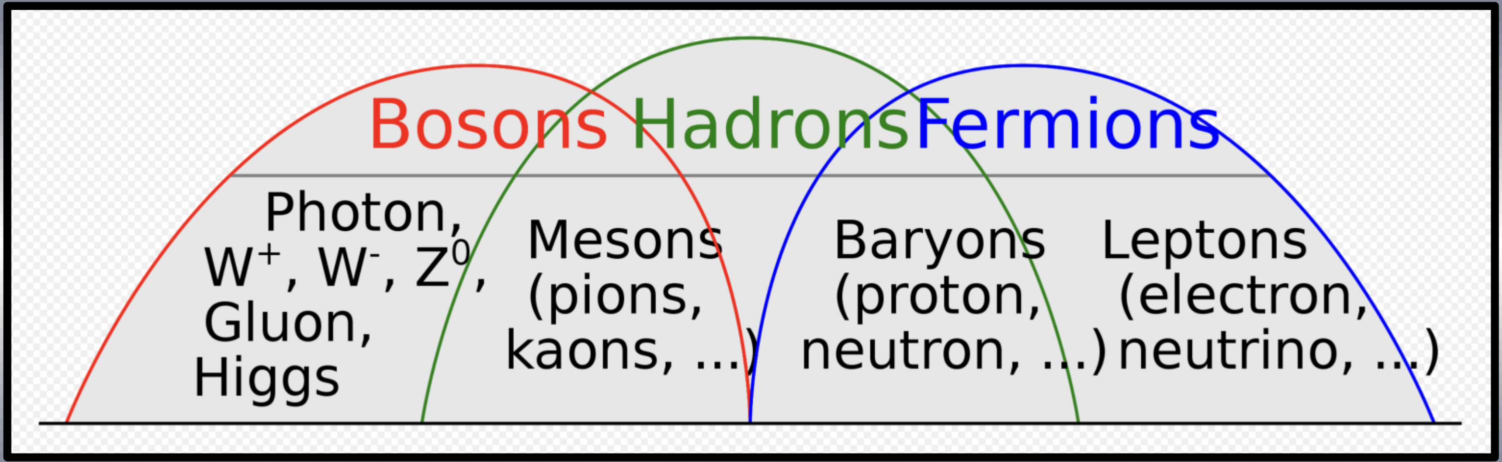
Forging a Path for Understanding Nuclear Structure from Strong QCD

(JLab 2019: `Particle Picture') ⟷ (Seville 2022: `Field Focus')

Jerry P Draayer (Louisiana State U) & David S Kekejian (U North Carolina)

(Lessons learned from Low-energy NP) & (Learning from Medium/High-energy NP)

Mr. Wikipedia: Subatomic World



Bosons (integer spin – Pauli blocking OFF) form one of the two fundamental classes of subatomic particles, the other being Fermions (half-integer spin – Pauli blocking ON). All subatomic particles must be one or the other. Composite particles (Hadrons) may fall into either class depending on their composition.

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U.S. NSF & DOE & LSU Sponsored Research

- Abstract -

The focus of our program – and hence this paper – is to recast lessons learned over the last half of the 20th Century regarding the structure of atomic nuclei (primarily a particle-based picture) into a forward leaning 21st Century (field-theory-based framework) in anticipation that this will encourage the establishment of a natural bridge between the low-energy and medium-to-high energy nuclear physics communities. This has been and continues to be enabled by two major developments, the first technical (high-end computational facilities of the 90s) and the other analytical (underpinned by the so-called `no-core' *ab initio* – from first principles – shell-model theories). Early successes of the latter rest upon a realization that special symmetries and the associated algebraic methods that this enables are much better than previously anticipated. Consequently, this presentation includes a few-slide dive into some key group theory concepts, plus showing some early results that expose `simplicity within complexity' in atomic nuclei that has until now it seems been under appreciated!

Next Slide (same as) Final Slide – Captures Key Message

Status of our hunt for Symplicity within Complexity* in nuclear matter!

(*Charge received from Eugene Wigner, 1st year as Assistant Professor at LSU, S-1974)

Symplectic Effective Field Theory for Nuclear Structure Studies

(David Kekejian, Thesis Research @ LSU 10/21 & Currently Post Doc, UNC)

Generic (Scalar) Field Theory — **canonical quantization** → **Quantum (Scalar) Field Theory**

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) + \frac{1}{2} m^2 \varphi^2 \xrightarrow{\text{canonical quantization}} H = \sum_k E_k (b_k^+ b_k^- + \tfrac{1}{2})$$

$$\varphi(r,t) = \frac{1}{\sqrt{V}} \sum_k b_k^- \frac{1}{\sqrt{|2k^0|}} e^{-\iota k^\mu x_\mu} + \frac{1}{\sqrt{V}} \sum_k b_k^+ \frac{1}{\sqrt{|2k^0|}} e^{\iota k^\mu x_\mu}$$

$$\mathcal{L}^{(n)} = \frac{\alpha^n}{2(n+1)!} (\partial_\mu \varphi \partial^\mu \varphi^* + m^2 \varphi \varphi^*)^{n+1}$$

$$\mathcal{H}^{(n)} = \frac{\alpha^n}{2(n+1)!} (\dot{\varphi}\dot{\varphi}^* - \varphi' \cdot \varphi'^* + m^2 \varphi\varphi^*)^n ((2n+1)\dot{\varphi}\dot{\varphi}^* + \varphi' \cdot \varphi'^* - m^2 \varphi\varphi^*)$$

$$H^{(n)} \sim (\frac{\alpha}{V} \hbar\Omega)^n \times (g^2 Q \cdot Q)^n, (K \cdot K)^n, (gQ \cdot K)^n, (gK \cdot Q)^n$$

Where: $\frac{\alpha}{V} \hbar\Omega = \frac{\beta^2}{8N_\sigma}$ $\quad g = \frac{m^2}{\hbar^2\omega^2}$

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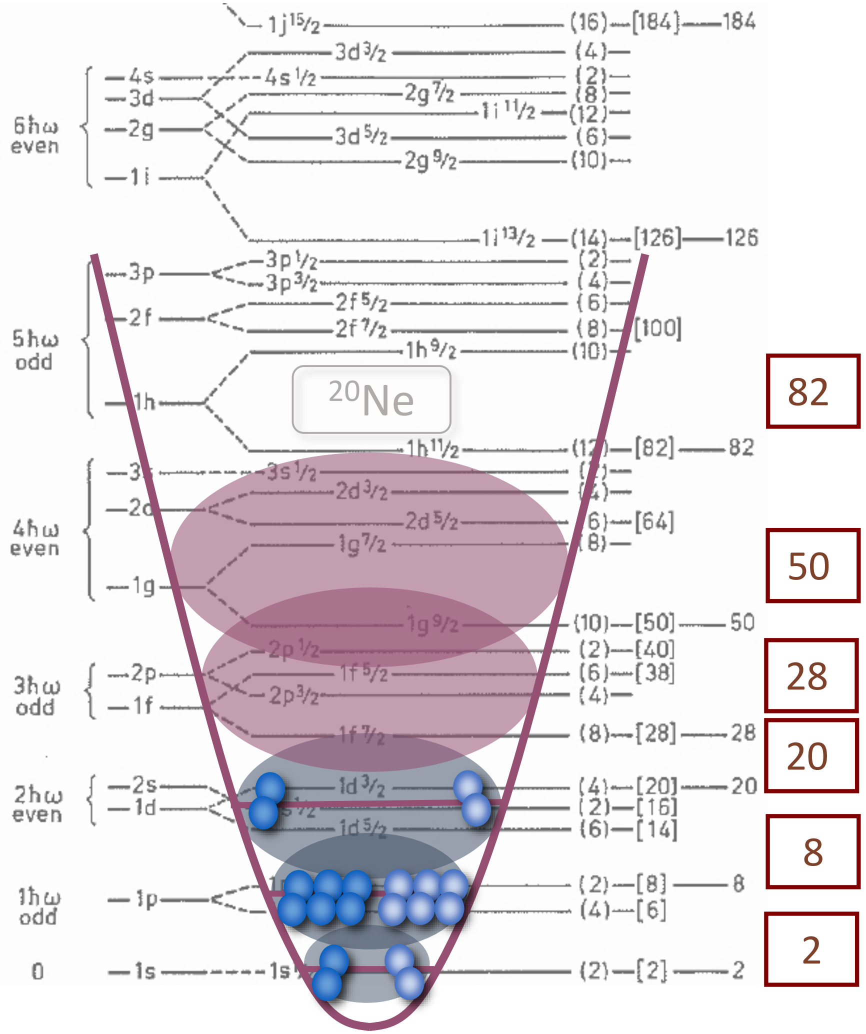
(DOI: 10.1103/PhysRevC.106.014304)

Kekejian (Thesis), Draayer & Mokeev, Roberts

Conclusion: Symplectic Symmetry emerges naturally from a quantum effective Field Theory!

