Nucleon self-energy including two-loop contributions

Nils Dirk Conrad
Ashot Gasparyan, Jambul Gegelia, Evgeny Epelbaum

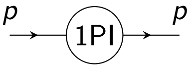
Institute of Theoretical Physics II - Hadron and Particle Physics
Ruhr University Bochum, Germany

November 10, 2022

Baryons 2022, 7-11 November, Seville

Contents

Nucleon self-energy – different methods

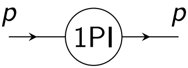
$\Sigma(p) =$ 

- Lattice Calculation (QCD) $\rightarrow$ best for higher quark masses
- EFTs: study low-energy QCD and its dependence on the pion mass M^2 around the chiral limit
  - HBChPT
    $\rightarrow$ non-relativistic (convergence problems)
  - IR (ChPT)
    $\rightarrow$ taking only the infrared-singular part (un-physical cuts)
  - EOMS (ChPT)
    $\rightarrow$ preserves proper analytic structures of the Green's functions

Nucleon self-energy $\rightarrow$ physical nucleon mass

The correction to the **nucleon mass** is given by the **self energy** Σ.

$$m_N = m^{\text{Bare}} + \Sigma(p^2 = m_N^2) = m^{\text{Bare}} +$$



$$\Big|_{p^2=m_N^2}$$

Calculate $\Sigma(p)$ up to $\mathcal{O}(q^6)$ $\rightarrow$ include two-loop diagrams

- [McGovern and Birse, Phys. Lett. B 446, 1999] in HBChPT: order five contributions can be absorbed in πN coupling constant
- [Schindler et al., Nuc.Phys.A 803.1, 2008] with IR and $1/m$ expansion:

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4 + k_5 M^5 \ln\frac{M}{\mu}$$
$$+ k_6 M^5 + k_7 M^6 \ln\frac{M}{\mu} + k_8 M^6 \ln^2\frac{M}{\mu} + k_9 M^6$$

with $k_i(m, g_A, F, l_3, l_4, c_{1-4}, d_{16}, d_{18}, e_i, \hat{g}_1)$

- With EOMS [Fuchs et al. Phys.Rev.D 68, 056005, 2003] we successfully renormalized Σ on-shell

Chiral Perturbation Theory

- Quantum Chromodynamics (QCD) $\hat{=}$ strong interaction
- Chiral Perturbation Theory (ChPT) $=$ EFT for low energies

Chiral Lagrangian up to chiral order $\mathcal{O}(q^4)$ in SU(2) [Fettes et al., APhy 283.2, 2000]:

$$\mathcal{L} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)}$$

$$\mathcal{L}_\pi^{(2)} = -\frac{1}{2} M^2 \vec{\pi} \cdot \vec{\pi} + \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{8\alpha - 1}{8F^2} M^2 \left(\vec{\pi} \cdot \vec{\pi}\right)^2 + \cdots + O(\pi^6)$$

$$\mathcal{L}_\pi^{(4)} = -\frac{(l_3 + l_4) M^4}{F^2} \left(\vec{\pi} \cdot \vec{\pi}\right) + \cdots + O(\pi^3)$$

$$\mathcal{L}_{\pi N}^{(1)} = -\bar{\Psi} m \Psi + i \bar{\Psi} \partial\!\!\!/ \Psi + \frac{g_A}{2F} \bar{\Psi} \gamma_5 \vec{\tau} \cdot \partial\!\!\!/ \vec{\pi} \Psi + \cdots + O(\pi^5)$$

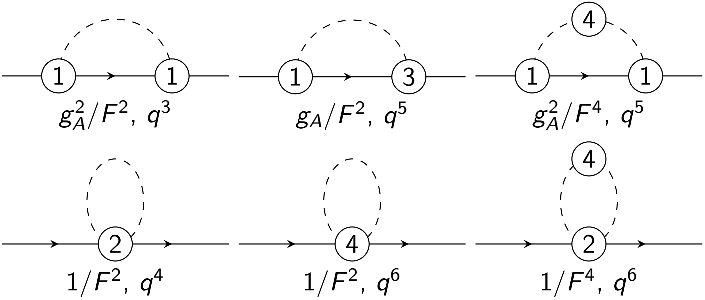
$$l_i \in \mathcal{L}_\pi^{(4)}, \quad c_i \in \mathcal{L}_{\pi N}^{(2)}, \quad d_i \in \mathcal{L}_{\pi N}^{(3)}, \quad e_i \in \mathcal{L}_{\pi N}^{(4)}$$

with $M/m < 1$ $(M = \mathcal{O}(q^1),\ m = \mathcal{O}(\Lambda),\ q/\Lambda < 1)$

