Explicit renormalization of nuclear chiral EFT and non-perturbative effects.

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in collaboration with E. Epelbaum

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Outline

- Explicit renormalization in nuclear EFT: motivation
- Finite cutoff scheme
- Infinite cutoff scheme
- Summary

EFT: systematic expansion. Power counting. Theoretical error estimation.

Expansion parameter: (soft scale)/(hard scale) $Q = \frac{q}{\Lambda_b}$

$$q \in \{|\vec{p}|\,, M_\pi\}\,, \qquad \Lambda_b \sim M_\rho$$

"Perturbative" calculation of the S-matrix, spectrum, etc.

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$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \dots$$

Contains bare parameters → Renormalization: power counting for renormalized quantities

Explicit renormalization of nuclear chiral EFT is a complicated matter.
Non-perturbative effects.

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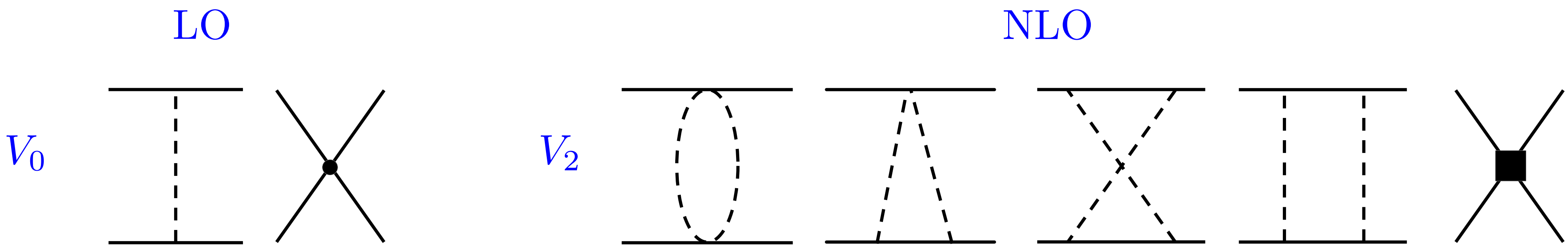
Recent progress: NN EFT at NLO

AG, Epelbaum, **PRC105**, 024001 (2022)

Power counting for NN chiral EFT

Weinberg, S., **NPB363**, 3 (1991)



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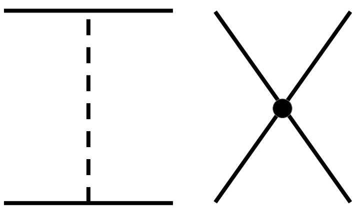
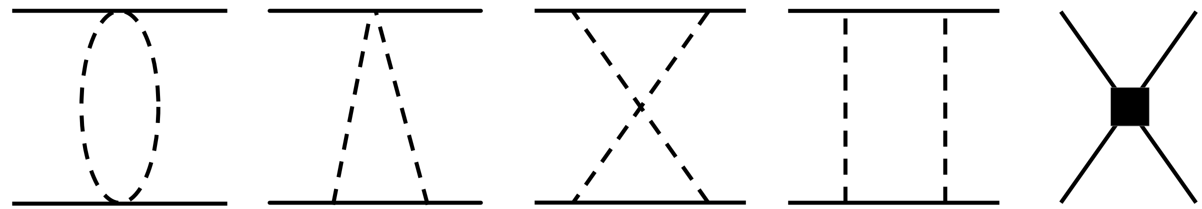
LO

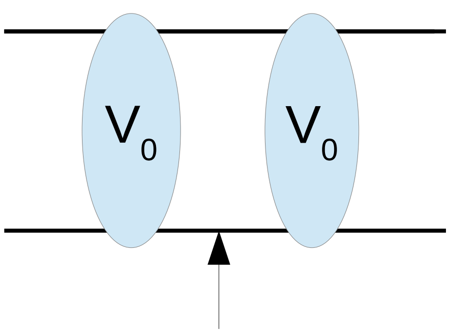
V_0



NLO

V_2





$$\sim \frac{m_N q}{\Lambda_b^2} \sim 1$$

Enhancement due to the infrared singularity: V_0 must be iterated

$$\text{LO: } T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \ldots$$

$$\text{NLO: } T_2 = V_2 + V_2 G V_0 + V_0 G V_2 + V_2 G V_0 G V_0 + \ldots$$

Regularization

The unrenormalized amplitude is divergent:

