Gravitational form factors of the Δ resonance in chiral EFT

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Gravitational form factors of the delta resonance in chiral EFT. Eur. Phys. J. C 82, 907 (2022)

Motivation

- The gravitational form factors (GFFs) can be measured in inclusive and exclusive processes like deeply virtual Compton scattering and hard meson production.

- The global mechanical properties of the Δ-resonance can be obtained as a linear response of the generally covariant action to the spacetime metric ($\sim \delta S / \delta g^{\mu\nu}$).

On the action in curved spacetime and EMT

- By " linear response of the generally covariant action to the spacetime metric " we mean generalizing the action to curved spacetime and then taking its functional variation with respect to metric and taking flat spacetime limit

- To do so, first for the action in flat spacetime we do following substitution:

$$S^n_{\text{flat}} = \int d^4x \, \mathscr{L}\left(\bar{\Psi}, \Psi, \bar{\Psi}_\mu, \Psi_\mu, D_\mu, \pi\right) \mapsto \int d^4x \, \sqrt{-g} \, \mathscr{L}\left(g_{\mu,\nu} \, , e^a_\mu, \bar{\Psi}, \Psi, \bar{\Psi}_\mu, \Psi_\mu, \nabla_\mu, \pi\right)$$

- In EFT the above substitution may not be sufficient. Since at each order, one has to find the most general and minimal Lagrangian, and in the generally covariant action there will be new field (metric field), which leads to new interactions.

- That is, after giving the metric field a specific chiral order, we have to investigate which new interactions can appear at a given order, that do not exist in flat spacetime.

On the action in curved spacetime and EMT

If, e.g., in the flat spacetime-Lagrangian such a term exists at the leading order: $\eta_{\mu\nu}\bar{\Psi}^{\mu}\Psi^{\nu}$,

then in curved spacetime we obtain $g_{\mu\nu}\bar{\Psi}^{\mu}\Psi^{\nu}$.

But at higher orders there are other possible terms like : $h_i\ R_{\mu\nu}\bar{\Psi}^{\mu}\Psi^{\nu}$.

On the action in curved spacetime and EMT

- That is, to obtain the linear response of the generally covariant action to the spacetime metric, we generalize the action in flat spacetime to curved spacetime and find terms that contain derivatives of the metric field, i.e.

$$S^n_{\text{flat}} = \int d^4x\ \mathscr{L}\left(\bar{\Psi}, \Psi, \bar{\Psi}_\mu, \Psi_\mu, D_\mu, \pi\right) \mapsto S^n_{\text{curved}}$$

$$\int d^4x\ \sqrt{-g}\ \mathscr{L}\left(g_{\mu,\nu}\ , e^a_\mu, \bar{\Psi}, \Psi, \bar{\Psi}_\mu, \Psi_\mu, \nabla_\mu, \pi\right) + \int d^4x\ \sqrt{-g}\ \mathscr{L}^{(n)}_{\text{grav}}\left(R, R^\alpha_{\mu\nu\beta}\ , R_{\mu\nu}\ , e^a_\mu, \bar{\Psi}, \Psi, \bar{\Psi}_\mu, \Psi_\mu, \nabla_\mu, \pi\right)$$

Must be constructed in the most general and minimal form up to order n.

On the action in curved spacetime and EMT

- The EMT given by

Bosonic fields

Fermionic fields

$$T^{(n)}_{\mu\nu}(g,\psi) = \frac{2}{\sqrt{-g}} \frac{\delta S^n_{\text{curved}}}{\delta g^{\mu\nu}}\Bigg|_{g=\eta} \sim \frac{1}{2e}\left[\frac{\delta S^n_{\text{curved}}}{\delta e^{a\mu}} e^a_\nu + \frac{\delta S^n_{\text{curved}}}{\delta e^{a\nu}} e^a_\mu\right]\Bigg|_{g=\eta}$$

