Consistency of the molecular picture of Ω(2012) with the latest Belle results

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Genaro Toledo, Wei-Hong Liang, Eulogio Oset

R. Pavao and E. Oset, Eur. Phys. J. C78, 857 (2018).
N. Ikeno, G. Toledo, and E. Oset, Phys. Rev. D 101, 094016 (2020).
N. Ikeno, W. H. Liang, G. Toledo, and E. Oset, Phys.Rev.D 106, 034022 (2022).

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Discovery of Ω(2012): Strangeness =-3

In **2018**, Belle reported a new state Ω(2012) state: [Phys. Rev. Lett. 121, 052003 (2018)]

$$M_{\Omega(2012)} = 2012.4 \pm 0.7 \pm 0.6\ \mathrm{MeV}$$
$$\Gamma_{\Omega(2012)} = 6.4^{+2.5}_{-2.0} \pm 1.6\ \mathrm{MeV}$$

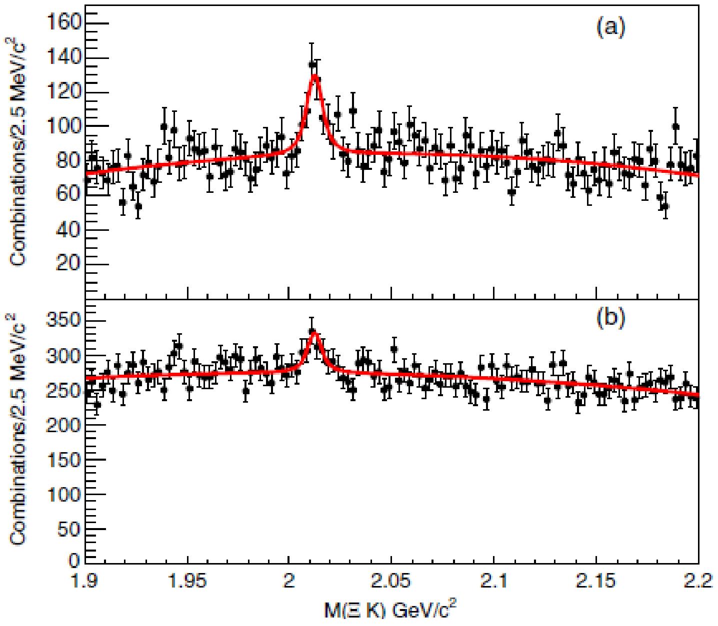


FIG. 2. The (a) $\Xi^0 K^-$ and (b) $\Xi^- K_S^0$ invariant mass distributions in data taken at the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ resonance energies.

This prompted many theoretical studies on the issue

- Quark model pictures

- Molecular pictures based on the meson-baryon interaction

- Only $\bar{K}\Xi^*(1530)$ state:
  - Y. H. Lin and B. S. Zou, Phys. Rev. D 98, 056013 (2018).

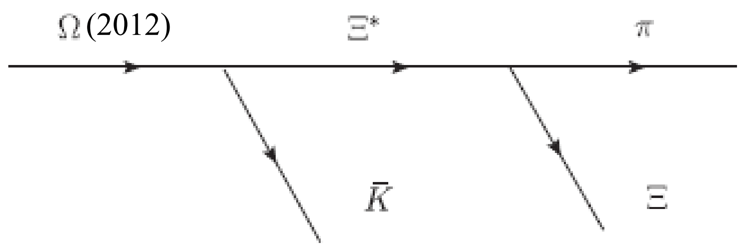
- Coupled channels $\bar{K}\Xi^*(1530)$, $\eta\Omega$, $\bar{K}\Xi$
  - M. P. Valderrama, Phys. Rev. D 98, 054009 (2018).
  - Y. Huang, M. Z. Liu, J. X. Lu, J. J. Xie, and L. S. Geng, Phys. Rev. D 98, 076012 (2018).
  - R. Pavao and E. Oset, Eur. Phys. J. C 78, 857 (2018).
  - M. V. Polyakov, H. D. Son, B. D. Sun, and A. Tandogan, Phys. Lett. B 792, 315 (2019). .....

Recent Belle data

- In **2019**, Belle showed a result of the $\Omega(2012)$ decay: [Phys. Rev. D 100, 032006(2019)]
  - Ratio R of the $\Xi\pi K$ width to the ΞK width is smaller than 11.9%.

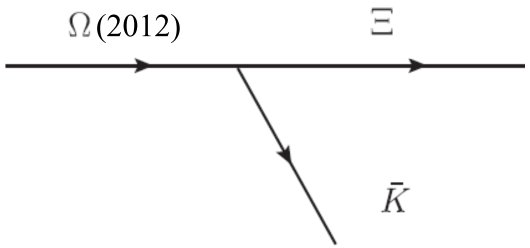
$$\mathcal{R}^{\Xi\pi K}_{\Xi K} = \frac{\mathcal{B}(\Omega(2012) \to \Xi(1530)(\to \Xi\pi)K)}{\mathcal{B}(\Omega(2012) \to \Xi K)} < 11.9\%$$

$\Omega(2012) \to \pi\bar{K}\Xi$



$\Rightarrow$ In **2022**, a recent reanalysis of data (different cut):
[arXiv:2207.03090 (2022)]

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = 0.97 \pm 0.24 \pm 0.07 \quad \sim 97\%$$



- In **2021**, $\Omega(2012)$ has been observed in the Ω_c decay [Phys. Rev. D 104, 052005(2021)]

$$\frac{\mathcal{B}(\Omega_c^0 \to \pi^+\Omega(2012)^-) \times \mathcal{B}(\Omega(2012)^- \to K^-\Xi^0)}{\mathcal{B}(\Omega_c^0 \to \pi^+K^-\Xi^0)} = (9.6 \pm 3.2 \pm 1.8)\%$$

Our study

We study the $\Omega(2012)$ state with the coupled channels approach:

$\Omega(2012)$ is dynamically generated as a molecular state from the interaction of the $\bar{K}\Xi^*$ and $\eta\Omega$ coupled channels

=> We like to show the consistency of the molecular picture with Belle results

$$\mathcal{R}^{\Xi\pi K}_{\Xi K} = \frac{\mathcal{B}(\Omega(2012) \to \Xi(1530)(\to \Xi\pi)K)}{\mathcal{B}(\Omega(2012) \to \Xi K)} < 11.9\%$$ (2019 ver.)