$$\text{LO: } T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \cdots = \sum_{n=0}^{\infty} T_0^{[n]}\,, \qquad T_0^{[n]} \sim p^n$$

$$\text{NLO: } T_2 = \sum_{m,n=0}^{\infty} (V_0 G)^m\, V_2\, (G V_0)^n = \sum_{m,n=0}^{\infty} T_2^{[m,n]}\,, \qquad T_2^{[m,n]} \sim p^{m+n+2}$$

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Two alternative schemes:
Finite cutoff
Infinite cutoff

Finite cutoff scheme

$\Lambda \approx \Lambda_b$

Cutoff dependence gets weaker when chiral order increases

Λ_b -chiral expansion breakdown scale

Phenomenological success (NN): $\geq N^4LO$

P. Reinert, H. Krebs, and E. Epelbaum, **EPJA54**, 86 (2018)
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Explicit renormalization: power counting?

Power counting. LO and NLO. Perturbative case.

Perturbative: the series in V_0 is convergent, but the number of iterations is arbitrary

$$\text{LO: } T_0^{[n]} = V_0 (G V_0)^n \sim \mathcal{O}(Q^0)$$

$$\Lambda \approx \Lambda_b : \int \frac{p^{n-1} dp}{(\Lambda_{\rm V})^n} \sim \left(\frac{\Lambda}{\Lambda_V}\right)^n \sim \left(\frac{\Lambda_b}{\Lambda_b}\right)^n \sim \mathcal{O}(Q^0)$$

G. P. Lepage, nucl-th/9706029
J. Gegelia, **JPG25**, 1681 (1999)

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$$\text{NLO: } T_2^{[m,n]} = (V_0 G)^m \, V_2 \, (G V_0)^n \sim \mathcal{O}(Q^0) \neq \mathcal{O}(Q^2)$$

Power-counting violating contributions from momenta: $p \sim \Lambda \, , p' \sim \Lambda$ in $V_2(p', p)$

Renormalization via the BPHZ procedure to all orders in V_0:
subtractions in all nested subdiagrams.
Power-counting breaking terms are absorbed by LO contact terms

$$\mathbb{R}(T_2^{[m,n]}) \sim \mathcal{O}(Q^2)$$

AG, E.Epelbaum, **PRC 105**, 024001 (2022)

Non-perturbative LO. Fredholm formula

LO: $T_0 = \frac{N_0(p)}{D(p)}$ NLO: $T_2(p) = \frac{N_2(p)}{D(p)^2}$

Convergent series in V_0: $N_0 = \sum_{i=0}^{\infty} N_0^{[i]}\,,\ N_2 = \sum_{i,j=0}^{\infty} N_2^{[i,j]}\,,\ D = \sum_{i=0}^{\infty} D^{[i]}$

→ Matching to the perturbative case

(Quasi-) bound states: $D(p) \sim 0$

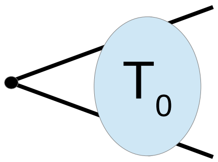
→ Enhancement at threshold: $T_0(p) \sim \mathcal{O}(Q^{-1})$

The same treatment for the non-perturbative (in V_0) counter terms: $\delta T_2 = (1 + T_0 G)\delta V_0^{ct}(1 + G T_0)$

Subtractions at NLO in the non-perturbative case

The series for ***R(T₂^[m,n])*** can be summed explicitly

$$\mathbb{R}\left(T_2\right)(p) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)(p) = T_2(p) - \delta C_0 \psi(p)^2$$

$\psi(p)$ =1+ 

$$\delta C_0 = \frac{T_2(0)}{\psi(p=0)^2}$$

$$\boxed{\mathbb{R}\left(T_2\right)(p=0) = 0}$$

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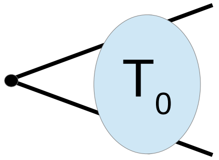
$\psi(p)$ =1+ 

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Potential problems:

$$\psi(p=0) = 0 \rightarrow \delta C_0 = \infty,$$
$$\psi(p) \neq \psi(p=0)$$

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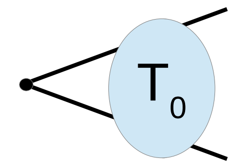
$\psi(p)$=1+ 

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Renormalizability constraints on the the short-range part of the LO potential:

$$\psi_{p=0} \not\approx 0$$

More constraints at higher orders!