N. D. Birrell and P. C. W. Davies, "Quantum Fields in Curved Space," Cambridge Univ. Press, Cambridge, UK, 1984

- Remember: $e^a_\mu e^b_\nu \eta_{ab} = g^{\mu\nu}$, $e^a_\mu e^b_\nu g^{\mu\nu} = \eta^{ab}$.

$$\gamma^\mu V_\mu \mapsto e^\mu_a \, \gamma^a V_\mu$$

Action in curved spacetime up to second chiral order

$$S^{(2)}_{\text{curved}} = S^{(1)}_{\pi\Delta} + S^{(1)}_{\pi \mathrm{N}} + S^{(1)}_{\pi N\Delta} + S^{(2)}_{\pi} + S^{(2)}_{\pi\Delta,a} + S^{(2)}_{\pi\Delta,b}\,,$$

$$S^{(1)}_{\pi\Delta} = -\int d^4x\sqrt{-g}\Big[g^{\mu\nu}\bar{\Psi}^i_\mu\, i\gamma^\alpha \overleftrightarrow{\nabla}_\alpha \Psi^i_\nu - m_\Delta\, g^{\mu\nu}\bar{\Psi}^i_\mu\Psi^i_\nu - g^{\lambda\sigma}\left(\bar{\Psi}^i_\mu i\gamma^\mu \overleftrightarrow{\nabla}_\lambda \Psi^i_\sigma + \bar{\Psi}^i_\lambda i\gamma^\mu \overleftrightarrow{\nabla}_\sigma \Psi^i_\mu\right) + i\bar{\Psi}^i_\mu\gamma^\mu\gamma^\alpha\gamma^\nu \overleftrightarrow{\nabla}_\alpha \Psi^i_\nu + m_\Delta \bar{\Psi}^i_\mu\gamma^\mu\gamma^\nu\Psi^i_\nu$$
$$+\frac{g_1}{2}\, g^{\mu\nu}\bar{\Psi}^i_\mu u_\alpha\gamma^\alpha\gamma_5\Psi^i_\nu + \frac{g_2}{2}\bar{\Psi}^i_\mu\left(u^\mu\gamma^\nu + u^\nu\gamma^\mu\right)\gamma_5\Psi^i_\nu + \frac{g_3}{2}\bar{\Psi}^i_\mu u_\alpha\gamma^\mu\gamma^\alpha\gamma_5\gamma^\nu\Psi^i_\nu\Big]\,,$$

$$S^{(1)}_{\pi \mathrm{N}} = \int d^4x\sqrt{-g}\left\{\bar{\Psi}\, i\gamma^\mu \overleftrightarrow{\nabla}_\mu \Psi - m\bar{\Psi}\Psi + \frac{g_A}{2}\bar{\Psi}\gamma^\mu\gamma_5 u_\mu\Psi\right\}, \quad S^{(1)}_{\pi N\Delta} = -\int d^4x\sqrt{-g}\; g_{\pi N\Delta}\; \bar{\Psi}_{\mu,i}\left(g^{\mu\nu} - \gamma^\mu\gamma^\nu\right) u_{\nu,i}\Psi + \text{H.c.}\,,$$

$$S^{(2)}_{\pi} = \int d^4x\sqrt{-g}\left\{\frac{F^2}{4} g^{\mu\nu}\,\mathrm{Tr}(D_\mu U (D_\nu U)^\dagger) + \frac{F^2}{4}\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\right\}, \quad S^{(2)}_{\pi\Delta,a} = \int d^4x\sqrt{-g}\, a_1\, \bar{\Psi}^i_\mu\Theta^{\mu\alpha}(z)\langle\chi_+\rangle\, g_{\alpha\beta}\Theta^{\beta\nu}(z')\Psi^i_\nu\,,$$

$$S^{(2)}_{\pi\Delta,b} = \int d^4x\sqrt{-g}\Big[h_1 R\; g^{\alpha\beta}\bar{\Psi}^i_\alpha\Psi^i_\beta + h_2 R\;\bar{\Psi}^i_\alpha\gamma^\alpha\gamma^\beta\Psi^i_\beta + ih_3 R\left(g^{\alpha\lambda}\bar{\Psi}^i_\alpha\gamma^\beta \overrightarrow{\nabla}_\lambda\Psi^i_\beta - g^{\beta\lambda}\bar{\Psi}^i_\alpha\gamma^\alpha \overleftarrow{\nabla}_\lambda\Psi^i_\beta\right) + \cdots\,,$$

