Heavy baryon spectroscopy in a quark-diquark approach

Speaker: André Torcato

Supervisors:

Prof. Dr. Gernot Eichmann[1]
Prof.ª Dr.ª Ana Arriaga[2,3]

1: University of Graz
2: Laboratório de Instrumentação e Física Experimental de Partículas (LIP)
3: Faculdade de Ciências da Universidade de Lisboa (FCUL)

Event:

BARYONS 2022 (10/11)

Acknowledgments:

Academic institutions of Lic. & MSc

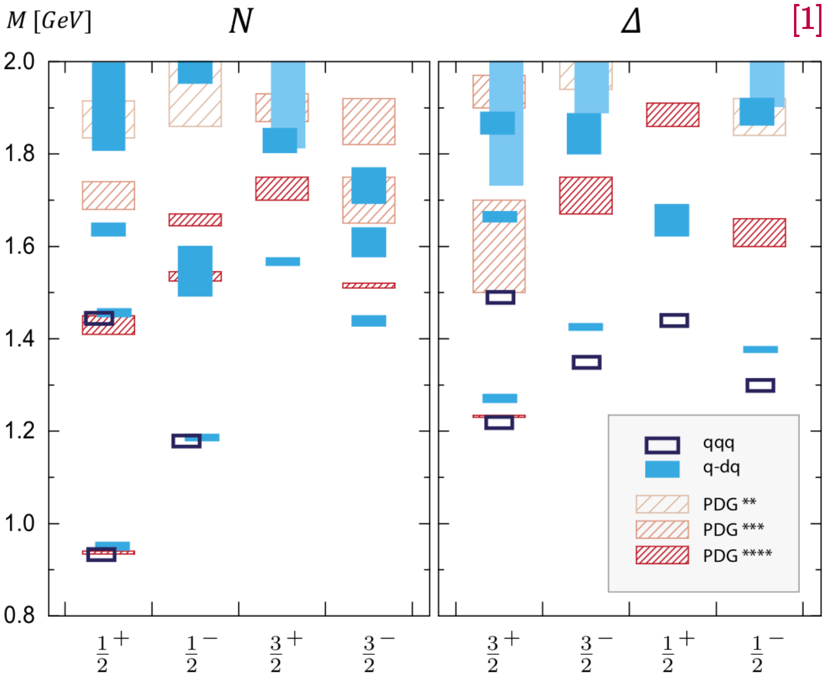
This work is supported by national funds through FCT - Fundação para a Ciência e a Tecnologia, I.P., under project CERN/FIS-PAR/0023/2021 & Multiquarks - EXPL IF/00898/2015

Objective

[1]: G. Eichmann, C. S. Fischer, H. Sanchis-Alepuz, arXiv:1607.05748
[2]: pdg.lbl.gov

Problem

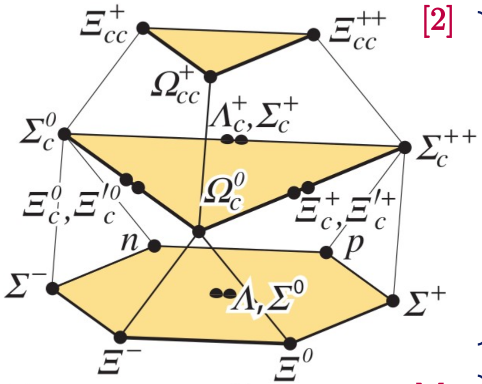
No unified description for all baryons
- Low-energy QCD is non-perturbative
- Three-body eq. is difficult to solve



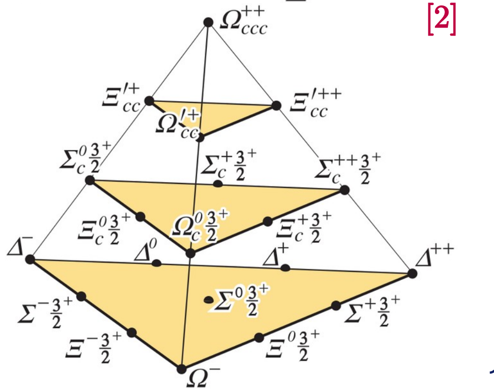
Flavors: $\{n, s, c(, b)\}$

Three valence quarks: $q_1 q_2 q_3$

Spin multiplets:



$J = \frac{1}{2}$



$J = \frac{3}{2}$

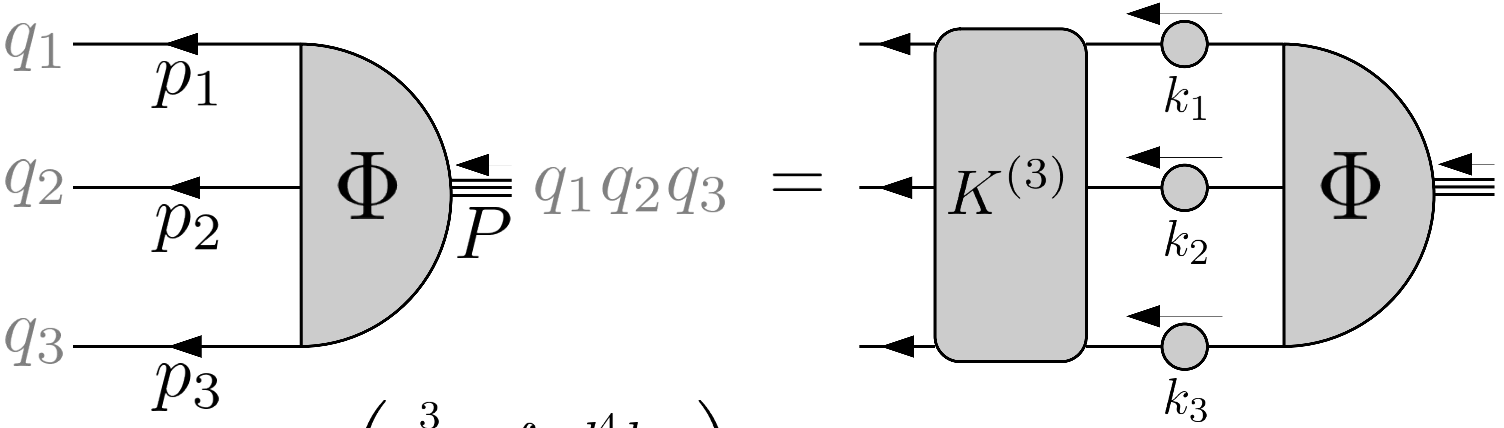
Objective

Use quark-diquark (q-qq) model to calculate light, strange & charmed baryon masses. Success in this approach: e.g.: [1]

Methodology

[3]: G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, arXiv:1606.09602; [4-5]: G. Eichmann, arXiv:0909.0703, arXiv:2202.13378; [6]: M. Yu. Barabanov, et. al., arXiv:2008.076309; [7]: J. C. R. Bloch, et. al., arXiv:nucl-th/9907120; [8]: R. T. Cahill, et. al., Phys. Rev. D 36; [9-10]: P. Maris, Few-Body Sys. 32, 35; [11]: M. Oettel, arXiv:nucl-th/0012067

Bethe-Salpeter amplitude (BSA) & equation (BSE) of a baryon



The baryon & the quarks inhabit Dirac, flavor and color spaces:

$\Phi_{\alpha\beta\delta;\gamma}$ $\quad K^{(3)}_{\alpha\beta\delta;\alpha'\beta'\delta'}$

$(G_0^{(3)})_{\alpha\beta\delta;\alpha'\beta'\delta'}$

$$\Phi\left(\{p\};P\right) = \left(\prod_{a=1}^{3}\int\frac{d^4k_a}{(2\pi)^4}\right) K^{(3)}\left(\{p\},\{k\}\right) G_0^{(3)}\left(\{k\}\right)\Phi\left(\{k\};P\right)$$

Dominance of quark pair interactions

$$K^{(3)} \approx \sum_{a=1}^{3} K_a^{(2)}$$

a : spectator quark (q)

b, c : interacting quark pair (qq)

$a, b, c = \{1,2,3\}_{\text{Cyclical}}$

Color multiplets for qq: $3 \otimes 3 = \bar{3} \oplus 6$

Color multiplets for q-qq:

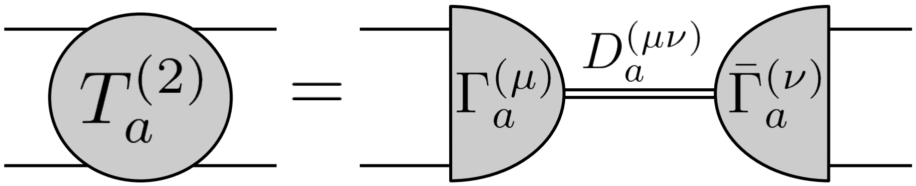
$3 \otimes (\bar{3} \oplus 6) = (1 \oplus 8) \oplus (8 \oplus 10)$

Methodology

[3]: G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, arXiv:1606.09602; [4-5]: G. Eichmann, arXiv:0909.0703, arXiv:2202.13378; [6]: M. Yu. Barabanov, et. al., arXiv:2008.076309; [7]: J. C. R. Bloch, et. al., arXiv:nucl-th/9907120; [8]: R. T. Cahill, et. al., Phys. Rev. D 36; [9-10]: P. Maris, Few-Body Sys. 32, 35; [11]: M. Oettel, arXiv:nucl-th/0012067

Quark-diquark model ansatz

1 $K_a^{(2)} \xrightarrow[\text{to}]{\text{related}} T_a^{(2)}$ via Dyson eq.



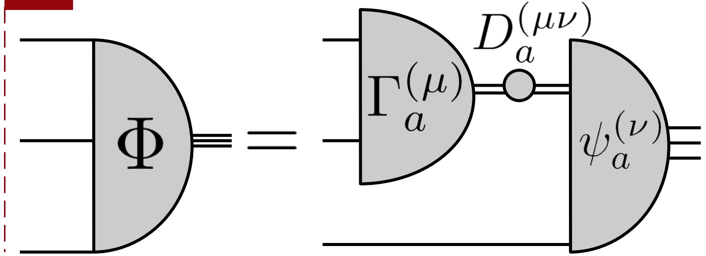
SC qq (~ PS q$\bar{\text{q}}$):

No L.i. [Lorentz ind.]

AV qq (~ V q$\bar{\text{q}}$):

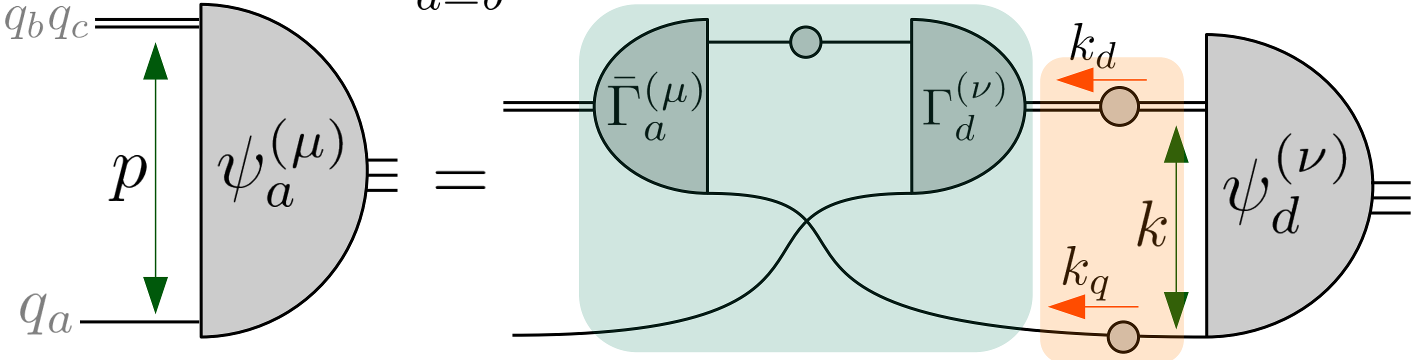
$\mu, \nu = \{1,\ldots,4\}$

2 q-qq BSA related to qqq BSA



Quark-diquark BSE

$$\psi^{(\mu)}_{a;\alpha\gamma}(p,P) = \sum_{d=b}^{c} \int \frac{d^4k}{(2\pi)^4} K^{(\mu\rho)}_{ad;\alpha\beta}(p,k,P)\, G^{(\rho\nu)}_{d;\beta\delta}(k_d,k_q)\, \psi^{(\nu)}_{d;\delta\gamma}(k,P)$$



Baryon's rest frame

$P = iM(0\ 0\ 0\ 1)^T$

All momenta expressed by:

$p, k, \eta P$

Methodology

[3]: G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, arXiv:1606.09602; [4-5]: G. Eichmann, arXiv:0909.0703, arXiv:2202.13378; [6]: M. Yu. Barabanov, et. al., arXiv:2008.076309; [7]: J. C. R. Bloch, et. al., arXiv:nucl-th/9907120; [8]: R. T. Cahill, et. al., Phys. Rev. D 36; [9-10]: P. Maris, Few-Body Sys. 32, 35; [11]: M. Oettel, arXiv:nucl-th/0012067

Color structure

$$\Gamma^{(\mu)}_{\text{Color}} = \frac{\epsilon_{ABE}}{\sqrt{2}}$$

Antisymmetric anti-triplet:

$3 \otimes 3 = \bar{3} \oplus 6$

$$\psi^{(\mu)}_{\text{Color}} = \frac{\delta_{AB}}{\sqrt{3}}$$

Re-call the ansatz:

$$\Phi = \Gamma^{(\mu)} D^{(\mu\nu)} \psi^{(\nu)}$$

Dirac structure

1 General form: $\sum_{i=1}^{N_{\text{Dirac}}} f_i\left(p^2, p\cdot P\ , P^2 = -M^2\right) \sigma_i\left(p, P\right)$

Dirac dressing functions — f_i

Dirac tensor structure — $\sigma_i(p,P)$

E.g.: (SC qq)-q for J=1/2:

$$\sigma_i\left(p, P\right) = \tau_i\left(p, P\right) \Lambda_+\left(P\right)$$

2 $\Gamma^{(\mu)}_{\text{Dirac}}$

- $J^P = 0^+$: SC qq [4 tensors; 0 L.i.]
- $J^P = 1^+$: AV qq [12 tensors; 1 L.i.]