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = 0.97 \pm 0.24 \pm 0.07$$ (reanalysis: 2022 ver.)

$$\frac{\mathcal{B}(\Omega_c^0 \to \pi^+\Omega(2012)^-) \times \mathcal{B}(\Omega(2012)^- \to K^-\Xi^0)}{\mathcal{B}(\Omega_c^0 \to \pi^+K^-\Xi^0)} = (9.6 \pm 3.2 \pm 1.8)\%$$

✓ We calculate the mass, width, and decay ratio R of $\Omega(2012)$

✓ We also study a mechanism for $\Omega_c \to \pi^+\Omega(2012) \to \pi^+(K^-\Xi^0)$ reaction

Formalism: Coupled channels approach

R. Pavao and E. Oset, EPJC78(2018)
N. Ikeno, G. Toledo, E. Oset, PRD(2020)

3 channels: $\boxed{\bar{K}\Xi^*, \eta\Omega \text{ (}s\text{-wave)}, \ \bar{K}\Xi \text{ (}d\text{-wave)}}$

- Bethe-Salpeter equation:

$$T = [1 - VG]^{-1} V$$

- Transition potential: Ω^* $J^p = 3/2^-$

$$\begin{array}{cc} \begin{array}{ccc} \bar{K}\Xi^* & \eta\Omega & \bar{K}\Xi \end{array} & \\ V = \begin{pmatrix} \boxed{\begin{matrix} 0 & 3F \\ 3F & 0 \end{matrix}} & \begin{matrix} \alpha q_{\rm on}^2 \\ \beta q_{\rm on}^2 \end{matrix} \\ \begin{matrix} \alpha q_{\rm on}^2 & \beta q_{\rm on}^2 \end{matrix} & 0 \end{pmatrix} & \begin{array}{c} \bar{K}\Xi^* \\ \eta\Omega \\ \bar{K}\Xi \end{array} \end{array}$$

$$F = -\frac{1}{4f^2}(k^0 + k'^0) \qquad q_{\rm on} = \frac{\lambda^{1/2}\left(s, m_{\bar{K}}^2, m_{\Xi}^2\right)}{2\sqrt{s}}$$

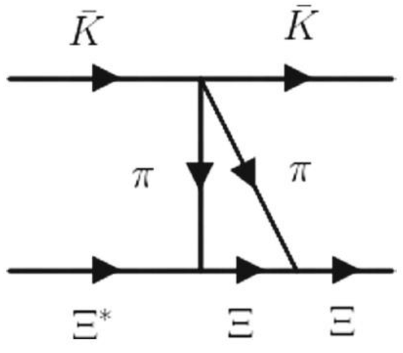
k^0, k'^0 the energies of initial and final states

-the diagonal potential is null
-the non-diagonal potential is nonzero.

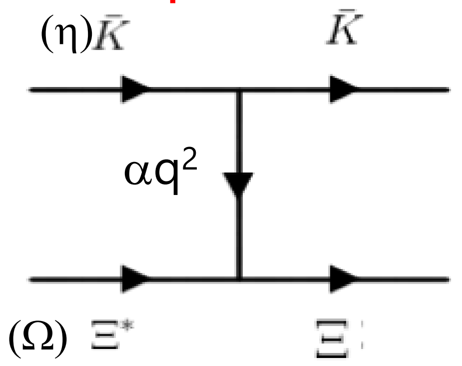
- s-wave potentials between $\bar{K}\Xi^*$ and $\eta\Omega$:
  taken from chiral Lagrangian of

  S. Sarkar, E. Oset, M.J. Vicente Vacas, NPA750(2005) 294
  E.E. Kolomeitsev, M.F.M. Lutz, PLB 585 (2004) 243–252

- d-wave potential between $\bar{K}\Xi$ and $\bar{K}\Xi^*$ or $\eta\Omega$:
  described in terms of α, β: free parameters



A possible d-wave diagram for the $\bar{K}\Xi^*$ -> $\bar{K}\Xi$ transition



We do not make a model
Estimates done by M. P. Valderrama, PRD98,054009 (2018).

$G_{K^-\Xi^*}$ function accounting for $\Xi^* \to \pi\Xi$ decay

- Meson-Baryon loop function G:

$$G(\sqrt{s}) = \begin{pmatrix} G_{\bar{K}\Xi^*}(\sqrt{s}) & 0 & 0 \\ 0 & G_{\eta\Omega}(\sqrt{s}) & 0 \\ 0 & 0 & G_{\bar{K}\Xi}(\sqrt{s}) \end{pmatrix}$$

q_{max}: cut off parameter

For s-wave channel

$$G_i(\sqrt{s}) = \int_{|\boldsymbol{q}|<q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_i(\boldsymbol{q})} \frac{M_i}{E_i(\boldsymbol{q})} \frac{1}{\sqrt{s} - \omega_i(\boldsymbol{q}) - E_i(\boldsymbol{q}) + i\epsilon}$$

for $i = \bar{K}\Xi^*, \eta\Omega$

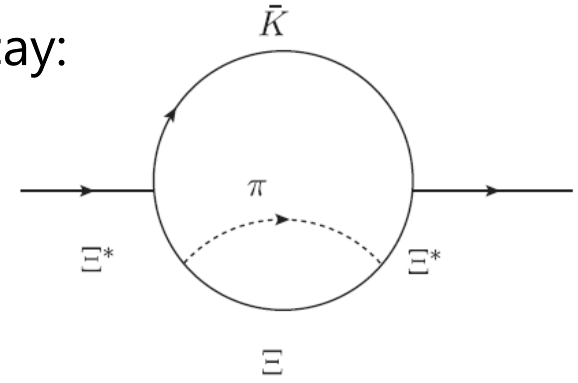
For d-wave channel

$$G_{\bar{K}\Xi}(\sqrt{s}) = \int_{|\boldsymbol{q}|<q'_{\max}} \frac{d^3q}{(2\pi)^3} \frac{(q/q_{\rm on})^4}{2\omega_{\bar{K}}(\boldsymbol{q})} \frac{M_\Xi}{E_\Xi(\boldsymbol{q})} \frac{1}{\sqrt{s} - \omega_{\bar{K}}(\boldsymbol{q}) - E_\Xi(\boldsymbol{q}) + i\epsilon}$$