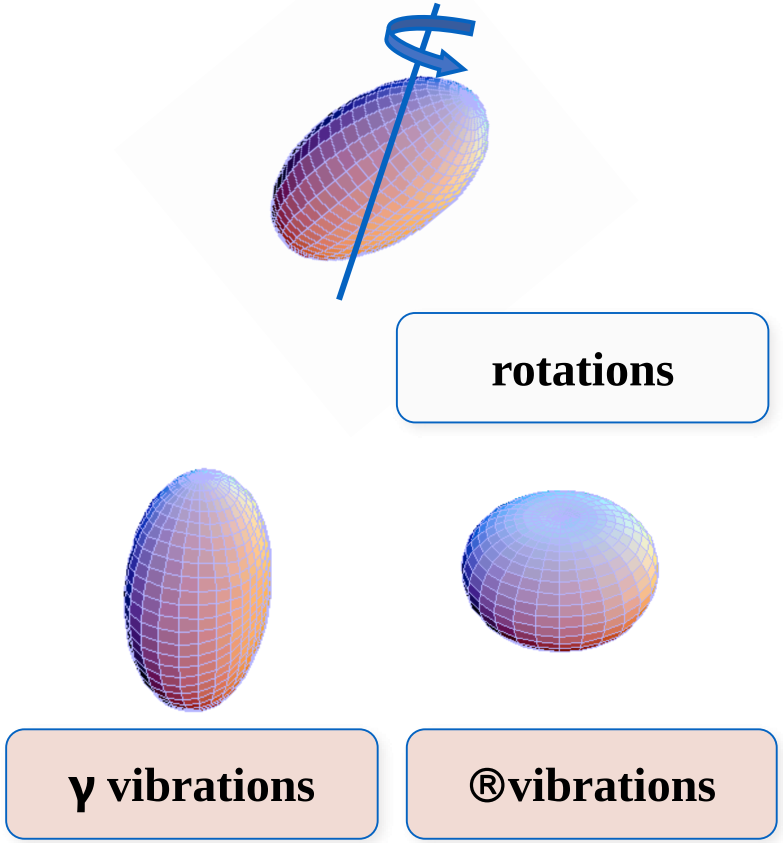
Shell Model

Collective Model



Maria Goeppert-Mayer & J. Hans D. Jensen (~'50)
"…for their discoveries concerning nuclear shell structure …"
(1963 Nobel Prize)

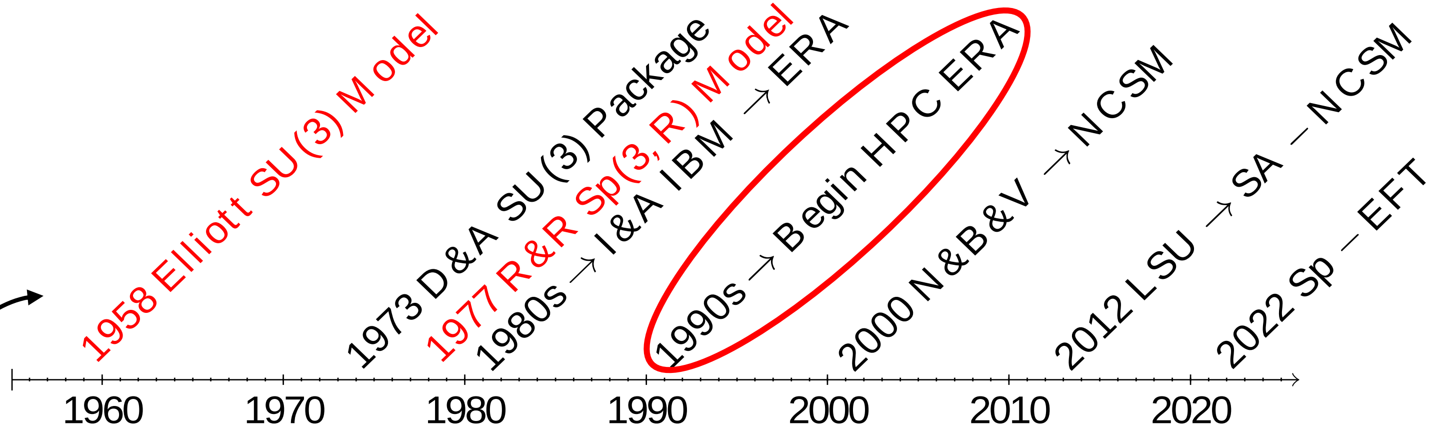
A.N. Bohr, B.R. Mottelson & J. Rainwater ('50s)
"… for the discovery of the connection between collective motion and particle motion in atomic nuclei and the development of the theory of the structure of the atomic nucleus based on this connection …"
(1975 Nobel Prize)



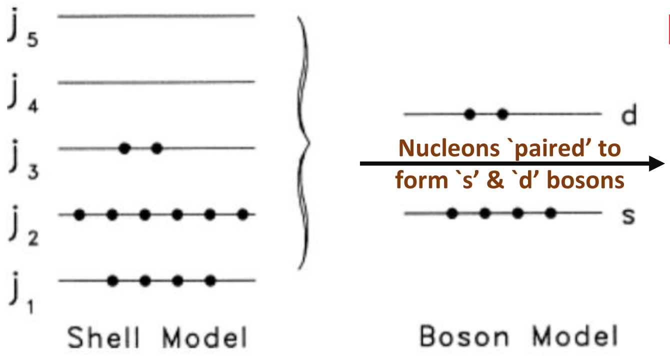
Shell Model (Particle Picture) versus (Field Focused) Dynamics

Caricature of an Evolving Nuclear Structure Landscape

50s	Structureless Point Particles (proton & neutrons)
60s	3D Harmonic Oscillator `Box' (rotations & vibrations)
70s	Various Algebraic Theories (Fermion & Boson)
80s	Phenomenological versus Realistic Interactions
90s	**Supercomputers enabled a Transformational Change**
00s	Open (No-Core) Models with `ab initio' Interactions
10s	Algebraic theories extended via non-compact structures
20s	Inter-nucleon Fields drive Nuclear Structure



Shell Model Approach
(Elliott & Harvey plus Rowe & Rosensteel …)
Has both **fermion** and field theory features.



Interacting Boson Model
(Iachello & Arima plus Many IBM Disciples …)
Has both **boson** and field theory features.

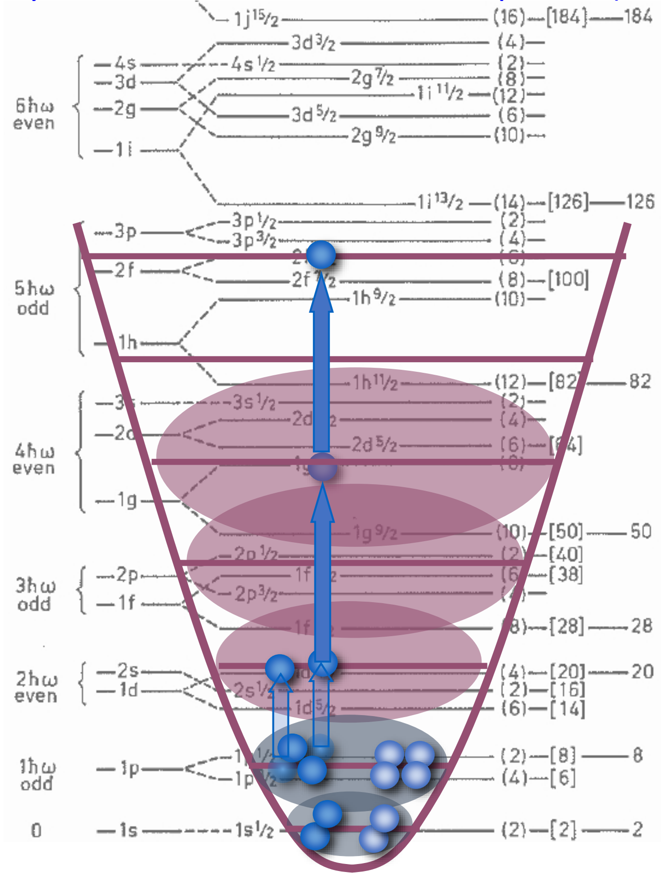
non-compact <-- **Symmetry Platforms** --> **compact only**

Blue `Basis States' Elliott SU(3); R&R Fermion Model	Sp(3,R) (non-compact) ⊃SU(3) (3D-oscillator) ⊃SO(3) (3D-rotations)	U(6) (pairing modes) & SU(3) (rotor modes) ⊃SO(3) (3D-rotations)	**Red `Basis States' IBM: Interacting Boson Model**

Shell Model | Advances | Collective Model

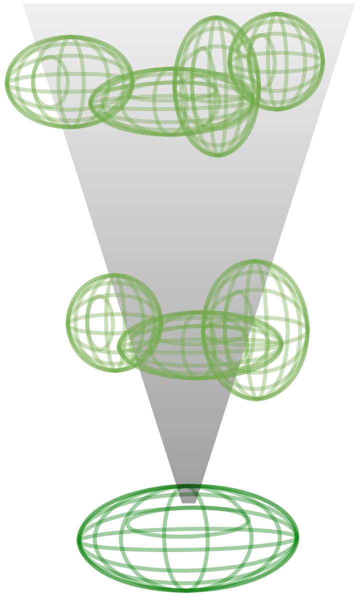
No-Core Shell Model (NCSM)*

(Vary, Navratil, & Barrett, ~2000 to present)