$\psi^{(\mu)}_{\text{Dirac}}$

- $J^P = 1/2^+$: (SC qq)-q [2 tensors; 0 L.i.]
  (AV qq)-q [6 tensors; 1 L.i.]
- $J^P = 3/2^+$: (AV qq)-q [8 tensors; 2 L.i.]

3 Orbital angular momentum:

$\tau_i^{(\mu(,\nu))} \propto 1$	$\tau_i^{(\mu(,\nu))} \propto p$	$\tau_i^{(\mu(,\nu))} \propto p^2$	$\tau_i^{(\mu(,\nu))} \propto p^3$
s-wave	p-wave	d-wave	f-wave

Methodology

[3]: G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, arXiv:1606.09602; [4-5]: G. Eichmann, arXiv:0909.0703, arXiv:2202.13378; [6]: M. Yu. Barabanov, et. al., arXiv:2008.076309; [7]: J. C. R. Bloch, et. al., arXiv:nucl-th/9907120; [8]: R. T. Cahill, et. al., Phys. Rev. D 36; [9-10]: P. Maris, Few-Body Sys. 32, 35; [11]: M. Oettel, arXiv:nucl-th/0012067

Flavor structure

Premise: Construct matrices that uniquely identify each baryons' flavor composition

1st step: Quark flavor states & basis:

Flavor state is: $R\,|I; I_3; Y; C\rangle$

$$u \to \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad d \to \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad s \to \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad c \to \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

E.g.:

$$d \to |\tfrac{1}{2}; -\tfrac{1}{2}; 0; 0\rangle$$

$$s \to |0; 0; -\tfrac{2}{3}; 0\rangle$$

2nd step: Diquark flavor states & matrices

Irreducible representations of $S_2 \otimes SU(4)_F : 4 \otimes 4 = 6_{\mathrm{A}} \oplus 10_{\mathrm{S}}$

Matrix representation: $\mathrm{q_b}$ is a column vector. $\mathrm{q_c}$ is a row vector. Analogous to mesons ($\mathrm{q\bar{q}}$)

What we do: SC qq are [.,.] & AV qq are {.,.}. Reasoning:

- Pauli principle of total qq BSA
- Leading Dirac tensor for each qq (SC is antisym.; AV is sym.)

For 6_{A} : $[q_b, q_c]$

For 10_{S}: $\{q_b, q_c\}$

E.g.: $6_{\mathrm{A}}\,|\tfrac{1}{2}; -\tfrac{1}{2}; -\tfrac{1}{3}; 0\rangle = \frac{1}{\sqrt{2}}[d, s]$

$$\to \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & \frac{1}{\sqrt{2}} & 0\\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

Methodology

[3]: G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, arXiv:1606.09602; [4-5]: G. Eichmann, arXiv:0909.0703, arXiv:2202.13378; [6]: M. Yu. Barabanov, et. al., arXiv:2008.076309; [7]: J. C. R. Bloch, et. al., arXiv:nucl-th/9907120; [8]: R. T. Cahill, et. al., Phys. Rev. D 36; [9-10]: P. Maris, Few-Body Sys. 32, 35; [11]: M. Oettel, arXiv:nucl-th/0012067

Flavor structure

3rd step: Baryon flavor states & matrices

$\mathcal{A}$ $\mathcal{D}_1 , \mathcal{D}_2$ $\mathcal{S}$ | The analogy of [.,.] & {.,.}

Irreducible representations of $S_3 \otimes SU(4)_F : 4 \otimes 4 \otimes 4 = \bar{4}_\mathrm{A} \oplus 20_\mathrm{MA} \oplus 20_\mathrm{MS} \oplus 20_\mathrm{S}$

(SC qq)-q   (SC qq)-q & (AV qq)-q   (AV qq)-q

1. We obtain 64 different flavor states
2. We organize them in the 20 combos. of {u,d,s,c}
3. We organize these combos. in the 21 baryons built with {n,s,c} [E.g.: nns is Λ , Σ and Σ^*]
4. We calculate the matrix representations and put the diquark matrices in evidence

Trace of the q-qq BSE

Color: $\frac{\delta_{BA}}{\sqrt{3}} \frac{\epsilon_{AED}}{\sqrt{2}} \frac{\epsilon_{CEB}}{\sqrt{2}} \frac{\delta_{CD}}{\sqrt{3}} = -1$

What about flavor?

Dirac: Basis is built to be **orthonormal**

Dressing functions are expanded in **Chebyshev** series and are computed dynamically. Number of Chebyshev moments (including 0$^{\text{th}}$ degree): N_Cheb

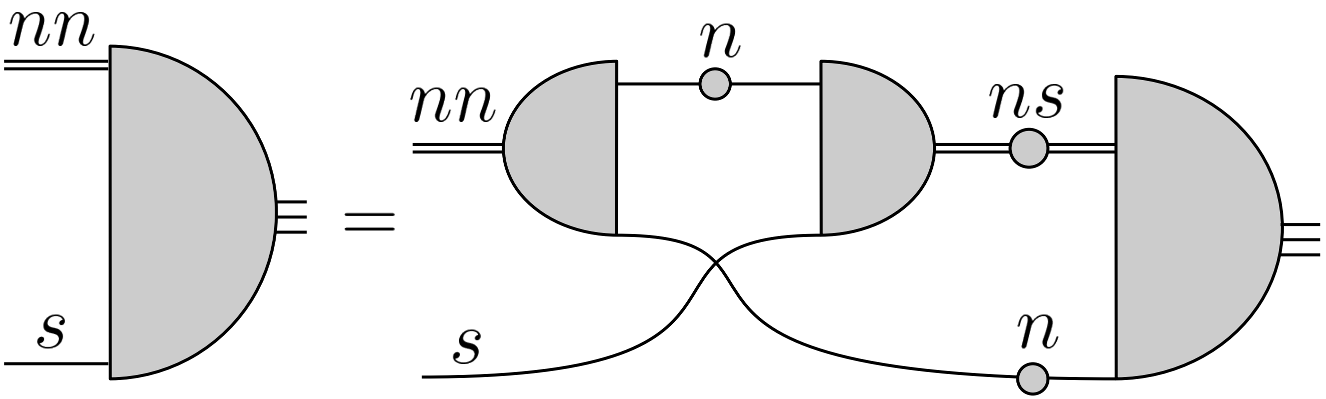
Methodology

[3]: G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, arXiv:1606.09602; [4-5]: G. Eichmann, arXiv:0909.0703, arXiv:2202.13378; [6]: M. Yu. Barabanov, et. al., arXiv:2008.076309; [7]: J. C. R. Bloch, et. al., arXiv:nucl-th/9907120; [8]: R. T. Cahill, et. al., Phys. Rev. D 36; [9-10]: P. Maris, Few-Body Sys. 32, 35; [11]: M. Oettel, arXiv:nucl-th/0012067

Flavor factors

E.g.: $\Sigma \ [nns]$

q-qq possibilities:
1) n-ns
2) s-nn



q-qq transition: s-nn to n-ns

The other transitions are:
n-ns to s-nn
n-ns to n-ns

q-qq transition as a matrix:

$$\begin{pmatrix} nn_{\mathrm{SC}} \to ns_{\mathrm{SC}} & nn_{\mathrm{SC}} \to ns_{\mathrm{AV}} \\ nn_{\mathrm{AV}} \to ns_{\mathrm{SC}} & nn_{\mathrm{AV}} \to ns_{\mathrm{AV}} \end{pmatrix} \equiv \mathrm{Mat}(nn \to ns)$$

All q-qq transitions:

$$\begin{pmatrix} 0 & \mathrm{Mat}(nn \to ns) \\ \mathrm{Mat}(ns \to nn) & \mathrm{Mat}(ns \to ns) \end{pmatrix}$$

Solution strategy: q-qq BSE as an eigenvalue problem

$$\psi_{\mathrm{state}}\left(P^2\right) = \lambda_{\mathrm{state}} \, \mathrm{Mat}\left[K\left(P^2\right) G\left(P^2\right)\right] \psi_{\mathrm{state}}\left(P^2\right) \quad \text{w/ } P^2 = -M^2_{\mathrm{state}}$$

Build a sample of baryon masses: $\{M_{\mathrm{min}},\ldots,M_{\mathrm{max}}\}$
Input them in the equation and calculate the eigenvalues of the matrix
If the eigenvalue equals 1, we recuperate the original BSE, meaning:

$$\lambda_{\mathrm{state}} = 1 \Rightarrow M_{\mathrm{state}}$$ is physical

Methodology

[3]: G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, arXiv:1606.09602; [4-5]: G. Eichmann, arXiv:0909.0703, arXiv:2202.13378; [6]: M. Yu. Barabanov, et. al., arXiv:2008.076309; [7]: J. C. R. Bloch, et. al., arXiv:nucl-th/9907120; [8]: R. T. Cahill, et. al., Phys. Rev. D 36; [9-10]: P. Maris, Few-Body Sys. 32, 35; [11]: M. Oettel, arXiv:nucl-th/0012067

Final considerations

Explicit expressions of quark & diquark propagators, diquark amplitude, quark & diquark masses were obtained with RL interaction in the Alkofer-Watson-Weigel form:

$\alpha(p^2) \propto e^{-(p/\Lambda)^2}$ | Current q masses $m_n \simeq 3$ MeV, $m_s \simeq 93$ MeV, $m_c \simeq 1270$ MeV | $S, D \propto \frac{1}{p^2}$ $\quad \Gamma \propto e^{-p^2}$

Dimension of the matrix:

$N_{\text{Dirac}} \times N_{\text{Cheb}} \times N_{\text{Momenta}}$ | E.g.: nns baryon until 5$^{\text{th}}$ degree moment with 36 momenta $= 16 \times 6 \times 36 = 3456$

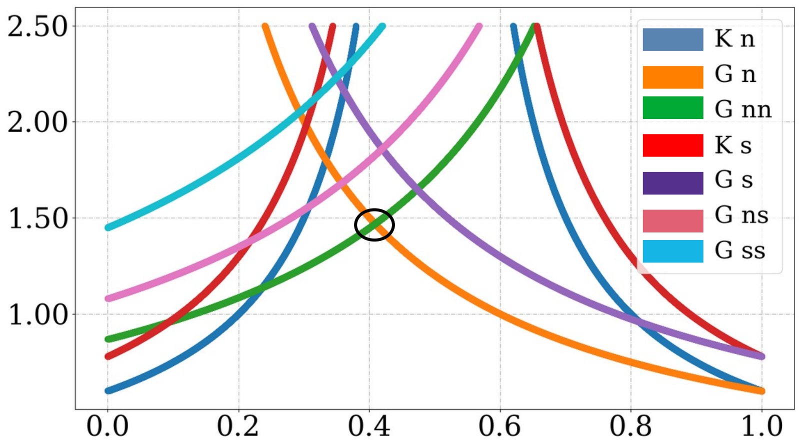
Maximum baryon mass of our sample:

Any momentum r can be put into the form:

$r^\mu = g(p^\mu, k^\mu) + h(\eta) P^\mu$ w/ $r^\mu \in \mathbb{C}$

$\Rightarrow \text{IMG}[r^2] \geq -h(\eta)^2 M^2$

Propagator poles define the value of $\text{IMG}[r^2]$



Plot: $h(\eta)$ vs. η

At the value of $\eta = 0.4087$ we get:

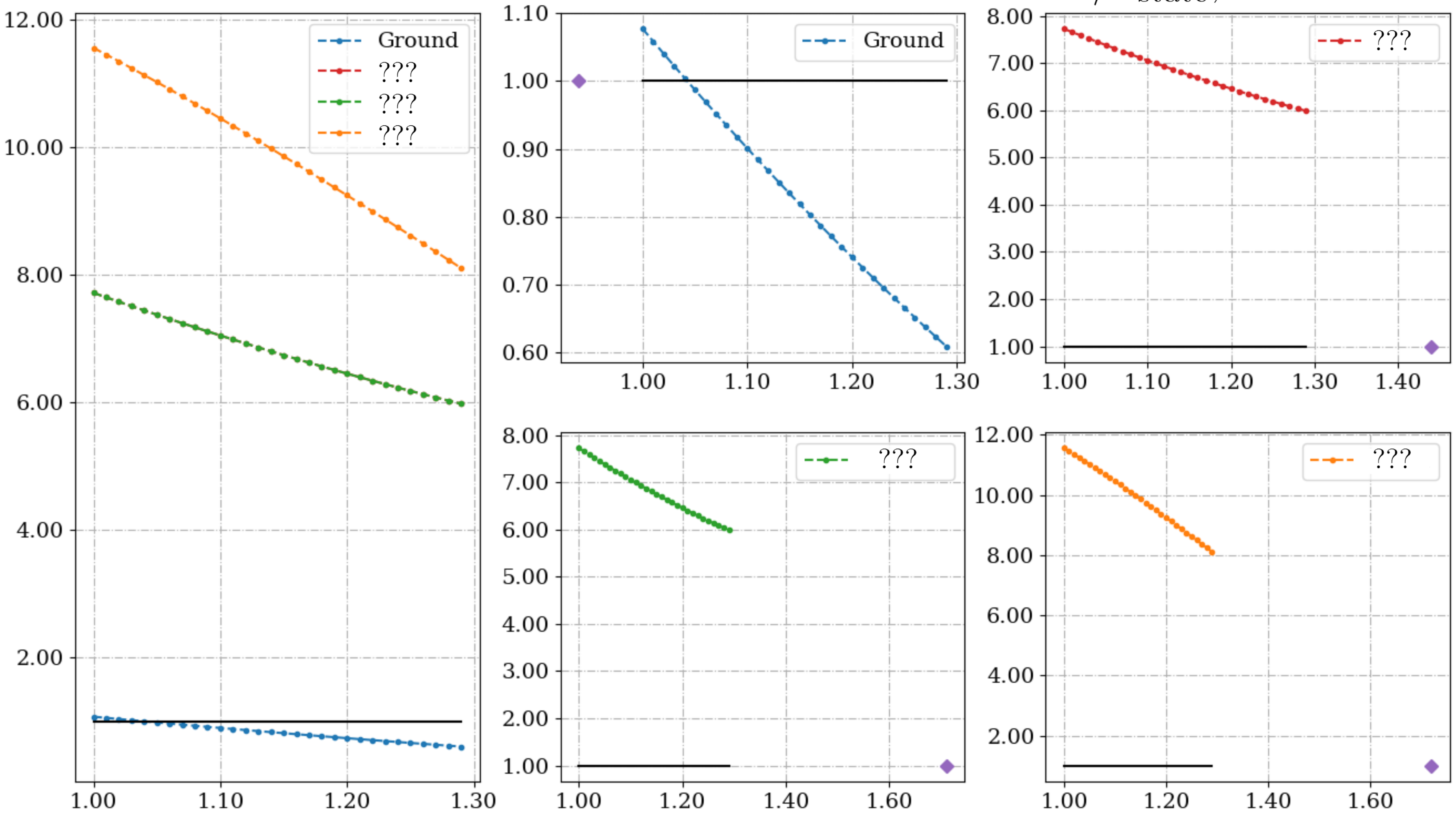
$M_{\max} = 1.4697$ GeV

Eigenvalue spectrum

PRELIMINARY

[2]: pdg.lbl.gov

Vertical Axis: $1/\lambda_{\text{state}}$; Horizontal axis: M [GeV]