Finite-cutoff scheme

In perturbative NN channels (P-waves and higher)
renormalization always works, at least formally

In non-perturbative channels ($^1S_0$, $^3S_1$-$^3D_1$, $^3P_0$)
Renormalization works under additional constraints on the LO potential (its short range part)

Infinite cutoff ($\Lambda >> \Lambda_b$) scheme, "RG invariant"

$\Lambda \to \infty$:

Cutoff independence for each chiral order individually!

A. Nogga, R. Timmermans,
U. van Kolck, **PRC72**, 054006 (2005)
M.P. Valderrama, **PRC84**, 064002 (2011)
B. Long, C. Yang, **PRC84**, 057001 (2011)

All positive powers of Λ cancel:

$$T \approx 1 + \Lambda + \Lambda^2 + \cdots = \frac{1}{1-\Lambda}$$

Motivation: singular potentials

W. Frank, D. J. Land and R. M. Spector,
Rev. Mod. Phys. 43, 36 (1971)

Criticism

E. Epelbaum, J. Gegelia, **EPJA41**, 341 (2009)
E. Epelbaum, AG, J. Gegelia,U.-G. Meißner, **EPJA54**, 186 (2018)

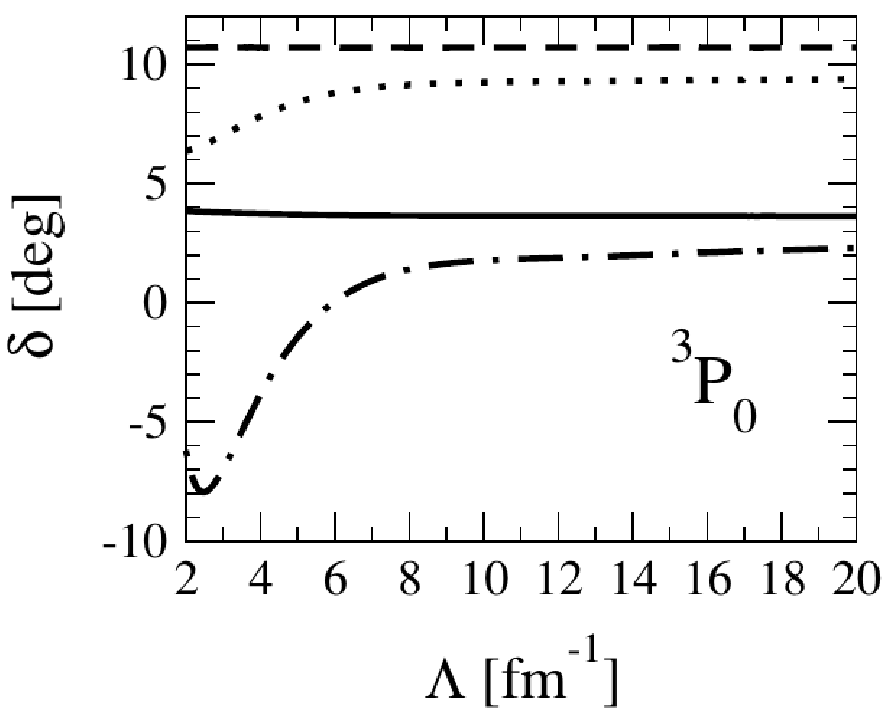
Infinite cutoff ("RG-invariant") scheme. LO. $^3P_0$

Seems to work at least for the nucleon-nucleon scattering at LO

$$V^{(0)}(p',p) = V_{1\pi}(p',p) + C_0^{(0)}(\Lambda)p'p$$

Renormalization condition: $\delta^{(0)}(E_0) = \delta_{\exp}(E_0)\,,\qquad E_0 = 50\,\mathrm{MeV}$