- We take into account the Ξ^* mass distribution due to its width for $\Xi^* \to \pi\Xi$ decay:

  $G_{K^-\Xi^*}$ is convolved with the Ξ^* mass distribution: $\Omega(2012) \to \pi K\Xi$ decay

$$\tilde{G}_{\bar{K}\Xi^*}(\sqrt{s}) = \frac{1}{N}\int_{M_{\Xi^*}-\Delta M_{\Xi^*}}^{M_{\Xi^*}+\Delta M_{\Xi^*}} d\tilde{M} \left(-\frac{1}{\pi}\right) \mathrm{Im}\left(\frac{1}{\tilde{M} - M_{\Xi^*} + i\frac{\Gamma_{\Xi^*}}{2}}\right) G_{\bar{K}\Xi^*}(\sqrt{s}, m_{\bar{K}}, \tilde{M})$$



=> Comparison of the $\Omega(2012)$ with/without convolution gives us the estimate of $\Omega(2012)$ decay width into $\bar{K}\Xi$ and $\pi\bar{K}\Xi$ decay channels :

$$R = \frac{\Gamma_{\Omega^* \to \pi\bar{K}\Xi}}{\Gamma_{\Omega^* \to \bar{K}\Xi}} = \frac{\Gamma_{\Omega^*,\rm con} - \Gamma_{\Omega^*,\rm non}}{\Gamma_{\Omega^*,\rm non}}$$

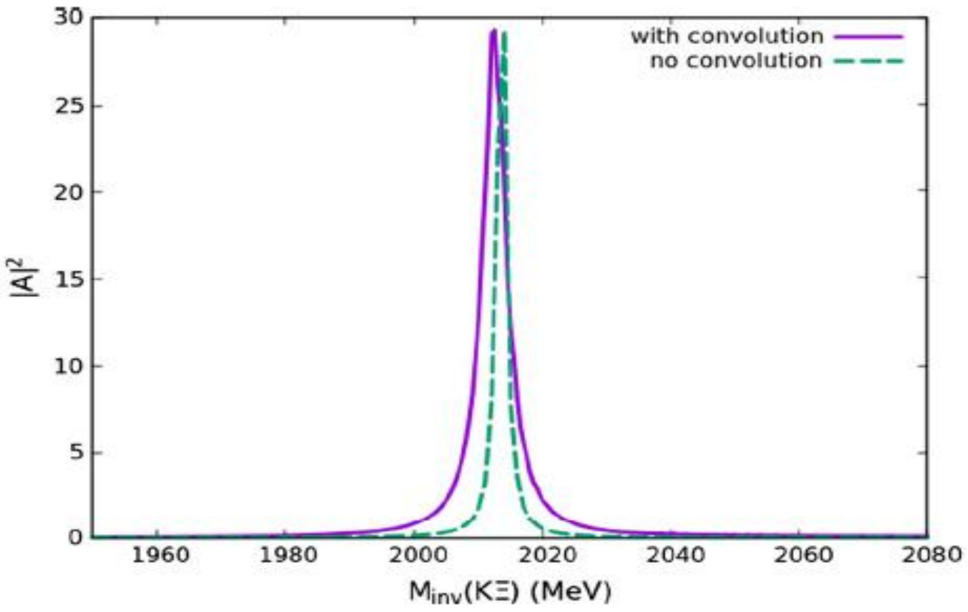
$\Gamma_{\rm con}$: $G_{K^-\Xi^*}$ with convolution (accounts for $K\Xi$ and $\pi K\Xi$ decays)
$\Gamma_{\rm non}$: $G_{K^-\Xi^*}$ without convolution (only for $K\Xi$ decay)

Calculated Ω(2012) mass, width, and decay ratio

We make a fit to the experimental data by changing the α, β, q_{max} parameters.

$$M_{\Omega(2012)} = 2012.4 \pm 0.7 \pm 0.6\ \mathrm{MeV} \qquad \Gamma_{\Omega(2012)} = 6.4^{+2.5}_{-2.0} \pm 1.6\ \mathrm{MeV}$$

- Result by R. Pavao and E. Oset, EPJC78(2018)



α (MeV^{-3})	β (MeV^{-3})	$q_{\max} = q'_{\max}$ (MeV)
4.0×10^{-8}	1.5×10^{-8}	735

- Result with convolution

$$m_{\Omega^*} = 2012.37\ \mathrm{MeV},$$
$$\Gamma_{\Omega^*} = 6.24\ \mathrm{MeV}.$$

- Result without convolution

$$m_{\Omega^*}^{(\text{no conv.})} = 2013.5\ \mathrm{MeV},$$
$$\Gamma_{\Omega^*}^{(\text{no conv.})} = 3.2\ \mathrm{MeV}.$$

$$R = \frac{\Gamma_{\Omega^* \to \pi \bar{K} \Xi}}{\Gamma_{\Omega^* \to \bar{K} \Xi}} = \frac{\Gamma_{\Omega^*,\mathrm{con}} - \Gamma_{\Omega^*,\mathrm{non}}}{\Gamma_{\Omega^*,\mathrm{non}}} = 0.95$$

=> Good agreement with the latest Belle result $\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = 0.97 \pm 0.24 \pm 0.07$

Limitation of calculated ratio R

We also find an acceptable solution in terms of natural values for the q_{max}, α, β parameters which reproduce fairly well the experimental data in 2019

$$R = \frac{\Gamma_{\Omega}(\pi \bar{K} \Xi)}{\Gamma_{\Omega,\bar{K}\Xi}} < 11.9\,\%$$

- Results by N. Ikeno, G. Toledo, and E. Oset, PRD101, 094016 (2020)