Baryon

N (nnn)

Colors

: experimental data from [2]

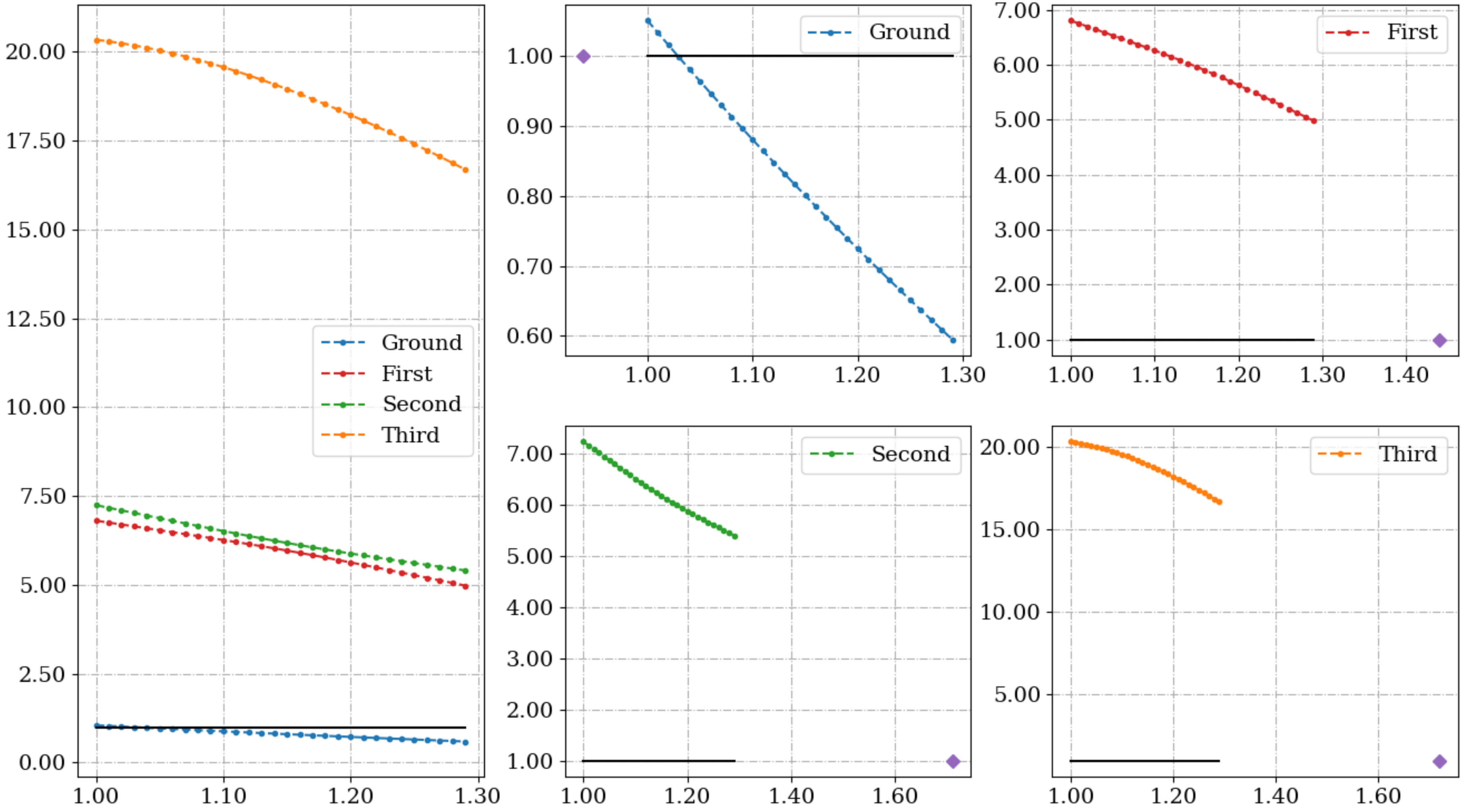
There exist complex eigenvalues – what to do?

Eigenvalue spectrum

PRELIMINARY

[2]: pdg.lbl.gov

Vertical Axis: $1/\lambda_{\text{state}}$; Horizontal axis: M [GeV]



Baryon

N (nnn)

Colors

: experimental data from [2]

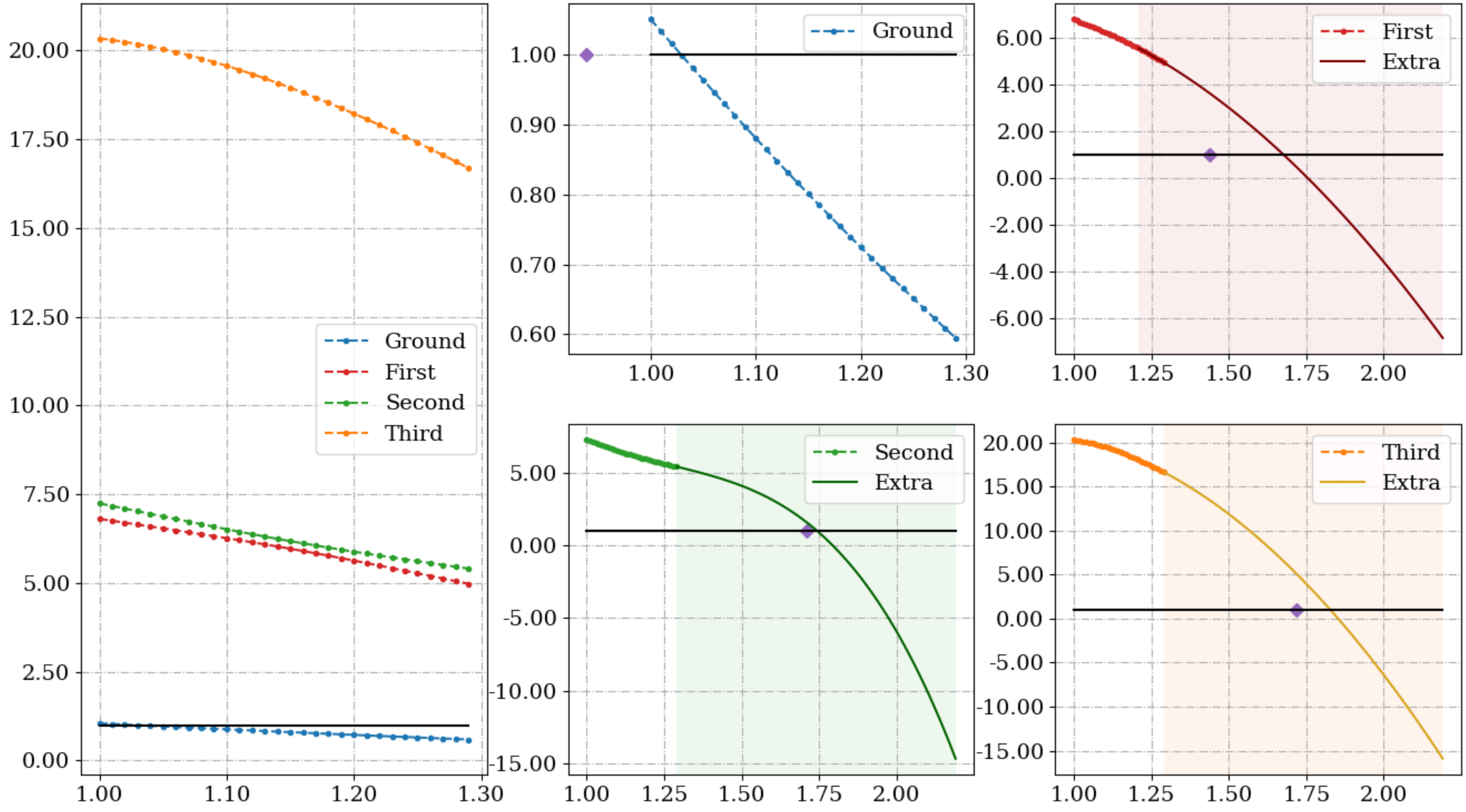
Answer: Spectral reconstruction (SR) of G with positive EVs only

Eigenvalue spectrum

PRELIMINARY

[2]: pdg.lbl.gov

Vertical Axis: $1/\lambda_{\text{state}}$; Horizontal axis: M [GeV]



Baryon

N (nnn)

Colors

: experimental data from [2]

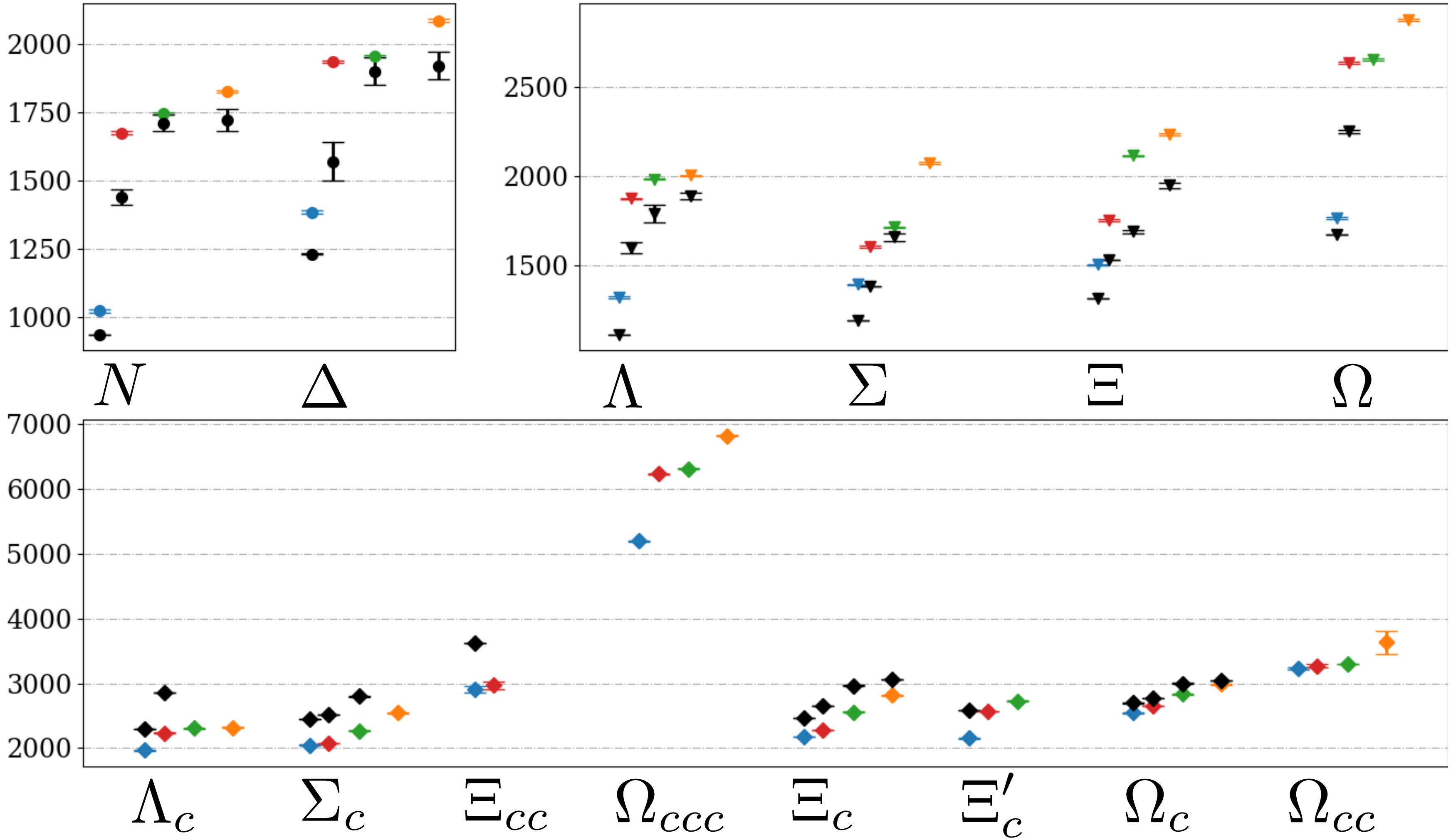
For states that did not cross EV = 1 inside mass grid, we extrapolate

Mass Spectrum

PRELIMINARY

[2]: pdg.lbl.gov
(**** & ***, J = 1/2+, 3/2+ states)

