Proton GPDs from Lattice QCD: Fast and Accurate

Shohini Bhattacharya *et al.*, arXiv:2209.05373

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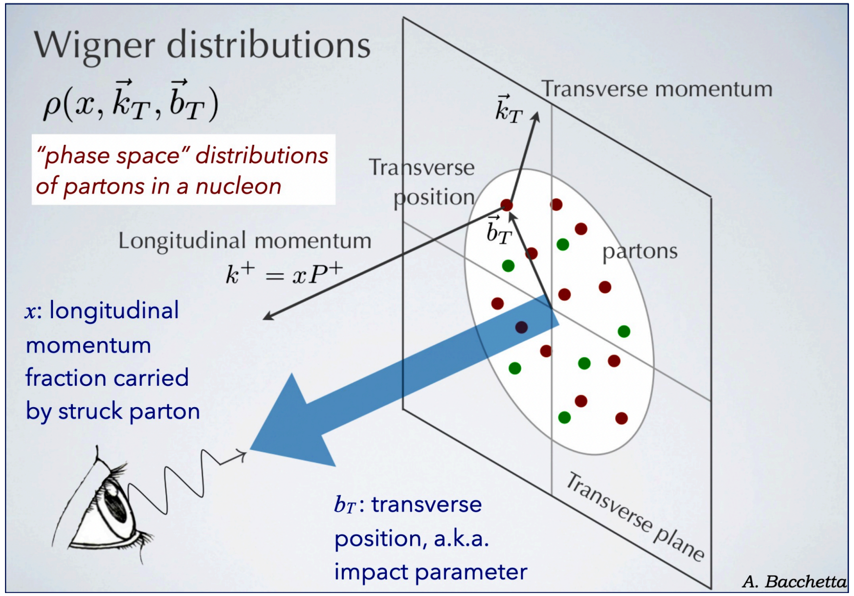
the goal

GPDs from first-principle lattice QCD

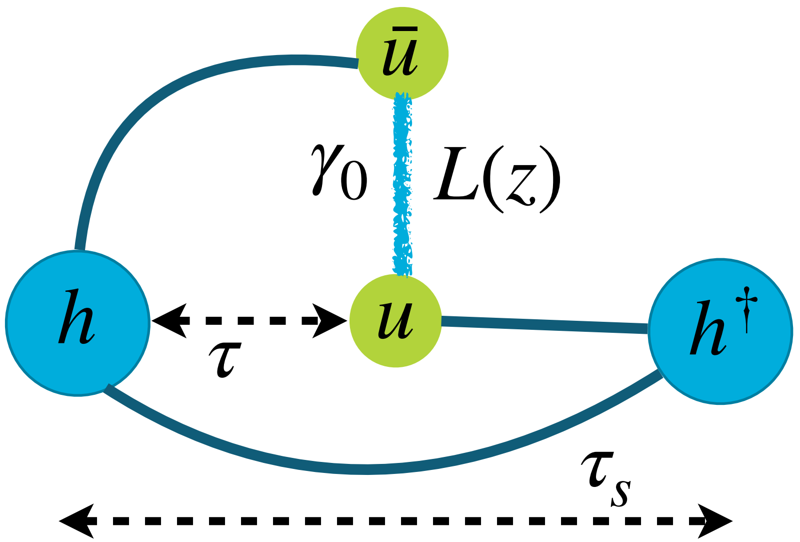
- x dependence
- t-dependence



the way it was …

Huey-Wen Lin, Tuesday

momenta transfer: $P_z^s - \frac{\mathbf{\Delta}^s}{2}, \pm S_x$



$P_z^s + \frac{\mathbf{\Delta}^s}{2}, \pm S_x$ $\equiv F_0^s$

symmetric

$\mathscr{H}_0^s, \mathscr{E}_0^s$: pseudo-/quasi-GPD

$$F_0^s = \bar{u}\left[\gamma_0 \mathscr{H}_0^s + \frac{i\sigma^{0\mu}\Delta_\mu^s}{2m}\mathscr{E}_0^s\right]u$$

$\mathscr{H}_0^s, \mathscr{E}_0^s$ + pQCD matching + $z^2 \to 0 / P_z \to \infty$

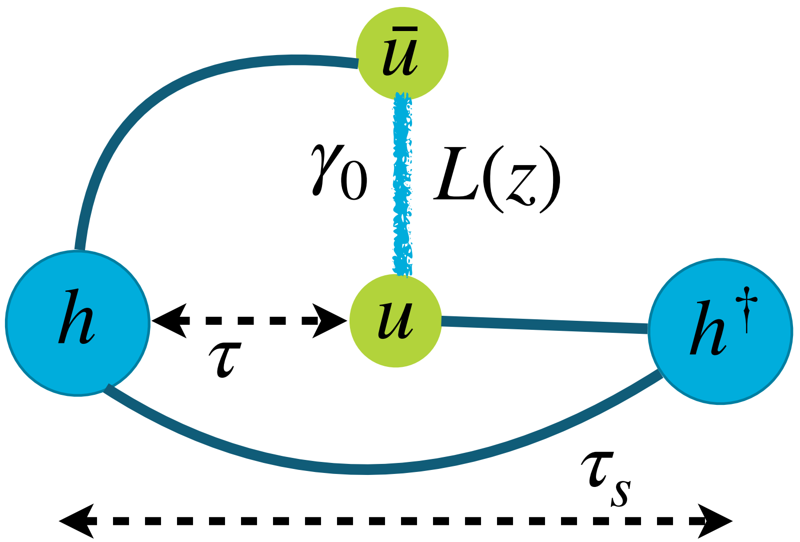
H, E: light-cone GPD

N / q	U	L	T
U	H		E_T
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	H_T $\tilde{H}_T$