*Realistic interaction (local or not; NN, NNN,…)

- In principle, exact solutions, up to N_{max}
- Successful description up through ^{16}O



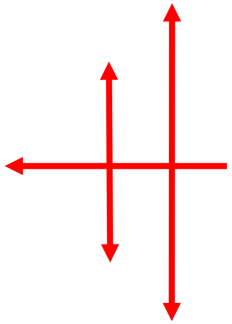
Symplectic Model – NCSpM*

(Rowe & Rosensteel~1980s to Present)

- Sp(3,R) group structure
- SU(3) - symmetry adapted basis
- Schematic interactions

Typical Vertical Symplectic Slice

(monopole/quadrupole excitations, including 2p2h, 4p4h, …)



$0\hbar\Omega$ **Elliott's SU(3)**

*Multi-shell generalization of Elliott Model

- No spurious center-of-mass motion
- Fully microscopic, equals NCSM if all symplectic slices included
- Reproduces rotational bands & B(E2) rates without effective charge

‘Symmetry Adapted’ / SA-NCSM Campaign

Timeline: 5 (2002-06) + 5 (2007-2011) + 5 (2012-20XX)

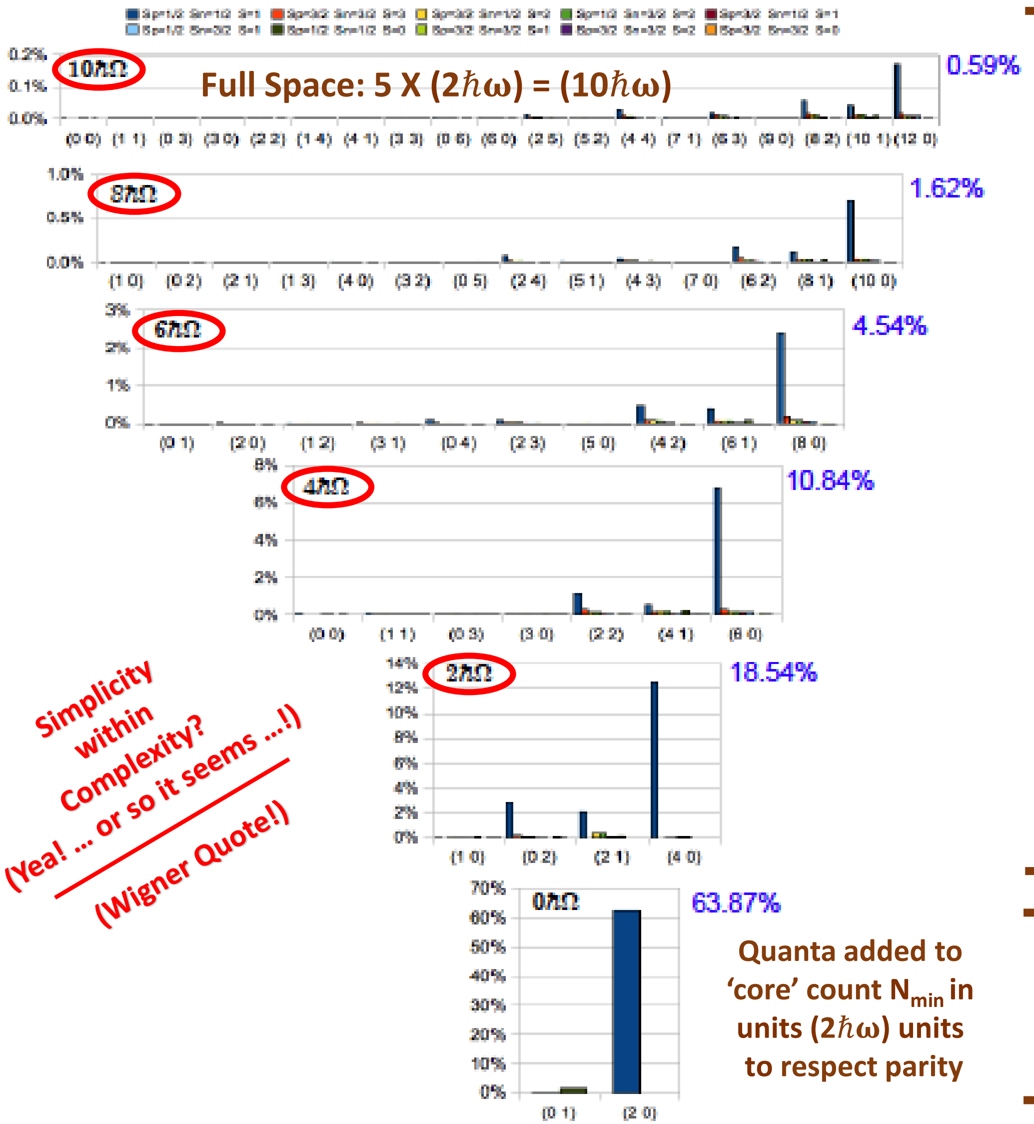
Goal –

Reproduce and predict properties of heavy as well as light nuclei, starting with and building upon QCD/EFT informed and inspired interactions …

Plan –

- ✓ **Exploit existing capabilities to evaluate probability of success and level of effort required to develop a full-blown symmetry adapted NCSM**
- ✓ **Develop a symmetry adapted no-core shell model code that capitalizes on exact and approximate (partial) symmetries of nuclei (SA-NCSM)**
  - Exploit existing NCSM technology to prove efficacy of method, revealing (or not) any inherent limitations
  - Explore need (or not) for renormalization, winnowing the space to physically relevant and tractable subspaces, etc.
  - Evaluate extensibility of theory and its characteristics by taking full advantage of emerging computational resources
- ✓ **Study the emergence of collective phenomena, tracking their evolution to and from fundamental (*ab initio*) features of the interaction**
  - Apply the theory to study of extreme processes known to be very important to understanding nuclei and nuclear systems
  - Ultimately develop a `user-friendly’ code (desktop version) for easy applications and teaching as well as training purposes
  - Extend the theory to nuclear reactions - coupling to continuum and apply the results to important astrophysical processes

First Results for ^{6}Li with N_{add} = 10 --



Simplicity within Complexity? (Yea! … or so it seems …!) (Wigner Quote!)

Quanta added to 'core' count N_{min} in units $(2\hbar\omega)$ units to respect parity

N_{add}

N_{min}

… Proof of Principle …

Answer: > 85% of Physics In < 1% of the "extended" space

JISP16 '*bare' interaction in N_{add} => 10 'space'; i.e., 5 x $(2\hbar\omega)$ = $(10\hbar\omega)$ to N_{min} 'core'

*Bonn, Argonne, Idaho, N3LO (optimized), etc. … all yield close results … (only change at the 1-2% level)

… Team Work …

Many 'helps' along the way … e.g., James Vary making his NCSM available to us, Mark Caprio (ND) visiting LSU on a sabbatical, along with quality input from Anna Hayes (LANL) & various other collaborators from Bulgaria, China, Mexico, and so on. Also, to NSF for a PetaApps award, and DOE for an EPSCoR grant, plus SURA for release time and financial assistance!

Simply Add 3D-HO Quanta to 'core'!

Typically … 75% -> 95% Leading Sp-IR

Many examples published, so …

Why does it work?

The rest of the story follows …

Hint: The Physics is in the Fields!

Symplectic Symmetry: The Dynamical Symmetry of 3D-HO

Sp(3,R) ⊃ U(3) ⊃ [U(1) X SU(3)] ⊃ SU(3) ⊃ SO(3)

- Sp(3,R): `New' Stuff R&R (>70s) Adds N_{add}
- U(3): Deformation (Q) & Quantum Count of `Core' Configs
- [U(1) X SU(3)]: Quantum (3D-HO) Count ($N_{(\lambda_i,\mu_i)}$) of unique `Core' Configs
- SU(3): Deformation (Q) of unique (λ_i,μ_i) `Core' Configs
- SO(3): Rotational (L) Momentum (K,L,M)

Non-compact Group 'Extension' of U(3) … adds multiple [$(\lambda,\mu) = (2,0)$ / ($2\hbar\omega$)] 3D–HO excitations (N_{add}) to the initial 'core' configuration(s) count (N_{min}) for total 'field' count: $N_{tot} = N_{add} + N_{min}$ (Quanta).

In principle, N_{add} can run from 0 to infinity (∞), but practically, as shown on previous slide, results converge at ~10, which sets the scale of the extension; i.e., ~200 MeV [10 x ($2\hbar\omega$)] energy units.

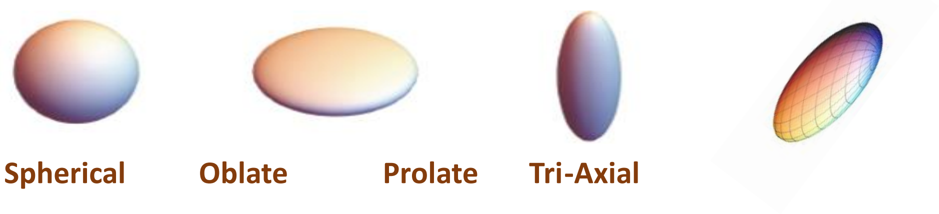
Digging deeper into the group structure: Number of 'generators' of the respective groups listed above follows:

Sp(3,R) ⊃ U(3) ⊃ [U(1) X SU(3)] ⊃ SU(3) ⊃ SO(3)

Missing Dynamics! ← 21 = 12 + 9 → Elliott Plus!