	Set 1	Set 2	Set 3
$q_{\max}(\bar{K}\Xi^*)$ [MeV]	735	775	735
$q_{\max}(\eta\Omega)$ [MeV]	735	710	750
α [10^{-8} MeV^{-3}]	−8.7	−8.7	−11.0
β [10^{-8} MeV^{-3}]	18.3	18.3	20.0
R	10.9 %	10.4 %	10.9 %

Similar results in J. Lu, C. Zeng, E. Wang, J. Xie, and L. Geng, Eur. Phys. J. C, 361 (2020)

=> The molecular picture was pushed to the limit and showed that the ratio R could be made as small as 10%, but not smaller than 10%

Couplings g_i of different channels

- The couplings g_i of the Ω(2012) to the different channels are obtained from the residue of the T-matrix of the pole in the second Riemann sheet. Close to the pole

$$T_{ij} = \frac{g_i g_j}{z - z_R} \; (z, \text{complex energy}; \; z_R, \text{complex pole position})$$

$$g_i^2 = \lim_{z \to z_R} (z - z_R) T_{ii}; \quad g_j = g_i \frac{T_{ij}}{T_{ii}} |_{z = z_R}.$$

- We also show the wave function at the origin for the s-wave states, wf($g_i G_i$), and the probability of each channel $-g_i^2 \frac{\partial G_i}{\partial \sqrt{s}}$

$$\sum_i (-) g_i^2 \frac{\partial G}{\partial \sqrt{s}} = 1,$$

	$\bar{K}\Xi^*$ (2027)	$\eta\Omega$ (2220)	$\bar{K}\Xi$ (1812)
g_i	$1.86 - i0.02$	$3.52 - i0.46$	$-0.42 + i0.12$
g_i (Pavao, Oset)	$2.01 + i0.02$	$2.84 - i0.01$	$-0.29 + i0.04$
$\mathrm{wf}_i(g_i G_i)$	$-34.05 - i1.10$	$-30.66 + i3.67$	…
$-g_i^2 \frac{\partial G_i}{\partial \sqrt{s}}$	$0.57 + i0.10$	$0.25 - i0.06$	…

- D. Gamermann, J. Nieves, E. Oset, and E. R. Arriola, PRD 81, 014029 (2010).
- F. Aceti, L.R. Dai, L.S. Geng, E. Oset and Y. Zhang, EPJA50, 57 (2014)
- S. Weinberg, Phys. Rev. 130, 776 (1963).
- T. Hyodo, D. Jido and A. Hosaka, PRC85,015201 (2012).
- T. Hyodo, Int. J. Mod. Phys. A28, 1330045 (2013).
- T. Sekihara, PRC104, 035202 (2021), etc.

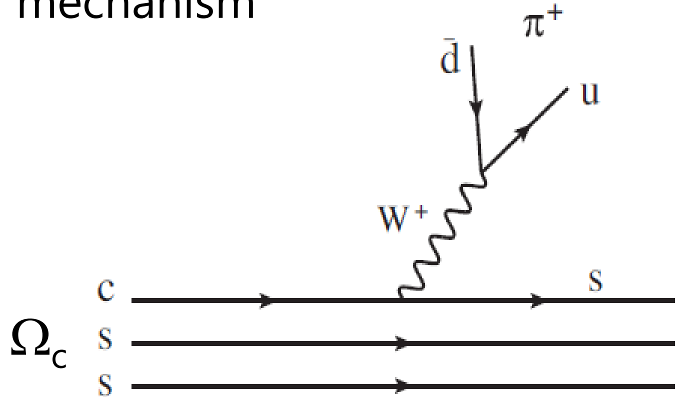
=> The strength of the *wf* and the probability dominates for the $\bar{K}\Xi^*$ state.

Note, however, that the $\eta\Omega$ channel is required to bind Ω^* state since the diagonal potential of the $\bar{K}\Xi^*$ channel is zero and hence cannot produce any bound state by itself.

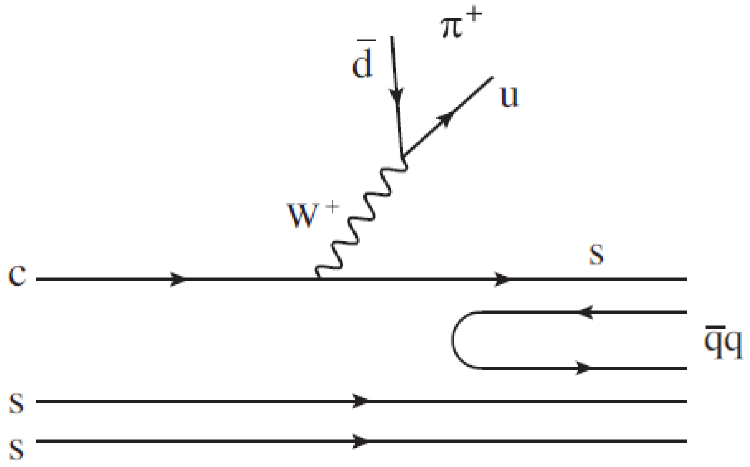
The $\Omega_c \to \pi^+\Omega(2012) \to \pi^+(K^-\Xi^0)$ reaction

- Belle data in 2021 [PRD104, 052005(2021)] : $\dfrac{\mathcal{B}(\Omega_c^0 \to \pi^+\Omega(2012)^-) \times \mathcal{B}(\Omega(2012)^- \to K^-\Xi^0)}{\mathcal{B}(\Omega_c^0 \to \pi^+K^-\Xi^0)} = (9.6 \pm 3.2 \pm 1.8)\%$
- We study a mechanism for $\Omega_c \to \pi^+\Omega(2012)$ production through an external emission Cabibbo favored weak decay mode, where the $\Omega(2012)$ is dynamically generated from the interaction of $\bar{K}\Xi^*$ and $\eta\Omega$, with $\bar{K}\Xi$ as the main decay channel. N. Ikeno, W. H. Liang, G. Toledo, and E. Oset, PRD 106, 034022 (2022).