$T_{\mu\nu}$

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Gravitational form factors of Δ resonance

S. Cotogno, *et al.* Phys. Rev. D 101, no.5, 056016 (2020), [arXiv:1912.08749 [hep-ph]].

$$\langle p_f, s_f | T^{\mu\nu} | p_i, s_i \rangle =$$

$$-\bar{u}_{\alpha'}(p_f, s_f)\Bigg[\frac{P^\mu P^\nu}{m}\left(\eta^{\alpha'\alpha}F_{1,0}(t) - \frac{\Delta^{\alpha'}\Delta^\alpha}{2m_\Delta^2}F_{1,1}(t)\right) + \frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{4m}\left(\eta^{\alpha'\alpha}F_{2,0}(t) - \frac{\Delta^{\alpha'}\Delta^\alpha}{2m_\Delta^2}F_{2,1}(t)\right)$$

$$+\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{4m}\left(\eta^{\alpha'\alpha}F_{2,0}(t) - \frac{\Delta^{\alpha'}\Delta^\alpha}{2m_\Delta^2}F_{2,1}(t)\right) + \frac{i}{2m_\Delta}P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho\left(\eta^{\alpha'\alpha}F_{4,0}(t) - \frac{\Delta^{\alpha'}\Delta^\alpha}{2m_\Delta^2}F_{4,1}(t)\right)$$

$$-\frac{1}{m_\Delta}\left(\eta^{\alpha(\mu}\Delta^{\nu)}\Delta^{\alpha'} + \eta^{\alpha'(\mu}\Delta^{\nu)}\Delta^\alpha - 2\eta^{\mu\nu}\Delta^\alpha\Delta^{\alpha'} - \Delta^2\eta^{\alpha(\mu}\eta^{\nu)\alpha'}\right)F_{5,0}(t)\Bigg]u_\alpha(p_i, s_i)$$

With $P = (p_i + p_f)/2, \Delta = p_f - p_i, t = \Delta^2$ and $A^{(\alpha}B^{\beta)} = A^\alpha B^\beta + A^\beta B^\alpha$

Tree-order contributions to the GFFs

$$F_{1,0,\mathrm{tree}}(t) = 1 - \frac{t}{m_\Delta^2} + \frac{t\left(2h_5 m_\Delta + 2h_{10} - h_{13}\right)}{m_\Delta} - \frac{\left(-2h_6 + 2h_{11} + h_{12}\right)t^2}{2m_\Delta^2},$$

$$F_{1,1,\mathrm{tree}}(t) = -4 - 4m_\Delta\left(h_{12}m_\Delta - 2h_{10} + h_{13}\right) + \left(4h_6 - 2\left(2h_{11} + h_{12}\right)\right)t,$$

