On the $Z_{cs}(3985)$ and $X(3960)$ states

Towards HQSS and $SU(3)$ multiplet descriptions

CSIC
Consejo Superior de Investigaciones Científicas

Vniversitat de València

IFIC
Institut de Física Corpuscular

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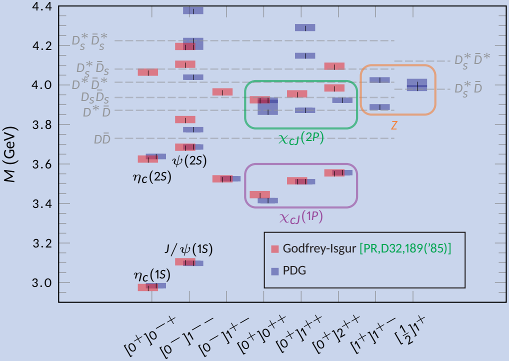
Generalitat Valenciana

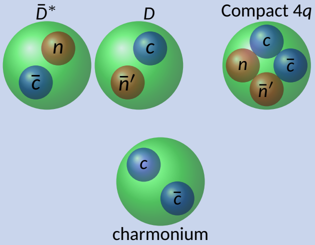
Gobierno de España
Ministerio de Ciencia e Innovación

STRONG-2020

Outline

Quark model in the charmonium sector





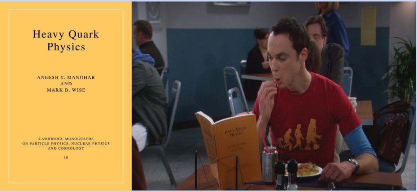
Appropriate tool: Weinberg's compositeness
[Weinberg PR,137,B672('65)]
[MA, Nieves, EPJ,C82,724('22)]

- $\chi_{cJ}(1P)$ well established, "very CQM model" state.
- $X(3872)$ discovered by Belle [PRL,91,262001('03)] (also 2003!)
- $J^{PC} = 1^{++}$ and $\Gamma \simeq 1\,\text{MeV}$ established by LHCb (*e.g.* [JHEP,08(2020),123])
- $\chi_{cJ}(2P)$ Not established. Influence of open thresholds? $X(3872)$ a molecular state, $4q$,...?
- Z_c/Z_{cs} states have $I = 1$ or $1/2$, clearly "tetraquarks" ($c\bar{c}u\bar{d}$, ...)
- Many theoretical and lattice and experimental works: can't cite them properly here!
  (many references in [PR,D106,094002('22)])

HQSS and flavour $SU(3)$ LO lagrangian

- $SU(3)$ light flavour symmetry:
  $H_a^{(Q)} \sim (Q\bar{u}, Q\bar{d}, Q\bar{s}) \sim (D^0, D^+, D_s^+)$
- HQSS: $H_a^{(Q)} = \frac{1+\not{v}}{2}\left(P_{a\mu}^{*(Q)}\gamma^\mu - P_a^{(Q)}\gamma_5\right)$
  (with $v \cdot P_a^{*(Q)} = 0$)

  [Grinstein *et al.*, NP,B380('92); Alfiky *et al.*, PL,B640('06), ...]



- **$H\bar{H} \longrightarrow H\bar{H}$** LO lagrangian ($S$-wave contact interactions):

$$\begin{aligned}\mathcal{L}_{4H} &= \frac{1}{4}\,\mathrm{Tr}\left[\bar{H}^{(Q)a}H_b^{(Q)}\gamma_\mu\right]\mathrm{Tr}\left[H^{(\bar{Q})c}\bar{H}_d^{(\bar{Q})}\gamma^\mu\right]\left(F_A\,\delta_a^b\delta_c^d + F_A^\lambda\,\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right)\\ &+ \frac{1}{4}\,\mathrm{Tr}\left[\bar{H}^{(Q)a}H_b^{(Q)}\gamma_\mu\gamma_5\right]\mathrm{Tr}\left[H^{(\bar{Q})c}\bar{H}_d^{(\bar{Q})}\gamma^\mu\gamma_5\right]\left(F_B\,\delta_a^b\delta_c^d + F_B^\lambda\,\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right),\end{aligned}$$