Microscopic picture for the weak decay $\Omega_c \to \pi^+ sss$ through external emission mechanism



Hadronization of an ss pair



$$sss \to \sum_i s\bar{q}_i q_i ss = \sum_i P_{3i} q_i ss$$

where $P \equiv \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}$

$$sss \to K^- \underline{uss} + \bar{K}^0 \underline{dss} - \frac{\eta}{\sqrt{3}} \underline{sss}$$

=> We obtain $\bar{K}\Xi^*$, $\bar{K}\Xi$, $\eta\Omega$

⇒ Our decay channels require three particles in the final state

⇒ Hadronization must occur to produce the extra particle.

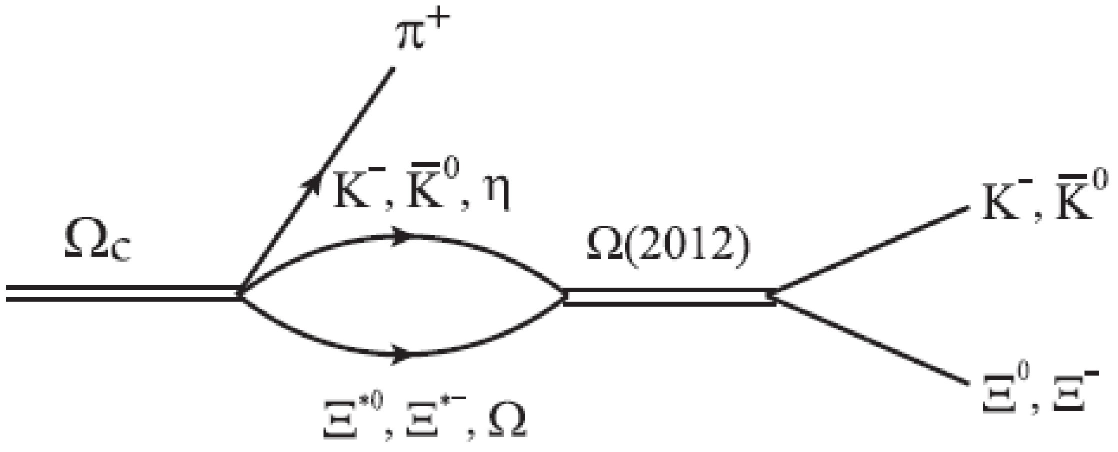
- Weak interaction vertices:

$\mathcal{L}_{W,\pi} \sim W^\mu \partial_\mu \phi$. $\quad \mathcal{L}_{\bar{q}Wq} \sim \bar{q}_{\text{fin}} W_\mu \gamma^\mu (1-\gamma_5) q_{\text{in}}$

$V_P = C(q^0 + \vec{\sigma} \cdot \vec{q})$ C: unknown constant

K. Miyahara, et al., PRC95, 035212 (2017) :
V. R. Debastiani, et al., PRD97, 094035 (2018)

The $\Omega_c \to \pi^+\Omega(2012) \to \pi^+(K^-\Xi^0)$ reaction



$\pi^+ K\Xi^*$ and $\pi^+\eta\Omega$ are produced
=> $K\Xi^*$ and $\eta\Omega$ interact and produce $\Omega(2012)$
=> Later $\Omega(2012)$ decays into $K\Xi$

The $\bar{K}\Xi$ mass distribution for Ω_c decay:

$$\frac{d\Gamma_{\text{signal}}}{dM_{\text{inv}}(K^-\Xi^0)} = \frac{1}{(2\pi)^3}\frac{M_\Xi}{M_{\Omega_c}} p_\pi \tilde{p}_{K^-} \sum |t|^2,$$

Amplitude t:

$$t = W(K^-\Xi^{*0})G_{\bar{K}\Xi^*}(M_{\text{inv}})\left(-\frac{1}{\sqrt{2}}\right)g_{R,\bar{K}\Xi^*}\frac{1}{M_{\text{inv}} - M_R + i\frac{\Gamma_R}{2}}\left(-\frac{1}{\sqrt{2}}g_{R,\bar{K}\Xi}\right)$$
$$+ W(\bar{K}^0\Xi^{*-})G_{\bar{K}\Xi^*}(M_{\text{inv}})\left(-\frac{1}{\sqrt{2}}\right)g_{R,\bar{K}\Xi^*}\frac{1}{M_{\text{inv}} - M_R + i\frac{\Gamma_R}{2}}\left(-\frac{1}{\sqrt{2}}g_{R,\bar{K}\Xi}\right)$$
$$+ W(\eta\Omega)G_{\eta\Omega}(M_{\text{inv}})g_{R,\eta\Omega}\frac{1}{M_{\text{inv}} - M_R + i\frac{\Gamma_R}{2}}\left(-\frac{1}{\sqrt{2}}g_{R,\bar{K}\Xi}\right)$$

$$p_{\pi^+} = \frac{\lambda^{1/2}(M^2_{\Omega_c}, m^2_\pi, M^2_{\text{inv}}(K^-\Xi^0))}{2M_{\Omega_c}}$$

$$\tilde{p}_{K^-} = \frac{\lambda^{1/2}(M^2_{\text{inv}}(K^-\Xi^0), m^2_{K^-}, M^2_{\Xi^0})}{2M_{\text{inv}}(K^-\Xi^0)}$$

R stands for $\Omega(2012)$ resonance

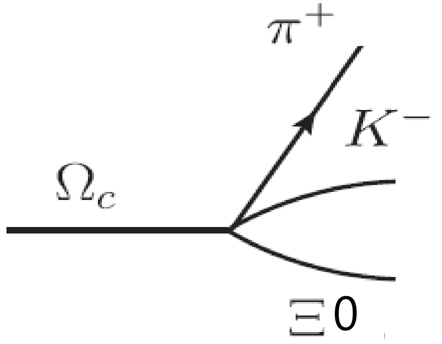
$g_{R,\bar{K}\Xi^*}$, $g_{R,\eta\Omega}$, $g_{R,\bar{K}\Xi}$: Couplings to $\Omega(2012)$

W: Weight for the matrix elements of Ω_c ↑↑↑ going to π^+ and the different final states

We use the same values of g, q_{max} obtained before

Background for $\Omega_c \to \pi^+ K^- \Xi^0$ without going through the $\Omega(2012)$

