



On the definition of electromagnetic and gravitational local spatial densities

RUB

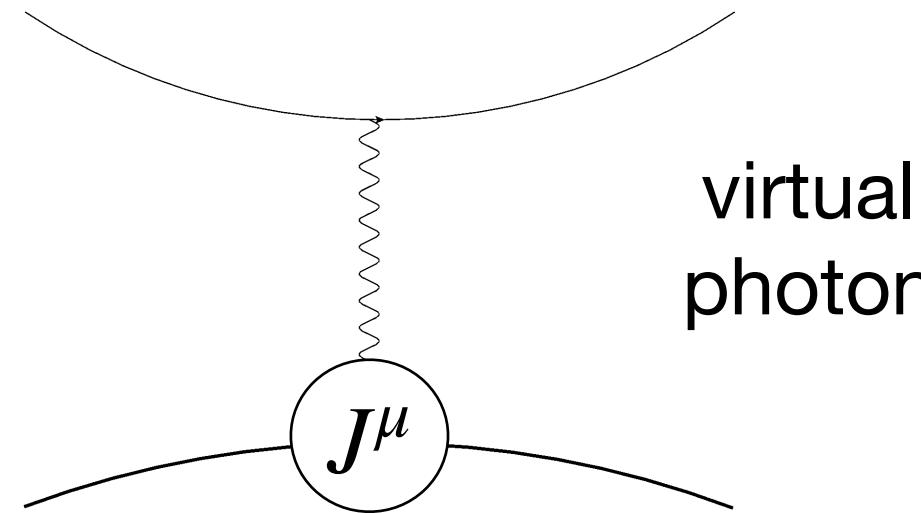
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Based on:

Epelbaum, Gegelia, Lange, Mei^ßner, Polyakov [Phys.Rev.Lett.129, 012001] (2022)
Panteleeva, Epelbaum, Gegelia, Mei^ßner [PhysRevD.106, 056019] (2022)
and
Panteleeva, Epelbaum, Gegelia, Mei^ßner ['in preparation'] (2022)

Structure of a particle

Electromagnetic structure



$$\frac{d\sigma}{d\Omega} \propto F_1(q^2), F_2(q^2)$$

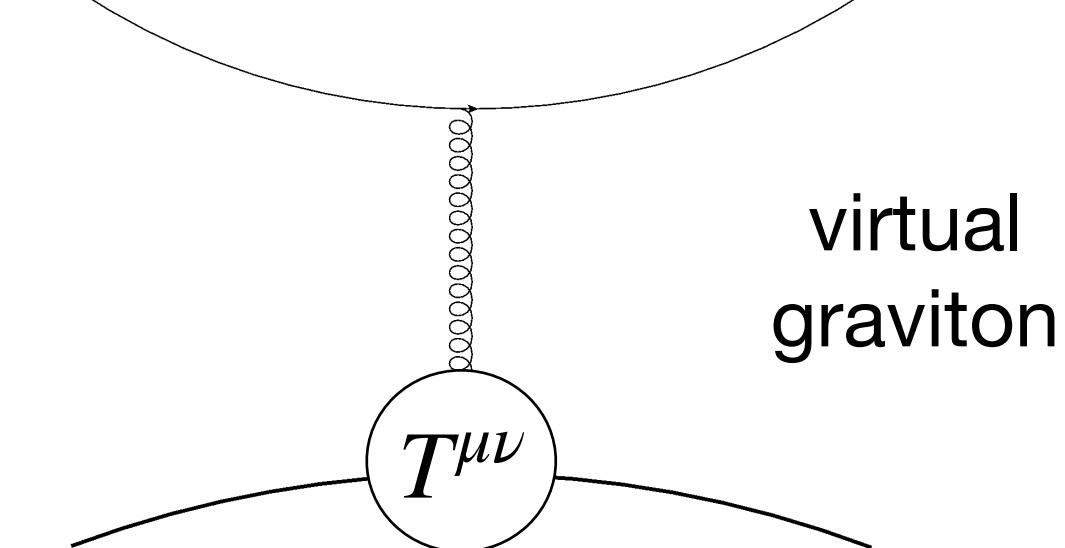
For spin-1/2

$$\langle p', s' | \hat{j}^\mu(0) | p, s \rangle = \bar{u}(p', s') \left[\gamma^\mu F_1(q^2) + \frac{1}{2} i \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p, s)$$

can be extracted from experiment

- charge
- charge radius
- spatial distributions

Gravitational structure



Gravity couples to matter due to EMT

$$T_{\mu\nu}(x) \sim \frac{\delta S_M}{\delta g^{\mu\nu}(x)}$$

can be extracted from DVCS

$$\langle p' | \hat{T}^{\mu\nu}(0) | p \rangle = \bar{u} \left[A(q^2) \frac{P_\mu P_\nu}{m} + i J(q^2) \frac{P_\mu \sigma_{\nu\alpha} q^\alpha + P_\nu \sigma_{\mu\alpha} q^\alpha}{2m} + D(q^2) \frac{q_\mu q_\nu - \eta_{\mu\nu} q^2}{4m} \right] u$$