- **Only 4 constants**, any linear combination can be used

$$\mathcal{C}_{0a} = F_A + \frac{10F_A^\lambda}{3}, \qquad \mathcal{C}_{1a} = F_A - \frac{2}{3}F_A^\lambda,$$
$$\mathcal{C}_{0b} = F_B + \frac{10F_B^\lambda}{3}, \qquad \mathcal{C}_{1b} = F_B - \frac{2}{3}F_B^\lambda.$$

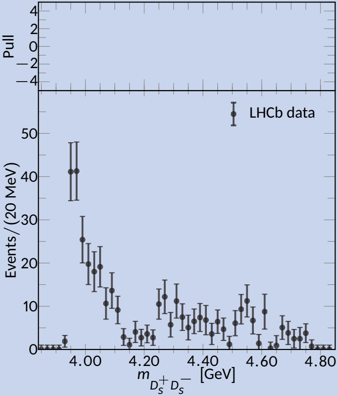
$D_s^+ D_s^-$ interaction and $B^+ \to D_s^+ D_s^- K^+$ decay [Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)]

- Scattering amplitude: $T^{-1}(E) = V^{-1} - G(E)$
  - $V = 4m_{D_s}^2 \frac{C_{0a} + C_{1a}}{2}$
  - $G(E)$: loop functions, once-subtracted DR, $G(E_{th}) = G_\Lambda(E_{th})$
- Simple production model:

  $$T_B(E) = P + P\,G(E)T(E) = P\frac{1}{1 - V\,G(E)}$$
- $$\frac{d\Gamma}{dE} = \frac{1}{(2\pi)^3}\frac{k\,p}{4m_B^2}\,|T_B(E)|^2$$

[LHCb: 2210.15153, 2211.05034]

[LHC Seminar (July 5th, 2022)]



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- Fit: two solutions (virtual or bound), in both:
  $M_{X(3960)} = 3928(3)$ MeV
  $2M_{D_s} - M_{X(3960)} = 8(3)$ MeV
- LHCb: $M = 3956(5)(11)$ MeV, $\Gamma = 43(13)(7)$ MeV
- [Prelovsek *et al.*, JHEP 06,035('20)]: Bound state $B = 6.2^{+2.0}_{-3.8}$ MeV (*cf.* also [Bayar, Feijoo, Oset, 2207.08490])

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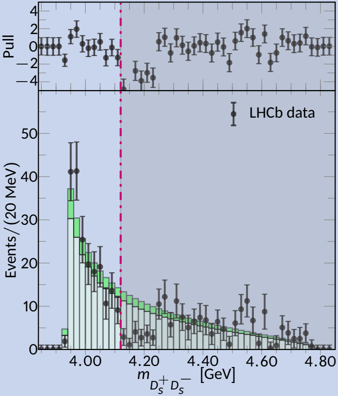

	Vir. (S-I)		Bou. (S-II)	
	$\Lambda = 0.5$ GeV	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV	$\Lambda = 1.0$ GeV
$C_{D_s\bar{D}_s}$ (fm^2)	$-0.74^{+0.04}_{-0.04}$	$-0.46^{+0.02}_{-0.02}$	$-3.36^{+0.56}_{-1.02}$	$-0.91^{+0.05}_{-0.06}$

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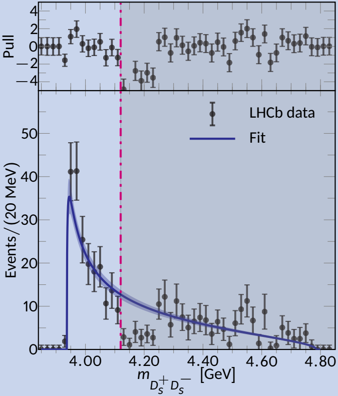

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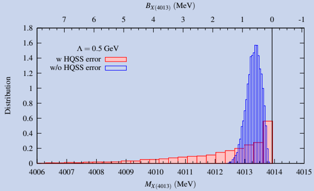
Fixing constants

- Lagrangian $\mathcal{L}$ with HQSS and light-flavour $SU(3)$ symmetry has 4 constants (C_{0a}, C_{0b}, C_{1a}, C_{1b})
- Some relations can be independently useful. Some examples:
  1. $X(3872)$ and X_{2++}