Sp(3,R)	U(3)	[U(1) X SU(3)]	SU(3)	SO(3)
21 = 12 + 9	9	[1 + 8]	[5 + 3 = 8]	3

Elipsoidal Shapes as related to (λ,μ) values



Spherical	Oblate	Prolate	Tri-Axial
(0,0)	(0,μ)	(λ,0)	(λ,μ)

Mapping 3-D Rotor to SU(3)

(β, γ) Bohr-Mottelson

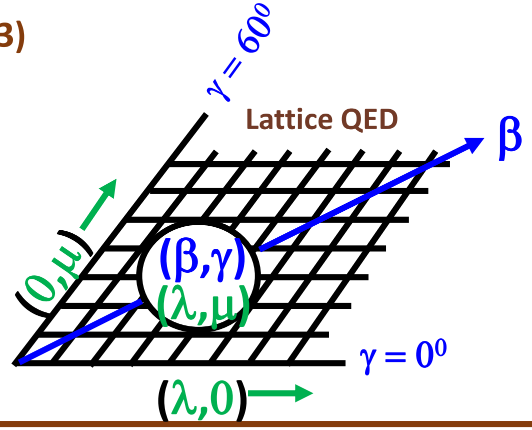
(λ, μ) Elliott (1958)

$\beta^2 \sim C_2$

(2nd Order SU(3) Invariant)

$\beta^3 \cos(3\gamma) \sim C_3$

(3rd Order SU(3) Invariant)



Symplectic Symmetry: Spectrum Generating Algebra (3D-HO)

The 21 generators of Sp(3,R) - characterized on the last slide in terms of their group/subgroup properties - can be recast in terms of 'field operators/tensors' built from distinct quadratic forms in the coordinates (q_i) and momenta (p_i) of the system:

$$Q_{ij} = \sum_{n}^{A} q_{in} q_{jn} \qquad K_{ij} = \sum_{n}^{A} p_{in} p_{jn} \qquad L_{ij} = \sum_{n}^{A} (q_{in} p_{jn} - q_{jn} p_{in}) \qquad S_{ij} = \sum_{n}^{A} (q_{in} p_{jn} + p_{in} q_{jn})$$

Quadrupole (Shape) `Tensor' (6)	Kinetic Energy `Tensor' (6)	Orbital Angular Momentum Operator (3)	Internal Dynamics `Circulation' / `Vorticity' (6)

Important observations: First of all, note that these 'field tensors/operators' are 'particle neutral' since the sum runs over all particles, signaling a shift in focus from 'particles' to the 'field' (single frequency) through which particles 'communicate' with one another. Second, the `isotropy of space' means the field can only be represented in terms of `rotational scalars'. And Third, this suggests that an understanding the 'field dynamics' of such a system may be simpler than heretofore appreciated; specifically, it depend upon the size of the set of 'rotational scalars' that one can construct, which is a relatively small number - 5 total of operator rank 4 or less. Specifically, there are 2 rank 2 tensors (L^2 and C_2), and as well 2 rank 3 tensors [C_3 and $X_3 = (LxQxL)^0$], plus 1 rank 4 tensor ($X_4 = [(LxQ)x(QxL)]^0$). Of these, 3 are group invariants: [L^2 for SO(3)] and the 2 invariants of SU(3) [C_2 and C_3] augmented by 2 special non-invariant scalars (X_3 and X_4) that split states in an IR of SU(3), but do not break SU(3) symmetry. To illustrate the simplicity and beauty of exploiting such a framework, the Hamiltonian for a 3D-HO Triaxial Rotor follows:

$$H_{(\text{triaxial rotor})} = a_1(L_1)^2 + a_2(L_2)^2 + a_3(L_3)^2 \Leftarrow (L_1)^2/2I_1 + (L_2)^2/2I_2 + (L_3)^2/2I_3 \Rightarrow b_1(L^2) + b_2(X_3) + b_3(X_4)$$

(Leschber & Draayer, https://doi.org/10.1016/0370-2693(87)90829-X)

where the $a_i = 1/[2I_i]$ are moments of inertia of a triaxial rotor along it's i-th symmetry axes, with the 1-to-1 mapping between the a's and b's and analytic results for matrix elements of X_3 and X_4 all known.

Symplectic Symmetry: Independent Rotational Scalars (3D-HO)

If H is a subgroup of G ($G \supset H$), finding the independent H scalars one that one can build from the generators of G is a well-defined group theoretical problem; namely, establishing the Integrity Basis (IB) for the ($G \supset H$) structure. In our case, G is U(3) and H is SO(3), and the IB is well-known. Also, due to the group commutation properties, all such scalars can be expressed in terms of a subset of scalars, the so-called generators of the IB. For the case of SU(3) ⊃ SO(3) the result of such an analysis, given in terms of their tensorial rank (number of generators involved), is as follows:

***Tensorial Rank of Rotational Scalars in U(3) ⊃ SU(3) Reduction**

Rank 1 Tensors: N [total number of oscillator quanta]
Rank 2 Tensors: L^2 [angular momentum squared: $(L \times L)^0$]
C_2 [2^{nd} order invariant of SU(3)]
N^2 [2^{nd} order N]
Rank 3 Tensors: C_3 [3^{rd} order invariant of SU(3)]
X_3 {3^{rd} order non-invariant* SU(3) scalar: $(LxQxL)^0$}
$(N \times L^2)$
$(N \times C_2)$
N^3 [3^{rd} order N]
Rank 4 Tensors: X_4 {4^{th} order non-invariant* SU(3) scalar: $[(LxQ)x(QxL)]^0$}
$L^2 \times L^2 = (L^2)^2$ (square of L^2)
$C_2 \times C_2 = (C_2)^2$ (square of C_2)
$L^2 \times C_2$ = Product of L^2 & C_2
$(N^2 \times L^2)$
$(N^2 \times C_2)$
$(N \times X_3)$
N^4 [4^{th} order N]

*Nuclear Physics A439 (1985) 61-85 (Draayer & Rosensteel)

*Identifying subgroup scalars [SO(3) in our case], within a larger group [U(3) for us] is best done in two steps with the first being the establishment of the so-called Integrity Basis for this group-subgroup system, from which all others can be given as polynomials of the IB set. The results given in the table to the left lists all such tensorial forms - up to and including rank 4 terms - 5 \`tensors' if powers of N are excluded; 17 if included!

[Note: If limited to no more than rank 3 tensors, the count drops to 5 \`tensors' ($X_4$ dropped) but with this the ability to reproduce stable triaxial shapes is lost. The latter suggests that our EFT friends might need to assess the importance of including $4^{th}$ order terms in their analyses to gain even better (i.e., \`ab initio ') results for next generation nuclear physics studies!]

Simple Question: How do Nucleons `talk' to one another? *

Old School: Shell Model approach: $\mathcal{H}|\ \Psi > = E|\ \Psi >$, find 'matrix solution' if this can be achieved.

- Last half of 20th Century – Robust collection of methodologies proffered but usually computationally challenged,
- Early 21st Century – Open systems (NCSM & SA-NCSM) – yields reasonable results – inclusive of 'ab inito' input,
- The dominance of deformation across nuclides uncovered within the Sp(3,R) ⊃ U(3) ⊃ SU(3) ⊃ SO(3) framework!