Vertical Axis: M [MeV]; Horizontal axis: Baryon



Shapes

●: {n} flavor
▼: {n,s} flavors
◆: {n,s,c}

Colors

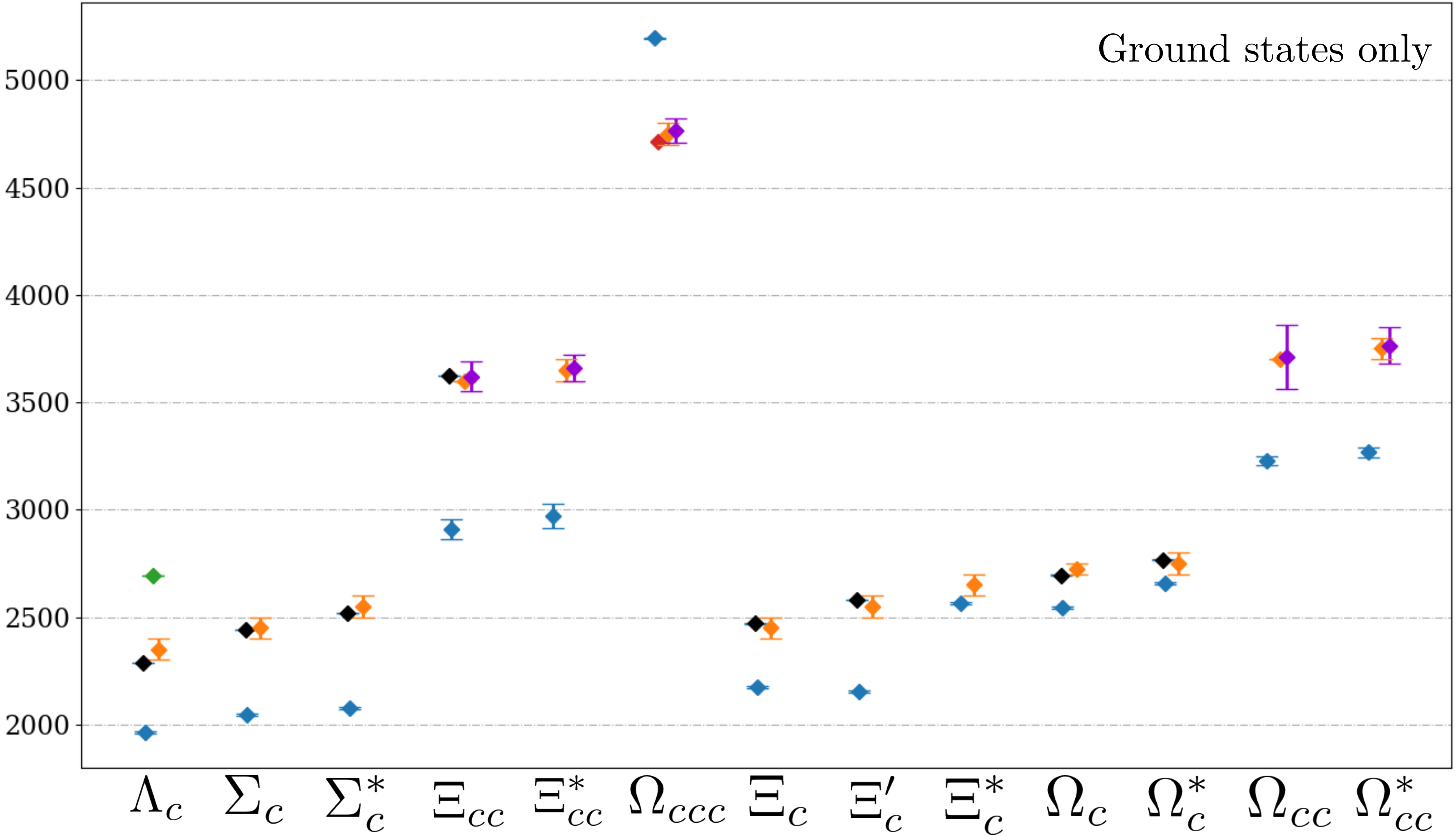
: fundamental state
: first excited
: second excited
: third excited
: experimental data from [2]

Charmed baryons

PRELIMINARY

[2]: pdg.lbl.gov

Vertical Axis: M [MeV]; Horizontal axis: Baryon



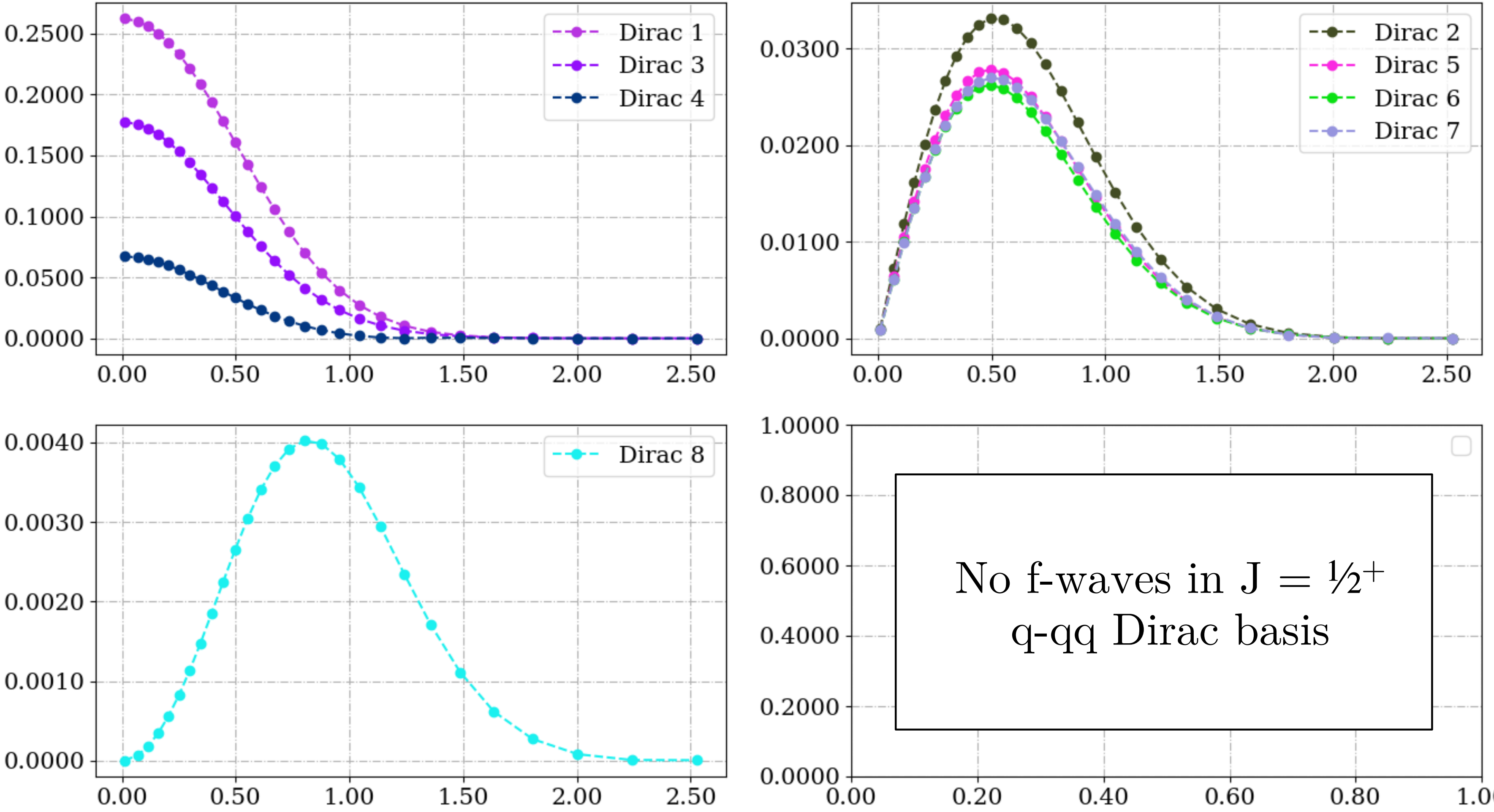
Colors

- : our calculations
- : theoretical data from [12]
- : lattice data from [13]
- : lattice data from [14]
- : average lattice data from [15-22]
- : experimental data from [2]

Cheb. 0^{th} moment

PRELIMINARY

Vertical Axis: $\left|\psi_{i0}\left(p_k^2; -M_{\text{Ground}}^2\right)\right|$; Horizontal axis: p_k [GeV]



Baryon

N (nnn)

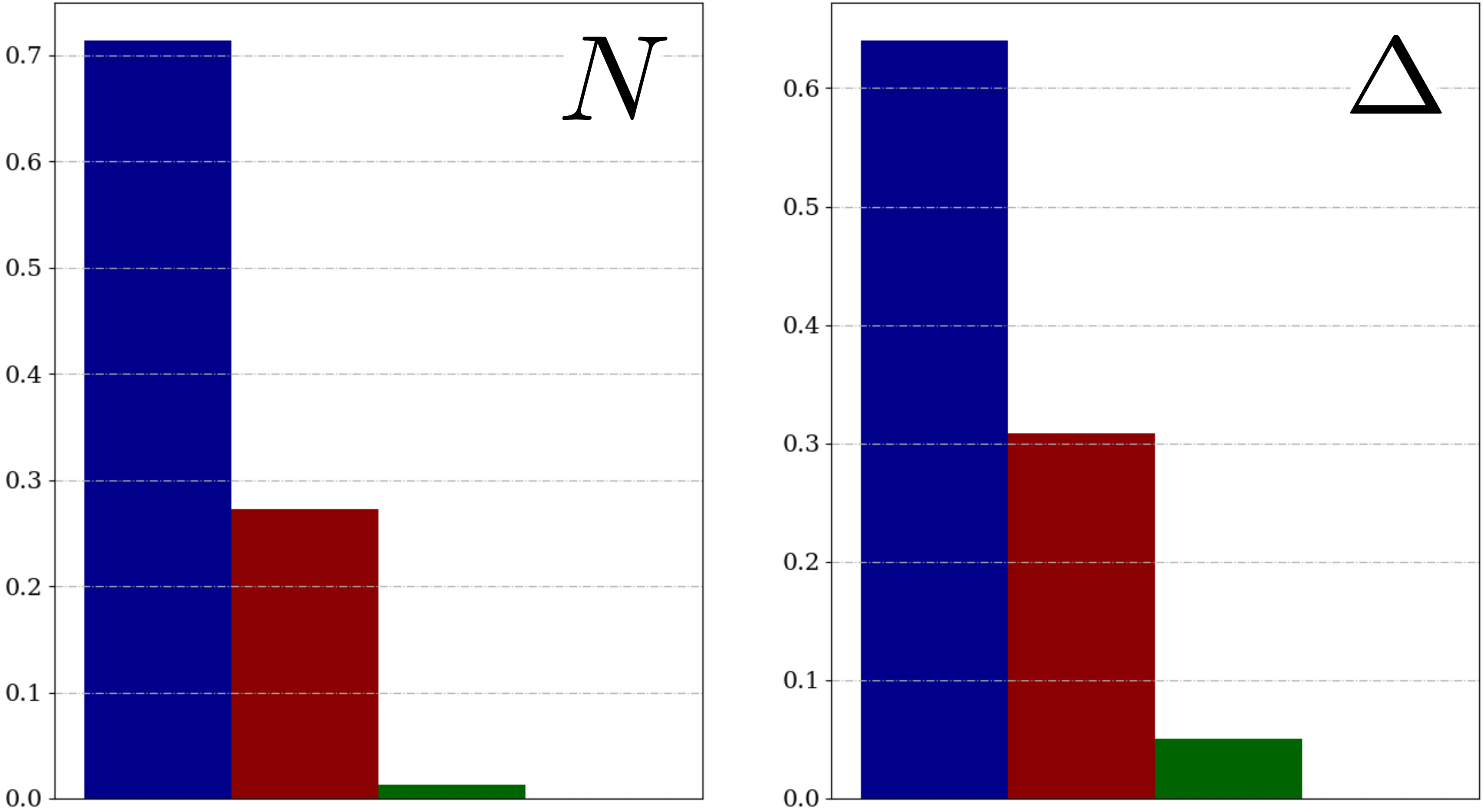
Labels

Dirac i $\Leftrightarrow$ τ_i

Partial waves

PRELIMINARY

Vertical Axis: % contribution; Horizontal axis: Baryon



Colors

: s-waves

: p-waves

: d-waves

: f-waves

Details

Evaluated at ground state's physical mass

Preliminary conclusions

1. Overestimation of light and strange baryon masses
2. Underestimation of charmed baryon masses
3. General agreement of relative mass differences between states, from our calculations and experimental results
4. s-waves dominate but p-waves contribute significantly

Outlook

1. BSA and partial wave contributions for excited states
2. Further study of the extrapolation schemes
3. Implementation of full RL instead of AWW
4. Move to three-body calculations and beyond RL

Thank you!