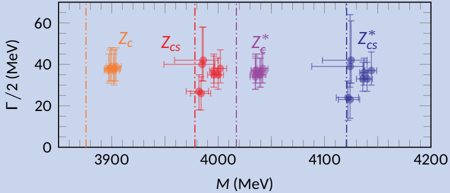
[MA, Guo, Hidalgo-Duque, Nieves, Pavón, EPJ,C75,547('15)]



$$\left.\begin{array}{l}\langle D\bar{D}^*;\,0(1^{++})|\,\hat{T}\,|D\bar{D}^*;\,0(1^{++})\rangle\\ \langle D^*\bar{D}^*;\,0(2^{++})|\,\hat{T}\,|D^*\bar{D}^*;\,0(2^{++})\rangle\end{array}\right\} = C_{0a} + C_{0b}$$

  2. $Z_c, Z_c', Z_{cs}, Z_{cs}'$

[Du, MA, Guo, Nieves, PR,D105,074018('22); see below]



$$\left.\begin{array}{l}\langle D\bar{D}^*;\,1(1^{+-})|\,\hat{T}\,|D\bar{D}^*;\,1(1^{+-})\rangle\\ \langle D_s\bar{D}^*;\,\tfrac{1}{2}(1^{+})|\,\hat{T}\,|D_s\bar{D}^*;\,\tfrac{1}{2}(1^{+})\rangle\\ \langle D^*\bar{D}^*;\,1(1^{+-})|\,\hat{T}\,|D^*\bar{D}^*;\,1(1^{+-})\rangle\\ \langle D_s^*\bar{D}^*;\,\tfrac{1}{2}(1^{+})|\,\hat{T}\,|D_s^*\bar{D}^*;\,\tfrac{1}{2}(1^{+})\rangle\end{array}\right\} = C_{1a} - C_{1b}$$

Fixing all constants

[Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)]

- $X(3960)$: fixes $C_{D_s\bar{D}_s} = (C_{0a} + C_{1a})/2$ (as previously seen)
- $Z_c(3900)$: fixes $C_{1Z} = C_{1a} - C_{1b}$
  - Assume virtual state $M = 3813^{+28}_{-21}$ MeV ([2201.08253; 1512.03638] from a fit to BESIII data)
- $X(3872)$: fixes $\begin{cases} C_{0X} = (C_{0a} + C_{0b})/2 \\ C_{1X} = (C_{1a} + C_{1b})/2 \end{cases}$
  - Experimental information:

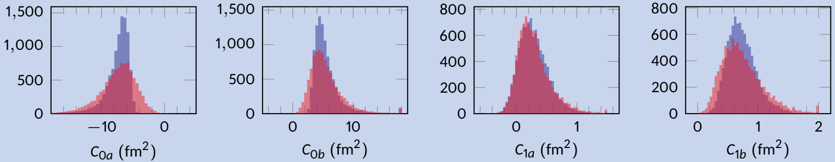
$$\begin{cases} \text{[LHCb, 2204.12597]} & R^{\rm exp}_{X(3872)} = 0.29(4) \\ \text{[LHCb, PR,D 102,092005('20)]} & B^{\rm exp}_{X(3872)} = [-150, 0]\ \text{keV} \longleftarrow M^{\rm exp}_{X(3872)} = 3871.69^{+0.00\,+0.05}_{-0.04\,-0.13}\ \text{MeV} \end{cases}$$

  - Theoretically: [0911.4407; 1210.5431; 1504.00861]

$$V = \frac{1}{2}\begin{pmatrix} C_{0X} + C_{1X} & C_{0X} - C_{1X} \\ C_{0X} - C_{1X} & C_{0X} + C_{1X} \end{pmatrix}, \quad T = (\mathbb{I} - V\,G)^{-1}\,V\,.$$

$$R_{X(3872)} = \frac{\hat\Psi_n - \hat\Psi_c}{\hat\Psi_n + \hat\Psi_c}, \quad \frac{\hat\Psi_n}{\hat\Psi_c} = \frac{1 - (2m_{D^+}m_{D^{*-}})\,G_2\,(C_{0X} + C_{1X})}{(2m_{D^+}m_{D^{*-}})\,G_2\,(C_{0X} - C_{1X})} = \frac{(2m_{D^0}m_{D^{*0}})\,G_1\,(C_{0X} - C_{1X})}{1 - (2m_{D^0}m_{D^{*0}})\,G_1\,(C_{0X} + C_{1X})},$$