E_{lab}=10-190 MeV



A. Nogga, R. Timmermans,
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Infinite cutoff scheme at NLO. ${}^3P_0$

B. Long, C. J. Yang, **PRC84**, 057001 (2011)

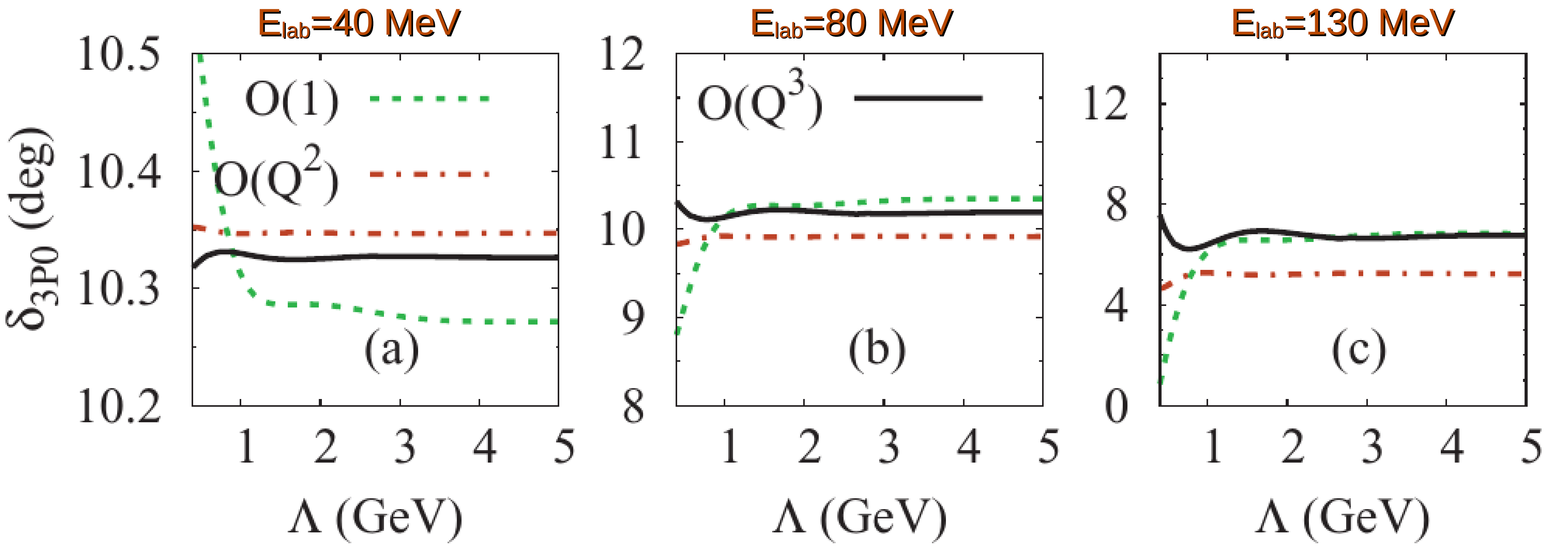
$$V^{(2)}(p',p) = V_{2\pi}(p',p) + C_0^{(2)}(\Lambda)p'p + C_2^{(2)}(\Lambda)p'p(p^2+p'^2)$$

Perturbative NLO: $T^{(2)} = \left[\mathbb{1} + T^{(0)}G\right]V^{(2)}\left[\mathbb{1} + GT^{(0)}\right]$

Additional renormalization conditions to fix C0 and C2:

$\delta^{(2)}(E_0) = 0\,,$ $\quad E_0 = 50\text{MeV}$

$\delta^{(2)}(E_1) = \delta_{\text{exp}}(E_1) - \delta^{(0)}(E_1)\,,$ $\quad E_1 = 25\text{MeV}$