References

References

[1]: G. Eichmann, C. S. Fischer, H. Sanchis-Alepuz, “On light baryons and their excitations” (2016), Phys. Rev. D 94: 094033, arXiv:1607.05748

[2]: pdg.lbl.gov

[3]: G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, “Baryons as relativistic three-quark bound states” (2016), Prog. Part. Nucl. Phys. 91: 1-100, arXiv:1606.09602

[4]: G. Eichmann, “Hadron properties from QCD bound-state equations” (2009), PhD Thesis, arXiv:0909.0703

[5]: G. Eichmann, “Theory introduction to baryon spectroscopy” (2022), Few Body Sys. 63: 57, arXiv:2202.13378

[6]: M. Yu. Barabanov, et. al., “Diquark correlations in hadron physics: origin, impact and evidence” (2021), Prog. Part. Nucl. Phys. 116: 103835, arXiv:2008.07630

References

[7]: J. C. R. Bloch, C. D. Roberts, S. M. Schmidt, A. Bender, M. R. Frank, “Nucleon form factors and a nonpointlike diquark” (1999), Phys. Rev. C 60: 062201, arXiv:nucl-th/9907120

[8]: R. T. Cahill, C. D. Roberts, J. Praschifka, “Calculation of diquark masses in QCD” (1987), Phys. Rev. D 36: 2804

[9]: P. Maris, “Effective masses of diquarks” (2002), Few-Body Sys. 32: 41-52

[10]: P. Maris, “Electromagnetic properties of diquarks” (2004), Few-Body Sys. 35: 117-127

[11]: M. Oettel, “Baryons as relativistic bound states of quark and diquark” (2000), PhD Thesis, arXiv:nucl-th/0012067

[12]: R. N. Faustov, V. O. Galkin, “Triple heavy baryon spectroscopy in the relativistic quark model” (2021), Phys. Rev. D 105: 014013, arXiv:2111.07702

References

[13]: K. Watanabe, "Quark-diquark potential and diquark mass from Lattice QCD" (2022), Phys. Rev. D 105: 074510, arXiv:2111.15167

[14]: K. U. Can, H. Bahtiyar, G. Erkol, P. Gubler, M. Oka, "Spectrum of the charmed baryons in 2+1-flavor lattice QCD" (2019), JPS Conf. Proc. 26: 022028

[15]: PACS-CS Collaboration, "Charmed baryons at the physical point in 2+1 flavor lattice QCD" (2013), Phys. Rev. D 87: 094512, arXiv:1301.4743

[16]: C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis, G. Koutsou, "Baryon spectrum with Nf = 2+1+1 twisted mass fermions" (2014), Phys. Rev. D 90: 074501, arXiv:1406.4310

[17]: L. Liu, H.-W. Lin, K. Orginos, A. Walker-Loud, "Singly and doubly charmed J=1/2 baryon spectrum from lattice QCD" (2009), Phys. Rev. D 81: 094505, arXiv:0909.3294

References

[18]: R. A. Briceno, H.-W. Lin, D. R. Bolton, "Charmed baryon spectroscopy from lattice QCD with Nf = 2+1+1 flavors" (2012), Phys. Rev. D 86: 094504, arXiv:1207.3536

[19]: M. Padamath, R. G. Edwards, N. Mathur, M. Peardon, "Spectroscopy of triply-charmed baryons from lattice QCD" (2013), Phys. Rev. D 90: 074504, arXiv:1307.7022

[20]: Z. S. Brown, W. Detmold, S. Meinel, K. Orginos, "Charmed bottom baryon spectroscopy from lattice QCD" (2014), Phys. Rev. D 90: 094507, arXiv:1409.0497

[21]: G. Bali, S. Collins, P. Perez-Rubio, "Charmed baryon spectroscopy and light flavor symmetry from lattice QCD" (2015), Phys. Rev. D 92: 034504, arXiv:1503.08440

[22]: C. Alexandrou, C. Kallidonis, "Low lying baryon masses using Nf=2 twisted mass clover-improved fermions directly at the physical point" (2017), Phys. Rev. D 96: 034511, arXiv:1704.02647

Extra slides

QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \overline{\psi}(\not{D} + M)\psi + \frac{1}{4}F_a^{\mu\nu}F_a^{\mu\nu} + \frac{(\partial^\mu A_a^\mu)^2}{2\xi} + (\partial^\mu \bar{c}_a)(D_{ab}^\mu c_b)$$

$$\not{D} = \not{\partial} + i g_S \not{A} \text{ w/ } A^\mu(x) = A_a^\mu(x) t_a$$

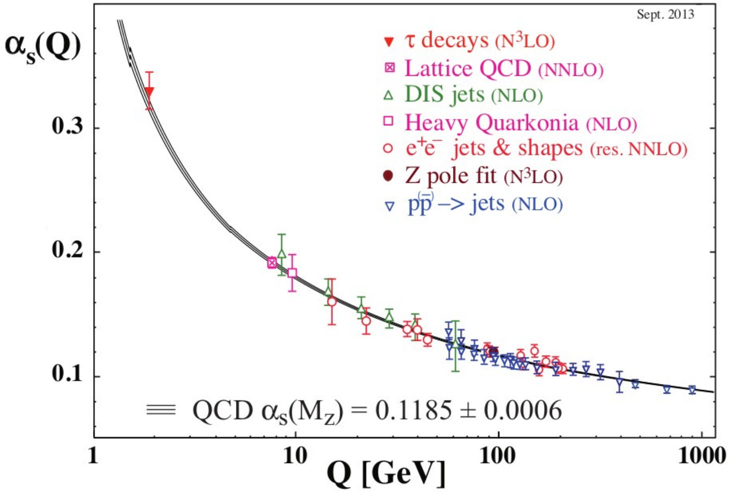
Color generator (a=1,...,8)

Strong coupling constant

$$g_S = \sqrt{4\pi\alpha_S}$$

$$F^{\mu\nu} = \frac{i}{g_S}[D^\mu, D^\nu] \text{ w/ } F^{\mu\nu} = F_a^{\mu\nu} t_a$$

[2]



$$S_{\rm QCD} = \int d^4x \, \mathcal{L}_{\rm QCD}(\psi, \bar{\psi}, A, c, \bar{c})$$

$$S_J = \int d^4x \, [J_\psi(x)\psi(x) + \ldots + J_{\bar{c}}(x)\bar{c}(x)]$$

$$\mathcal{Z}[J_\psi, \ldots, J_{\bar{c}}] = \int D[\psi, \ldots, \bar{c}] e^{-S_{QCD}} e^{-S_J}$$

Correlation functions

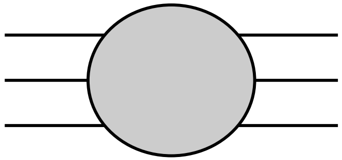
n-point function

$$G(x_1,\ldots,x_n) = \langle 0|\, T\phi(x_1)\ldots\phi(x_n)\, |0\rangle = \frac{1}{\mathcal{Z}_0}\left[\frac{\partial}{\partial J_\phi(x_1)}\cdots\frac{\partial}{\partial J_\phi(x_n)}\mathcal{Z}[\{J_\phi\}]\right]_{\{J_\phi\}=0}$$

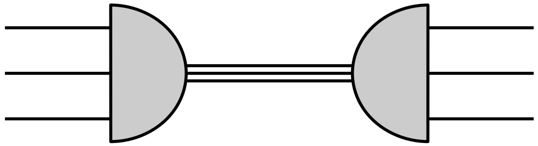
The fields acting as operators are generic: they can be any of the QCD fields. Also, they do not have to all be the same – generic field of x_i can be different from generic field of x_{i+1}

Function of interest: 6-point function written as

$$G^{(3)} = \langle 0|\, T\psi(x_1)\psi(x_2)\psi(x_3)\bar{\psi}(y_1)\bar{\psi}(y_2)\bar{\psi}(y_3)\, |0\rangle \longrightarrow$$



$$\xrightarrow[P^2\to -M^2]{\text{For}} \frac{\langle 0|\, T\psi(x_1)\psi(x_2)\psi(x_3)\, |\lambda\rangle \langle \lambda|\, T\bar{\psi}(y_1)\bar{\psi}(y_2)\bar{\psi}(y_3)\, |0\rangle}{P^2 + M^2} \longrightarrow$$

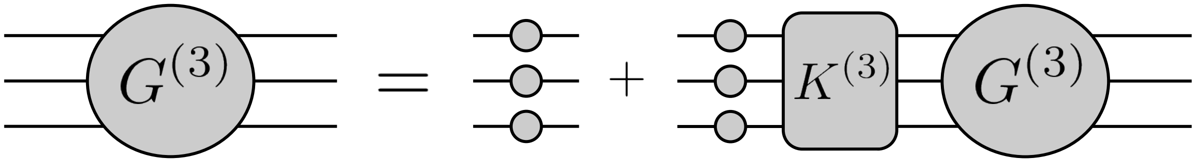


We assume a Fock space with a complete set of asymptotic states: the baryons, three valence quark bound states. These states are written as $|\lambda\rangle$ with well-defined $P = (\vec{P}, iP^0)$

QCD Lagrangian

Dyson eq. for the 6-point function

$$G^{(3)} = G_0^{(3)} + G_0^{(3)} K^{(3)} G^{(3)}$$



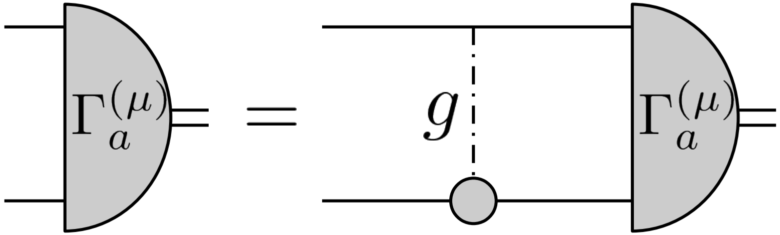
By substituting the pole behavior of P^2, we obtain

$$\Psi = G_0^{(3)} K^{(3)} \Psi \text{ w/ } \Psi = \langle 0 | T\psi(p_1)\psi(p_2)\psi(p_3) | \lambda \rangle$$

$$\Rightarrow \Phi = K^{(3)} G_0^{(3)} \Phi \Rightarrow \Phi = \tilde{K}^{(3)} \Phi \text{ w/ } \Psi = G_0^{(3)} \Phi$$

This is the BSE of page 03/18

Then it proceeds as pages 03/18-04/18. The diquark amplitude has its own BSE, with a kernel respecting the RL approximation



Categorizing q-qq BSA

Term of interest

$\psi_{im}\left(p_k^2, P_l^2 = -M_{l;\text{state}}^2\right)$ | q-qq BSA, the eigenvector

Subindices

i (or j) | q-qq Dirac tensor index

m (or n) | Chebyshev moment index

k | Momentum grid index

l | Baryon mass sample grid index

state | (Ground, 1^{st} EX, 2^{nd} EX, ...)

Dependencies

p (or k) | Relative initial (or internal) momentum

P | Baryon momentum

q-qq BSA Dirac structure

(SC qq)+q

$$\psi_{\text{Dirac}} = \sum_{i=1}^{2} f_i(p^2, p\cdot P)\tau_i(p,P)\Lambda_+(P)$$

$$\tau_1 = \frac{\mathbb{1}}{\sqrt{2}} \qquad \tau_2 = -\frac{i}{\sqrt{2}}\not{p}$$

Some definitions

$$T_P^{\mu\rho} = \delta^{\mu\rho} - \hat{P}^\mu \hat{P}^\rho \qquad b_T^\mu = T_P^{\mu\nu} b^\nu$$

$$v = \frac{\hat{p} - z\hat{P}}{\sqrt{1-z^2}} \qquad \Lambda_+(P) = \mathbb{1} + \frac{\not{P}}{iM}$$

(AV qq)+q

$$\psi^\mu_{\text{Dirac}} = \sum_{i=3}^{8} f_i(p^2, p\cdot P)\tau_i^\mu(p,P)\gamma^5\Lambda_+(P)$$

$$\tau_3^\rho = \frac{1}{\sqrt{6}}\gamma_T^\rho \qquad \tau_6^\rho = \frac{i}{\sqrt{2}}\hat{P}^\rho\not{p}$$

$$\tau_4^\rho = \frac{1}{\sqrt{2}}\hat{P}^\rho\mathbb{1} \qquad \tau_7^\rho = \frac{i}{\sqrt{2}}v^\rho\mathbb{1}$$

$$\tau_5^\rho = \frac{i}{4}[\gamma_T^\rho, \not{p}] \qquad \tau_8^\rho = \frac{1}{\sqrt{12}}(v^\rho\not{p} - \gamma_T^\rho)$$

q-qq BSA Dirac structure

(AV qq)+q

$$\psi^{\mu\nu}_{\text{Dirac}} = \sum_{i=1}^{8} f_i(p^2, p\cdot P)\tau_i(p,P)^{\mu\rho}\mathcal{P}^{\rho\nu}(P)$$

Some definitions

$$\mathbb{P}^{\mu\nu}(P) = \Lambda_+(P)\, T_P^{\mu\rho} T_P^{\sigma\nu}\left(\delta^{\rho\sigma} - \frac{1}{3}\gamma^\rho\gamma^\sigma\right)$$

$$\tau_1^{\mu\rho} = \frac{1}{2}\delta^{\mu\rho}\mathbb{1}$$

$$\tau_2^{\mu\rho} = -\frac{1}{2\sqrt{5}}\left(2i\gamma_T^\mu v^\rho - 3i\delta^{\mu\rho}\not{v}\right)$$

$$\tau_3^{\mu\rho} = \frac{\sqrt{3}}{2}\hat{P}^\mu v^\rho \not{v}$$

$$\tau_4^{\mu\rho} = -\frac{\sqrt{3}i}{2}\hat{P}^\mu v^\rho \mathbb{1}$$

$$\tau_5^{\mu\rho} = \frac{1}{2}v^\rho\left(\gamma_T^\mu \not{v}\right)$$

$$\tau_6^{\mu\rho} = \frac{i}{2}\gamma_T^\mu v^\rho$$

$$\tau_7^{\mu\rho} = -\tau_7^{\mu\rho} - \tau_3^{\mu\rho} + \frac{3}{2}v^\mu v^\rho \mathbb{1}$$

$$\tau_8^{\mu\rho} = -\frac{1}{2\sqrt{5}}\left(i\delta^{\mu\rho}\not{v} + i\gamma_T^\mu v^\rho - 5iv^\mu v^\rho \not{v}\right)$$

qq BSA Dirac structure

J=0$^+$: SC qq

$$\tau_1 = \mathbb{1} \quad \tau_2 = \hat{\not{k}}_d \quad \tau_3 = y' \left(\hat{\not{k}}_r\right)_T \quad \tau_4 = i[\hat{\not{k}}_r, \hat{\not{k}}_d]$$

$$\left[\Theta^0_{i;\alpha\beta'}\left(k_r, k_d\right)\right]_{\mathrm{Dirac}} = \sum_{l=1}^{4} h_{il}\left(k_r^2, k_r \cdot k_d, k_d^2\right)\left\{i\gamma^5 \tau_l\left(k_r, k_d\right) C\right\}_{\alpha\beta'}$$