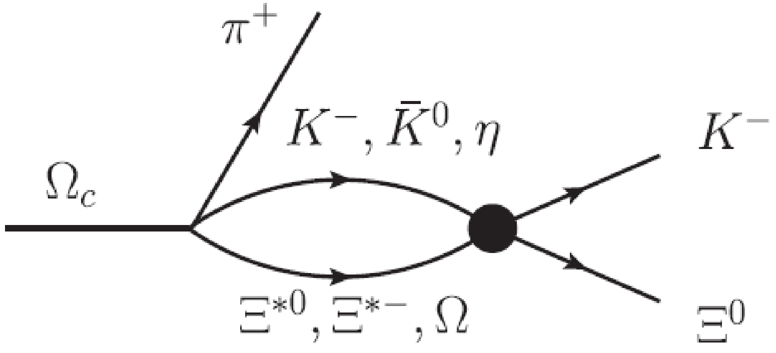
(1) Direct reaction:



$$\frac{d\Gamma^{(1)}_{\mathrm{bac}}}{dM_{\mathrm{inv}}(K^-\Xi^0)} = \frac{1}{(2\pi)^3}\frac{M_\Xi}{M_{\Omega_c}} p_\pi \tilde{p}_{K^-} \sum |\bar{t}^{(1)}_{\mathrm{bac}}|^2$$

$$\sum |\bar{t}^{(1)}_{\mathrm{bac}}|^2 = C^2 \frac{4}{27}\vec{q}^{\,2}. \qquad q = p_{\pi^+} = \frac{\lambda^{1/2}(M^2_{\Omega_c}, m^2_\pi, M^2_{\mathrm{inv}}(K^-\Xi^0))}{2M_{\Omega_c}}$$

(2) Reaction through intermediate states:



$$\frac{d\Gamma^{(2)}_{\mathrm{bac}}}{dM_{\mathrm{inv}}(K^-\Xi^0)} = \frac{1}{(2\pi)^3}\frac{M_\Xi}{M_{\Omega_c}} p_{\pi^+} \tilde{p}_{K^-} \sum |\bar{t}^{(2)}_{\mathrm{bac}}|^2$$

$$\begin{aligned} t^{(2)}_{\mathrm{bac}} = {} & W(K^-\Xi^{*0}) \cdot G_{\bar{K}\Xi^*}(M_{\mathrm{inv}}) \cdot \left(-\frac{1}{\sqrt{2}}\right) \alpha \vec{p}^{\,2}_{\bar{K}} \left(-\frac{1}{\sqrt{2}}\right) \\ & + W(\bar{K}^0\Xi^{*-}) \cdot G_{\bar{K}\Xi^*}(M_{\mathrm{inv}}) \cdot \left(-\frac{1}{\sqrt{2}}\right) \alpha \vec{p}^{\,2}_{\bar{K}} \left(-\frac{1}{\sqrt{2}}\right) \\ & + W(\eta\Omega) \cdot G_{\eta\Omega}(M_{\mathrm{inv}}) \cdot \beta \vec{p}^{\,2}_{\bar{K}} \left(-\frac{1}{\sqrt{2}}\right) \end{aligned}$$

$$V = \begin{array}{cc} & \begin{array}{ccc} \bar{K}\Xi^* & \eta\Omega & \bar{K}\Xi \end{array} \\ & \begin{pmatrix} 0 & 3F & \alpha q^2_{\mathrm{on}} \\ 3F & 0 & \beta q^2_{\mathrm{on}} \\ \alpha q^2_{\mathrm{on}} & \beta q^2_{\mathrm{on}} & 0 \end{pmatrix} \begin{array}{c} \bar{K}\Xi^* \\ \eta\Omega \\ \bar{K}\Xi \end{array} \end{array}$$

$$V_{\bar{K}\Xi^* \to \bar{K}\Xi} = \alpha \vec{q}^{\,2}_{\bar{K}},$$

$$V_{\eta\Omega \to \bar{K}\Xi} = \beta \vec{q}^{\,2}_{\bar{K}}$$

α, β : Potential parameters

We use the same values obtained before

Calculated total $\pi^+K^-\Xi^0$ production

	R_1	R_2	R_1+R_2
Set 1	0.23	0.16	0.39
Set 2	0.21	0.12	0.33
Set 3	0.21	0.20	0.41
Set 4	0.45	0.17	0.62

Calculated ratios with the parameter sets of g, q_{max}, α, and β, obtained before

N. Ikeno, G. Toledo, E. Oset, PRD(2020)
R. Pavao and E. Oset, EPJC78(2018)

- We define ratios:

$$R_1 \equiv \frac{\Gamma^{(1)}_{\rm bac}}{\Gamma_{\rm signal}} = \frac{\Gamma^{(1)}_{\rm bac}/\Gamma_{\Omega_c}}{\Gamma_{\rm signal}/\Gamma_{\Omega_c}} = \frac{\mathcal{B}^{(1)}_{\rm bac}}{\mathcal{B}_{\rm signal}} \qquad R_2 \equiv \frac{\Gamma^{(2)}_{\rm bac}}{\Gamma_{\rm signal}} = \frac{\Gamma^{(2)}_{\rm bac}/\Gamma_{\Omega_c}}{\Gamma_{\rm signal}/\Gamma_{\Omega_c}} = \frac{\mathcal{B}^{(2)}_{\rm bac}}{\mathcal{B}_{\rm signal}}$$

=> $\mathcal{B}^{(1)}_{\rm bac} = R_1\mathcal{B}_{\rm signal}$, => $\mathcal{B}^{(2)}_{\rm bac} = R_2\mathcal{B}_{\rm signal}$

- Total $\pi^+K^-\Xi^0$ production stemming from the molecular picture

$$\mathcal{B}[\Omega_c \to \pi^+K^-\Xi^0]_{\rm th} = \mathcal{B}_{\rm signal} + \mathcal{B}^{(1)}_{\rm bac} + \mathcal{B}^{(2)}_{\rm bac} = \mathcal{B}_{\rm signal}(1+R_1+R_2)$$