the way it was

Huey-Wen Lin, Tuesday

momenta transfer: $P_z^s - \frac{\mathbf{\Delta}^s}{2}, \pm S_x$



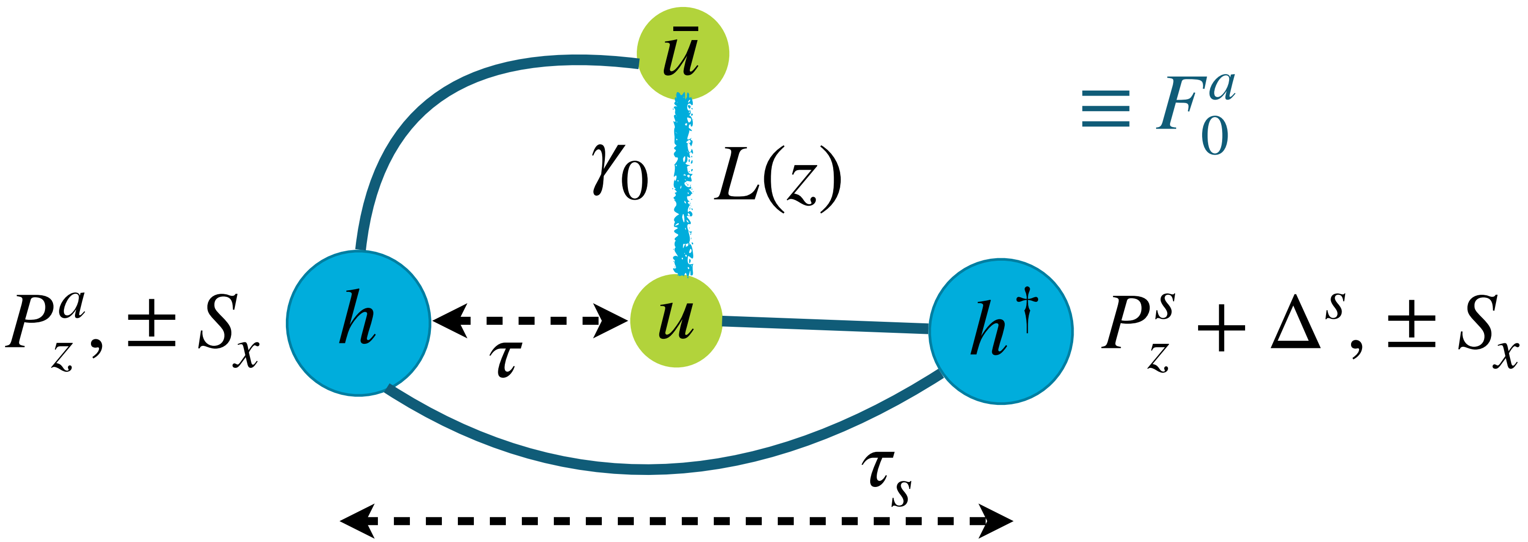
$P_z^s + \frac{\mathbf{\Delta}^s}{2}, \pm S_x$

$\equiv F_0^s$

symmetric

- need a separate calculation for each $\Delta^2 = -t$
- each calculation is $2\times$ costlier than asymmetric momenta transfer

the way we wanted …



momenta transfer:
asymmetric

$\mathscr{H}_0^a, \mathscr{E}_0^a$: pseudo-/quasi-GPD

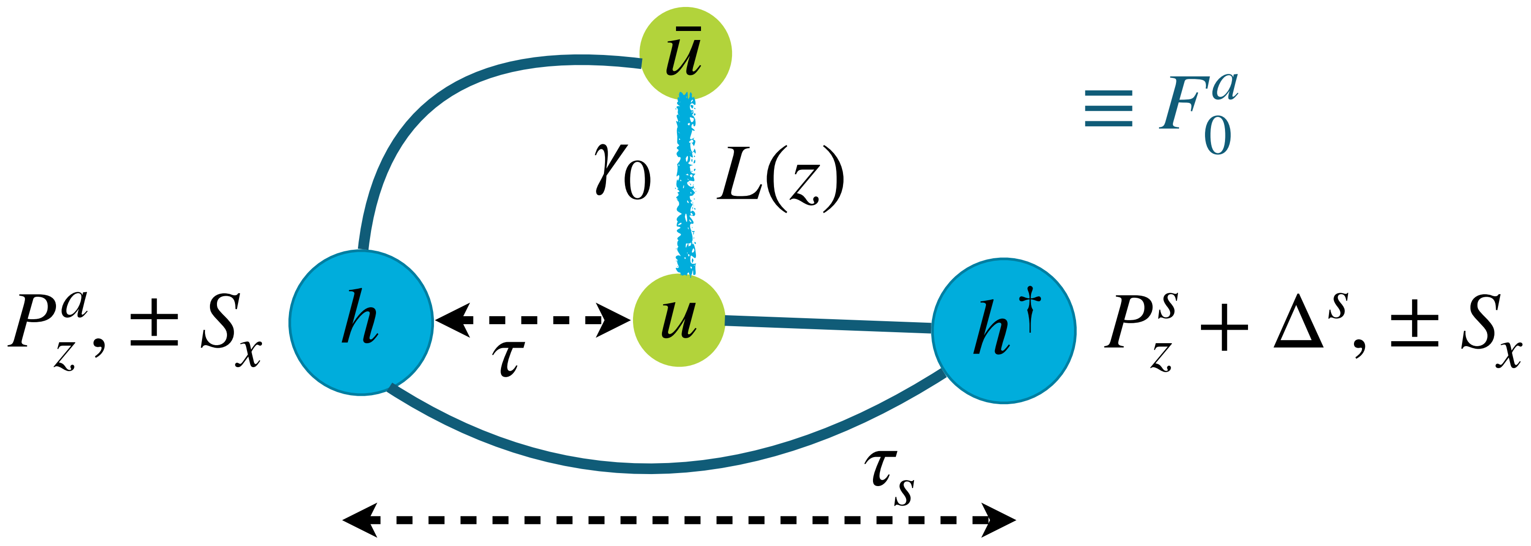
$$F_0^a = \bar{u}\left[\gamma_0 \mathscr{H}_0^a + \frac{i\sigma^{0\mu}\Delta_\mu^a}{2m}\mathscr{E}_0^a\right] u$$