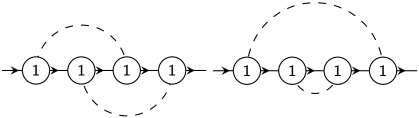
Diagrams - tree level and one-loop

Tree level

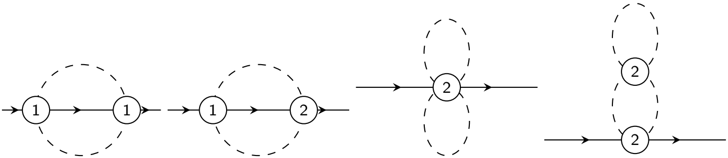
One-loop



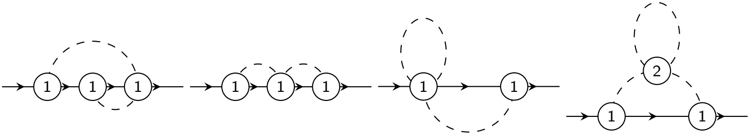
Diagrams - two-loop



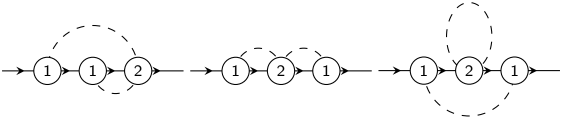
$\frac{g_A^4}{F^4}$, q^5



$\frac{c_i^{0-1}}{F^4}$, q^{5-6}



$\frac{g_A^2}{F^4}$, q^5

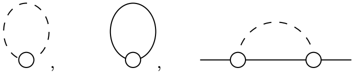


$c_i \frac{g_A^2}{F^4}$, q^6

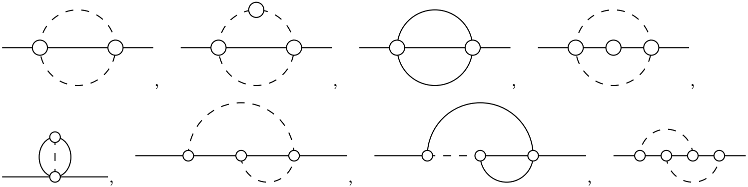
Reduction to master integrals

1. Feynman rules give mathematical expressions for the diagrams
2. Add suitable zeros to reduce the tensor rank of the integrals
3. Reduce to scalar integrals in $d + \ldots$ dimensions [Tarasov, NPhyB 502.1, 1996].
4. Express with a set of "Master Integrals" (TARCER [Mertig and Scharf, CPhyC 111.1, 1998])

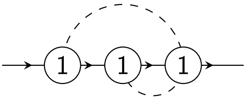
$T_{\pi}^{(1)}$, $T_{N}^{(1)}$, $T_{\pi N}^{(1)}$:



$T_{\mathrm{I}}^{(2)}$ - $T_{\mathrm{VIII}}^{(2)}$:



Reduction to master integrals: two-loop (d)+(e)



$$= -i\Sigma_g^{(2)} = -\frac{ig_A^2\left((2d-3)m^2-(d-2)M^2\right)}{2(3d-4)F^4 m}(T_\pi^{(1)})^2$$

$$-\frac{3ig_A^2}{8F^4m^3}\left(8m^4-3m^2(p^2-m^2)+4m^3(\not{p}-m)-3m(\not{p}-m)(p^2-m^2)+3(p^2-m^2)^2\right)(T_N^{(1)})^2$$

$$+\frac{ig_A^2\left(4(2d-3)m^2-(d-2)M^2\right)+(\dots)}{2(3d-4)F^4m}T_N^{(1)}T_\pi^{(1)}-\frac{3ig_A^2mM^2+(\dots)}{F^4}T_N^{(1)}T_{\pi N}^{(1)}$$

$$-\frac{ig_A^2\left(\left(8d^2-32d+30\right)m^4+\left(-8d^2+33d-32\right)m^2M^2-\left(d^2-5d+6\right)M^4\right)+(\dots)}{(d-2)(3d-4)F^4m}T_{\text{I}}^{(2)}$$

$$+\frac{4ig_A^2M^2\left(m^2-M^2\right)\left((4d-6)m^2+(d-2)M^2\right)+(\dots)}{(d-2)(3d-4)F^4m}T_{\text{II}}^{(2)}$$

$$-\frac{ig_A^2m(p^2-m^2)(\dots+\dots)}{2(d-2)(3d-4)F^4}T_{\text{IV}}^{(2)}-\frac{3ig_A^2mM^2+(\dots)}{F^4}T_{\text{V}}^{(2)}-\frac{3ig_A^2mM^4+(\dots)}{F^4}T_{\text{VI}}^{(2)}$$