<- Unknown constant C is implicitly included in B_{signal}

$$\frac{\mathcal{B}[\Omega_c \to \pi^+K^-\Xi^0]_{\rm th}}{\mathcal{B}[\Omega_c \to \pi^+K^-\Xi^0]_{\rm exp}} = \boxed{\frac{\mathcal{B}_{\rm signal}}{\mathcal{B}[\Omega_c \to \pi^+K^-\Xi^0]_{\rm exp}}}(1+R_1+R_2) = (9.6 \pm 3.2 \pm 1.8)(1+R_1+R_2)\%$$

Ratio measured by Belle

= 12.2–15.7%
with 38% uncertainty

=> Based on the molecular picture, we obtain only about 12–20% of the total production with the three modes evaluated to produce $\pi^+K^-\Xi^0$, one resonant and two nonresonant

- There are two other sources we did not consider: ~85% of the total production

$\Omega_c \to \Xi^0\bar{K}^{*0} \to \Xi^0K^-\pi^+$ (PDG data)

$\Omega_c \to \pi^+\Omega^* \to \pi^+K^-\Xi^0$ (Quark model)

K. L. Wang, Q. F. Lü, J. J. Xie, and X. H. Zhong, arXiv: 2203.04458

✓ The consistency of the molecular picture with all data

Summary

- We have studied the molecular picture for the $\Omega(2012)$ state with the coupled channels $\bar{K}\Xi^*$, $\eta\Omega$, $\bar{K}\Xi$
- We also have studied the $\Omega_c \to \pi^+\Omega(2012) \to \pi^+K^-\Xi^0$ reaction
- We found that the obtained results are consistent with all Belle results
  - $\Omega(2012)$ mass, width, and the ratio R of the $\Xi\pi K$ width to the ΞK width
  - All three channels account for about (12–20)% of the total $\Omega_c \to \pi^+K^-\Xi^0$ decay rate
- We should note that the molecular structure of $\Omega(2012)$ is mostly a $\bar{K}\Xi^*$ bound state. However, it requires the interaction with the $\eta\Omega$ channel to bind, while neither the $\bar{K}\Xi^*$ nor the $\eta\Omega$ states would be bound by themselves.
- New information on experimental data is most welcome.

Two other sources we did not consider

$$\boxed{\Omega_c \to \Xi^0 \bar{K}^{*0} \to \Xi^0 K^- \pi^+}$$

($\Gamma_{7,}$ Γ_8 of PDG data)

$$\frac{\Gamma_8}{\Gamma_7} = \frac{\mathcal{B}[\Xi^0 \bar{K}^{*0} \to \Xi^0 K^- \pi^+]}{\mathcal{B}[\Xi^0 K^- \pi^+]} = \frac{0.68 \pm 0.16}{1.20 \pm 0.18} = \underline{0.57 \pm 0.16},$$

=> This means that about 60% of the $\Omega_c \to \pi^+ K^- \Xi^0$ decay comes from the $\Xi^0 K^{*-} \to \pi^+ K^- \Xi^0$ decay, which is not a part of our calculation

We find a total fraction of

$$\boxed{0.82 \pm 0.16 \simeq 66 - 98\%.}$$

$$\boxed{\Omega_c \to \pi^+ \Omega^* \to \pi^+ K^- \Xi^0}$$

Ω^* are any kind of excited $\Omega(sss)$ states

We can rely upon a theoretical Quark model calculation

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$$\frac{\mathcal{B}[\Omega_c \to \pi^+ \Omega(1^2P_{3/2^-}) \to \pi^+ K^- \Xi^0]}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]} \simeq 0.08,$$

$$\frac{\mathcal{B}[\Omega_c \to \pi^+ \Omega(1^2P_{1/2^-}) \to \pi^+ K^- \Xi^0]}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]} \simeq 0.11,$$

$$\frac{\mathcal{B}[\Omega_c \to \pi^+ \Omega(1^4D_{1/2^+}) \to \pi^+ K^- \Xi^0]}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]} \simeq 0.04$$

$$\frac{\mathcal{B}[\Omega_c \to \pi^+ \Omega(1^4D_{3/2^+}) \to \pi^+ K^- \Xi^0]}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]} \simeq 0.02$$

Summing all these contributions, we find a fraction of

$$\frac{\mathcal{B}[\Omega_c \to \pi^+ \Omega^* \to \pi^+ K^- \Xi^0]}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]} \simeq \underline{0.25}$$

W: Weight for the matrix elements of $\Omega_c \uparrow\uparrow\uparrow$ going to π^+ and the different final states

$$K^-\Xi^{*0}(S_z = 3/2):\ W = \frac{1}{\sqrt{3}} C(q^0 + q_z),$$

$$\bar{K}^0\Xi^{*-}(S_z = 3/2):\ W = \frac{1}{\sqrt{3}} C(q^0 + q_z),$$

$$\eta\Omega(S_z = 3/2):\ W = -\frac{1}{\sqrt{3}} C(q^0 + q_z),$$

$$K^-\Xi^{*0}(S_z = 1/2):\ W = \frac{1}{3} C q_+,$$

$$\bar{K}^0\Xi^{*-}(S_z = 1/2):\ W = \frac{1}{3} C q_+,$$

$$\eta\Omega(S_z = 1/2):\ W = -\frac{1}{3} C q_+,$$

$$K^-\Xi^0(S_z = 1/2):\ W = \frac{\sqrt{2}}{3} C q_+$$

$$\bar{K}^0\Xi^-(S_z = 1/2):\ W = -\frac{\sqrt{2}}{3} C q_+,$$

$$V_P = C(q^0 + \vec{\sigma}\cdot\vec{q})$$

the matrix element of

$$\vec{\sigma}\cdot\vec{q} = \sigma_+ q_- + \sigma_- q_+ + \sigma_z q_z,$$

$$\sigma_+ = \frac{1}{2}(\sigma_x + i\sigma_y), \qquad \sigma_- = \frac{1}{2}(\sigma_x - i\sigma_y),$$

$$q_+ = q_x + iq_y, \qquad q_- = q_x - iq_y,$$

q^0 is the energy of the π^+ and q its three-momentum

We can take the z direction in the π^+ direction, and when integrating over the π^+ angles, we get the angle averaged values of q_z^2, $q^0 q_z$, and $|q_+|^2$,

$$q_z^2 \to \frac{1}{3}\vec{q}^2, \qquad q^0 q_z \to 0, \qquad |q_+|^2 = q_x^2 + q_y^2 \to \frac{2}{3}\vec{q}^2.$$

internal emission

We can also have $\bar{K}^0\pi^+\Xi^-$, $\bar{K}^0\pi^+\Xi^{*-}$ production via internal emission, although suppressed by a color factor around 1/3.