$\mathscr{H}_0^a, \mathscr{E}_0^a$ + pQCD matching + $z^2 \to 0/P_z \to \infty$

H, E: light-cone GPD

the way we wanted

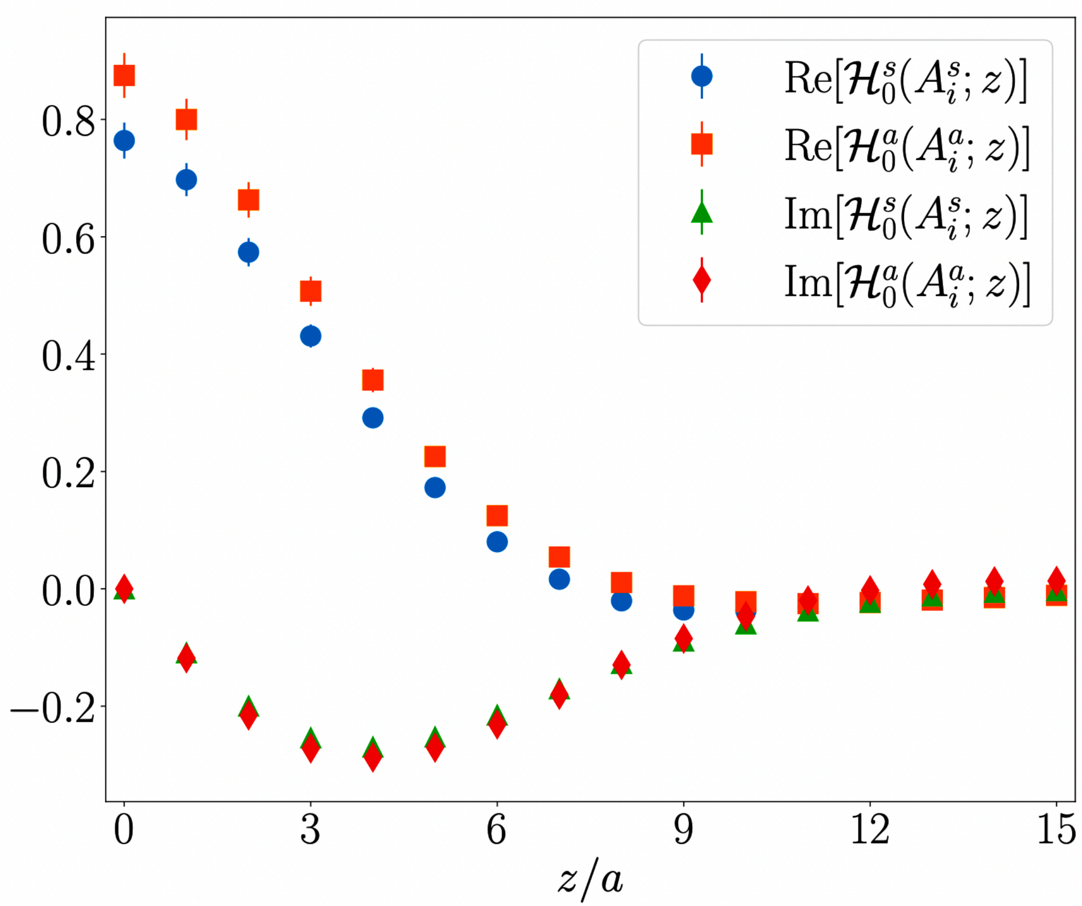
momenta transfer:
asymmetric

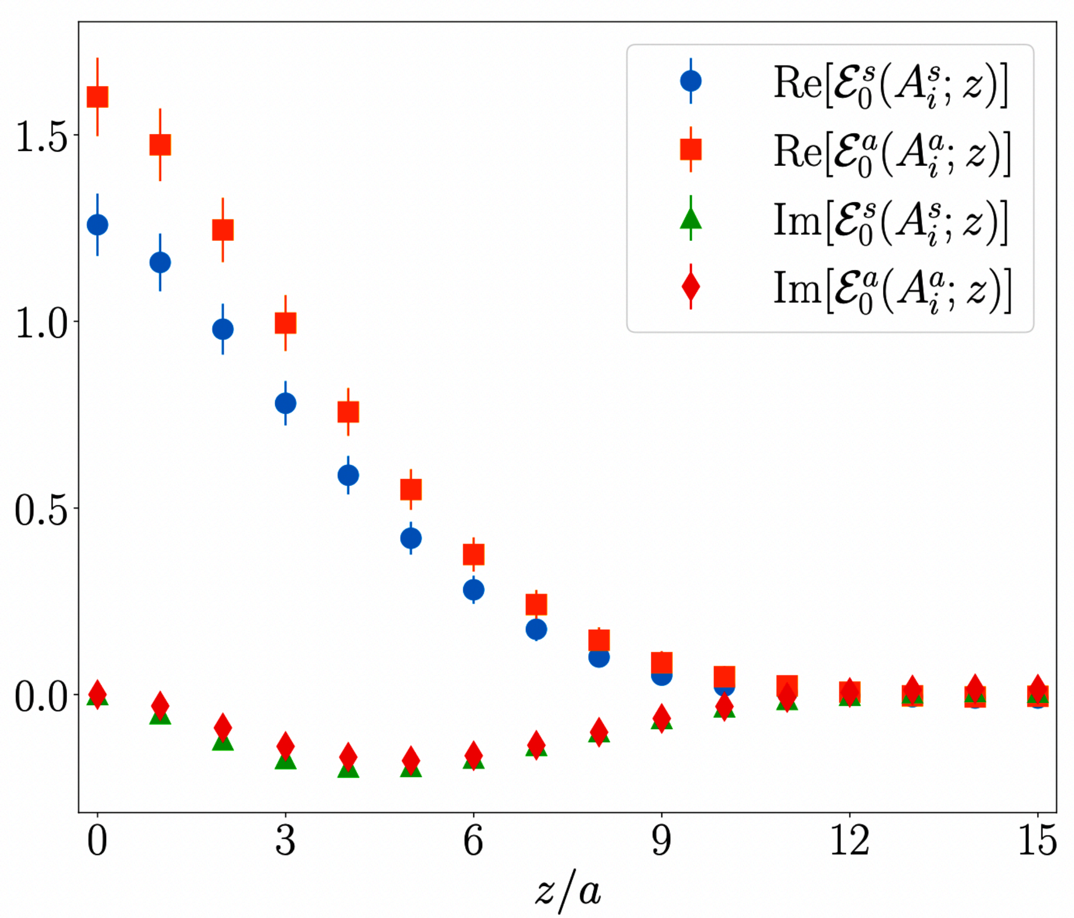


- multiple Δ^2 within a single calculation
- each calculation is $2\times$ faster than symmetric frame