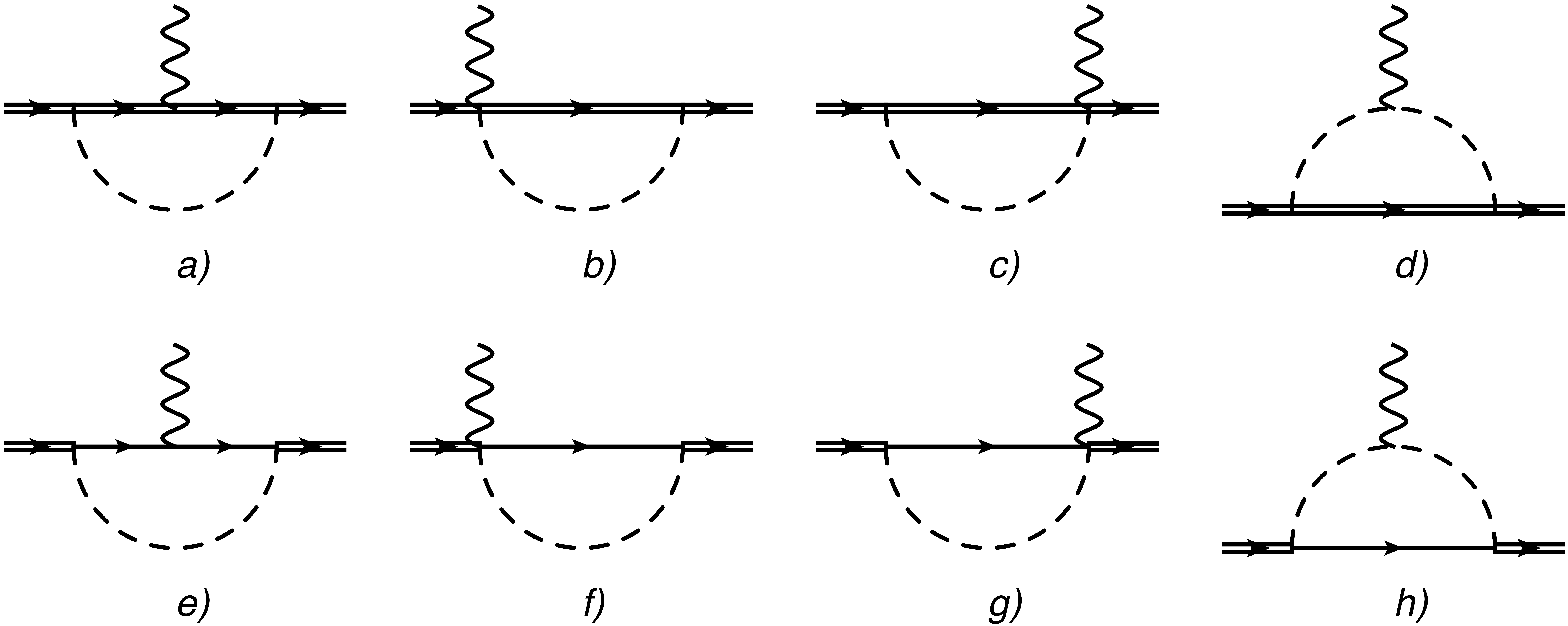
$$F_{2,0,\mathrm{tree}}(t) = -2 - 4\left(2h_1 - 2h_{10} + h_{13}\right)m_\Delta + \left(2h_6 - 2h_{11} - h_{12}\right)t,\ \ F_{2,1,\mathrm{tree}}(t) = 0,$$

$$F_{4,0,\mathrm{tree}}(t) = \frac{3}{2} - \frac{t}{2m_\Delta^2} + t\left(\frac{h_{10}}{m_\Delta} - \frac{h_{13}}{2m_\Delta} + h_5 - h_6 + h_{11} + \frac{h_{12}}{2}\right) - \frac{\left(-2h_6 + 2h_{11} + h_{12}\right)t^2}{4m_\Delta^2},$$

$$F_{4,1,\mathrm{tree}}(t) = -2 - 2m_\Delta\left(h_{12}m_\Delta - 2h_{10} + h_{13}\right) + \left(2h_6 - 2h_{11} - h_{12}\right)t,$$

$$F_{5,0,\mathrm{tree}}(t) = -\frac{1}{2} + \frac{1}{2}\left(h_4 + 4h_{10} - h_{13}\right)m_\Delta + \frac{1}{4}\left(2h_6 - 2h_{11} - h_{12}\right)t,$$

One-loop contributions to the GFFs



One-loop diagrams contributing to the one-particle matrix elements of the EMT for the delta resonances. Dashed, solid and double lines correspond to pions, nucleons and delta resonances, respectively. The wiggly line indicates the EMT insertion.