Some definitions $C = \gamma^4\gamma^2$

J=1$^+$: AV qq

$$\left[\Theta^\rho_{i;\alpha\beta'}\left(k_r, k_d\right)\right]_{\mathrm{Dirac}} = \sum_{l=5}^{16} h_{il}\left(k_r^2, k_r \cdot k_d, k_d^2\right)\left\{i\tau^\rho_l\left(k_r, k_d\right) C\right\}_{\alpha\beta'}$$

$$\tau^\rho_5 = \gamma^\rho \qquad \tau^\rho_6 = \gamma^\rho \hat{\not{k}}_d \qquad \tau^\rho_7 = i\hat{k}^\rho_r \qquad \tau^\rho_8 = y'\hat{k}^\rho_r \hat{\not{k}}_d \qquad \tau^\rho_9 = y'\gamma^\rho \left(\hat{\not{k}}_r\right)_T$$

$$\tau^\rho_{10} = i\hat{k}^\rho_r \hat{\not{k}}_d - \frac{i}{2}\gamma^\rho[\hat{\not{k}}_r, \hat{\not{k}}_d]$$

$$\tau^\rho_{11} = \hat{k}^\rho_r \left(\hat{\not{k}}_r\right)_T - \frac{1}{3}\left(\hat{\not{k}}_r\right)^2_T \gamma^\rho$$

$$\tau^\rho_{12} = \frac{1}{3}\left(\hat{\not{k}}_r\right)^2_T \gamma^\rho \hat{\not{k}}_d - \frac{1}{2}\hat{k}^\rho_r[\hat{\not{k}}_r, \hat{\not{k}}_d]$$

Diquark flavor states

$$4 \otimes 4 = 6_{\mathrm{A}} \oplus 10_{\mathrm{S}}$$

- Diquark flavors: $q_1\,,\ q_2$
- Matrix representations:
  - $\mathrm{Mat}'(q_b q_c)$
    - $R\,|I;\, I_3;\, Y_C;\, C\rangle$
  - $\mathrm{Mat}'(q_b q_c)^\dagger$
    - $\langle I;\, I_3;\, Y_C;\, C|\, R$

$$\{q_1\,, q_2\} = q_1 q_2 + q_2 q_1$$

$$[q_1\,, q_2] = q_1 q_2 - q_2 q_1$$

- Normalization constant for these expressions:

$$\frac{1}{\sqrt{\mathrm{Tr}\left\{\mathrm{Mat}'(q_b q_c)\mathrm{Mat}'(q_b q_c)^\dagger\right\}}}$$

Which ones are SC and AV?

$$\Theta^{(\rho)}_{n;\alpha\beta'}(k_r\,, k_d) = -\Theta^{(\rho)}_{n;\beta'\alpha}(-k_r\,, k_d)$$

Total BSA & color term are antisymmetric
Therefore, flavor $\otimes$ Dirac is symmetric

$$\left[\Theta^{(\mu)}(q, P)\right]_{\mathrm{Dirac}} \equiv \pm \left[\Theta^{(\mu)}(-q, P)\right]^T_{\mathrm{Dirac}}$$

SC diquarks: Leading term is $+$ Dirac solution
Flavor states are 6_{A}
AV diquarks: Leading term is $-$ Dirac solution
Flavor states are 10_{S}

Baryon flavor states

$$4 \otimes 4 \otimes 4 = 4 \otimes (6_A \oplus 10_S) = \overline{4}_A \oplus 20_{M_S} \oplus 20_{M_A} \oplus 20_S$$

- 20_S : $\mathcal{S} = f_1^+ + f_2^+ + f_3^+ = \frac{1}{2}\left(\hat{\mathbb{1}} + \hat{P}_{12}\right)\left[\hat{\mathbb{1}} + \hat{P}_{123} + \hat{P}_{123}^2\right] f_{123}$

- $\overline{4}_A$: $\mathcal{A} = f_1^- + f_2^- + f_3^- = \frac{1}{2}\left(\hat{\mathbb{1}} - \hat{P}_{12}\right)\left[\hat{\mathbb{1}} + \hat{P}_{123} + \hat{P}_{123}^2\right] f_{123}$

- 20_{M_A} :
  - $a_1 = f_2^- - f_3^- = \frac{1}{2}\left(\hat{\mathbb{1}} - \hat{P}_{12}\right)\left[\hat{P}_{123} - \hat{P}_{123}^2\right] f_{123}$
  - &/or
  - $a_2 = \frac{-1}{\sqrt{3}}\left(f_2^- + f_3^- - 2f_1^-\right) = \frac{-1}{2\sqrt{3}}\left(\hat{\mathbb{1}} - \hat{P}_{12}\right)\left[\hat{P}_{123} + \hat{P}_{123}^2 - 2.\mathbb{1}\right] f_{123}$

- 20_{M_S} :
  - $s_1 = \frac{-1}{\sqrt{3}}\left(f_2^+ + f_3^+ - 2f_1^+\right) = \frac{-1}{2\sqrt{3}}\left(\hat{\mathbb{1}} + \hat{P}_{12}\right)\left[\hat{P}_{123} + \hat{P}_{123}^2 - 2.\mathbb{1}\right] f_{123}$
  - &/or
  - $s_2 = f_2^+ - f_3^+ = \frac{1}{2}\left(\hat{\mathbb{1}} + \hat{P}_{12}\right)\left[\hat{P}_{123} - \hat{P}_{123}^2\right] f_{123}$

How to build f_{123} and permutations

$$f_{123} = (q_a q_b q_c)_{123} = q_a q_b q_c$$

e.g: $(uds)_{123} = uds$

$$f_{231} = (q_a q_b q_c)_{231} = q_c q_a q_b$$

e.g: $(uds)_{231} = sud$

Quark remainder matrices

E.g.: $20_{M_S}\,|1;\,I_3;\,1;\,1\rangle = \{20_{M_S}\,|1;\,1;\,1;\,1\rangle\ \ 20_{M_S}\,|1;\,0;\,1;\,1\rangle\ \ 20_{M_S}\,|1;\,-1;\,1;\,1\rangle\}$

$$= \left\{ \frac{1}{\sqrt{3}}\left[\left(\frac{1}{\sqrt{2}}\{u,c\}\right)u - \sqrt{2}(uu)c\right]\ \ \frac{1}{\sqrt{2}}\left[\left(\frac{1}{\sqrt{2}}\{u,c\}\right)d - \left(\frac{1}{\sqrt{2}}\{d,c\}\right)u\right]\ \ \frac{1}{\sqrt{3}}\left[\left(\frac{1}{\sqrt{2}}\{d,c\}\right)d - \sqrt{2}(dd)c\right] \right\}$$

$$= \left(\frac{1}{\sqrt{2}}\{u,c\}\right)\left\{\frac{1}{\sqrt{3}}u\ \ \frac{1}{\sqrt{2}}d\ \ 0\right\} + \left(\frac{1}{\sqrt{2}}\{d,c\}\right)\left\{0\ \ -\frac{1}{\sqrt{2}}u\ \ \frac{1}{\sqrt{3}}d\right\} + (uu)\left\{-\sqrt{2}c\ \ 0\ \ 0\right\} + (dd)\left\{0\ \ 0\ \ -\sqrt{2}c\right\}$$

$$\to \mathrm{Mat}\left[\left(\frac{1}{\sqrt{2}}\{u,d\}\right)\right]\cdot \mathrm{Mat}\left[\left\{\frac{1}{\sqrt{3}}u\ \ \frac{1}{\sqrt{2}}d\ \ 0\right\}\right] + \dots$$

$$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The matrix representation of the flavor state is obtained by employing the matrix representations of the diquark flavor states and the remaining quark terms, i.e, the quark remainders

Flavor factors

$$\begin{pmatrix}
\text{Mat}(nn \to nn) & \text{Mat}(nn \to ns) & \text{Mat}(nn \to ss) & \text{Mat}(nn \to nc) & \text{Mat}(nn \to sc) & \text{Mat}(nn \to cc) \\
\text{Mat}(ns \to nn) & \text{Mat}(ns \to ns) & \text{Mat}(ns \to ss) & \text{Mat}(ns \to nc) & \text{Mat}(ns \to sc) & \text{Mat}(ns \to cc) \\
\text{Mat}(ss \to nn) & \text{Mat}(ss \to ns) & \text{Mat}(ss \to ss) & \text{Mat}(ss \to nc) & \text{Mat}(ss \to sc) & \text{Mat}(ss \to cc) \\
\text{Mat}(nc \to nn) & \text{Mat}(nc \to ns) & \text{Mat}(nc \to ss) & \text{Mat}(nc \to nc) & \text{Mat}(nc \to sc) & \text{Mat}(nc \to cc) \\
\text{Mat}(sc \to nn) & \text{Mat}(sc \to ns) & \text{Mat}(sc \to ss) & \text{Mat}(sc \to nc) & \text{Mat}(sc \to sc) & \text{Mat}(sc \to cc) \\
\text{Mat}(cc \to nn) & \text{Mat}(cc \to ns) & \text{Mat}(cc \to ss) & \text{Mat}(cc \to nc) & \text{Mat}(cc \to sc) & \text{Mat}(cc \to cc)
\end{pmatrix}$$

This is the generic flavor factor matrix. Each element is a 2x2 matrix, representing diquark transitions. The diquark on the left is the initial diquark of the BSE whereas the diquark on the right is the internal diquark of the BSE. Each element of the 2x2 matrix is a flavor factor:

$$\text{Mat}(q_b q_c \to q_a q_c) = \begin{pmatrix} [q_b q_c]_{\text{SC}} \to [q_a q_c]_{\text{SC}} & [q_b q_c]_{\text{SC}} \to [q_a q_c]_{\text{AV}} \\ [q_b q_c]_{\text{AV}} \to [q_a q_c]_{\text{SC}} & [q_b q_c]_{\text{AV}} \to [q_a q_c]_{\text{AV}} \end{pmatrix}$$

Since what we want are eigenvalues and eigenvectors, rows and columns of zeros can be discarded.

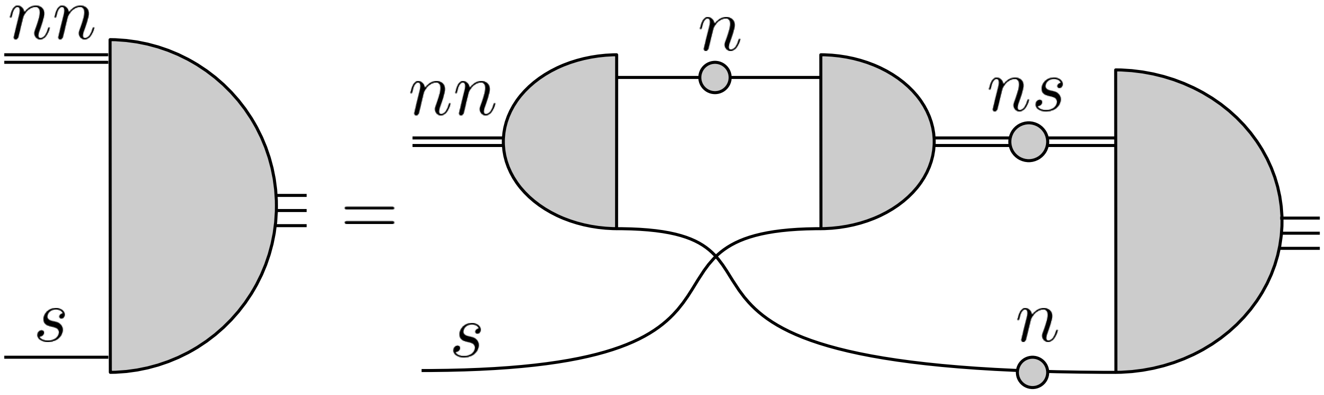
E.g:

$$\text{Mat}_F^{\Sigma} = \begin{pmatrix}
0 & \text{Mat}(nn \to ns) & 0 & 0 & 0 & 0 \\
\text{Mat}(ns \to nn) & \text{Mat}(ns \to ns) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \Rightarrow \begin{pmatrix} 0 & \text{Mat}(nn \to ns) \\ \text{Mat}(ns \to nn) & \text{Mat}(ns \to ns) \end{pmatrix}$$

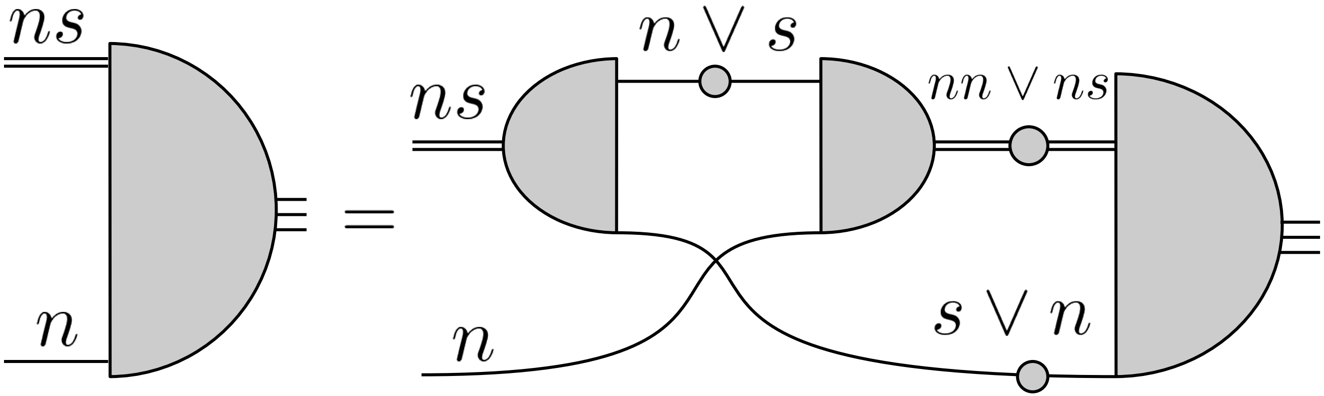
Flavor factors

E.g: nns flavor combination

s-nn



n-ns



$$\mathrm{Mat}(ns \to nn)$$

$$\begin{pmatrix} ns_{\mathrm{SC}} \to nn_{\mathrm{SC}} & ns_{\mathrm{SC}} \to nn_{\mathrm{AV}} \\ ns_{\mathrm{AV}} \to nn_{\mathrm{SC}} & ns_{\mathrm{AV}} \to nn_{\mathrm{AV}} \end{pmatrix}$$

$$ns_{\mathrm{SC}} \to nn_{\mathrm{AV}} \equiv x$$

Fix the ns diquark (either us or ds, the answer will be the same). Go over all possibel nn diquarks (uu, ud, dd). Dq are diquark matrices, Qr are quark remainder matrices

$$x\mathrm{Qr}_2^{\Sigma} = \sum_{j=\{7,8,11\}} \mathrm{Dq}_j \mathrm{Dq}_2^{\dagger} \mathrm{Qr}_j^{\Sigma}$$

$$\Rightarrow x = -\frac{1}{\sqrt{3}}$$

After obtaining all flavor factors, we perform a rotation on the matrix to make it symmetric