$\gtrsim 5\times$ faster access to t-dependence of GPD

the way it went …

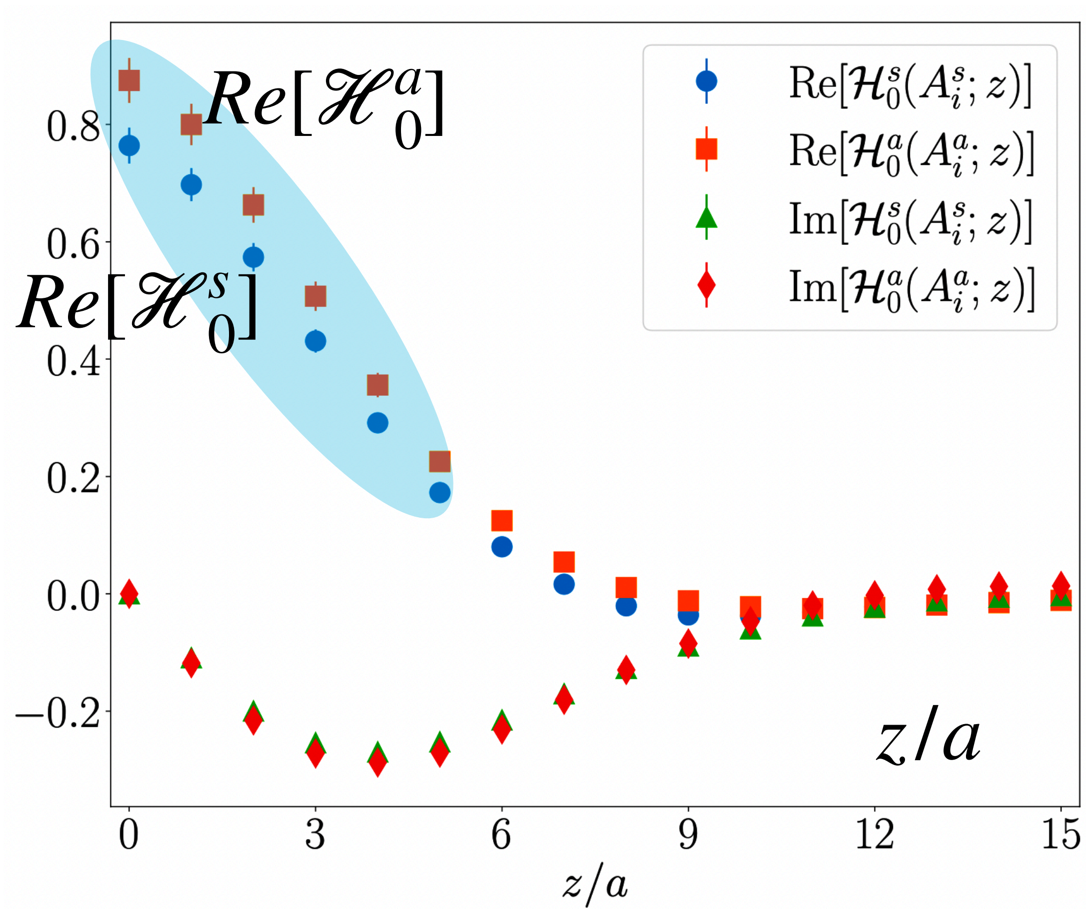


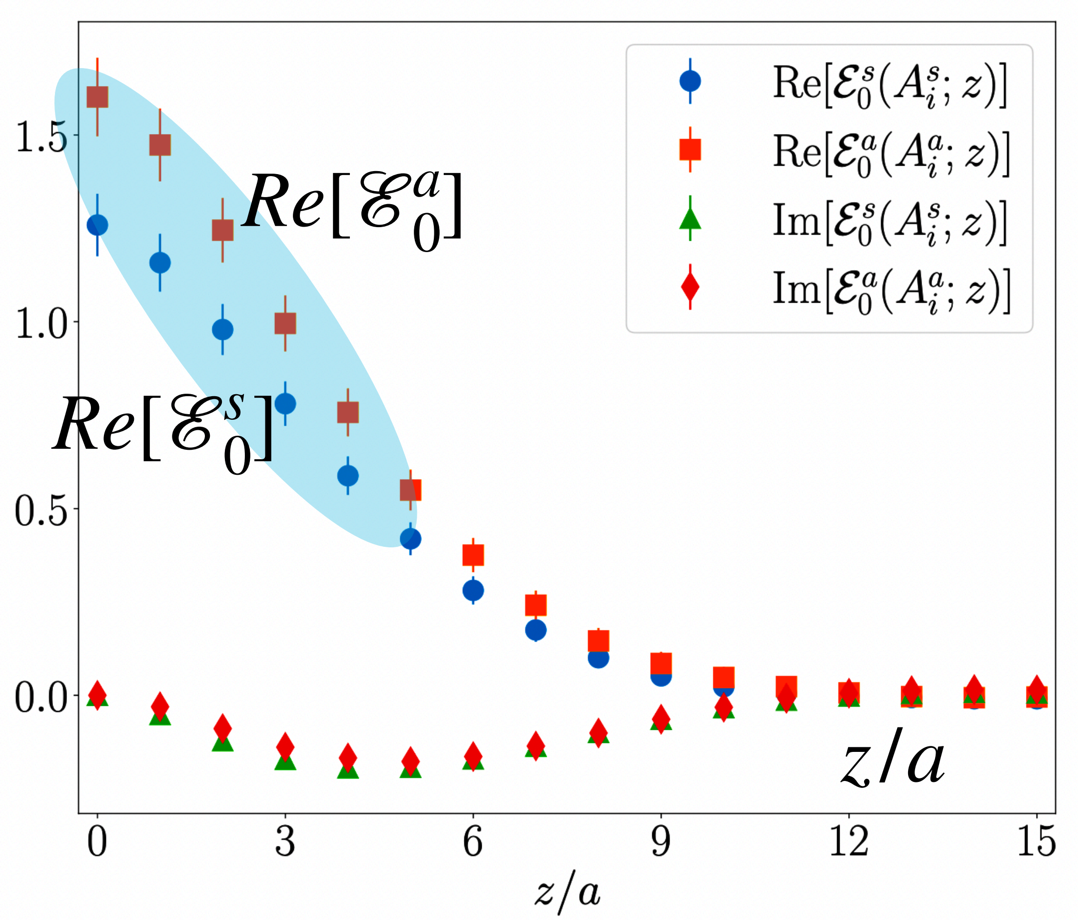


$P_z = 1.25$ GeV, $t \simeq -0.67$ GeV, $\xi = 0$

$m_\pi = 260$ MeV, $a = 0.093$ fm, $32^3 \times 64$, $N_f = 2 + 1 + 1$twisted mass fermions

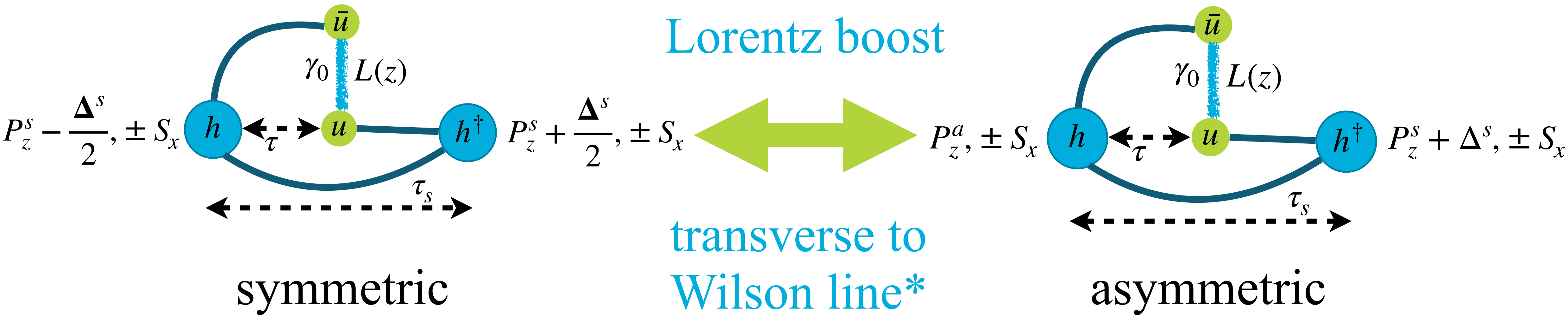
the way it went





frame-dependent power corrections

the way it went wrong



$$\boxed{F_0^s} \leftrightarrow \gamma F_0^a - \gamma\beta \boxed{F_\perp^a}$$

$\perp \equiv x, y$

frame-dependent power corrections

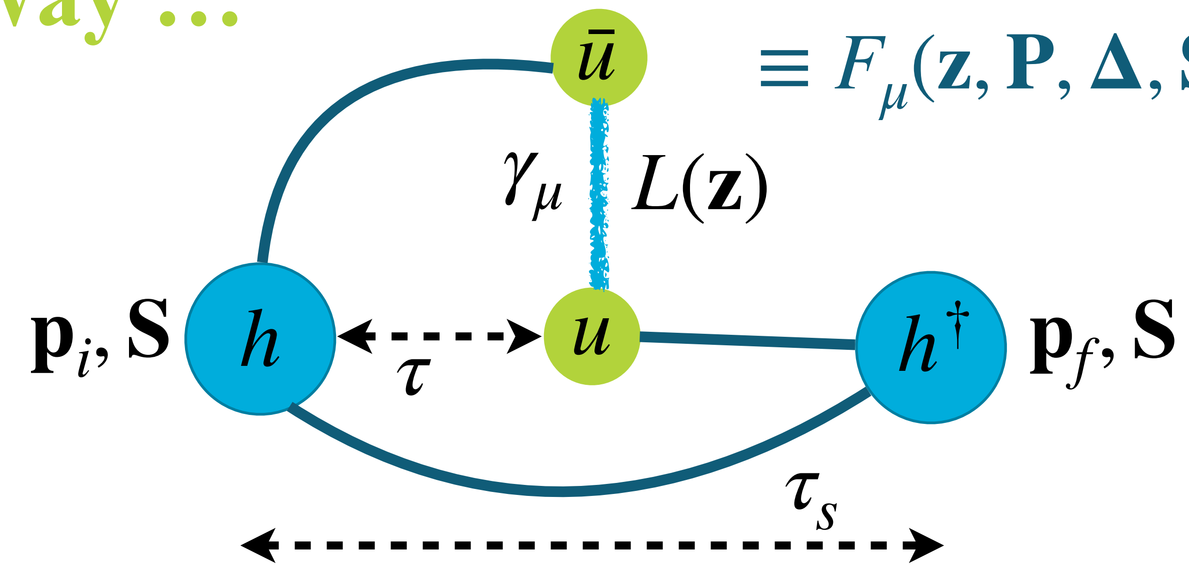
$P_z \to \infty \qquad F_0^s \leftrightarrow F_0^a$

* Euclidean lattice: the operator must remain space-like $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \sqrt{\frac{E_i^a + E_f^a}{2E_f^a}} \quad \beta = -\sqrt{\frac{E_i^a - E_f^a}{E_i^a + E_f^a}} < 0$

the new Lorentz invariant way …

$$\mathbf{P} = (\mathbf{p}_i + \mathbf{p}_f)/2$$

$$\mathbf{\Delta} = \mathbf{p}_f - \mathbf{p}_i$$



$\equiv F_\mu(\mathbf{z}, \mathbf{P}, \mathbf{\Delta}, \mathbf{S})$

Lorentz covariant parameterization:

$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\Big[\frac{P^\mu}{m}A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\Big]u(p_i,\lambda)$$