One-loop contributions to the GFFs

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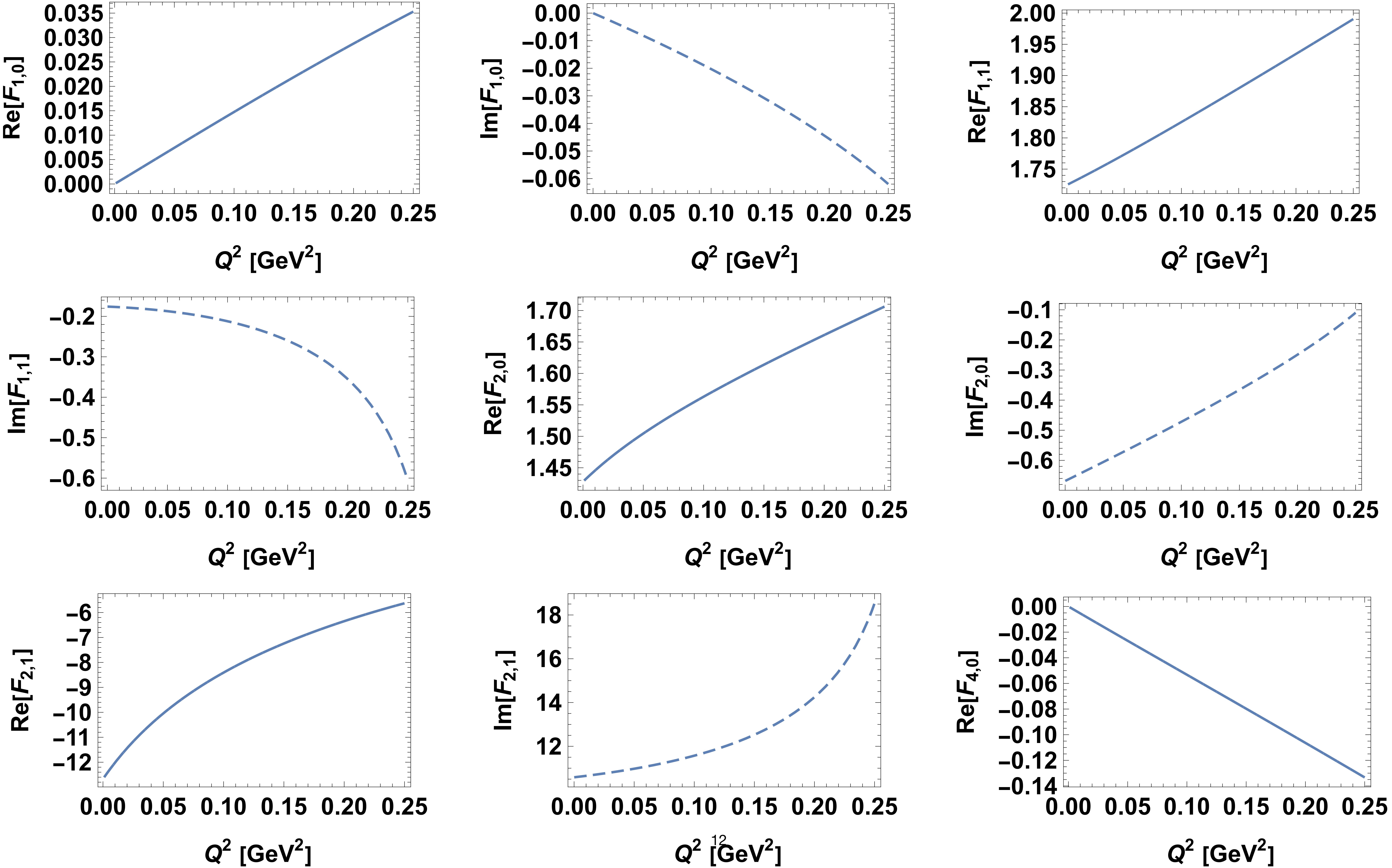
$$F_{i,j}(t) = F_{i,j}(0) + s_{F_{i,j}} t + \mathcal{O}(t^2)$$

These expressions are obtained as a function of M_π and they could be used for lattice extrapolations.