Baryon mass grid

To determine the mass grid threshold we start by localizing ourselves in the complex plane:

$(\mathrm{RE}\left\{r^2\right\}, \mathrm{IMG}\left\{r^2\right\})$ | Generic momentum of the dressing functions for propagators & diquark amplitude

We can also write the generic momentum as:

$$r^\mu = x^\mu + E(\eta)P^\mu$$

x^μ: Contains all momentum terms that do not depend on P^μ

$E(\eta)$: Expression dependent on momentum part. parameter

$$\Rightarrow r^2 = |x|^2 - (E(\eta))^2 M^2 + 2i|x|Mz_r E(\eta) \Rightarrow \boxed{r^2 = (|x| \pm iE(\eta)M)^2}$$

This parabola equation establishes the condition: $\mathrm{IMG}\left\{r^2\right\} \geqslant -(E(\eta))^2 M^2$

$$E(\eta)_q = (2\eta - 1)^2 \qquad \mathrm{IMG}\{q^2\} \geqslant -(2\eta-1)^2 M^2$$

$$E(\eta)_{k_q} = \eta^2 \quad \Rightarrow \quad \mathrm{IMG}\{k_q^2\} \geqslant -\eta^2 M^2$$

$$E(\eta)_{k_d} = (1-\eta)^2 \qquad \mathrm{IMG}\{k_d^2\} \geqslant -(1-\eta)^2 M^2$$

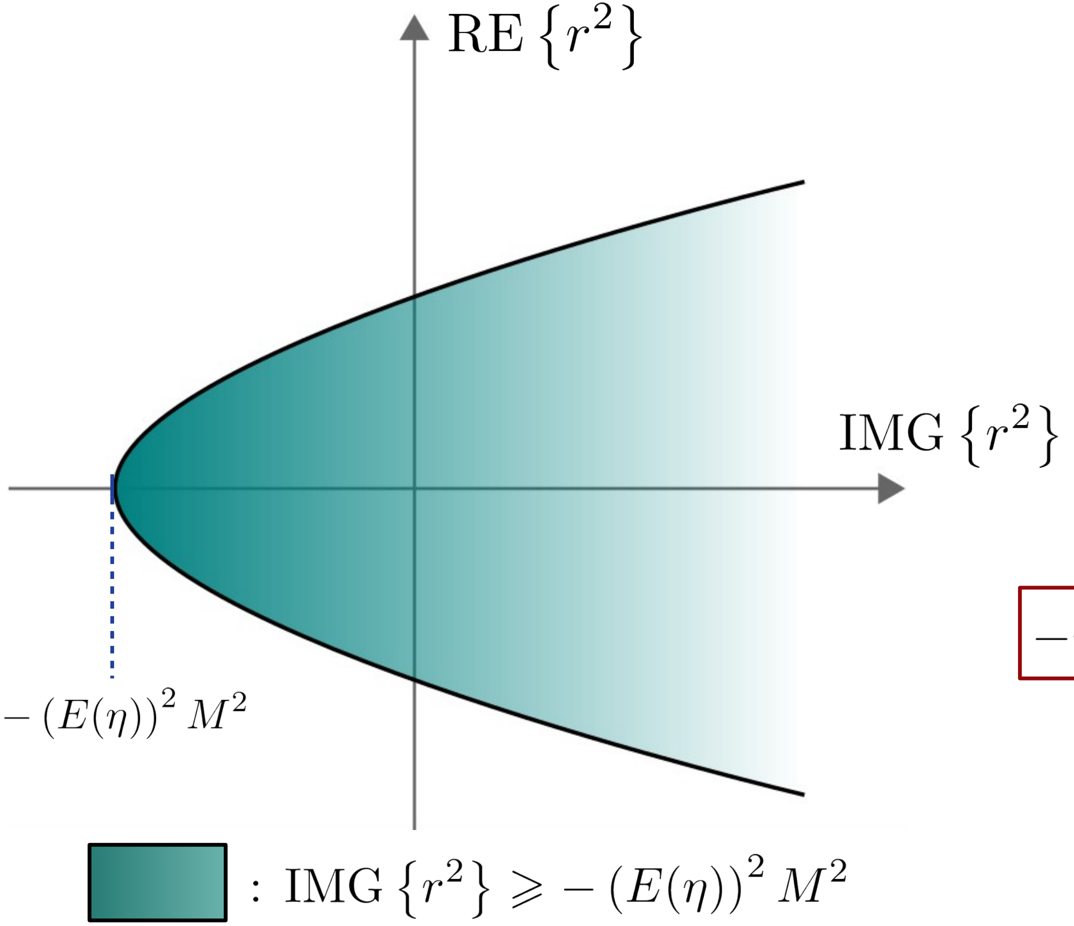
q : Quark in kernel momentum

k_q : Internal quark momentum

k_d : Internal diquark momentum

Baryon mass grid

Sketching the generic momentum parabola in the complex plane gets us:

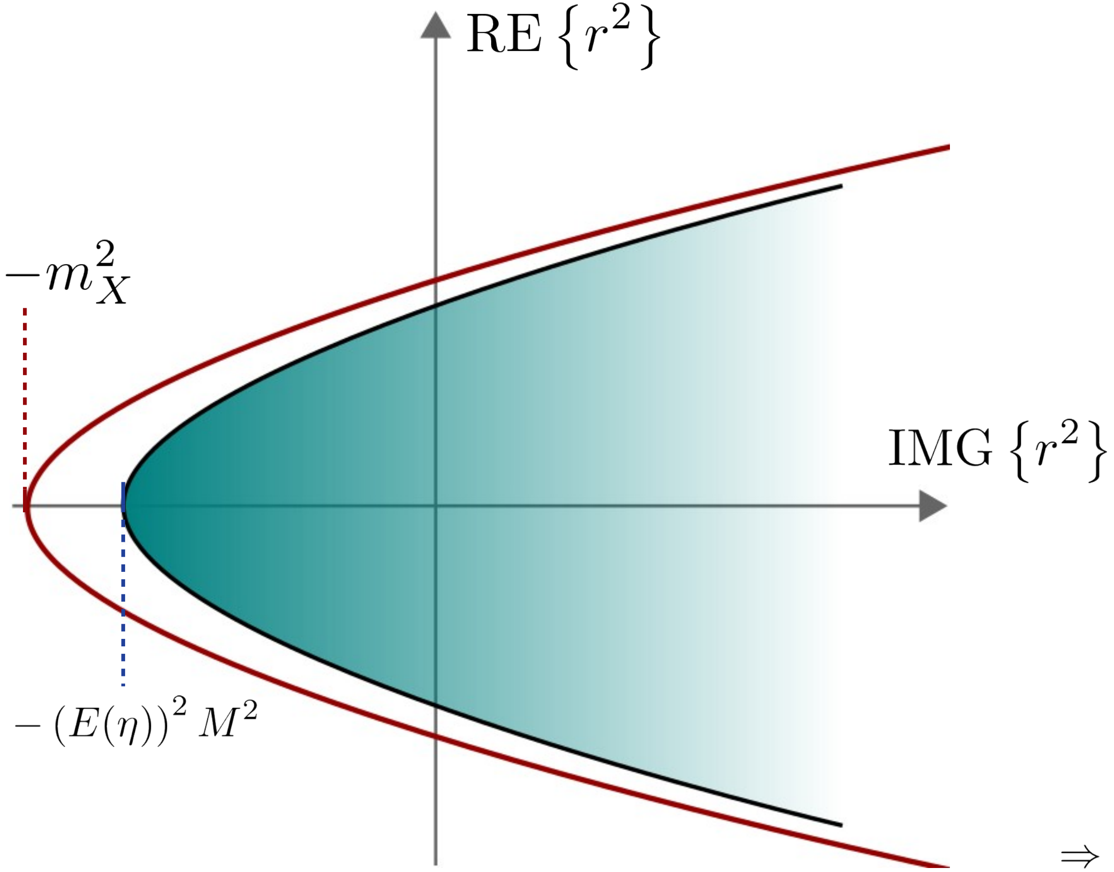


The apex of this generic momentum parabola cannot go beyond the apex of the limiting parabola, which is derived by the poles on r^2 set by the dressing functions. The only functions that have momenta poles are the propagators: S & D. Let us call this limiting parabola apex $-m_X^2$. Then, what we just said translaes to the condition:

$$-m_X^2 \leqslant -(E(\eta))^2 M^2 \Rightarrow m_X^2 \geqslant (E(\eta))^2 M^2$$

Baryon mass grid

Sketching the limiting parabola in the complex plane gets us:



Diquark propagator pole:

$$r^2_{\text{pole}} = -m^2_{\text{qq}} \Rightarrow m_X = m_{\text{qq}} \Rightarrow M \leqslant \frac{m_{\text{qq}}}{E(\eta)}$$

Quark propagator conjugate poles:

$$r^2_{\text{poles}} = -s_2\left(1 \pm i\frac{s_3}{s_2}\right) \Rightarrow r^2_{\text{poles}} = (-s_2 \pm i(-s_3))$$

Generic parabola equation with apex $-m_X^2$:

$$(t \pm i m_X)^2 = t^2 - m_X^2 \pm 2itm_X$$

$$\Rightarrow \begin{cases} t^2 - m_X^2 = -s_2 \\ \pm 2tm_X = \pm(-s_3) \end{cases}$$

$$m_X \equiv m_{\text{q}}$$

$$\Rightarrow m_q = \sqrt{\frac{s_2}{s}\left[1 + \sqrt{1 + \left(\frac{s_3}{s_2}\right)^2}\right]} \Rightarrow M \leqslant \frac{m_q}{E(\eta)}$$

Baryon mass grid

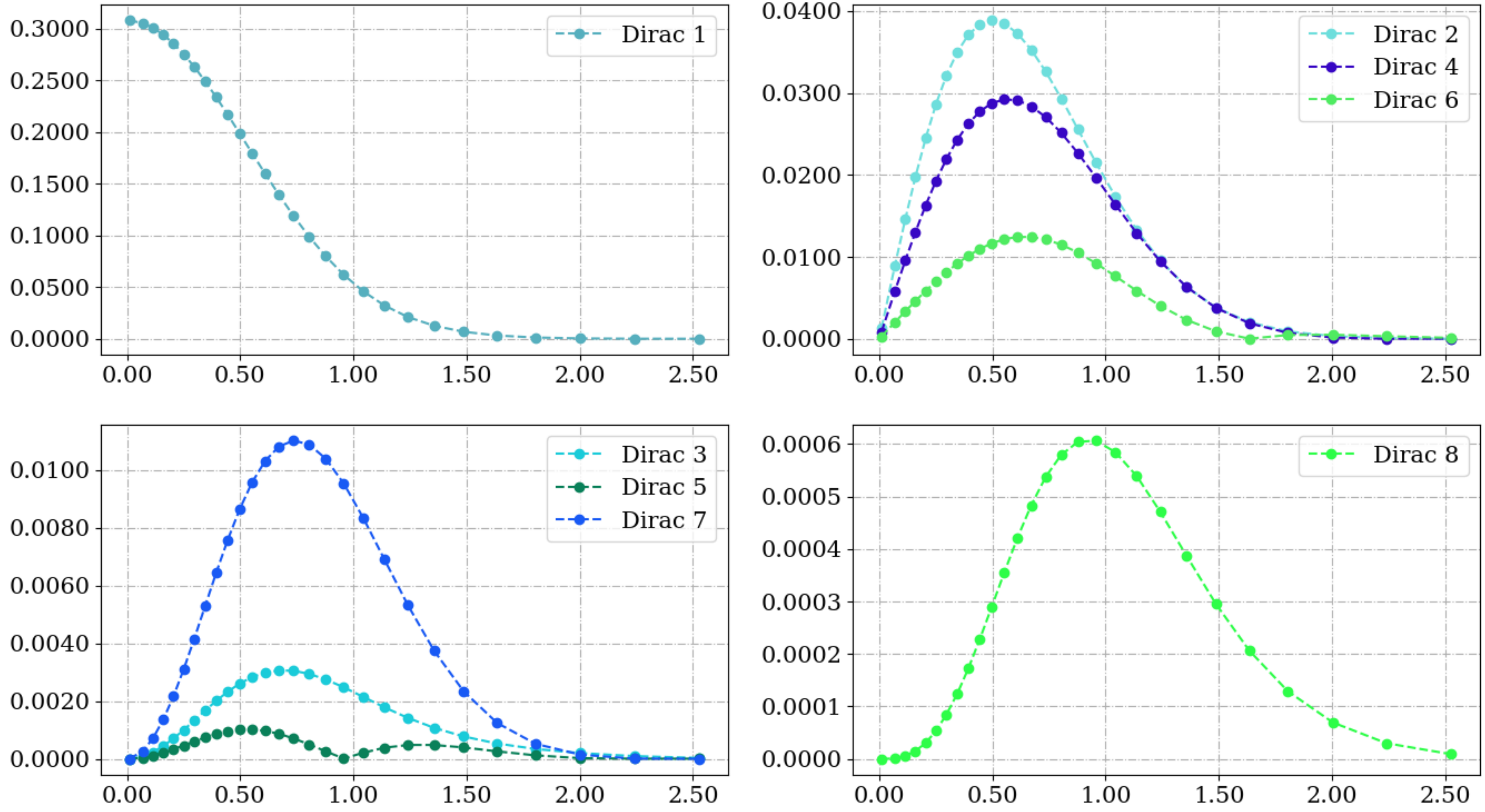
In summary, the three conditions for the baryon mass provided by the analysis of the limiting parabolas are:

$M \leqslant \dfrac{m_{\rm q}}{\lvert 2\eta - 1 \rvert}$	Quark propagator condition of the quark in the interaction kernel in the q-qq BSE	Momentum q
$M \leqslant \dfrac{m_{\rm q}}{\lvert \eta \rvert}$	Quark propagator condition of the quark in the propagator product in the q-qq BSE	Momentum k_q
$M \leqslant \dfrac{m_{\rm qq}}{\lvert 1 - \eta \rvert}$	Diquark propagator condition of the diquark in the propagator product in the q-qq BSE	Momentum k_d

The mass grid threshold of a given baryon is equal to the minimum mass threshold of the diquark transitions. For example, the single-strange baryon, as we saw, has 3 diquark transitions. Each of them has different flavors for the quarks and diquark with the above momenta, which has an impact in the masses. Therefore, the conditions for the baryon mass will differ based on the diquark transitions we consider.