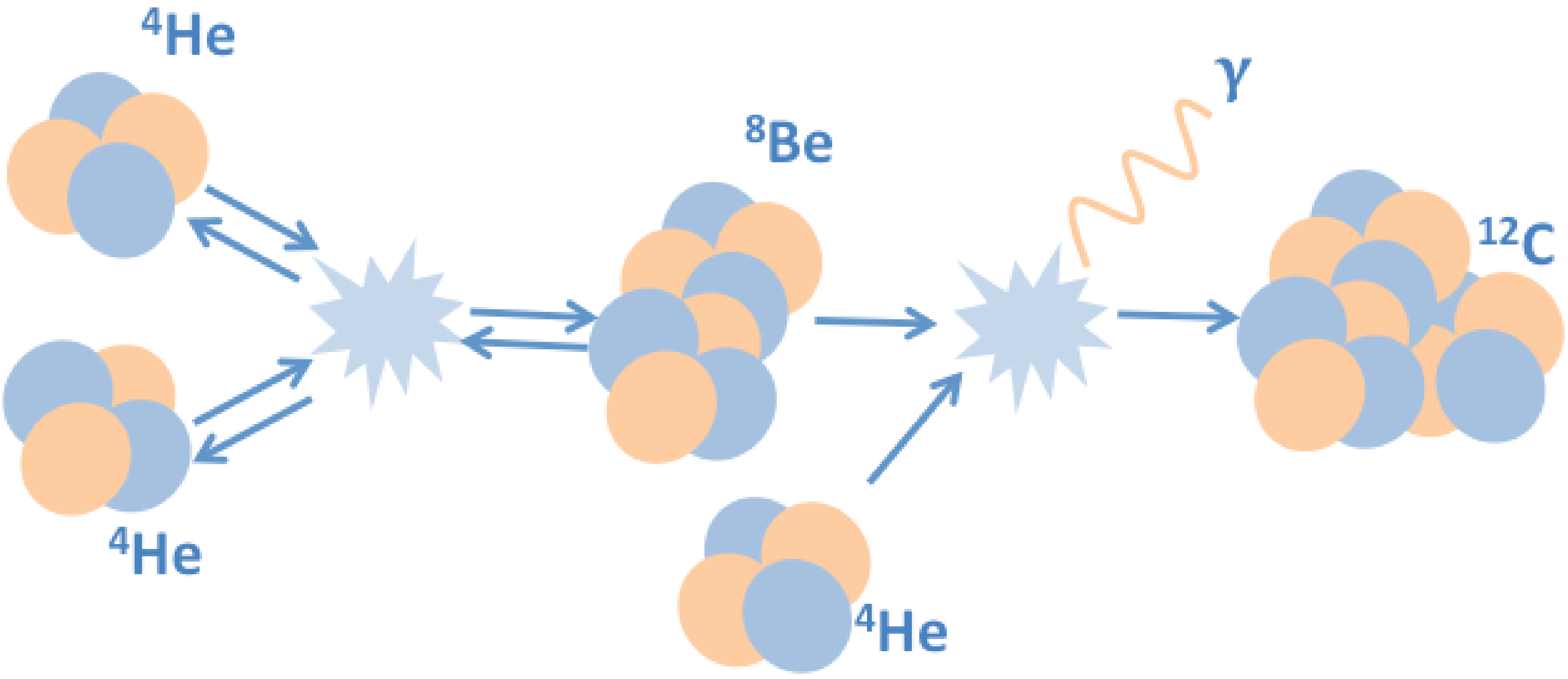
New School: Field Theory approach: $\mathcal{H}|\ \Psi > = E|\ \Psi >$, with a U(3) ⊃ SO(3) expansion of $\mathcal{H}$.

- The dominance of deformation means $\mathcal{H}$ is dominated by a very small number (5) of low-order scalar invariants,
- These low-order scalar invariants are ubiquitous, with analytic results for their matrix elements all well-known,
- This suggests (to me anyway) that an `ab initio' Chiral-EFT linkage might be simpler than previously anticipated,
- Also, with Chiral–EFT 'low-energy constants' in play, alignment with IB expansion coefficients seems to be likely,
- Not all invariants are of the Sp(3,R) type, mixing will occur (e.g., for pairing modes) so mode-mixing is required,
- The latter is a `work-in-progress', and a very necessary part of the symplectic picture - we'll keep you posted,
- Never lose sight of the fact that the goal is to `build-a-bridge' between the low-energy & medium-energy NP,
- Generally, never minimize the importance of symmetry - exposing `simplicity within complexity' is sound science!

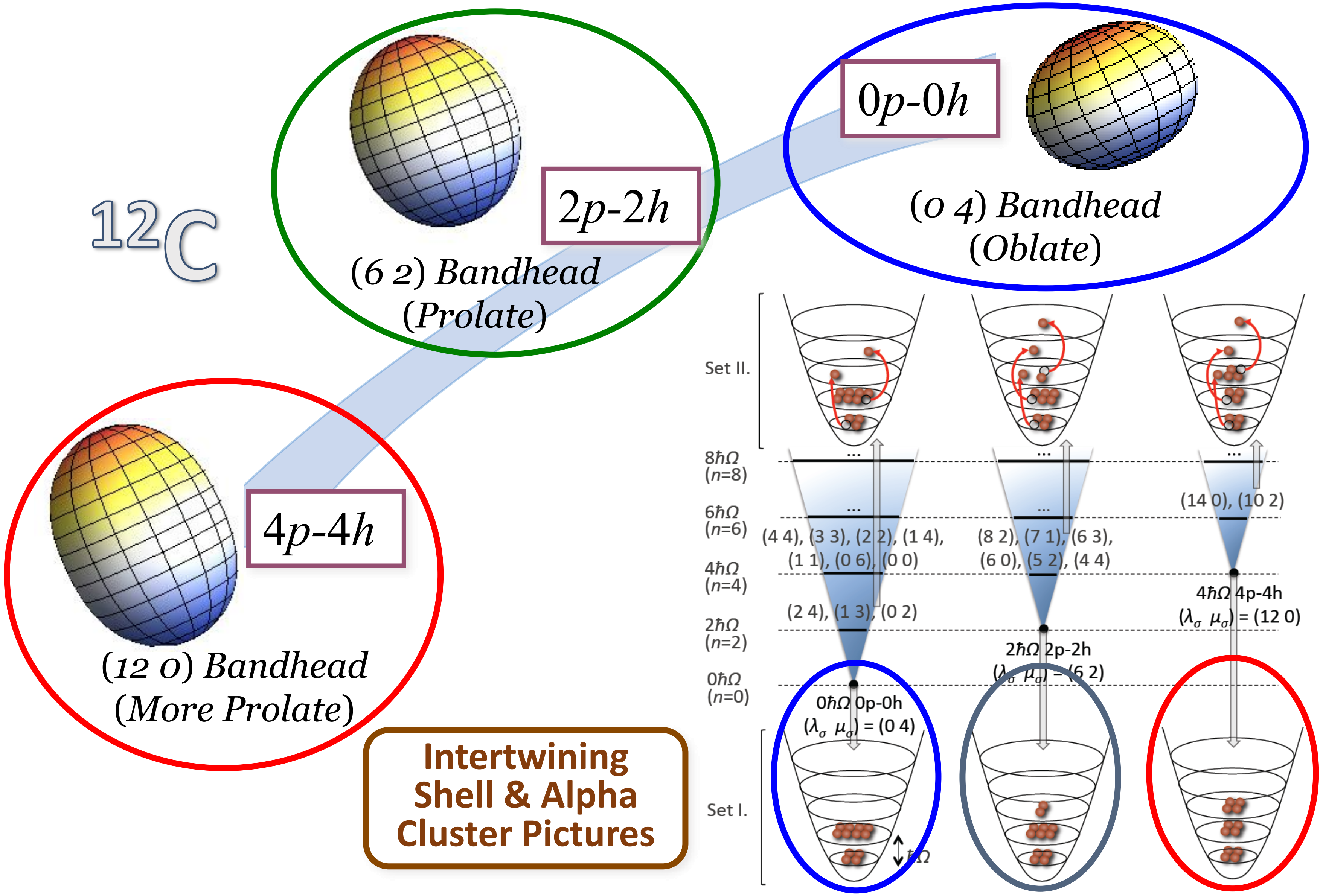
*I will now show results from some more `recent examples' applying the SA-NCSM - (Slides #10-#17)
and then I will close out with some additional remarks regarding `bridge building' - (Slides #19-#20)