In our calculations we applied dimensional regularization and used the program FeynCalc

To get rid of the power-counting violating contributions we split the bare low-energy parameters as the renormalized ones and counterterms. We specify the finite parts of counterterms by applying the EOMS scheme with renormalization scale $\mu = m_N$.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Re[F4,0]
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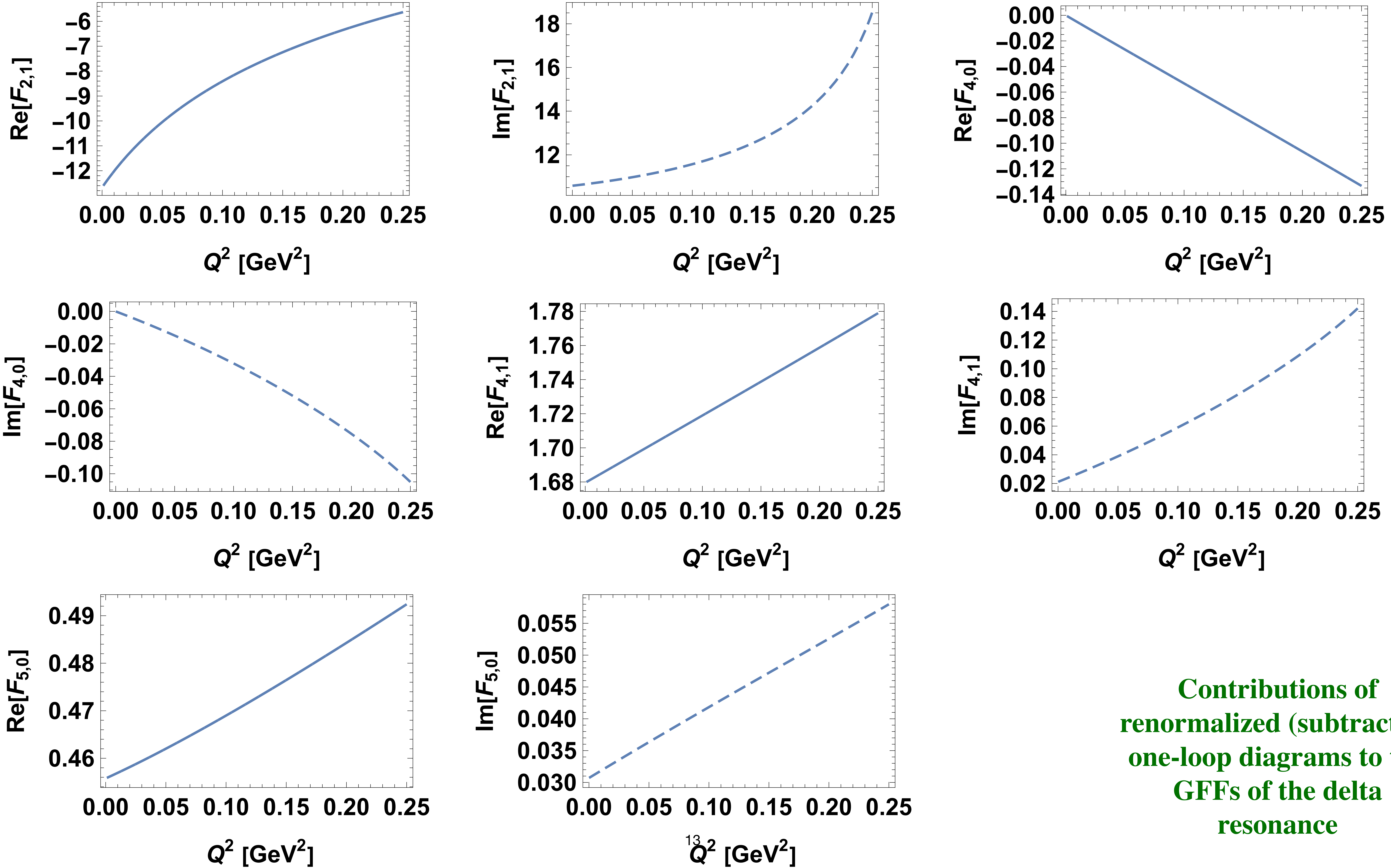