8 Lorentz invariant amplitudes $A_i\left(\mathbf{z}\cdot\mathbf{P}, \mathbf{z}\cdot\mathbf{\Delta}, \mathbf{\Delta}^2, \mathbf{z}^2\right)$

the new Lorentz invariant way …

From A's to GPD, Lorentz invariant mapping:

$$F^{+} = \bar{u}\left[\gamma^{+}\mathcal{H} + \frac{i\sigma^{+\mu}\Delta_{\mu}}{2m}\mathcal{E}\right]u$$

$$\boxed{\mathcal{H}} = A_1 + \left(\frac{\mathbf{\Delta}\cdot\mathbf{z}}{\mathbf{P}\cdot\mathbf{z}}\right)A_3$$

$$\boxed{\mathcal{E}} = -A_1 - \left(\frac{\mathbf{\Delta}\cdot\mathbf{z}}{\mathbf{\Delta}\cdot\mathbf{z}}\right)A_3 + 2A_5 + 2\left(\mathbf{P}\cdot\mathbf{z}\right)A_6 + 2\left(\mathbf{\Delta}\cdot\mathbf{z}\right)A_8$$

frame-dependent mapping: $\mathcal{H}_0^{s/a} = \sum \boxed{h_i^{s/a}} A_i$ $\mathcal{E}_0^{s/a} = \sum \boxed{e_i^{s/a}} A_i$

frame-dependent kinematic factors

the new Lorentz invariant way

$$\mathscr{H} = A_1 + \left(\frac{\mathbf{\Delta} \cdot \mathbf{z}}{\mathbf{P} \cdot \mathbf{z}}\right) A_3$$

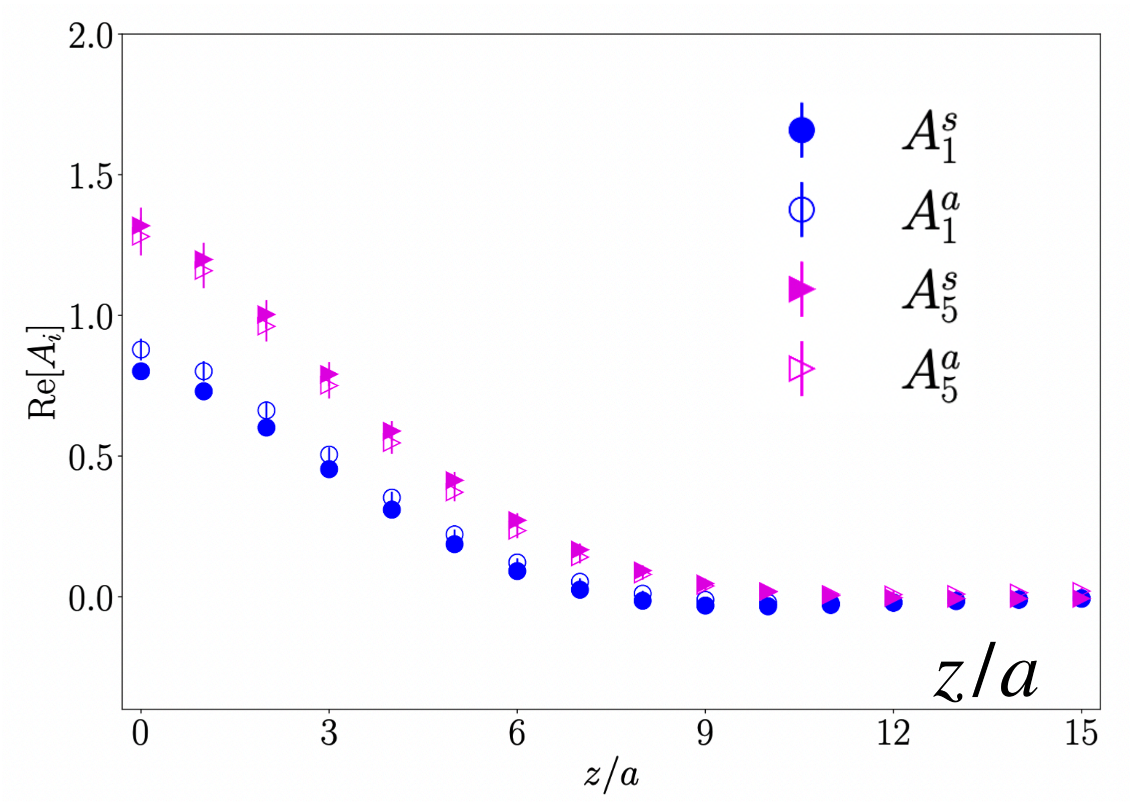
~~frame-dependent power corrections~~

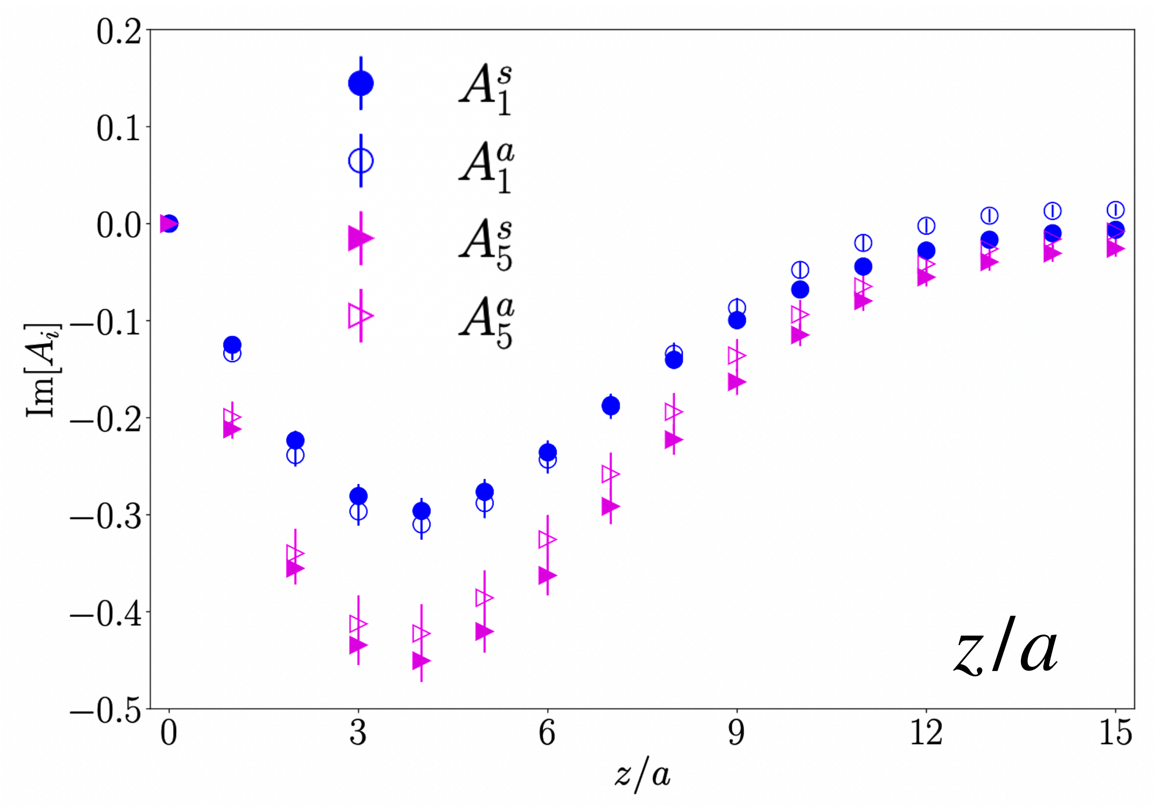
$$\mathscr{E} = -A_1 - \left(\frac{\mathbf{\Delta} \cdot \mathbf{z}}{\mathbf{\Delta} \cdot \mathbf{z}}\right) A_3 + 2A_5 + 2\left(\mathbf{P} \cdot \mathbf{z}\right) A_6 + 2\left(\mathbf{\Delta} \cdot \mathbf{z}\right) A_8$$