- mass
- spin
- D-term
- various radii
- spatial distributions

The form-factors are extracted from experiment
But how the FFs are related to the **spatial distributions**?

for non-relativistic (heavy) systems

[Hofstadter et. all,
Rev. Mod. Phys. 30, 482 (1958)]

[Sachs,
Phys. Rev. 126, 2256-2260 (1962)]

[M. Burkardt
Phys. Rev. D 66 (2002), 119903(E),
G. Miller
Phys. Rev. Lett. 99, 112001 (2007)
Phys. Rev. C 79, 055204 (2009)
Ann. Rev. Nucl. Part. Sci. 60 (2010), 1-25
Phys. Rev. C99, no.3, 035202 (2019),
A. Freese and G. Miller
Phys. Rev. D103, 094023 (2021)]
[R.L.Jaffe,
Phys. Rev. D103 no.1, 016017 (2021)]
[A. Freese and G. Miller,
arxiv.org/pdf/2210.03807(2022)]

Breit frame

$$Q^2 = -q^2$$

$$\rho(r) \equiv \int \frac{d^3 Q}{(2\pi)^3} G_E(Q^2) e^{-i \vec{Q} \cdot \vec{r}}$$



$$\rho(r) \equiv \langle \Psi | \hat{\rho}(\mathbf{r}, 0) | \Psi \rangle$$

$$F(Q^2) = \int d^3 r \rho(\mathbf{r}) e^{i \vec{Q} \cdot \vec{r}}$$

charge density
of proton

[M.V.Polyakov,
Phys. Lett.B 555, 57 (2003)]

$$T_{\mu\nu}(\mathbf{r}, s) = \frac{1}{2E} \int \frac{d^3 Q}{(2\pi)^3} e^{i \vec{Q} \cdot \vec{r}} \langle p', s' | \hat{T}_{\mu\nu}(0) | p, s \rangle$$

C. Lorce,
Phys. Rev. Lett. 125, no.23, 232002 (2020),

C. Lorce, P. Schweitzer and K. Tezgin,
Phys.Rev. D 106, 014012 (2022)

Y. Guo, X. Ji and K. Shiells,
Nucl. Phys. B 969, 115440 (2021),
C. Lorce, H. Moutarde and A. P. Trawinski,

Eur. Phys. J. C 79, no.1, 89 (2019).1, 016017 (2021)

Construction of electromagnetic densities for a spin-1/2 particle

Matrix element of electromagnetic current operator at t=0:

$$\langle p', s' | \hat{j}^\mu(\mathbf{x}, 0) | p, s \rangle = e^{-i(\mathbf{p}' - \mathbf{p}) \cdot \mathbf{x}} \bar{u}(p', s') \left[\gamma^\mu F_1(q^2) + \frac{1}{2} i \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p, s)$$

Normalised Heisenberg-picture state: $|\Phi, \mathbf{X}, s\rangle = \int \frac{d^3 p}{\sqrt{2E(2\pi)^3}} \phi(s, \mathbf{p}) e^{-i\mathbf{p}\cdot\mathbf{X}} |p, s\rangle$

$$j_\phi^\mu(\mathbf{r}) = \langle \Phi, \mathbf{X}, s' | \hat{j}^\mu(\mathbf{x}, 0) | \Phi, \mathbf{X}, s \rangle$$

ZAMF – zero average momentum frame, where $\langle \mathbf{p} \rangle = 0$ for $|\Phi, \mathbf{X}, s\rangle$

$$\mathbf{P} = (\mathbf{p}' + \mathbf{p})/2, \mathbf{q} = \mathbf{p}' - \mathbf{p}$$

$$j_\phi^\mu(\mathbf{r}) = \int \frac{d^3 P d^3 q}{(2\pi)^3 \sqrt{4EE'}} \bar{u}(p', s') \left[\gamma^\mu F_1((E - E')^2 - \mathbf{q}^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2} F_2((E - E')^2 - \mathbf{q}^2) \right] u(p, s) \times \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \phi^*\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$E = \sqrt{m^2 + \mathbf{P}^2 - \mathbf{P} \cdot \mathbf{q} + \mathbf{q}^2/4}$$

$$E' = \sqrt{m^2 + \mathbf{P}^2 + \mathbf{P} \cdot \mathbf{q} + \mathbf{q}^2/4}$$

$$F_1(0) = 1, F_2(0) = \kappa/m$$

$$q = p' - p$$

Profile function:
spherically symmetric

$$\phi(s, \mathbf{p}) = \phi(\mathbf{p}) = R^{3/2} \tilde{\phi}(R |\mathbf{p}|)$$