Contributions of renormalized (subtracted) one-loop diagrams to the GFFs of the delta resonance

Large distance asymptotics of the energy, spin, pressure and shear forces distributions

Local spatial densities for spin-3/2 systems (in preparation)

- The large distance power-like behavior of the distributions in the parametrically wide region $1/\Lambda_{\text{strong}} \ll r \ll 1/M_\pi$ is governed by the non-analytical terms of the GFFs in the chiral limit. The results read

$$F_{1,0}(t) \doteq -\frac{25g_1^2}{9216F^2m_\Delta}\, t\sqrt{-t}, \quad F_{1,1}(t) \doteq -\frac{5g_1^2m_\Delta}{1536F^2}\sqrt{-t}, \quad F_{2,0}(t) \doteq \frac{5g_1^2m_\Delta}{768F^2}\sqrt{-t},$$

$$F_{2,1}(t) \doteq \frac{5g_1^2m_\Delta^3}{384F^2}\,\frac{\sqrt{-t}}{t}, \quad F_{4,0}(t) \doteq -\frac{5g_1^2}{1728\pi^2F^2}\, t\log(-t/m_N^2), \quad F_{4,1}(t) \doteq 0,$$

$$F_{5,0}(t) \doteq -\frac{5g_1^2m_\Delta}{9216F^2}\sqrt{-t},$$

- We use the zero average momentum frame (ZAMF) to obtain the spatial density distributions of Δ

- E. Epelbaum, et al. Phys.Rev.Lett. 129 (2022), [arXiv:2201.02565 [hep-ph]].
- J. Y. Panteleeva, et al., Phys. Rev. D 106, no.5, 056019 (2022), [arXiv:2205.15061 [hep-ph]].

Large distance asymptotics of the energy distribution in ZAMF

Local spatial densities for spin-3/2 systems (in preparation)

$$t_{\phi}^{00}(s', s, r) = \int \frac{d^3P\, d^3q}{(2\pi)^3\sqrt{4EE'}}\, e^{-i\mathbf{q}\cdot\mathbf{r}} \langle p_f, s_f |\, T^{00}\, | p_i, s_i \rangle \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \phi^{\star}\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right)$$

$$= N_{\phi,R} \left\{ \rho_0^E(r)\, \delta_{s's} + \rho_2^E(r)\, Y_2^{kl}\left(\Omega_r\right) \hat{Q}_{s's}^{kl} \right\},$$

Monopole energy distribution: $\rho_0^E(r) = \frac{25 g_1^2}{1536 F^2 m_\Delta}\frac{1}{r^6} - \frac{10 g_1^2}{81\pi^2 F^2 m_\Delta^2}\frac{1}{r^7} + \frac{875 g_1^2}{6144 F^2 m_\Delta^3}\frac{1}{r^8}$ $\longrightarrow$ $\rho_0^E(r) > 0$

Large distance asymptotics of the spin distribution in ZAMF

Local spatial densities for spin-3/2 systems (in preparation)

$$t^{0i}_\phi(s',s,r) = \int \frac{d^3P\, d^3q}{(2\pi)^3\sqrt{4EE'}}\, e^{-i\mathbf{q}\cdot\mathbf{r}} \langle p_f, s_f | T^{0i} | p_i, s_i\rangle \phi\left(\mathbf{P}-\frac{\mathbf{q}}{2}\right)\phi^\star\left(\mathbf{P}+\frac{\mathbf{q}}{2}\right)$$
$$= N_{\phi,R}\left[\epsilon^{ikn}\hat{S}^k_{s's}Y_1^n\frac{1}{r}\rho_1^J(r) + \epsilon^{ikn}\hat{O}^{ktz}_{s's}Y_3^{ntz}\frac{1}{r}\rho_3^J(r)\right],$$

