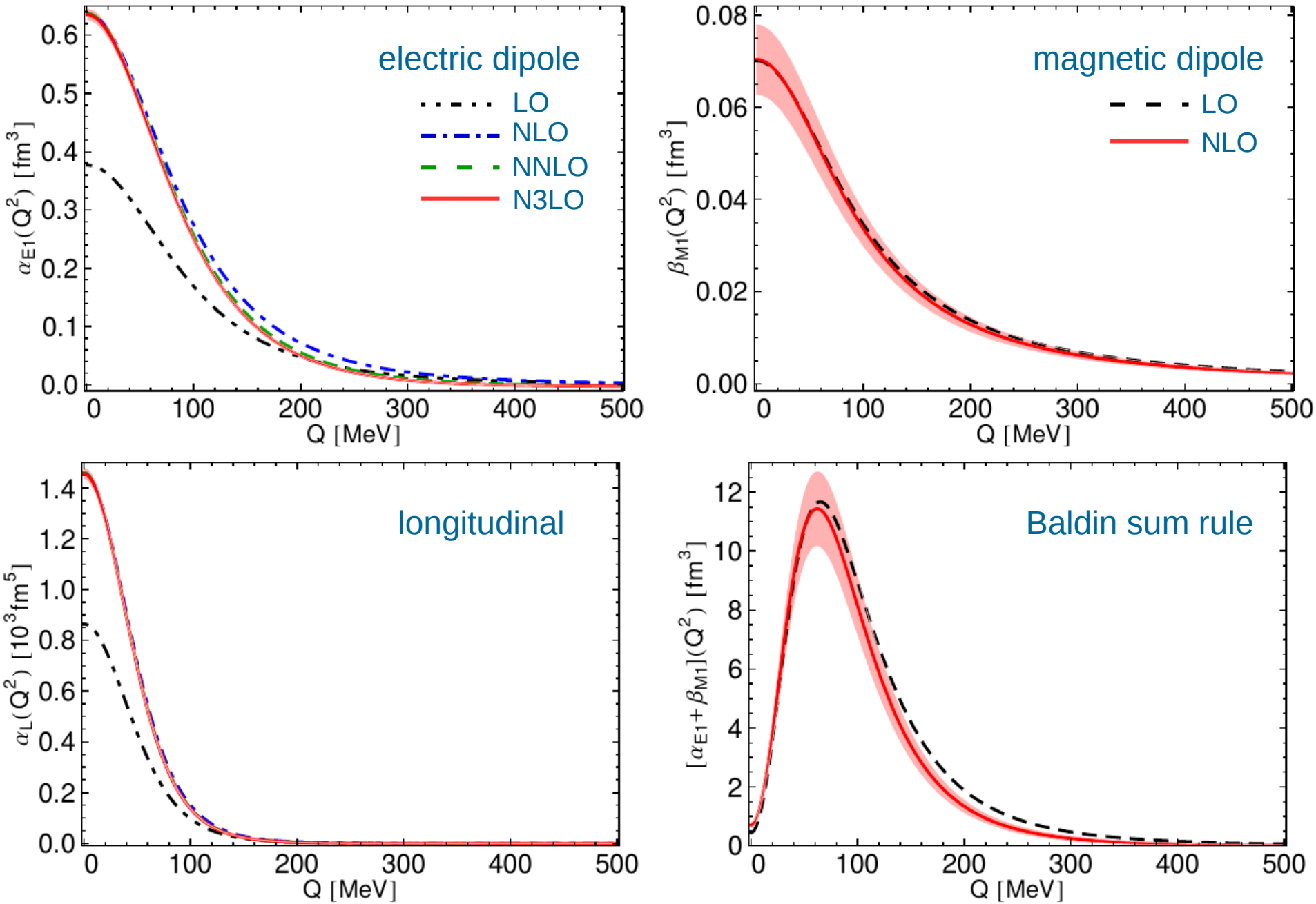
$$\alpha_{E1} = \frac{\alpha M}{32\pi\gamma^4} + \ldots$$

$$\beta_{M1} = -\frac{\alpha}{32M\gamma^2}\left[1 - \frac{16}{3}\mu_1^2 + \frac{32}{3}\mu_1^2 \frac{\gamma}{\gamma_s - \gamma}\right] + \ldots$$

$f_L = O(p^{-2}), \qquad f_T = O(p^0)$ in VVCS

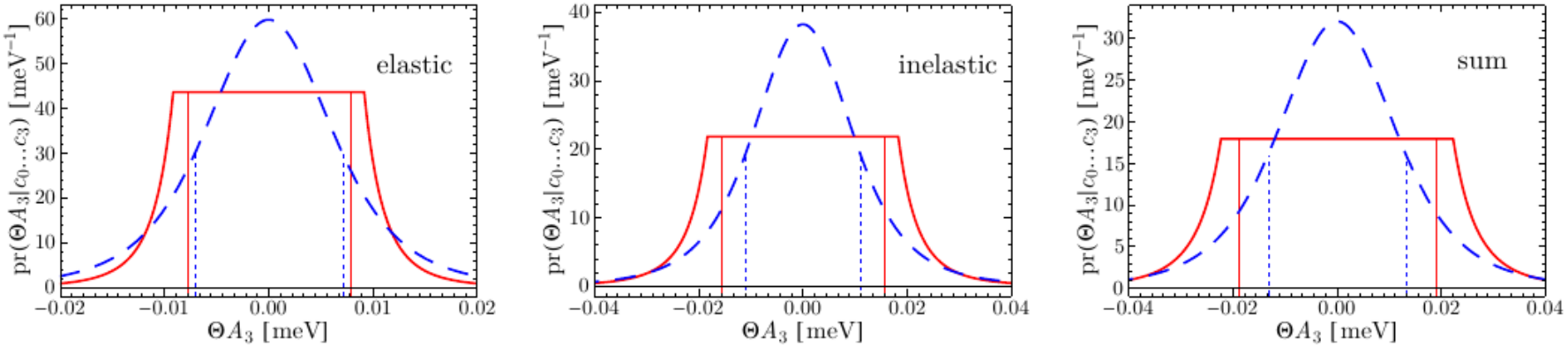
longitudinal $= O(p^{-2}), \qquad$ transverse $= O(p^2)$ in TPE

Deuteron VVCS: Generalised Polarisabilities



Bayesian Uncertainty Estimate

- Probabilty distribution functions corresponsing to the uncertainty estimate



VL, Hagelstein, Pascalutsa (2022)
Along the lines of Furnstahl, Klco, Phillips, Wesolowski (2015), Coello Perez, Papenbrock (2015)