Size of the packet

$$\int d^3 p |\phi(s, \mathbf{p})|^2 = 1$$

X – position of the charge and magnetisation centers

Current densities in static approximation

$$j_\phi^\mu(\mathbf{r}) = \int \frac{d^3P d^3q}{(2\pi)^3 \sqrt{4EE'}} \bar{u}(p', s') \left[\gamma^\mu F_1((E - E')^2 - \mathbf{q}^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2} F_2((E - E')^2 - \mathbf{q}^2) \right] u(p, s) \times \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \phi^*\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

the non-vanishing contribution for $R \rightarrow 0$ only in the region where $\mathbf{P} = \tilde{\mathbf{P}}/R$

[R.L. Jaffe, 2021]

taking $m \rightarrow \infty$ and after that $R \rightarrow 0$ using method of dimensional counting one obtains:

[J. Gegelia, G.Sh. Dzaparidze and K.Sh. Turashvili, Theor. Math. Phys. 101, 1313-1319 (1994)]

$$J_{\text{static}}^0(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} G_E(-\mathbf{q}^2) \equiv \rho_{\text{static}}^{\text{ch}}(r)$$

$$\mathbf{J}_{\text{static}}(\mathbf{r}) = \frac{\nabla_{\mathbf{r}} \times \boldsymbol{\sigma}}{2m} \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} G_M(-\mathbf{q}^2) \equiv \frac{\nabla_{\mathbf{r}} \times \boldsymbol{\sigma}}{2m} \rho_{\text{static}}^{\text{mag}}(r)$$

- coincide with Breit Frame expressions
- no dependence on wave packet
- valid for heavy systems with $\Delta \gg R \gg 1/m$
- this approximation is doubtful for light hadrons, $\Delta \lesssim 1/m$

[R.L. Jaffe, Phys. Rev. D103 no.1, 016017, (2021)]

Novel definition of the current densities

$$j_\phi^\mu(\mathbf{r}) = \int \frac{d^3P d^3q}{(2\pi)^3 \sqrt{4EE'}} \bar{u}(p', s') \left[\gamma^\mu F_1((E - E')^2 - \mathbf{q}^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2} F_2((E - E')^2 - \mathbf{q}^2) \right] u(p, s) \times \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \phi^\star\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

taking $R \rightarrow 0$ for arbitrary m , using method of dimensional counting (= strategy of regions):

$$\mathbf{J}(\mathbf{r}) = \frac{\nabla_{\mathbf{r}} \times \boldsymbol{\sigma}}{2m} \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int_{-1}^{+1} d\alpha \frac{1}{4} (1 + \alpha^2) m F_2[(\alpha^2 - 1)\mathbf{q}^2] \equiv \frac{\nabla_{\mathbf{r}} \times \boldsymbol{\sigma}}{2m} \rho_2(r) \quad \star$$

$$J^0(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int_{-1}^{+1} d\alpha \frac{1}{2} F_1[(\alpha^2 - 1)\mathbf{q}^2] \equiv \rho_1(r)$$

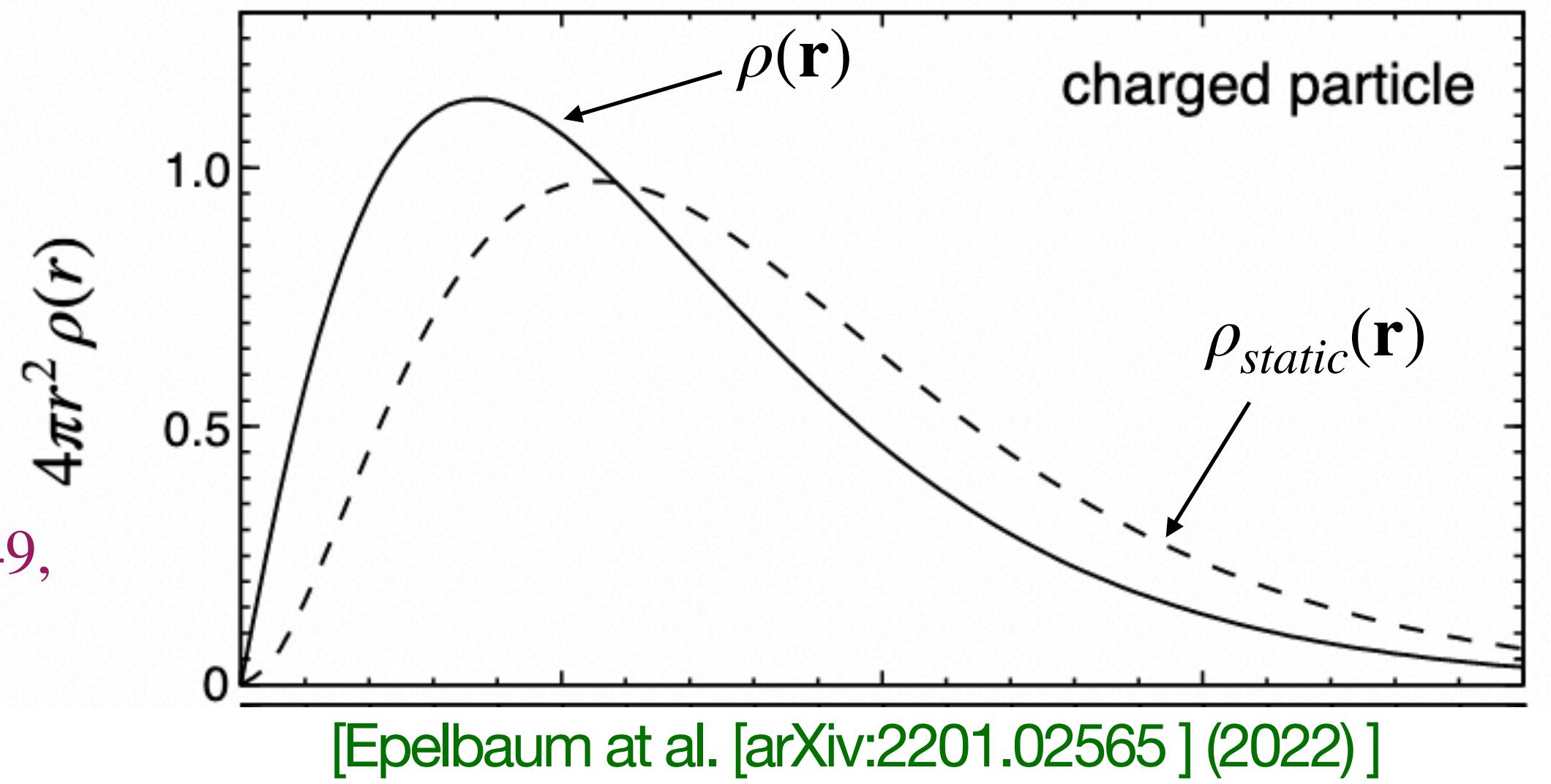
[G.N.Fleming, Physical Reality Math. Descrip., 357 (1974)]