$$\mathscr{H}_0^{s/a}, \mathscr{E}_0^{s/a} \xrightarrow{z^2 \to 0 / P_z \to \infty} \mathscr{H}, \mathscr{E}$$

the way it works …

A_i are frame independent*

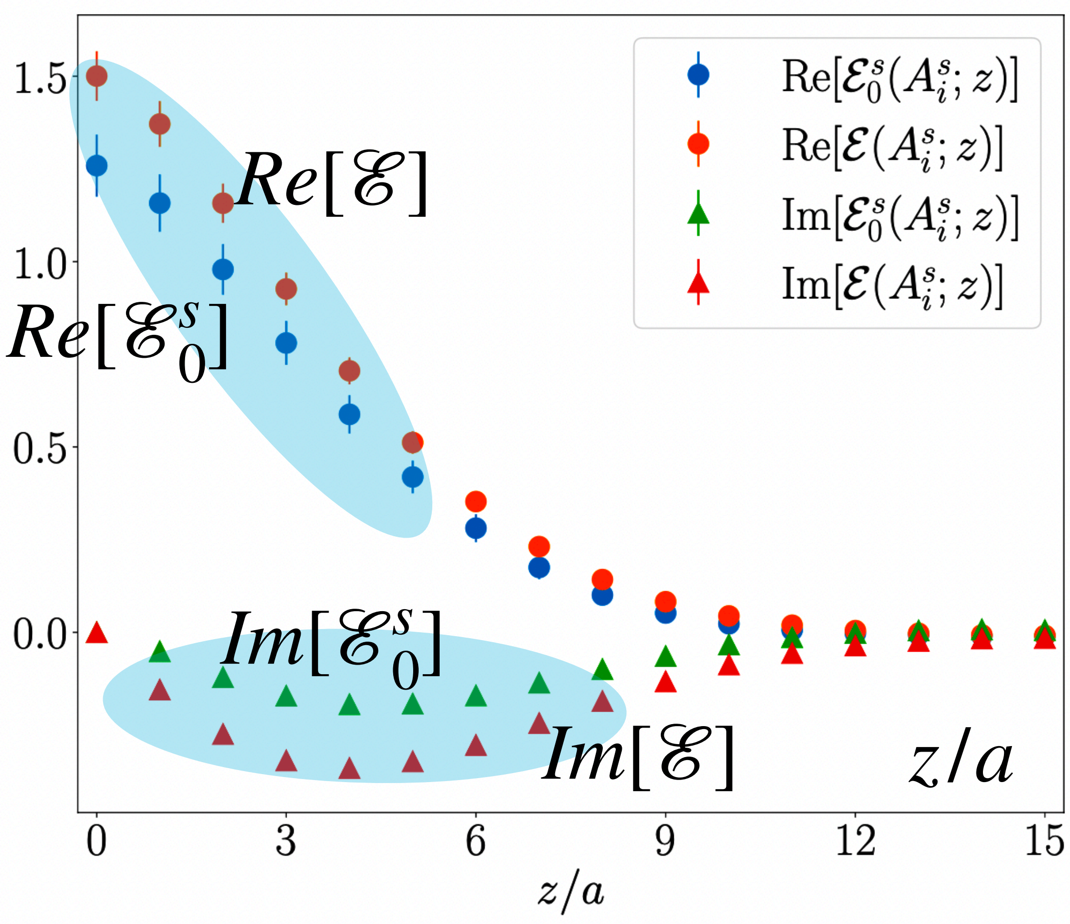




* A_i can be obtained in any frame from linear combinations of F_μ's with different proton polarizations