Strategy of regions

[Beneke and Smirnov, Nucl.Phys.B 522 321-344, 1998]

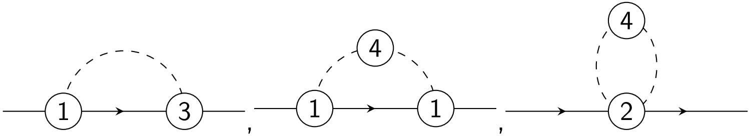
- $l_i \sim q$ or $l_i \gg q$
- Taylor series in small scale T_q

$$T_q\left[\int f(l_1,l_2,q)\mathrm{d}l_1\mathrm{d}l_2\right] = \int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\sim q\\ l_2\sim q}}\mathrm{d}l_1\mathrm{d}l_2 + \int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\gg q\\ l_2\gg q}}\mathrm{d}l_1\mathrm{d}l_2$$
$$+\left(\int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\sim q\\ l_2\gg q}}\mathrm{d}l_1\mathrm{d}l_2 + \int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\gg q\\ l_2\sim q}}\mathrm{d}l_1\mathrm{d}l_2\right) = S + R + M.$$

- Solution as chiral expansion in d-dimension ($\rightarrow$ massless propagators)
- IRR part
  - is analytic in M^2 (no M^3 or $\ln(M/m)$)
  - contain PCB terms

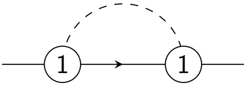
Renormalization - bare parameter shifts

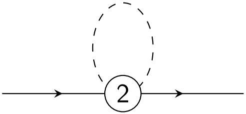
- Remove diagrams



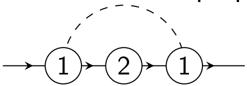
$$g_A^{\mathrm{B}} \to g_A^{\mathrm{B}} - 2(M^{\mathrm{B}})^2(2d_{16}^{\mathrm{B}} - d_{18}^{\mathrm{B}})$$

$$(M^{\mathrm{B}})^2 \to (M^{\mathrm{B}})^2 - \frac{2l_3^{\mathrm{B}}(M^{\mathrm{B}})^4}{(F^{\mathrm{B}})^2} \text{ and } F^{\mathrm{B}} \to F^{\mathrm{B}} - \frac{l_4^{\mathrm{B}}(M^{\mathrm{B}})^2}{F^{\mathrm{B}}}$$

in diagrams 

and 

- Include dressed propagator: $\tilde{m} = m + \Sigma_c = m - 4c_1 M^2 - 2\hat{e}M^4 + 2\hat{g}M^6$



$+ \ldots$

Renormalization - PCB and divergent

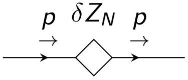
- EOMS: subtract only parts containing div. and PCB terms
  → by subtracting the LO (analytic) infrared regular contribution
- EOMS shifts [Siemens et al., PhyRevC 94, 014620, 2016]

$$\tilde{m}^B = \tilde{m}^R + \hbar\delta\tilde{m}^{(1)} + \hbar^2\delta\tilde{m}^{(2)}$$

$$\delta\tilde{m}^{(1)} = \frac{3i(g_A^R)^2\tilde{m}^R}{2(F^R)^2}T_N + \frac{3i(g_A^R)^2\tilde{m}^R(M^R)^2}{2(F^R)^2}T_{\pi N}^{\text{div+IRR}}(p^2 = m_N^2)$$
$$- \frac{3c_2^R}{128\pi^2(F^R)^2}(M^R)^4 - \frac{i(M^R)^2(24c_1^R - 3(c_2^R + 4c_3^R))}{4(F^R)^2}T_\pi^{\text{div}}$$

$$\delta\tilde{m}^{(2)} = \text{divergent and PCB terms that are analytic in } M^2$$

- Other corrections contributing: δM, δF, δg_A, δc_i, δe_i, δZ_N, no δZ_π
- From the free (and direct contact) Lagrangian the nucleon Z-factor gives



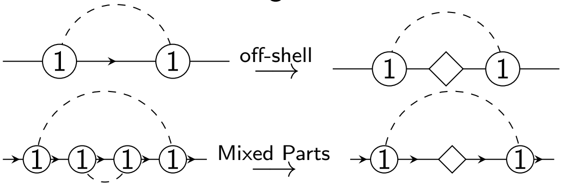
$$\hat{=} \quad i\delta Z_N\left(\not{p} - \tilde{m}^{\mathrm{R}}\right)$$