$$\sqrt{\langle r^2 \rangle_{\text{static}}} = \sqrt{6 \left(F'_1(0) + \frac{F_2(0)}{4m} \right)} \simeq 0.8409(4), \quad \sqrt{\langle r^2 \rangle} = \sqrt{4F'_1(0)} \simeq 0.62649,$$

$R \rightarrow 0$

$\Delta \gg R \gg 1/m$

[G. A. Miller, Phys. Rev. C
99, no.3, 035202 (2019).]



these densities can be rewritten as:

$$J^\mu(\mathbf{r}) = \frac{1}{4\pi} \int d\hat{\mathbf{n}} J_{\hat{\mathbf{n}}}^\mu(\mathbf{r})$$

$$\hat{\mathbf{n}} = \frac{\mathbf{P}}{|\mathbf{P}|}$$

$$J_{\hat{\mathbf{n}}}^0(\mathbf{r}) = \rho_{1,\hat{\mathbf{n}}}(\mathbf{r})$$

$$\mathbf{J}_{\hat{\mathbf{n}}}(\mathbf{r}) = \frac{1}{2m} \nabla_{\mathbf{r}} \times \boldsymbol{\sigma} \rho_{2,\hat{\mathbf{n}}}(\mathbf{r})$$

$$\rho_{i,\hat{\mathbf{n}}}(\mathbf{r}) = \rho_i(r_\perp) \delta(r_\parallel)$$

$$\rho_1(r_\perp) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{r}_\perp} F_1(-\mathbf{q}_\perp^2) ,$$

$$\rho_2(r_\perp) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{r}_\perp} m F_2(-\mathbf{q}_\perp^2)$$

- no dependence of the radial form of the wave packet
- **no dependence on Compton wavelength $1/m$**
 - > applicable also for light hadrons
 - > $J_{\text{static}}^\mu(\mathbf{r})$ doesn't emerge from $J^\mu(\mathbf{r})$ by $m \rightarrow \infty$
 - > non-commutativity $R \rightarrow 0$ and $m \rightarrow \infty$

[Epelbaum et al. [Phys.Rev.Lett. 129, 012001](2022)]

IMF densities

In moving frame:

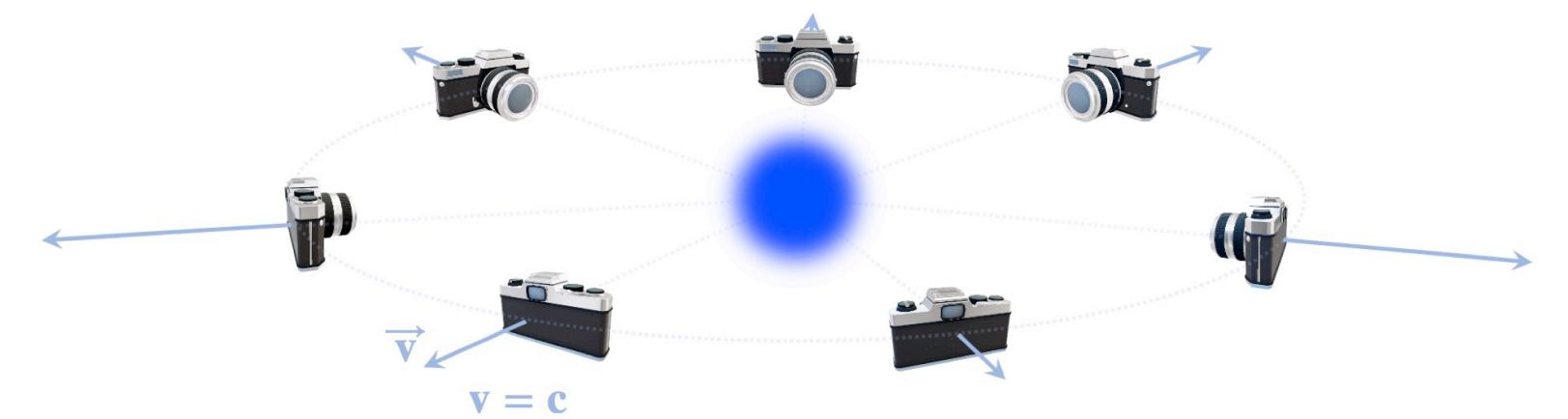
$$j_{\phi,v}^{\mu}(\mathbf{r}) = \langle \Phi, \mathbf{X}, s' | \hat{j}^{\mu}(\mathbf{r}, 0) | \Phi, \mathbf{X}, s \rangle_v$$

IMF with $v \rightarrow 1, \gamma \rightarrow \infty$

$$J_{ZAMF}^0(\mathbf{r}) = \frac{1}{4\pi} \int d\hat{\mathbf{v}} J_{IMF}^0(\mathbf{r}), \quad \mathbf{J}_{ZAMF}(\mathbf{r}) = 2 \times \frac{1}{4\pi} \int d\hat{\mathbf{v}} \mathbf{J}_{IMF}(\mathbf{r}).$$

- holographic relationship between ZAMF and IMF
- described only by intrinsic properties of system
- valid for any systems independently on the Compton wavelength

[Epelbaum et al. [Phys.Rev.Lett.129, 012001](2022)]



Gravitational spatial densities for spin-1/2

[Panteleeva, Epelbaum, Gegelia, Meißner, *'in preparation'*]

$$\langle p', s' | \hat{T}_{\mu\nu}(\mathbf{x}, 0) | p, s \rangle = e^{-i\mathbf{q}\cdot\mathbf{x}} \bar{u}(p', s') \left[A(q^2) \frac{P_\mu P_\nu}{m} + iJ(q^2) \frac{P_\mu \sigma_{\nu\alpha} q^\alpha + P_\nu \sigma_{\mu\alpha} q^\alpha}{2m} + D(q^2) \frac{q_\mu q_\nu - \eta_{\mu\nu} q^2}{4m} \right] u(p, s),$$

$$t_\phi^{\mu\nu}(\mathbf{r}) \equiv \langle \Phi, \mathbf{X}, s' | \hat{T}^{\mu\nu}(\mathbf{x}, 0) | \Phi, \mathbf{X}, s \rangle$$

$$t_\phi^{\mu\nu}(\mathbf{r}) = \int \frac{d^3P d^3q}{(2\pi)^3 \sqrt{4EE'}} \bar{u}(p', s') \left[A((E-E')^2 - \mathbf{q}^2) \frac{P_\mu P_\nu}{m} + iJ((E-E')^2 - \mathbf{q}^2) \frac{P_\mu \sigma_{\nu\alpha} q^\alpha + P_\nu \sigma_{\mu\alpha} q^\alpha}{2m} + D((E-E')^2 - \mathbf{q}^2) \frac{q^\mu q^\nu - \eta^{\mu\nu} q^2}{4m} \right]$$

$$\times u(p, s) \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \phi^\star\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

Novel gravitational spatial densities for spin-1/2

taking $R \rightarrow 0$ for arbitrary m , using method of dimensional counting one obtains:

$$t_\phi^{00}(\mathbf{r}) = N_{\phi,\infty} \int \frac{d^2 \hat{n} d^3 q}{(2\pi)^3} A(-\mathbf{q}_\perp^2) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$t_\phi^{0i}(\mathbf{r}) = N_{\phi,\infty} \int \frac{d^2 \hat{n} d^3 q}{(2\pi)^3} \left[\frac{iJ(-\mathbf{q}_\perp^2)}{2m} \left((\boldsymbol{\sigma}_\perp \times \mathbf{q}_\perp)^i + \hat{\mathbf{n}} \cdot (\boldsymbol{\sigma}_\perp \times \mathbf{q}_\perp) \hat{n}^i \right) \right] e^{-i\mathbf{q} \cdot \mathbf{r}}$$

$$t_\phi^{ij}(\mathbf{r}) = N_{\phi,\infty} \int \frac{d^2 \hat{n} d^3 q}{(2\pi)^3} \hat{n}^i \hat{n}^j A(-\mathbf{q}_\perp^2) e^{-i\mathbf{q}\cdot\mathbf{r}} + \frac{1}{2} N_{\phi,0} \int \frac{d^2 \hat{n} d^3 q}{(2\pi)^3} (q^i q^j - \delta^{ij} \mathbf{q}_\perp^2) D(-\mathbf{q}_\perp^2) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$N_{\phi,\infty} = \frac{1}{R} \int d\tilde{P} \tilde{P}^3 |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2$$

$$N_{\phi,0} = \frac{R}{2} \int d\tilde{P} \tilde{P} |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2$$