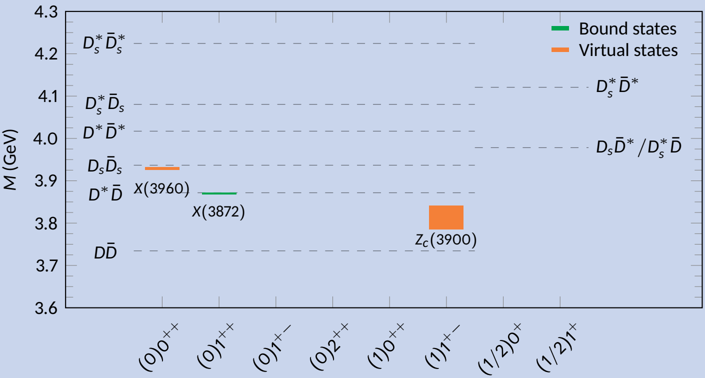
- $SU(3)$ and HQSS **breaking corrections** (30%) are taken into account for the LECs



Predictions (complete multiplet): $X(3960)$ as virtual

[Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)]

With the **four LECs** (C_{0a}, C_{0b}, C_{1a}, C_{1b}) fixed, poles can be looked for in every sector.



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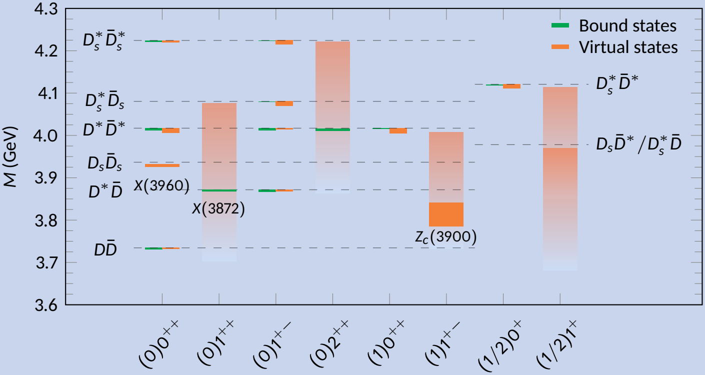

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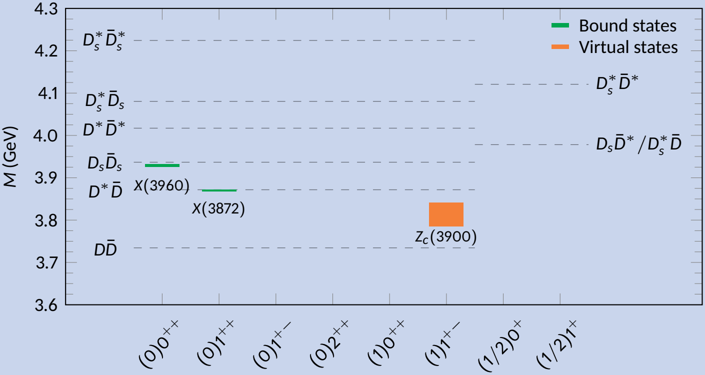

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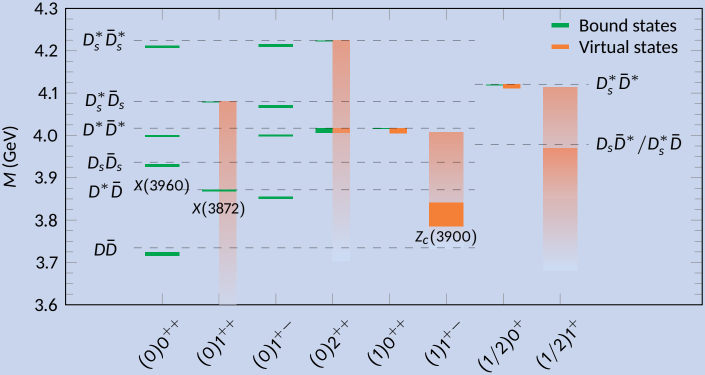
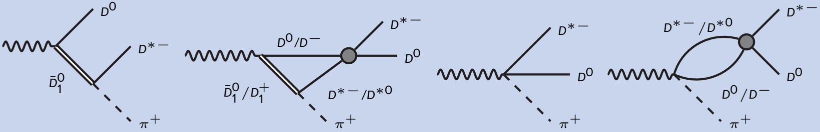


A closer look into $Z_c(3900)$ and $Z_{cs}(3985)$

[Du, MA, Guo, Nieves, PR,D105,074018('22)]