Renormalization – off-shell contributions

- choose $\tilde{m}^{\text{Bare}} = m_0 + \Sigma_c^R + \delta\tilde{m}$
  $\Rightarrow m_N = m_0 + \Sigma_c^R + \delta\tilde{m} + \Sigma_I$
- $\sqrt{p^2} = m_N = m_0 + \Sigma_c^R - \dfrac{3g_A^2 M^3}{32\pi F^2} + \mathcal{O}(M^4)\,,$
  Additional terms from $m_N - \tilde{m}^R \sim \mathcal{O}(M^3)$
- IRS

$$\frac{1}{(\not{p}+\not{l}) - \tilde{m} + i\epsilon} = \frac{1}{\not{l} + i\epsilon} + \underbrace{\frac{-1}{\left(\not{l} + i\epsilon\right)^2}\left(\not{p} - \tilde{m}\right)}_{\mathcal{O}(q^2)} + \mathcal{O}\left(\left(\not{p} - \tilde{m}\right)^2\right)$$

- Additional terms of higher order in $\hbar$



Renormalized nucleon self-energy

$$m_N = m_0 + \Sigma_c^R + \Sigma_l^R$$

Sketch of the renormalized loop contributions to the self-energy (as chiral expansion):

$$\Sigma_a^1 = -\frac{3g_A^2 M^3}{32\pi F^2} - \frac{3g_A^2 M^4}{64\pi^2 F^2 m_0} - \frac{3g_A^2 M^4 \ln\left(\frac{M}{m_0}\right)}{32\pi^2 F^2 m_0} + \frac{3g_A^2 M^5}{256\pi F^2 m_0^2} - \frac{9c_1 g_A^2 M^6}{16\pi^2 F^2 m_0^2} - \frac{3c_1 g_A^2 M^6 \ln\left(\frac{M}{m_0}\right)}{8\pi^2 F^2 m_0^2} + \frac{g_A^2 M^6}{128\pi^2 F^2 m_0^3} + \frac{27 g_A^4 M^6 (8c_1 m_0 - 3)\ln\left(\frac{M}{m_0}\right)}{2048\pi^4 F^4 m_0}$$

$$\Sigma_d^1 + \Sigma_e^1 = \dots$$

$$\Sigma_a^2 + \Sigma_b^2 = \dots$$

$$\Sigma_c^2 = \dots$$

$$\Sigma_d^2 + \Sigma_e^2 + \Sigma_f^2 = \dots$$

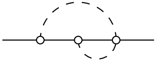
$$\Sigma_g^2 + \Sigma_h^2 + \Sigma_i^2 + \Sigma_j^2 = -\frac{21 g_A^2 M^5 \ln\left(\frac{M}{m_0}\right)}{512\pi^3 F^4} - \frac{3g_A^2 M^5\left(12\ln\left(\frac{M}{m_0}\right) - 1 + 3\ln(4)\right)}{2048\pi^3 F^4} + \frac{51 g_A^2 M^5}{1024\pi^3 F^4} - \frac{21 g_A^2 M^5 \ln(2)}{512\pi^3 F^4} - \frac{31 g_A^2 M^6}{2048\pi^2 F^4 m_0} + \frac{67 g_A^2 M^6}{768\pi^4 F^4 m_0} - \frac{9 g_A^2 M^6 \ln^2\left(\frac{M}{m_0}\right)}{512\pi^4 F^4 m_0} - \frac{g_A^2 M^6\left(48\ln^2\left(\frac{M}{m_0}\right) + 36\ln\left(\frac{M}{m_0}\right) + \pi^2 + 9\right)}{2048\pi^4 F^4 m_0} + \frac{3g_A^2 M^6 \ln\left(\frac{M}{m_0}\right)}{512\pi^4 F^4 m_0}$$

$$\Sigma_k^2 + \Sigma_l^2 + \Sigma_m^2 = \dots$$

Numeric

SecDec [Binoth and Heinrich, Nucl. Phys. B 585, 741-759, 2000]

pySecDec [Borowka et al., Comput. Phys. Commun. 222, 313-326, 2018]

- Numerical calculation of MIs via Sector Decomposition
- Example: $T_{\mathrm{VI}}^{(2)} =$ 

$$= \int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \int \frac{\mathrm{d}^d l_2}{(2\pi)^d} \left\{ \left[l_1^2 - M^2 \right] \left[l_2^2 - \tilde{m}^2 \right] \left[(l_2 - p)^2 - M^2 \right] \left[(l_1 - l_2)^2 - \tilde{m}^2 \right] \right\}^{-1}$$