- only overall normalisation of densities depends on the wave packet
 - _ in $t_\phi^{ij}(r)$ we keep the leading order terms for each form factor separately!
 - the reason: from the comparison with moving frame follows that EMT has two separate contributions

Gravitational spatial holographic densities

in moving frame:

$$t_v^{\mu\nu}(\mathbf{r}) = t^{\mu\nu}(\mathbf{r}) + t_2^{\mu\nu}(\mathbf{r})$$

flow tensor **stress tensor**

$$t_{ZAMF}^{00}(\mathbf{r}) = \frac{1}{4\pi\gamma} \int d^2\hat{\mathbf{v}} \ t_{IMF}^{00}(\mathbf{r})$$

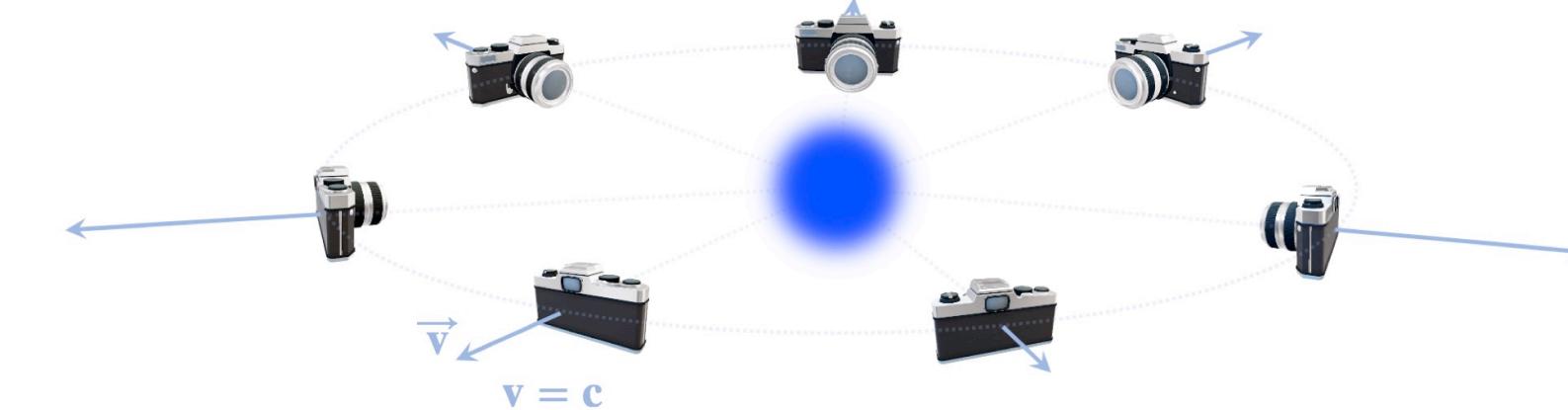
$$t_{ZAMF}^{0i}(\mathbf{r}) = \frac{2}{4\pi\gamma} \int d^2\hat{\mathbf{v}} \ t_{IMF}^{0i}(\mathbf{r})$$

$$t_{ZAMF}^{ij}(\mathbf{r}) = \frac{\gamma}{2\pi N_\infty} \int d^2\hat{\mathbf{v}} \ t_{IMF}^{ij}(\mathbf{r})$$

$\lim_{v \rightarrow 1} \int_{-1}^1 \frac{d\eta}{1+v\eta}$

- two contributions which characterise the movement of the system as a whole ($A(-\mathbf{q}_\perp^2)$ and $J(\mathbf{q}_\perp^2)$, flow tensor) and contributions corresponding to internal structure ($D(-\mathbf{q}_\perp^2)$, pure stress tensor)
[A. Freese, G. Miller “2021”]

- holographic interpretation of densities in ZAMF in terms of densities in IMF



[Panteleeva, Epelbaum, Gegelia, Meißner, ‘in preparation’]

Mass and energy distribution

$$t_\phi^{00}(\mathbf{r}) \equiv \langle \Phi, \mathbf{X}, s' | \hat{T}^{00}(\mathbf{r}, 0) | \Phi, \mathbf{X}, s \rangle$$

Interpretation

For sharply localised packet $R \rightarrow 0$ and arbitrary m

$$t_\phi^{00}(\mathbf{r}) = N_{\phi, \infty} \int d\hat{\mathbf{n}} \tilde{A}(r_\perp) \delta(r_\parallel)$$