- $\langle D\bar{D}^*;\, 1(1^{+-}) |\, \hat{T}\, | D\bar{D}^*;\, 1(1^{+-}) \rangle = \langle D_s\bar{D}^*;\, \frac{1}{2}(1^{+}) |\, \hat{T}\, | D_s\bar{D}^*;\, \frac{1}{2}(1^{+}) \rangle = C_{1a} - C_{1b} \equiv C_{1Z}$
- Constant V and single channel: only virtual or bound states (pole condition $V^{-1} = G$)
- Extension of the previous approach in two directions [MA, Guo, Hidalgo-Duque, Nieves, PL,B755,337('16)]:
  1. **Coupled channels** $\begin{cases} I = 1 & \left(\frac{1}{\sqrt{2}} \left[D\bar{D}^* - D^*\bar{D} \right],\ J/\psi\, \pi \right) \\ I = \frac{1}{2} & \left(\frac{1}{\sqrt{2}} \left[D_s\bar{D}^* - D_s^*\bar{D} \right],\ J/\psi\, K \right) \end{cases}$ (also necessary for $e^+e^- \to J/\psi\pi\pi$ data)
  2. **Energy dependence** $C_{1Z} \to C_{1Z} + b\, \frac{s - E_{\rm th}^2}{2E_{\rm th}}$
- Production mechanism (includes triangle singularity!) for $Y \to D^0\, \bar{D}^{*-}\, \pi^+$



$$\overline{|\mathcal{A}_2(s,t)|^2} = \left| \frac{1}{t - m_{D_1}^2} + I(s)T_{22}(s) \right|^2 q_\pi^4(s) + \frac{1}{2} \left| E_\pi(s) \frac{h_S}{h_D} \left[\frac{1}{t - m_{D_1}^2} + I(s)T_{22}(s) \right] + \beta \left[1 + G_2(s)T_{22}(s) \right] \right|^2$$

A closer look into $Z_c(3900)$ and $Z_{cs}(3985)$

[Du, MA, Guo, Nieves, PR,D105,074018('22)]

- $\langle D\bar{D}^*;\, 1(1^{+-})|\, \hat{T}\, |D\bar{D}^*;\, 1(1^{+-})\rangle = \langle D_s\bar{D}^*;\, \frac{1}{2}(1^{+})|\, \hat{T}\, |D_s\bar{D}^*;\, \frac{1}{2}(1^{+})\rangle = C_{1a} - C_{1b} \equiv C_{1Z}$
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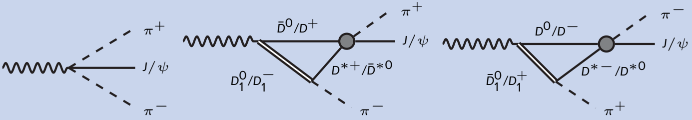


$$\overline{|\mathcal{A}_1(s,t)|^2} = |\tau(s)|^2 q_\pi^4(s) + |\tau(t)|^2 q_\pi^4(t) + \frac{3\cos^2\theta - 1}{2}\left[\tau(s)\tau(t)^* + \tau(s)^*\tau(t)\right] q_\pi^2(s) q_\pi^2(t)$$
$$+ \frac{1}{2}\left\{|\tau'(s)|^2 E_\pi^2(s) + |\tau'(t)|^2 E_\pi^2(t) + \left[\tau'(s)^*\tau'(t) + \tau'(s)\tau'(t)^*\right] E_\pi(s) E_\pi(t)\right\}$$
$$\tau(s) = \sqrt{2} I(s) T_{12}(s) + \alpha \qquad \tau'(s) = \sqrt{2} I(s) T_{12}(s) \times (h_S/h_D)$$

Fit to data

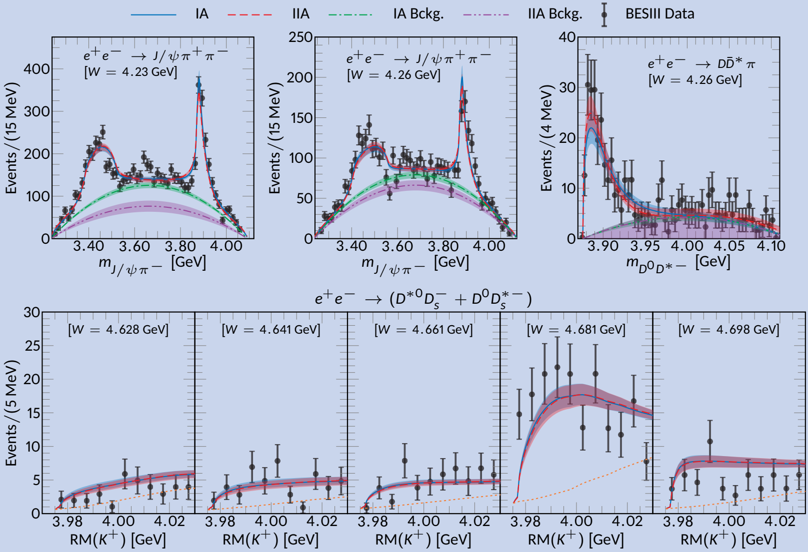
[Du, MA, Guo, Nieves, PR,D105,074018('22)]