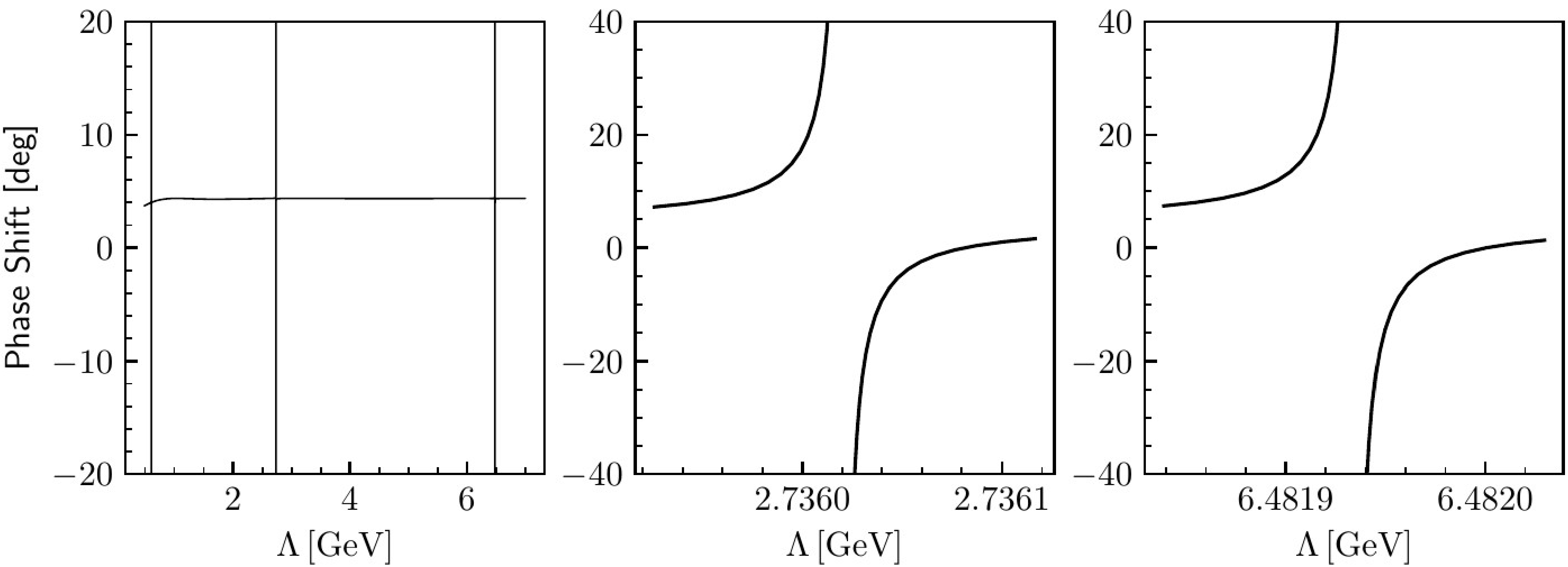
$^3P_0$ NLO phase shift at E_{lab}=130 MeV

AG, E.Epelbaum, *arXiv:nucl-th/2210.16225 (2022)*

Oscillations of the LO wave-function at short distances

↓

“Exceptionial cutoffs”



“Exceptional” cutoffs

$$T^{(2)}(E) = T_{2\pi}(E) + C_0^{(2)} T_{\mathrm{ct},0}(E) + C_2^{(2)} T_{\mathrm{ct},2}(E)$$

For some cutoffs:

$$\begin{vmatrix} T_{\mathrm{ct},0}(E_0) & T_{\mathrm{ct},2}(E_0) \\ T_{\mathrm{ct},0}(E_1) & T_{\mathrm{ct},2}(E_1) \end{vmatrix} = 0 \quad \Longrightarrow \quad C_0^{(2)}, C_2^{(2)} \to \infty$$