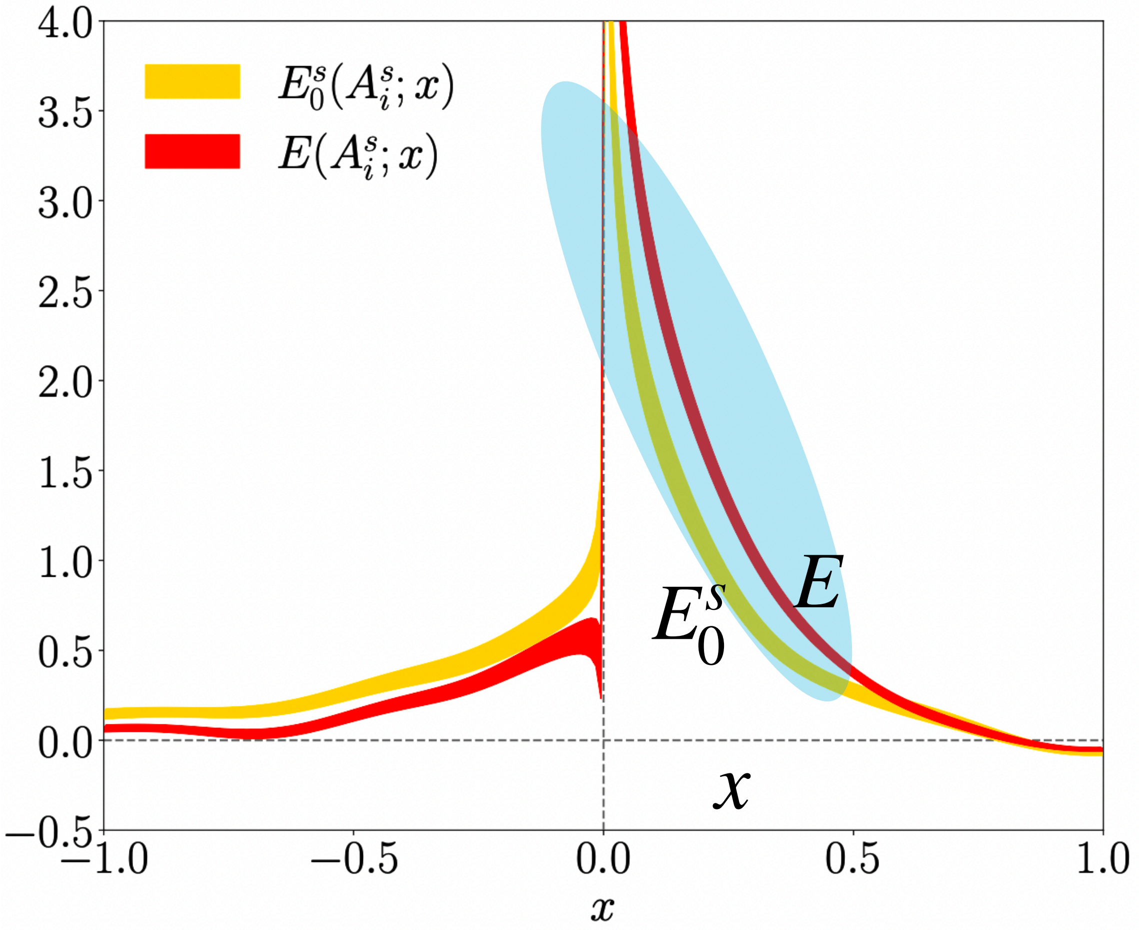
the way it works …

reduced frame-dependent power corrections in GPD matrix elements



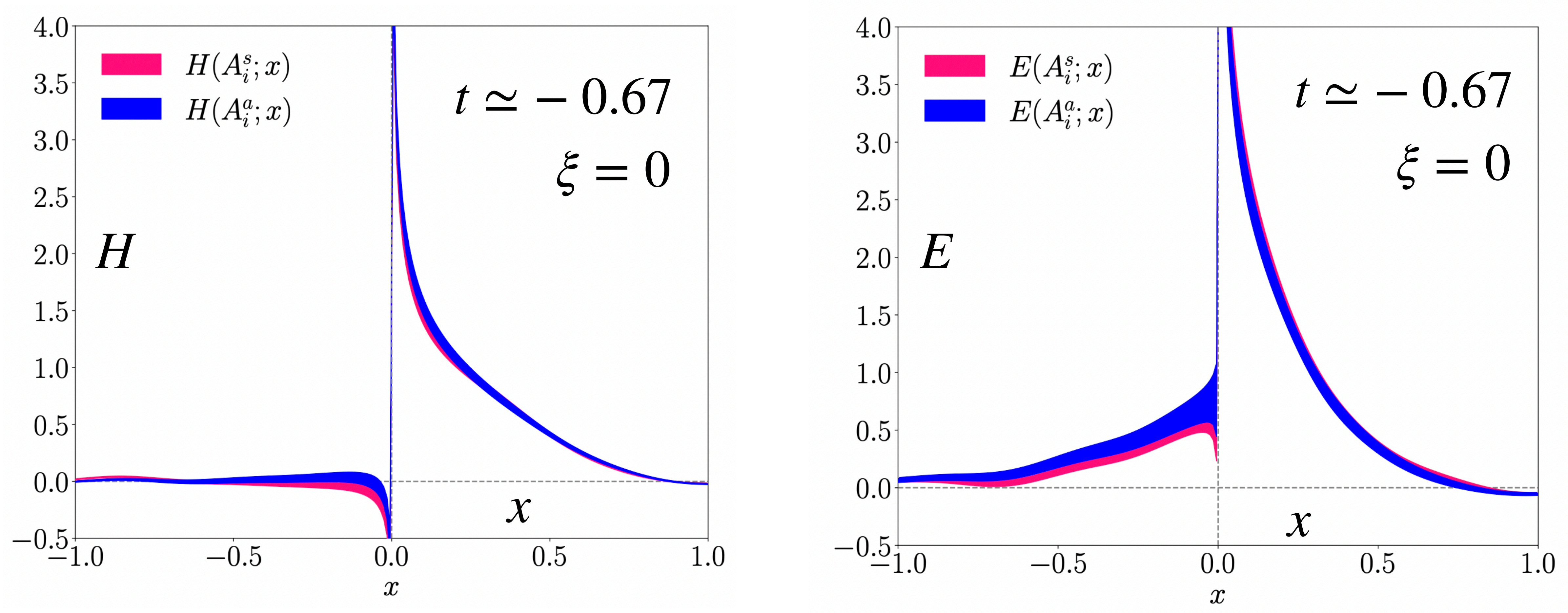
the way it works …

reduced frame-dependent power corrections in GPD



the way it works …

H and E GPD



the new way established …

a novel Lorentz invariant formalism for lattice calculations of GPD

- **faster:** $\gtrsim 5\times$ faster access to t-dependence of GPD
- **accurate:** reduces frame-dependent power corrections

long way to go …

- can be naturally extended to other twist-2 and higher-twist GPDs, including for spin-0 hadrons