- Fitted data:
  - $J/\psi\pi^-$ distribution in $e^+e^- \to J/\psi\pi^+\pi^-$ [BESIII, PRL,119('17)]
  - D^0D^{*-} distribution in $e^+e^- \to D^0D^{*-}\pi^+$ [BESIII,PR, D92('15)]
  - $e^+e^- \to \left(D^{*0}D_s^- + D^0D_s^{*-}\right)K^+$ [BESIII, PRL,126('21)]
- Some production/background/normalization constants are also fitted (not shown here)
- Four schemes:
  - A or B: $b = 0$ or free
  - I or II: $h_s = 0$ or not (D- or S- and D-waves)

Scheme	$D_1D^*\pi$	$a_2(\mu)$	χ^2/dof	C_{12} [fm^2]	C_Z [fm^2]	b [fm^3]
IA	D	-2.5	1.62	0.005(1)	$-0.226(10)$	0^*
		-3.0	1.62	0.005(1)	$-0.177(6)$	0^*
IIA	S+D	-2.5	1.83	0.006(1)	$-0.217(10)$	0^*
		-3.0	1.83	0.006(1)	$-0.171(6)$	0^*
IB	D	-2.5	1.24	0.007(4)	$-0.222(6)$	$-0.447(44)$
		-3.0	1.21	0.008(1)	$-0.177(4)$	$-0.255(30)$
IIB	S+D	-2.5	1.37	0.005(1)	$-0.203(7)$	$-0.473(45)$
		-3.0	1.27	0.005(1)	$-0.171(5)$	$-0.270(30)$

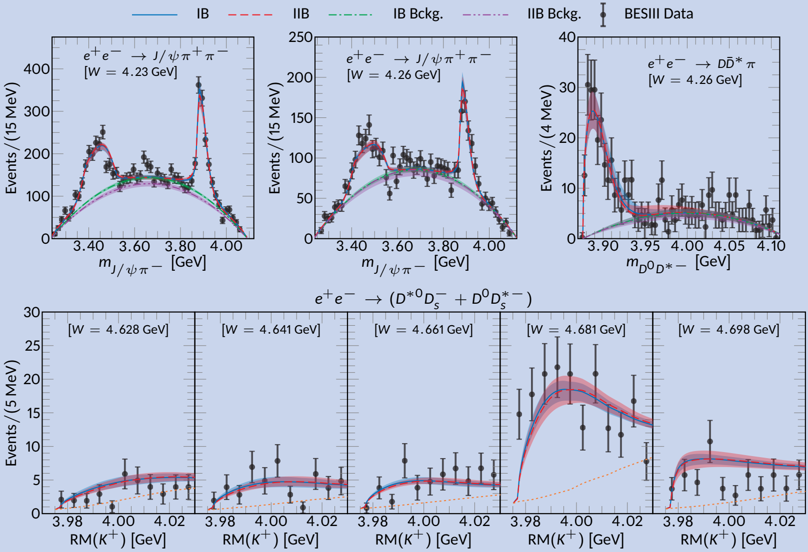
Fit to data (scheme A)

[Du, MA, Guo, Nieves, PR,D105,074018('22)]

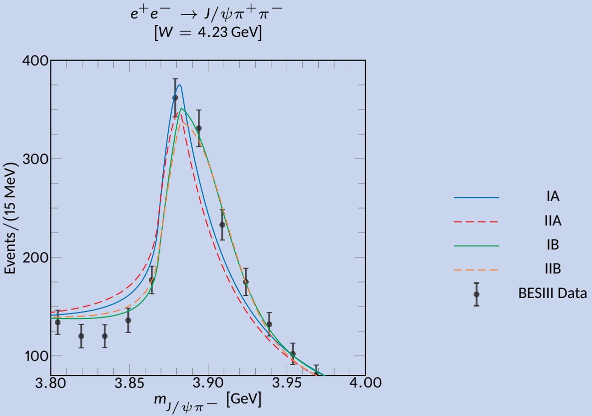


Fit to data (scheme B)