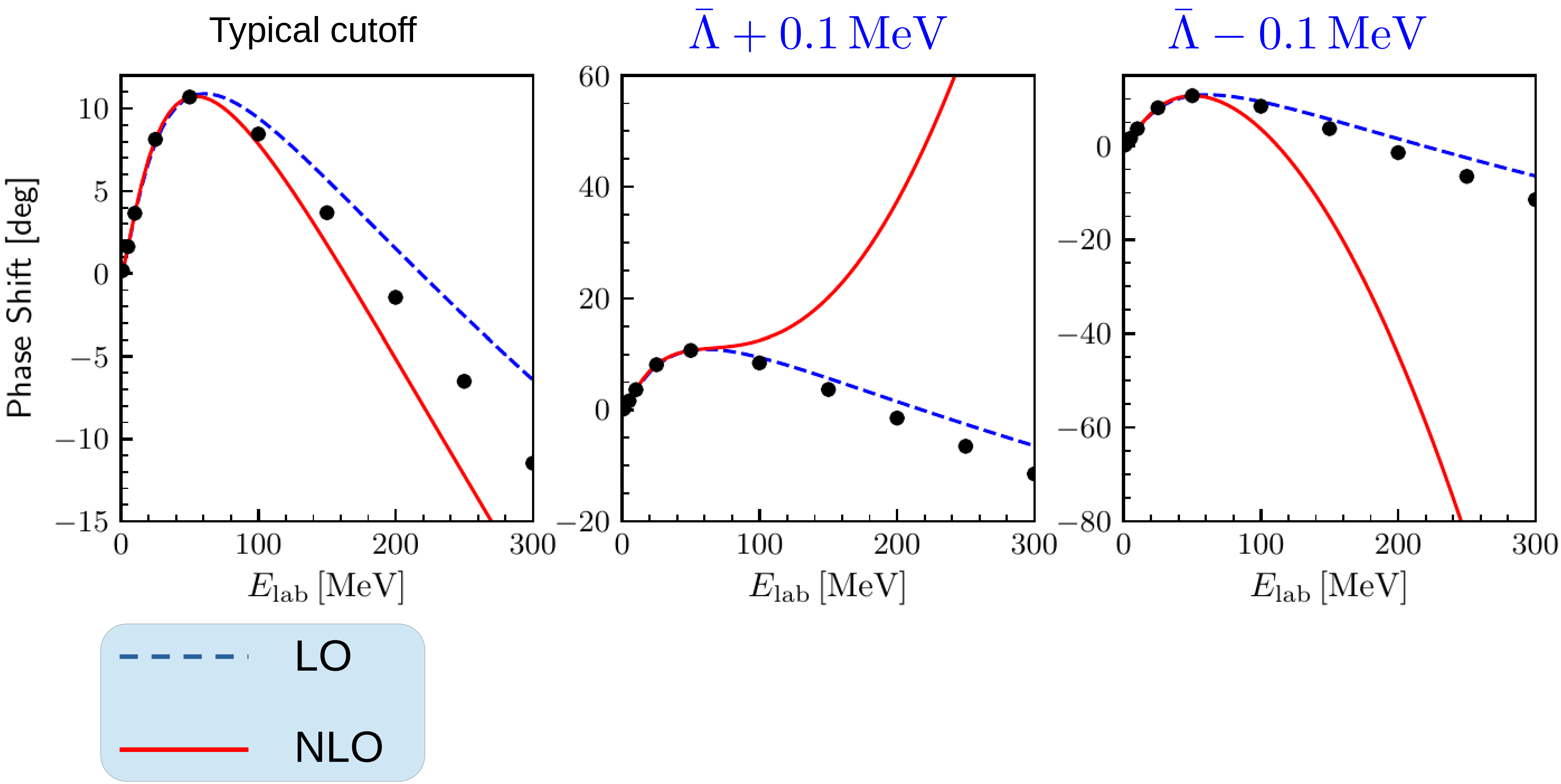
Residual cutoff dependence:

$$\delta T_{\mathrm{ct},i} \sim \left(\frac{p}{\Lambda}\right)^{\alpha}$$

is multiplied with an arbitrarily large number

$^3P_0$ phase shifts

"Exceptional" cutoff $\bar{\Lambda} \approx 12\,\mathrm{GeV}$



Infinite-cutoff scheme

“RG-invariant” scheme requires independence of the amplitude
from the form of a regulator and the value of the cutoff
for each chiral order individually

For a sufficiently general regulator,
there always exist “exceptional” cutoffs

↓

Renormalization breaks down

Summary

- ✔ Renormalization of NN Chiral EFT at NLO in the chiral expansion is understood
- ✔ Finite cutoff: renormalization works in perturbative channels. In nonperturbative channels the requirement of renormalizability imposes certain constraints on the LO potential
- ✔ In the infinite cutoff scheme, renormalization beyond LO does not work: “exceptional” cutoffs
- ✔ Extension to other systems (few- and many nucleon, electroweak currents) and higher orders is straightforward to analyze in a similar fashion