  - numbers: $\sqrt{p^2} = 910,\ m = 900,\ M = 150$
  - result (with $d = 4 - 2\varepsilon$):

$$\frac{1}{\varepsilon^2}: \quad 0.50000 + 0i \pm (0.00003 + 0.00003i)$$

$$\frac{1}{\varepsilon}: \quad -12.1009 + 0i \pm (0.0004 + 0.0010i)$$

$$\frac{1}{\varepsilon^0}: \quad 146.179 + 0i \pm (0.007 + 0.012i)$$

  - time: 3 minutes for uncertainty/result $\leq 10^{-4}$
  - ≈ 5 h for uncertainty/result $\leq 10^{-5}$

Summary and Outlook

Summary

- Expressed the self-energy in terms of master-integrals (including IRR)
- Applied Strategy of regions for two-loop calculation
- Confirmed EOMS corrections
- Computed $m_N(M_\pi)$
- Numerical computation of all MIs

Outlook:

- Finish analysis of the off-shell terms
- Derive the numerical result (subtracting finite PCB parts)
- Comparison with HB, IR and Lattice

Appendix: Strategy of regions

For $l_i \sim q$ substituting $l_i^\nu \to M q_i^\nu$

$$\frac{1}{(2\pi)^{2d}} M^{2d-2\alpha-2\beta-\gamma} \int\int \mathrm{d}^d q_1 \mathrm{d}^d q_2 \Big\{ [q_1 \cdot q_1 - 1]^\alpha [q_2 \cdot q_2 - 1]^\beta \times \left[2\,(p) \cdot (q_1 + q_2) + (q_1 + q_2) \cdot (q_1 + q_2)\, M + \frac{p \cdot p - m^2}{M}\right]^\gamma \Big\}^{-1}$$

Appendix: renormalized nucleon self-energy (on-shell)

With $m^R = m_N - \Sigma_c^R$ we get for
$\Sigma(p^2 = m_N^2) + \delta m$ as $1/m$ expansion (in EOMS renormalization):

$$
\frac{3c_1 M^4 \ln(M/m_N)}{4\pi^2 F^2} - \frac{3c_2 M^4 \ln(M/m_N)}{32\pi^2 F^2} - \frac{3c_3 M^4 \ln(M/m_N)}{8\pi^2 F^2}
$$

$$
+\frac{M^6}{32\pi^2 F^2}\Big\{ -12\ln(M/m_N)\ \text{Sum of } e_i + \ \text{Sum of } e_i \Big\}
$$

$$
-\frac{3g_A^2 M^3}{32\pi F^2} + \frac{3g_A^2 M^4}{32\pi^2 F^2 m_N} - \frac{3g_A^2 M^4 \ln(M/m_N)}{32\pi^2 F^2 m_N} + \frac{3g_A^2 M^5}{256\pi F^2 m_N^2} - \frac{g_A^2 M^6}{128\pi^2 F^2 m_N^3}
$$

$$
-\frac{M^6}{12288\pi^4 F^4 m_N}\Big\{ -144 m_N(6c_1 - c_2 - 4c_3)\ln^2(M/m_N) - 144 m_N(2c_1 - c_3)\ln(M/m_N)
$$

$$
-24\left(6+\pi^2\right)c_1 m_N + 18c_2 m_N + 3\pi^2 c_2 m_N + 72 c_3 m_N + 12\pi^2 c_3 m_N + 32 c_4 m_N + 60\pi^2 c_4 m_N - 30\Big\}
$$

$$
-\frac{9g_A^2 M^5 \ln(M/m_N)}{1024\pi^3 F^4} - \frac{g_A^2 M^6}{64\pi^2 F^4 m_N} + \frac{9g_A^2 M^6}{256\pi^4 F^4 m_N} - \frac{3g_A^2 M^6 \ln^2(M/m_N)}{256\pi^4 F^4 m_N} - \frac{5g_A^2 M^6 \ln(M/m_N)}{1024\pi^4 F^4 m_N}
$$

$$
+\frac{g_A^2 M^6}{18432\pi^4 F^4}\Big\{3\pi^2(36c_1 + 5c_2 + 119c_3 - 226c_4)
$$

$$
+162\ln(M/m_N)(3(8c_1 - c_2 - 4c_3)\ln(M/m_N)) + 2592c_1 - 438c_2 - 398c_3 + 2140c_4\Big\}
$$

$$
+\frac{21g_A^4 M^5 \ln(M/m_N)}{1024\pi^3 F^4} - \frac{g_A^4 M^5}{128\pi^3 F^4} - \frac{g_A^4 M^6\left(2592\ln^2(M/m_N) + 864\ln(M/m_N) + 239\left(8+3\pi^2\right)\right)}{98304\pi^4 F^4 m_N}
$$