Creation of ^{12}C in Hot Stars / Nucleosyntheses

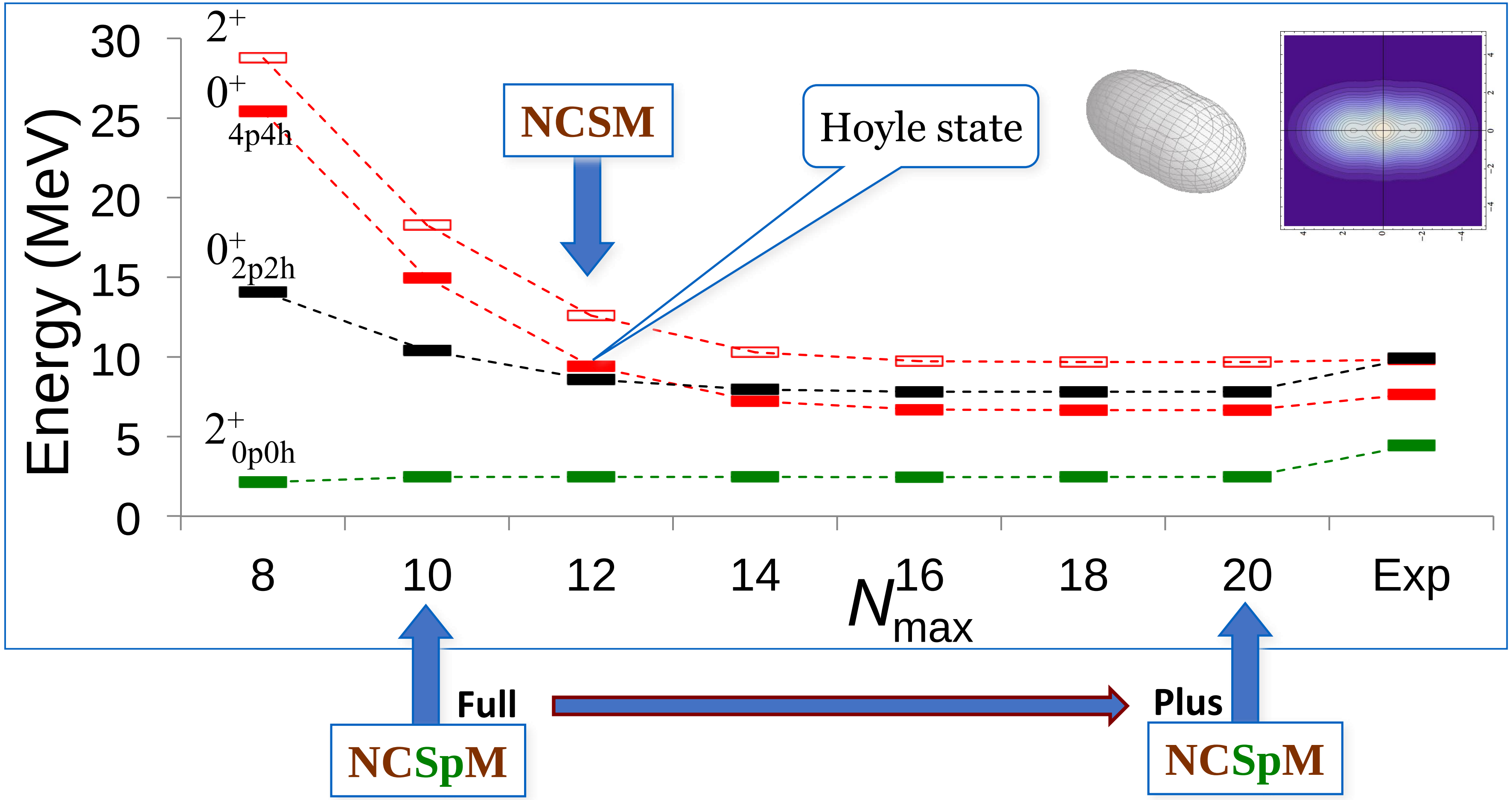
… The Elusive Hoyle State …



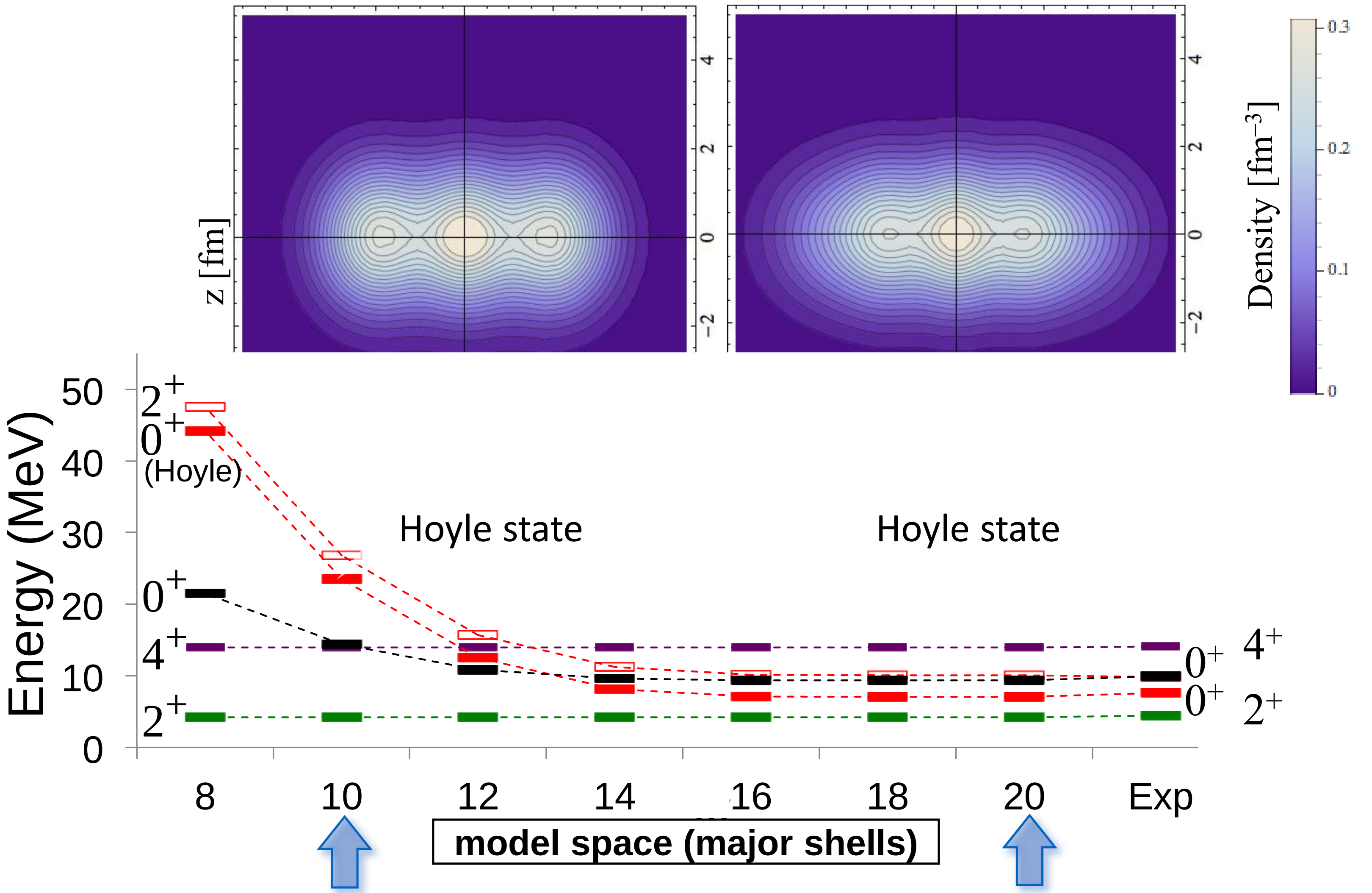
Three Primary `Slices' within a NCSpM Description



(N_{add} = Total Number of $2\hbar\omega$ Excitations above Ground State)



(Now with mixing at the band-head level turned on ... !)



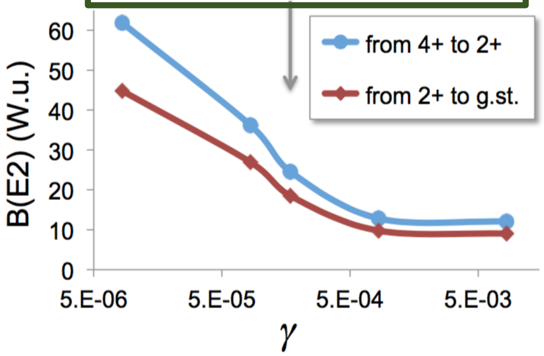
Standard ab initio NCSM

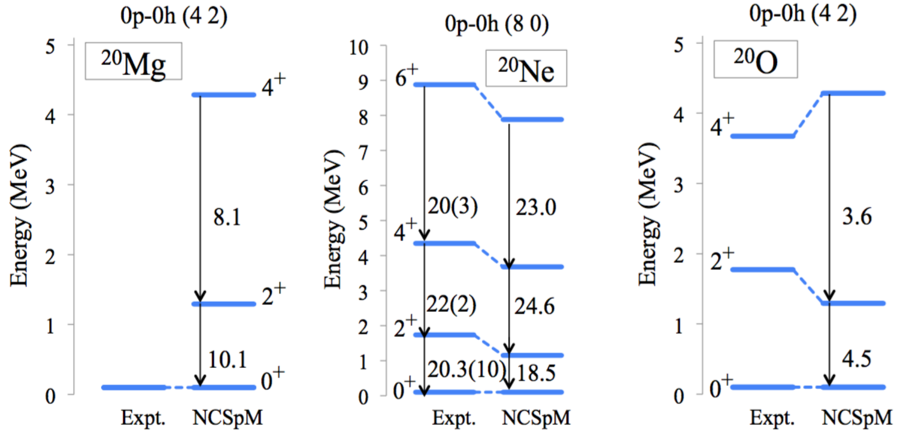
*(NCSpM & Band Mixing)

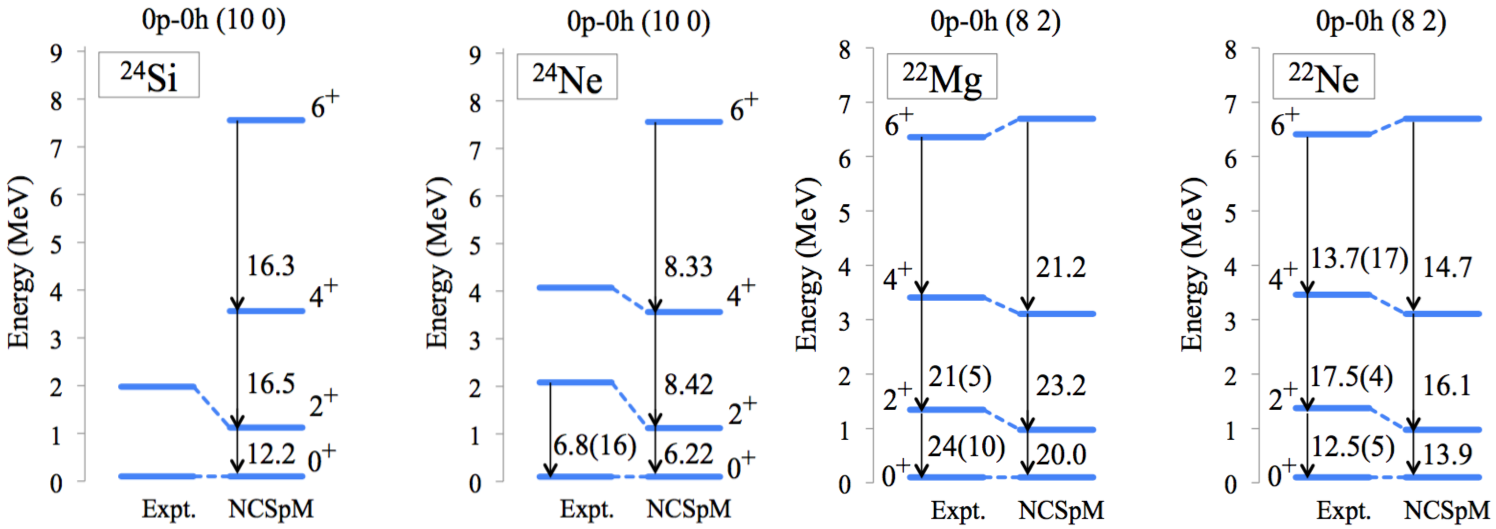
Medium Mass Nuclei (Gegory Tobin / REU Student)

Ne-20: grey arrow is gamma used

Almost "Laptop" Level Calculations!