Energy distribution

$$N_{\phi, \infty} = \frac{1}{R} \int d\tilde{\mathbf{P}} \tilde{\mathbf{P}}^3 |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2$$

for $R \rightarrow 0$ and $\mathbf{P} \sim 1/R$

$$\text{the energy } E = \sqrt{m^2 + \mathbf{P}^2} \sim \frac{1}{R}$$

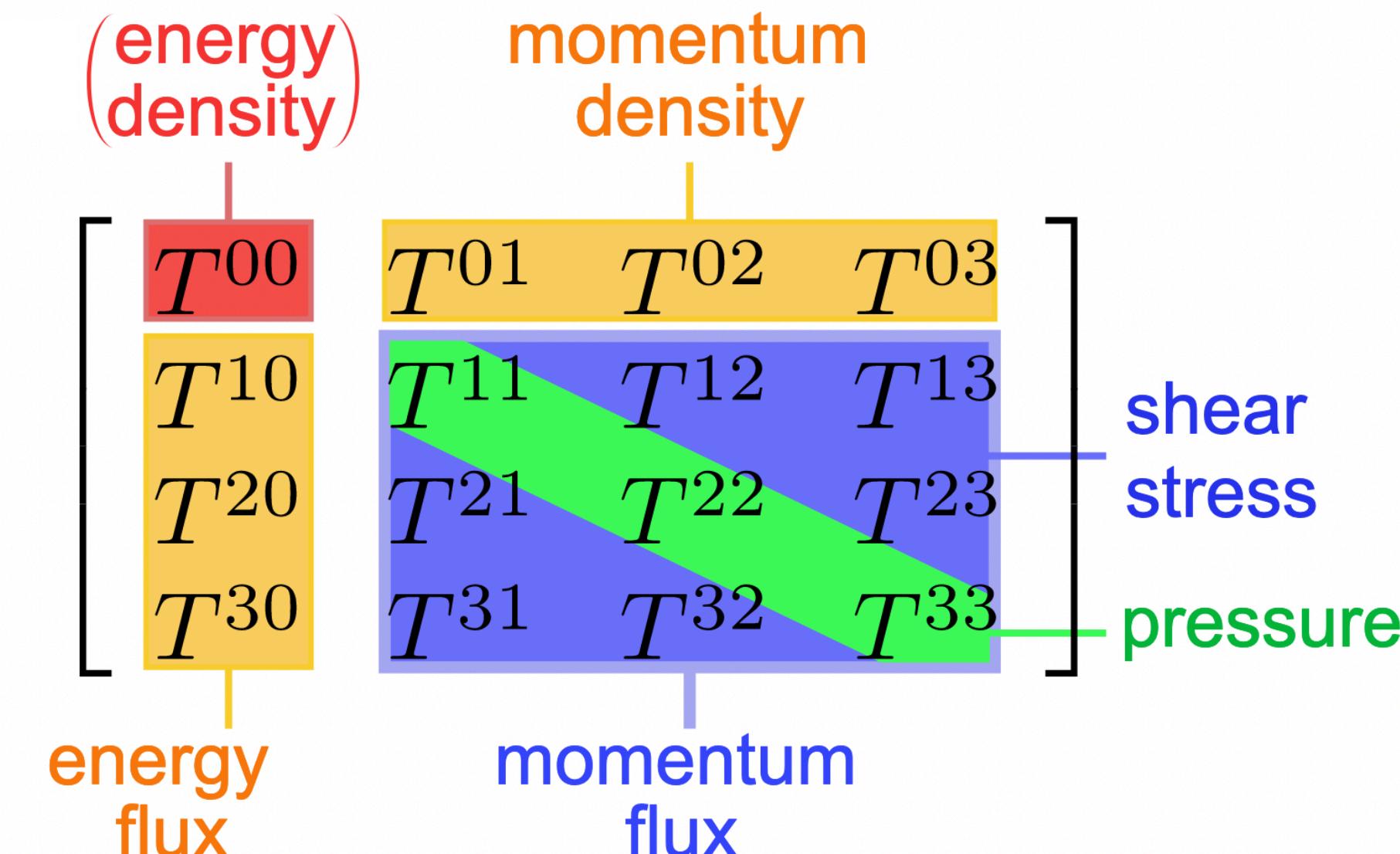
Static approximation ($m \rightarrow \infty, R \rightarrow 0$): $R \gg 1/m$

$$t_{\text{static}}^{00}(\mathbf{r}) = m \int \frac{d^3 q}{(2\pi)^3} A(-\mathbf{q}^2) e^{-i\mathbf{q} \cdot \mathbf{r}}$$