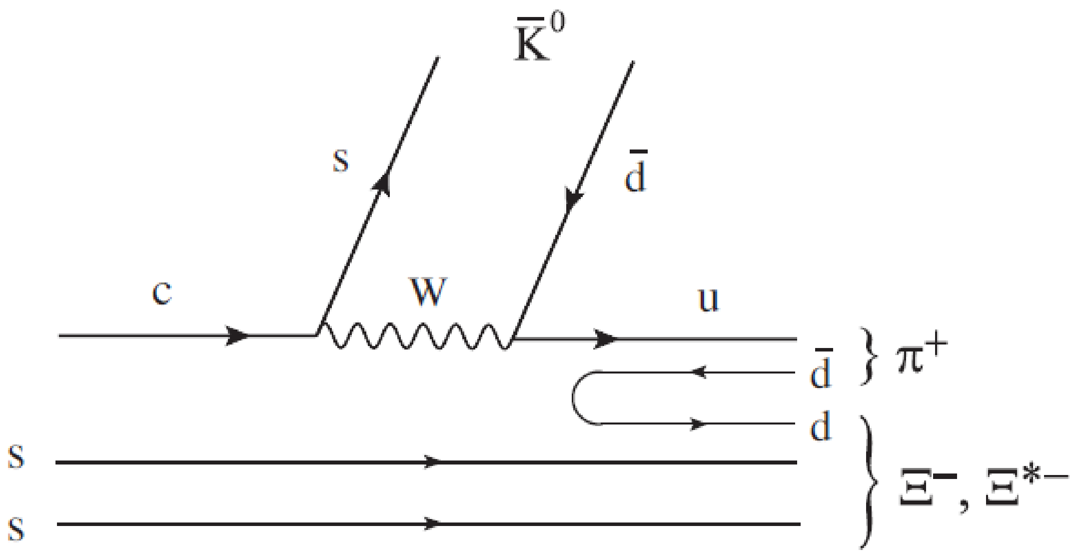

FIG. 4. Mechanism for internal emission for $\bar{K}^0\pi^+\Xi^-$, $\bar{K}^0\pi^+\Xi^{*-}$.

If we look at the $\pi^+K^-\Xi^0$ final state production, we just have an external emission

TABLE I. Values of the parameters $q_{\max}$, α, β and the resulting Γ_R ($R \equiv \Omega(2012)$) for three different sets of Ref. [14] (sets 1–3) and the one of Ref. [11] (set 4).

	Set 1	Set 2	Set 3	Set 4
$q_{\max}(\bar{K}\Xi^*)$ [MeV]	735	775	735	735
$q_{\max}(\eta\Omega)$ [MeV]	735	710	750	735
α [10^{-8} MeV^{-3}]	−8.7	−8.7	−11.0	4.0
β [10^{-8} MeV^{-3}]	18.3	18.3	20.0	1.5
$g_{R,\bar{K}\Xi^*}$	$1.86 - i0.02$	$1.79 + i0.02$	$1.88 + i0.04$	$2.01 + i0.02$
$g_{R,\eta\Omega}$	$3.52 - i0.46$	$3.79 - i0.53$	$3.55 - i0.67$	$2.84 - i0.01$
$g_{R,\bar{K}\Xi}$	$-0.42 + i0.12$	$-0.44 + i0.14$	$-0.42 + i0.22$	$-0.29 + i0.04$
Γ_R [MeV]	7.3	7.7	8.2	6.24

$$R_1 \equiv \frac{\Gamma^{(1)}_{\rm bac}}{\Gamma_{\rm signal}} = \frac{\Gamma^{(1)}_{\rm bac}/\Gamma_{\Omega_c}}{\Gamma_{\rm signal}/\Gamma_{\Omega_c}} = \frac{\mathcal{B}^{(1)}_{\rm bac}}{\mathcal{B}_{\rm signal}} \Rightarrow \mathcal{B}^{(1)}_{\rm bac} = R_1 \mathcal{B}_{\rm signal}$$

$$R_2 \equiv \frac{\Gamma^{(2)}_{\rm bac}}{\Gamma_{\rm signal}} = \frac{\Gamma^{(2)}_{\rm bac}/\Gamma_{\Omega_c}}{\Gamma_{\rm signal}/\Gamma_{\Omega_c}} = \frac{\mathcal{B}^{(2)}_{\rm bac}}{\mathcal{B}_{\rm signal}} \Rightarrow \mathcal{B}^{(2)}_{\rm bac} = R_2 \mathcal{B}_{\rm signal}$$

- Total $\pi^+K^-\Xi^0$ production stemming from the molecular picture

$$\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]_{\rm th} = \mathcal{B}_{\rm signal} + \mathcal{B}^{(1)}_{\rm bac} + \mathcal{B}^{(2)}_{\rm bac} = \mathcal{B}_{\rm signal}(1 + R_1 + R_2)$$

$$\frac{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]_{\rm th}}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]_{\rm exp}} = \boxed{\frac{\mathcal{B}_{\rm signal}}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]_{\rm exp}}}(1 + R_1 + R_2)$$

Ratio measured by Belle

$$= (9.6 \pm 3.2 \pm 1.8)(1 + R_1 + R_2)\%$$

= 12.2–15.7% with 38% uncertainty

TABLE II. Results for $R_1, R_2, R_1 + R_2$ with different sets of parameters shown in Table I.

	R_1	R_2	$R_1 + R_2$
Set 1	0.23	0.16	0.39
Set 2	0.21	0.12	0.33
Set 3	0.21	0.20	0.41
Set 4	0.45	0.17	0.62