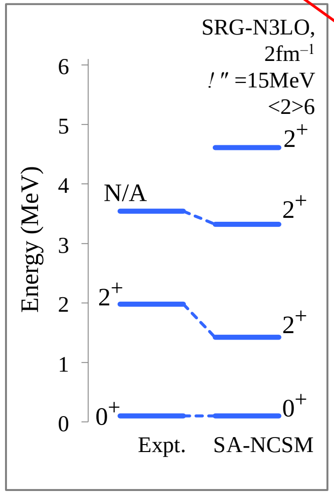




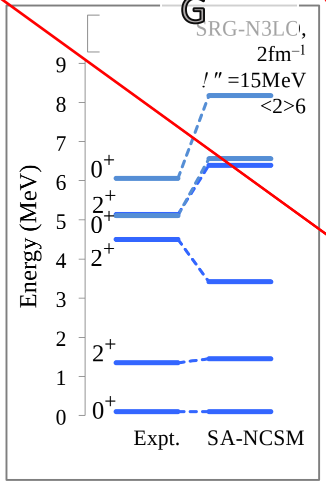
Further sd-shell Results [Robert Baker – GS – Ohio U (Athens)]

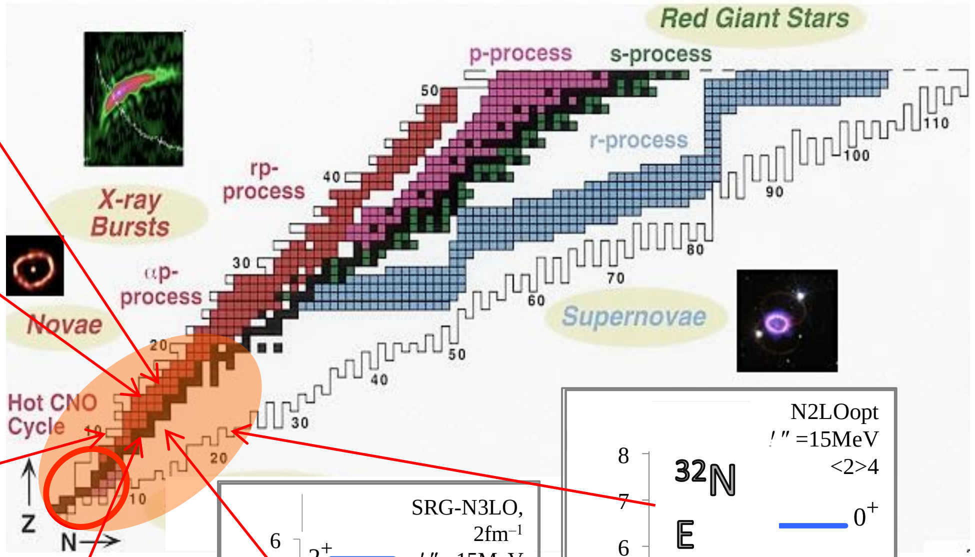
Selected (pre-thesis) Examples
(Now onto Beta Decay)

^{24}SI

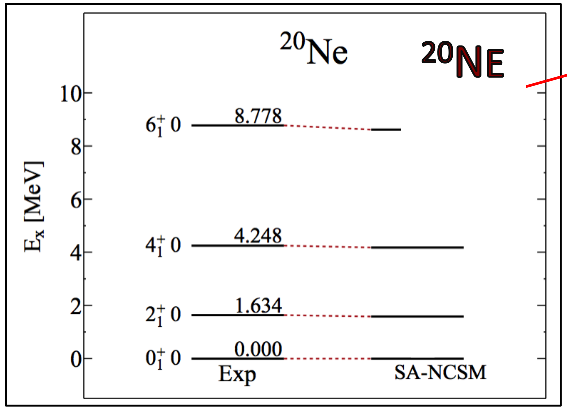


^{22}MG

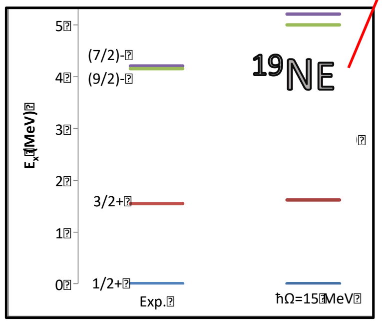




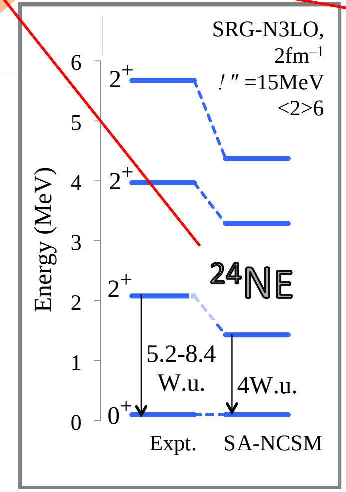
^{20}NE



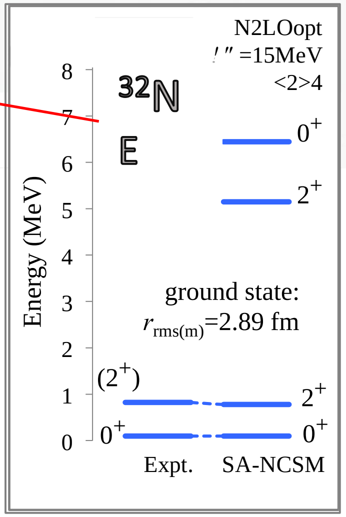
^{19}NE



^{24}NE

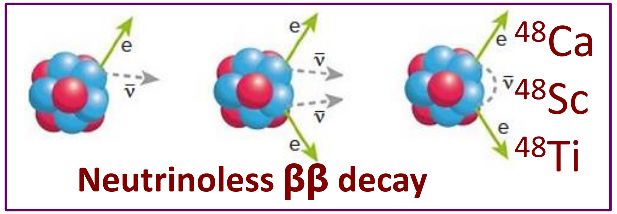


^{32}NE



Ab initio description (converged selected spaces)
($NNLO_{opt}$, ħΩ=15 MeV, 13 HO shells)

^{48}Ca



^{48}Ti

8 shells, N2LOopt

0^+

SA-NCSM (selected): 966,152

Complete model space: .... 3,162,511,819 **2^+**

SA-NCSM (selected): 3,055,554

Complete model space: 14,522,234,982

^{48}Ti, Q(2^+) [e fm^2]

Experiment .... -17.7

8 shells .... -19.3

(no effective charges)

8 shells, N2LOopt

0^+

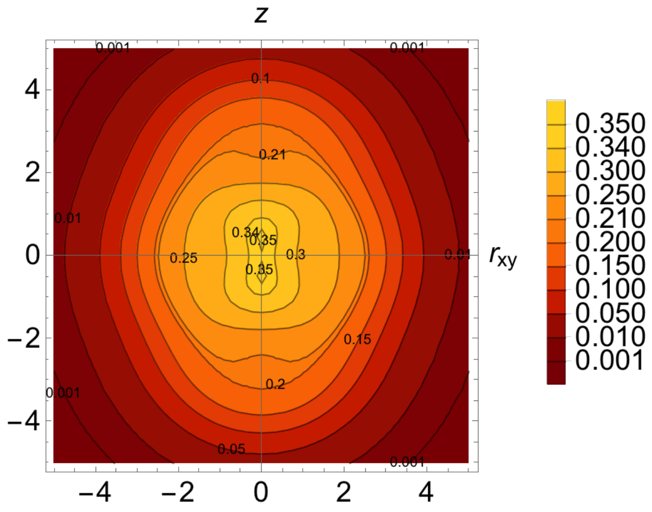
SA-NCSM (selected): 602,493

Complete model space: 24,694,678,414

2^+

SA-NCSM (selected): 1,178,834

Complete model space: 113,920,316,658



Symplectic Effective Field Theory for Nuclear Structure Studies

(David Kekejian, Thesis Research @ LSU 10/21 & Currently Post Doc, UNC)