Mass distribution

for $m \rightarrow \infty, R \gg 1/m,$
 $\mathbf{P} \sim 1/R \ll m$

$$E = \sqrt{m^2 + \mathbf{P}^2} \simeq m + O(\mathbf{P}^2/(2m))$$



Pressure and shear force distributions

$$t_{\phi}^{ij}(\mathbf{r}) \equiv \langle \Phi, \mathbf{X}, s' | \hat{T}^{ij}(\mathbf{x}, 0) | \Phi, \mathbf{X}, s \rangle = t_{\phi,0}^{ij}(\mathbf{r}) + t_{\phi,2}^{ij}(\mathbf{r})$$

Interpretation

For sharply localised packet ($R \rightarrow 0$ and arbitrary m)

Static approximation ($m \rightarrow \infty, R \rightarrow 0$): $R \gg 1/m$

$$t_{\phi,2}^{ij}(\mathbf{r}) = \frac{1}{2} N_{\phi,0} \int d^2 \hat{n} \frac{d^3 q}{(2\pi)^3} (q^i q^j - \delta^{ij} \mathbf{q}_\perp^2) D(-\mathbf{q}_\perp^2) e^{-i\mathbf{q} \cdot \mathbf{r}}$$

$$t_{static,2}^{ij}(\mathbf{r}) = \frac{1}{4m} \int \frac{d^3 q}{(2\pi)^3} (-\mathbf{q}^2 \delta^{ij} + q^i q^j) D(-\mathbf{q}^2) e^{-i\mathbf{q} \cdot \mathbf{r}}$$

pressure **shear force**

$$t_2^{ij}(r) = \delta^{ij} p(r) + \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) s(r)$$

$$p(\mathbf{r}) = \frac{N_{\phi,R,2}}{2} \int d^2 \hat{n} \left(\frac{1}{r_\perp^2} \frac{d}{dr_\perp} r_\perp^2 \frac{d}{dr_\perp} - \frac{1}{3} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \right) (\delta(r_\parallel) \tilde{D}[\mathbf{r}_\perp])$$

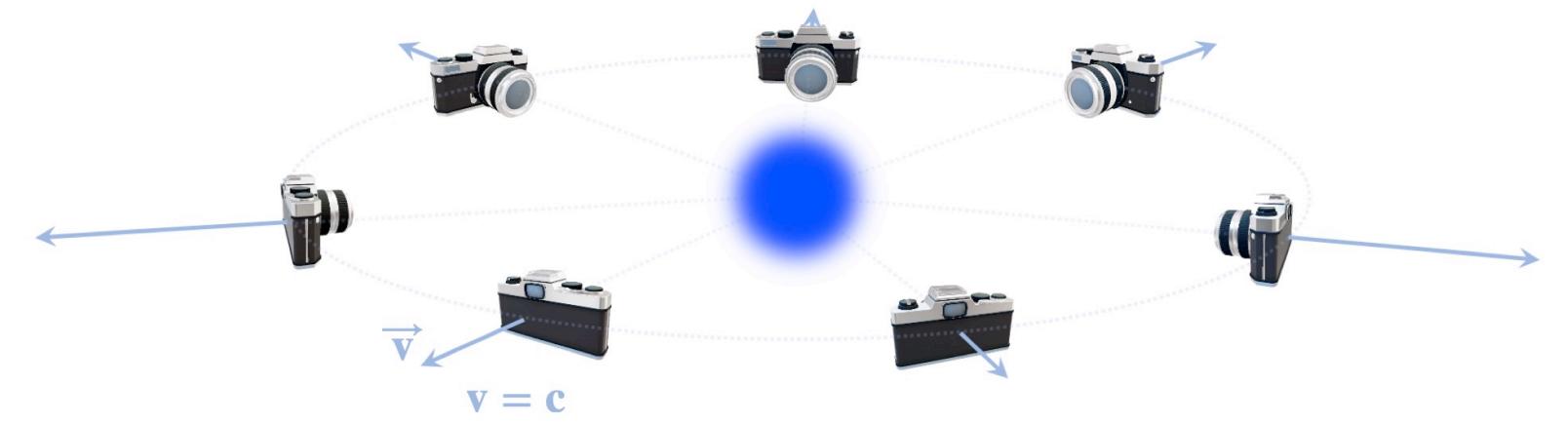
$$s(\mathbf{r}) = -\frac{N_{\phi,R,2}}{2} \int d^2 \hat{n} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} (\delta(r_\parallel) \tilde{D}[\mathbf{r}_\perp])$$

$$p_{static}(\mathbf{r}) = \frac{1}{6m} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}[\mathbf{r}]$$

$$s_{static}(\mathbf{r}) = -\frac{1}{4m} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{D}[\mathbf{r}]$$

Conclusions

- Novel definition of electromagnetic spatial densities in ZAMF using spherically symmetric wave packet
 - > applicable to system independently of the relation between Compton wavelength and other length scales
 - > independent of the radial form of the wave packet
- Generalisation for gravitational density distributions
 - > applicable to any system
 - > dependence on the wave packet only through a normalisation factors
- Holographic interpretation of the ZAMF densities in terms of the IMF densities



Thank you for your attention!