Chebyshev 0^{th} moment contributions

Vertical Axis: $\left|\psi_{i0}\left(p_k^2; -M^2_{\text{Ground}}\right)\right|$; Horizontal axis: p_k [GeV]



Baryon

Ω (sss)

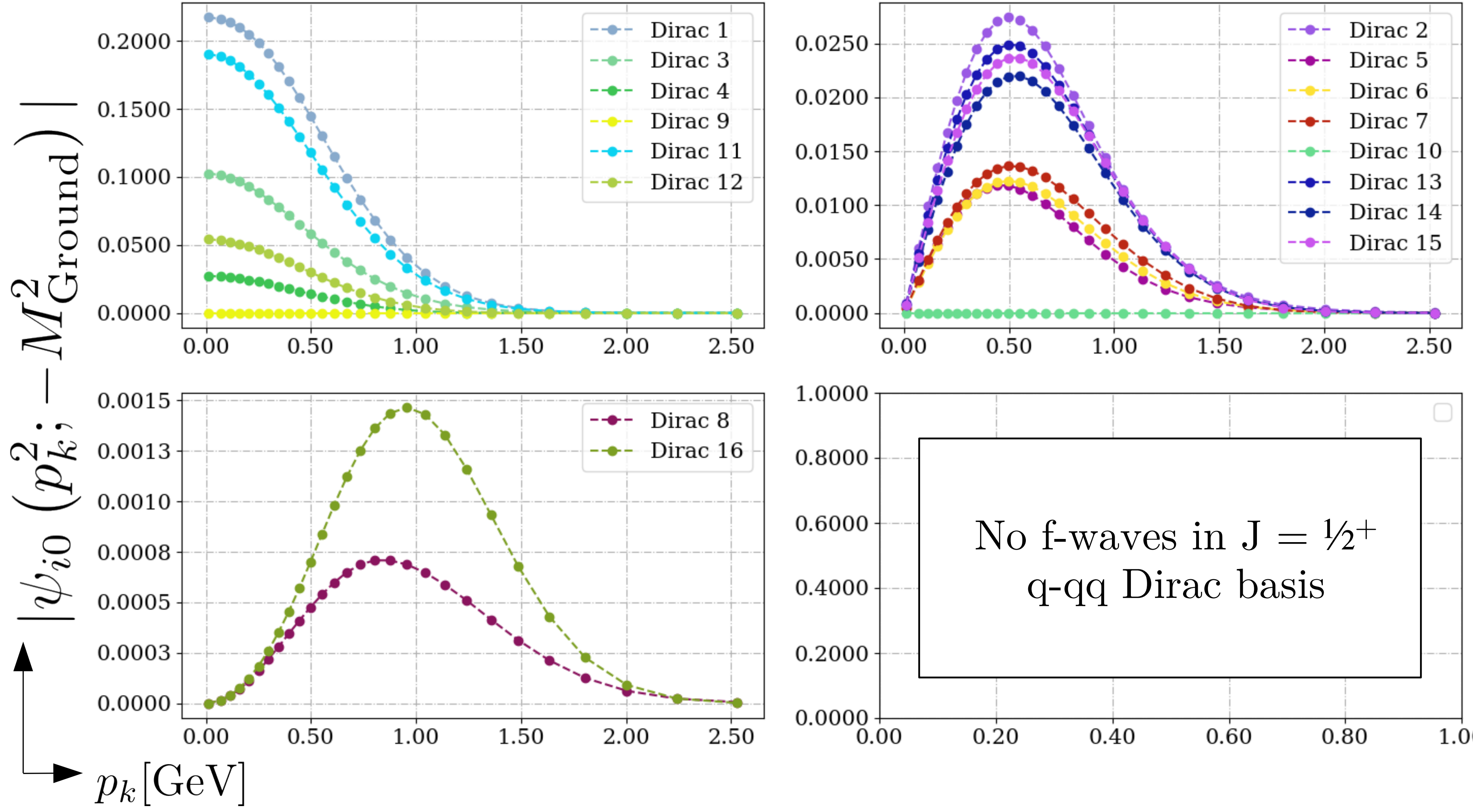
Labels

Dirac i

$\Leftrightarrow$

τ_i

Chebyshev 0^{th} moment contributions



Baryon

Ξ (nss)

Labels

Dirac i

$\Leftrightarrow$

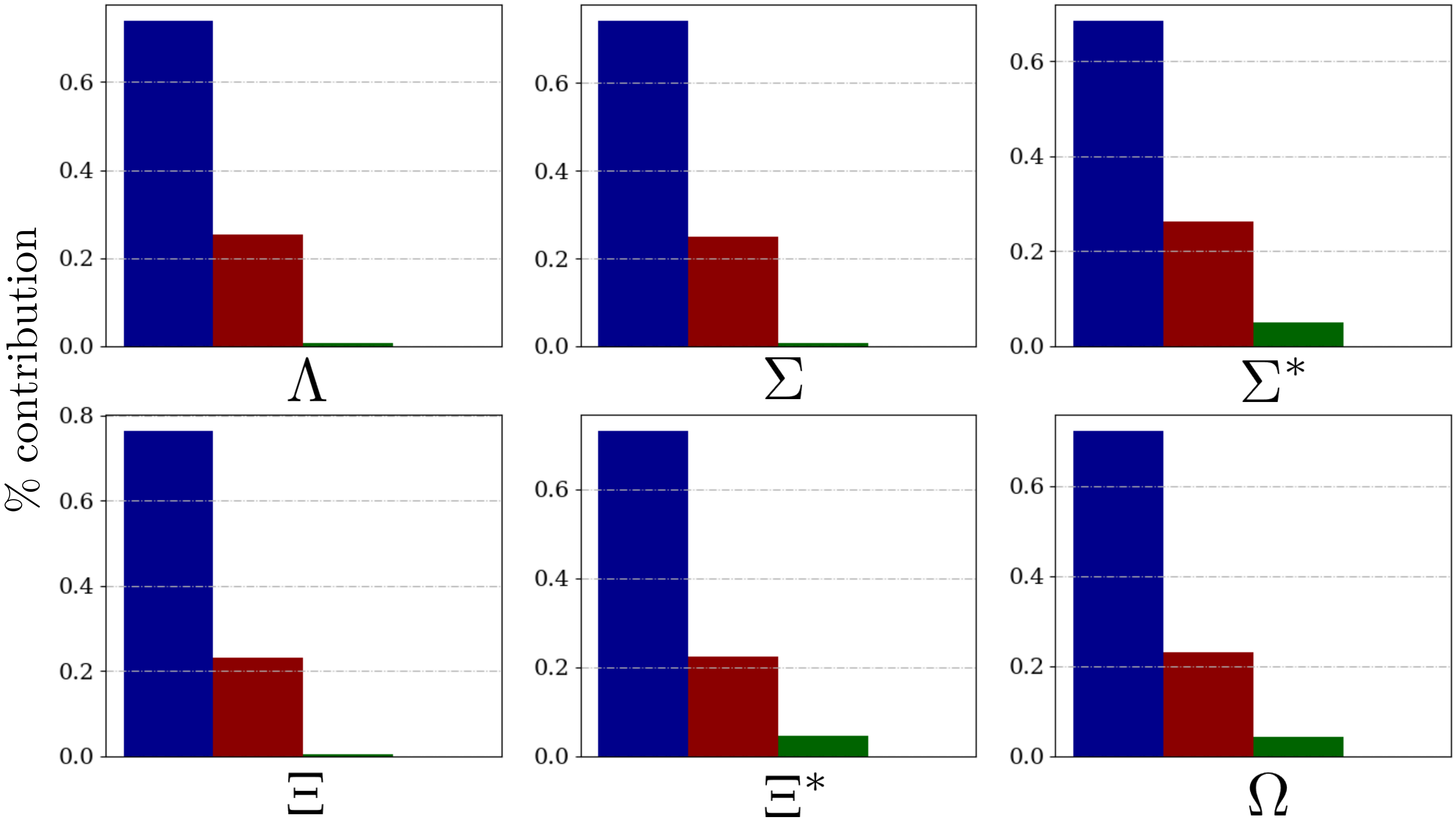
τ_i

When i > 8, then it refers to the other possible qq. In this case:

$i \leq 8$: ns

$i > 8$: ss

Partial wave contributions



Colors

: s-waves

: p-waves

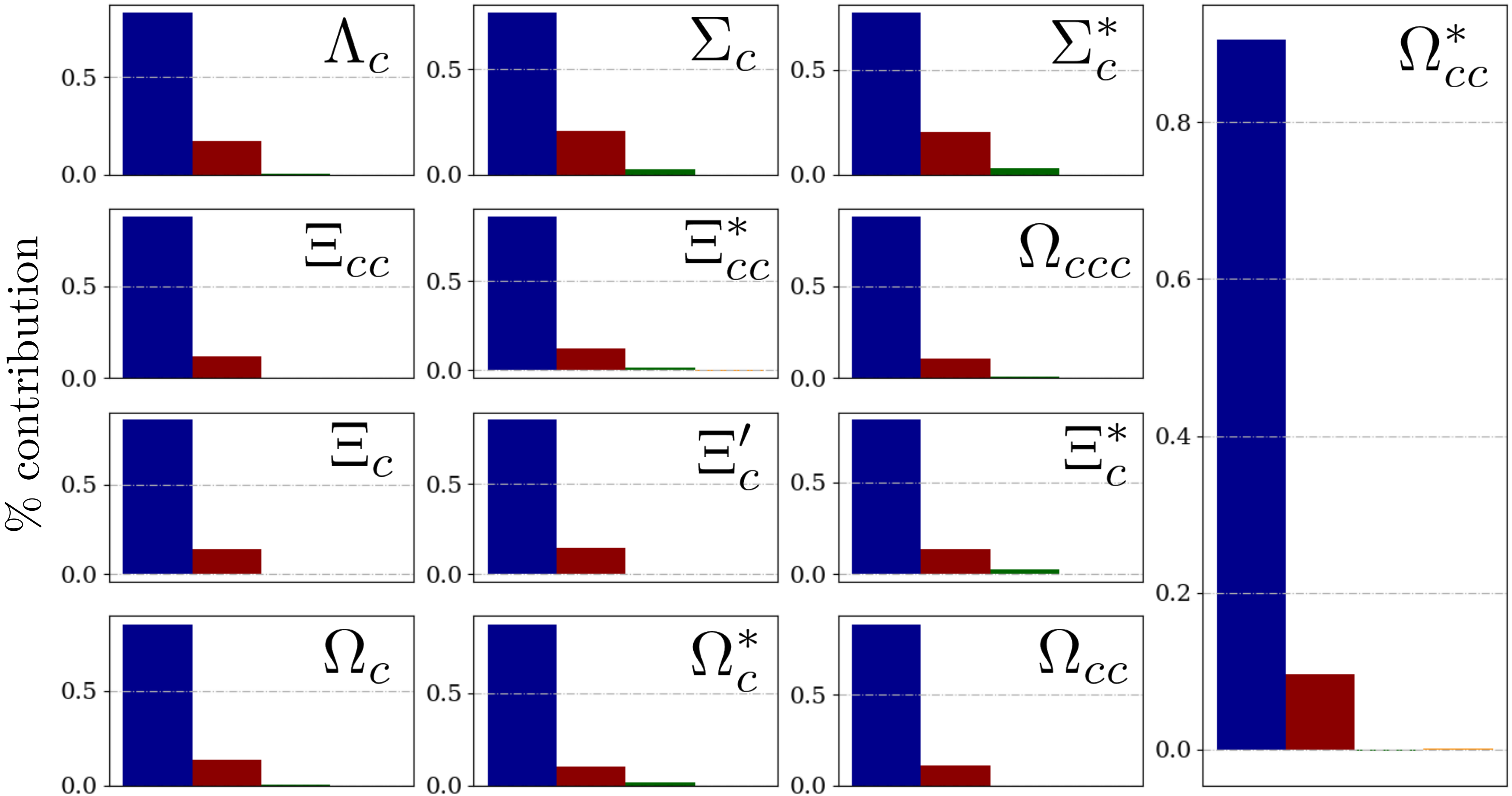
: d-waves

: f-waves

Details

Evaluated at ground state's physical mass

Partial wave contributions



Colors

: s-waves

: p-waves

: d-waves

: f-waves

Details

Evaluated at ground state's physical mass