Generic (Scalar) Field Theory — **Quantum (Scalar) Field Theory**

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) + \frac{1}{2} m^2 \varphi^2 \xrightarrow[\text{quantization}]{\text{canonical}} H = \sum_k E_k (b_k^+ b_k^- + \tfrac{1}{2})$$

$$\varphi(r,t) = \frac{1}{\sqrt{V}} \sum_k b_k^- \frac{1}{\sqrt{|2k^0|}} e^{-\iota k^\mu x_\mu} + \frac{1}{\sqrt{V}} \sum_k b_k^+ \frac{1}{\sqrt{|2k^0|}} e^{\iota k^\mu x_\mu}$$

$$\mathcal{L}^{(n)} = \frac{\alpha^n}{2(n+1)!} (\partial_\mu \varphi \partial^\mu \varphi^* + m^2 \varphi \varphi^*)^{n+1}$$

$$\mathcal{H}^{(n)} = \frac{\alpha^n}{2(n+1)!} (\dot{\varphi}\dot{\varphi}^* - \varphi' \cdot \varphi'^* + m^2 \varphi\varphi^*)^n ((2n+1)\dot{\varphi}\dot{\varphi}^* + \varphi' \cdot \varphi'^* - m^2 \varphi\varphi^*)$$

$$H^{(n)} \sim (\frac{\alpha}{V}\hbar\Omega)^n \times (g^2 Q \cdot Q)^n, (K \cdot K)^n, (gQ \cdot K)^n, (gK \cdot Q)^n$$

Where: $\frac{\alpha}{V}\hbar\Omega = \frac{\beta^2}{8N_\sigma}$ $\quad g = \frac{m^2}{\hbar^2\omega^2}$

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Kekejian (Thesis), Draayer & Mokeev, Roberts

Conclusion: Symplectic Symmetry emerges naturally from a quantum effective Field Theory!

Back to the Future: ~~Recent Examples and B~~ridge Building

Nuclear (LE-QED) Discoveries! | **Particle (HE-QCD) Challenges?**

20th Century Subatomic Physics

Great Minds and Creative Models
Numerous Prestigious Awards Won
Major New Facilities Commissioned
Simplicity within Complexity Good
Analytic Solutions Well Regarded

- 100 - Years | **- 100 - Years**

Physics Changed Technology
1990s – HPC/ERA – 2010s
Technology Changed Physics

21st Century Subatomic Physics

Bigger Computers! -> Better Results?

Symmetries Important

Partial Symmetries Expose Coherent Features

Lunch (~5 years back) at JLab: `Is the nucleon deformed (round)?'

Nucleons Communicate with one another via Scalar Fields!

Our job is to learn the grammar of that new language, perhaps in a `nucleon speakeasy' such as this meeting?

In any case, thanks to the organizers for their efforts in pulling this meeting together and letting me `speakeasy' (of perhaps `speakhard' for some … sorry about details!

A special thank you to Craig Roberts and Volker Mokeev who kindly took our earliest ramblings on this seriously and have continued to proffer guidance … sharing in a vision that there is value in seeking and finding simplicity within complexity!

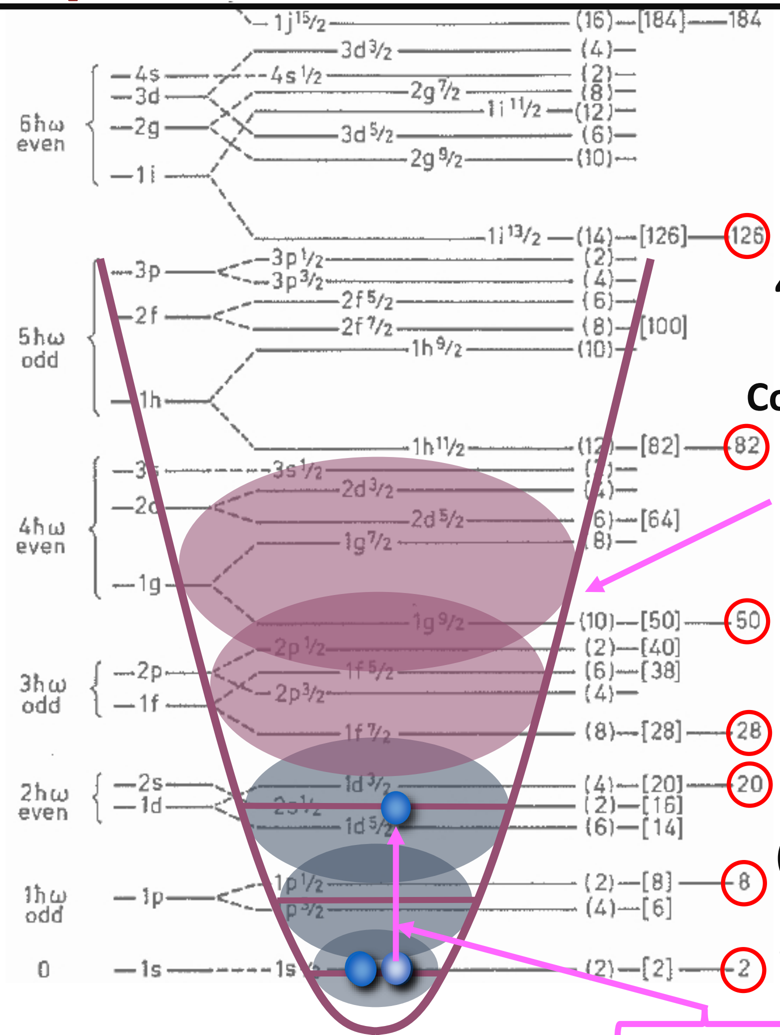
Thanks to One & All!
From the extended (non-compact)
LSU Team

- Exploit the KISS (Keep It Simple Stupid) Principle! –

H = T(kinetic Energy) + V(potential Energy)

V(potential Energy) = V(rotor: ~Q·Q) & V(pairs: ~N_s + ~N_d + …) + V(cross-couplings)

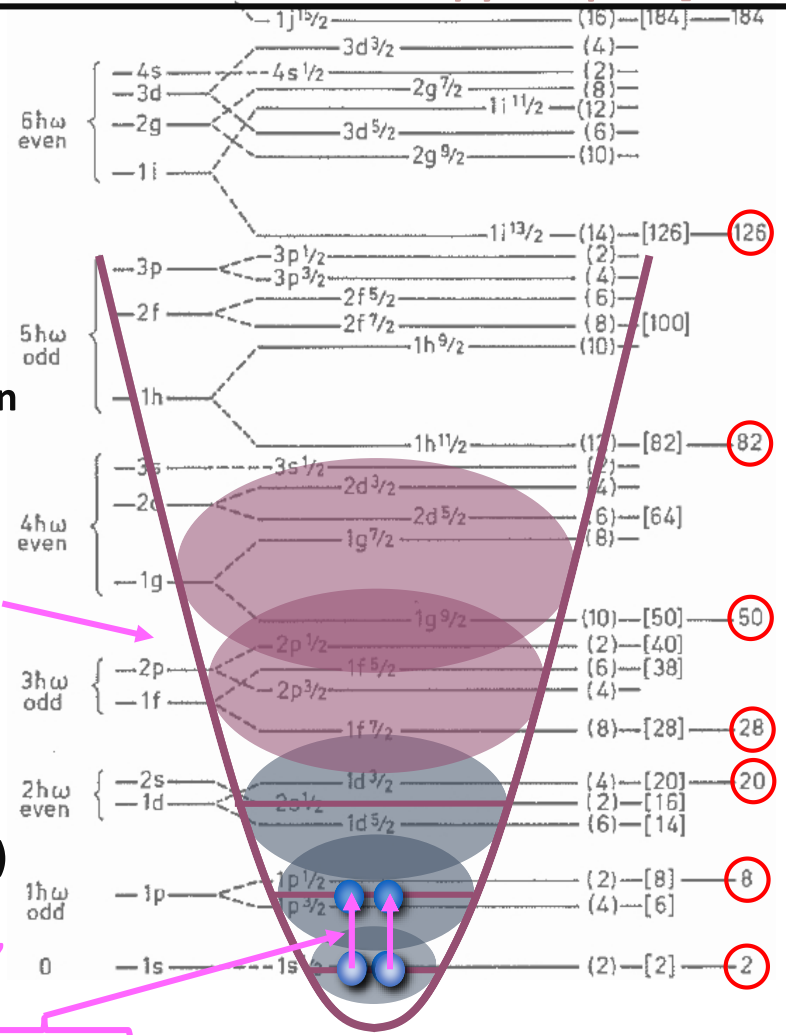
[Recall: Terms in "V" terms must be Rotational Scalars due to Isotropy of Space]



The Structure of the Deuteron really "Matters"

Competition to Rotate or Vibrate or Hybrid?

Pauli (Principle) also "Matters"



+ Parity (s, sd, sdg, …)

1p1h $2\hbar\omega$

2p2h $1\hbar\omega$

- Parity (p, pf, pfh, …)