The spin density is given by

$$J^i(r,s',s) = \epsilon^{ijk}r^j t^{0k}_\phi(s',s,r) = N_{\phi,R}\left\{\left(\frac{2}{3}\delta^{il}Y_0 - Y_2^{il}\right)\rho_1^J(r)\,\hat{S}^l_{s's} + \left[-Y_4^{iltz} + \frac{2}{35}\left(8\delta^{il}Y_2^{tz} + \delta^{it}Y_2^{lz} + \delta^{iz}Y_2^{lt}\right)\right]\rho_3^J(r)\,\hat{O}^{ltz}_{s's}\right\}.$$

Monopole spin distribution: $\rho_1^J(r) = \frac{5g_1^2}{162\pi^2F^2m_\Delta}\frac{1}{r^5} - \frac{125g_1^2}{3072F^2m_\Delta^2}\frac{1}{r^6} + \frac{4g_1^2}{27\pi^2F^2m_\Delta^3}\frac{1}{r^7} - \frac{1225g_1^2}{12288F^2m_\Delta^4}\frac{1}{r^8} - \frac{8g_1^2}{9\pi^2F^2m_\Delta^5}\frac{1}{r^9}$

Large distance asymptotics of the pressure and shear forces distributions in ZAMF

Local spatial densities for spin-3/2 systems (in preparation)

$$t^{ij}_{\phi}(s',s,r) = \int \frac{d^3P\, d^3q}{(2\pi)^3\sqrt{4EE'}}\, e^{-i\mathbf{q}\cdot\mathbf{r}} \langle p_f, s_f | T^{ij} | p_i, s_i\rangle \phi\left(\mathbf{P}-\frac{\mathbf{q}}{2}\right)\phi^{\star}\left(\mathbf{P}+\frac{\mathbf{q}}{2}\right) = t^{ij}_{\phi,0}(s',s,r) + t^{ij}_{\phi,2}(s',s,r)$$

describes the whole system

describes the internal structure

$$t^{ij}_{\phi,2}(s',s,r) = N_{\phi,R,2}\Bigg\{ p_0(r)\delta^{ij}\delta_{s's} + s_0(r)Y_2^{ij}\delta_{s's} + p_2(r)\hat{Q}^{ij}_{s's} + 2s_2(r)\left[\hat{Q}^{ik}_{s's}Y_2^{kj} + \hat{Q}^{jk}_{s's}Y_2^{ki} - \delta^{ij}\hat{Q}^{kl}_{s's}Y_2^{kl}\right] - \frac{1}{m^2}\hat{Q}^{kl}_{s's}\partial_k\partial_l\left[p_3(r)\delta^{ij} + s_3(r)Y_2^{ij}\right]\Bigg\},$$

M. V. Polyakov and B. D. Sun, Phys. Rev. D 100 (2019), [arXiv:1903.02738 [hep-ph]].

- Monopole pressure distribution: $p_0(r) = -\frac{25g_1^2}{2304F^2m_\Delta}\frac{1}{r^6} - \frac{75g_1^2}{1024F^2m_\Delta^3}\frac{1}{r^8}$
- Monopole shear forces distribution: $s_0(r) = \frac{5g_1^2}{96F^2m_\Delta}\frac{1}{r^6} + \frac{15g_1^2}{64F^2m_\Delta^3}\frac{1}{r^8}$

$$\frac{2}{3}s(r) + p(r) > 0$$

Irina A. Perevalova, Maxim V. Polyakov, and Peter Schweitzer. Phys. Rev. D 94, 054024.

Summary

- We generalized the effective chiral Lagrangian describing Δ interacting with pions and nucleons up to second chiral order to curved spacetime.

- We calculated the corresponding matrix element of EMT up to third chiral order in one-loop expansion parametrized in terms of GFFs.

- From our results of the GFFs we obtained the Large distance asymptotics of the energy, spin, pressure and shear force distributions in ZAMF. Our densities have similar behavior compared with the Breit-frame densities obtained in Skyrme model in: **J. Y. Kim and B. D. Sun, Eur. Phys. J. C 81, no.1, 85 (2021), [arXiv:2011.00292 [hep-ph]].**