[Du, MA, Guo, Nieves, PR,D105,074018('22)]



Not possible to distinguish different scenarios in $J/\psi\pi$ spectrum



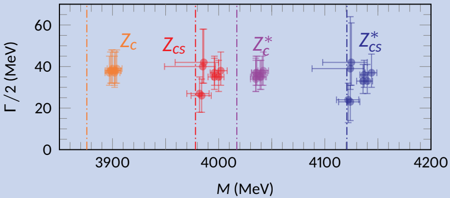
- The effect/peak produced by a **virtual pole** is always at threshold
- The peak of a **resonance** can be shifted from $\mathrm{Re}\sqrt{s}_{\text{pole}}$
- and in this case the effect is very close to threshold

$Z_{cs}^{(*)}$ and $Z_c^{(*)}$ poles

[Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)]

	$D_1D^*\pi$	$a_2(\mu)$	Z_c [MeV] Mass	$\Gamma/2$	Z_{cs} [MeV] Mass	$\Gamma/2$	Z_c^* [MeV] Mass	$\Gamma/2$	Z_{cs}^* [MeV] Mass	$\Gamma/2$
IA	D	-2.5	3813^{+21}_{-28}	vir.	3920^{+18}_{-26}	vir.	3962^{+19}_{-25}	vir.	4069^{+12}_{-16}	vir.
		-3.0	3812^{+22}_{-26}	vir.	3924^{+19}_{-23}	vir.	3967^{+19}_{-22}	vir.	4078^{+17}_{-13}	vir.
IIA	S+D	-2.5	3799^{+24}_{-33}	vir.	3907^{+22}_{-31}	vir.	3949^{+22}_{-30}	vir.	4057^{+20}_{-28}	vir.
		-3.0	3798^{+25}_{-31}	vir.	3911^{+17}_{-27}	vir.	3955^{+22}_{-27}	vir.	4067^{+19}_{-25}	vir.
IB	D	-2.5	3897^{+4}_{-4}	37^{+8}_{-6}	3996^{+4}_{-4}	37^{+8}_{-6}	4035^{+4}_{-4}	37^{+8}_{-6}	4137^{+4}_{-4}	36^{+7}_{-6}
		-3.0	3898^{+5}_{-5}	38^{+10}_{-7}	3996^{+5}_{-6}	35^{+9}_{-6}	4035^{+4}_{-5}	34^{+9}_{-6}	4136^{+5}_{-6}	33^{+8}_{-6}
IIB	S+D	-2.5	3902^{+6}_{-6}	38^{+9}_{-6}	4002^{+6}_{-6}	38^{+9}_{-7}	4042^{+5}_{-5}	38^{+9}_{-7}	4144^{+5}_{-6}	37^{+9}_{-7}
		-3.0	3902^{+5}_{-5}	37^{+9}_{-6}	4000^{+5}_{-6}	35^{+8}_{-7}	4039^{+5}_{-6}	35^{+8}_{-6}	4140^{+5}_{-6}	33^{+8}_{-6}

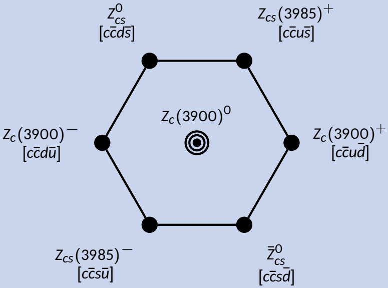
- Z_c exp.: $M = 3881.2(4.2)(52.7)$ MeV, $\Gamma/2 = 25.9(2.3)(18.0)$ MeV
- Z_{cs} exp.: $M = 3982.5(2.6)(2.1)$ MeV, $\Gamma/2 = 6.4(2.6)(1.5)$ MeV
- [Yang *et al.*, PR,D103('21)] (Z_{cs} and distribution)
- [Ikeno, Molina, Oset, PL,B814('21)] (Z_{cs} threshold effect)
- [JPAC, PL,B772('17)] Several possibilities for Z_c



- Both schemes (IB and IIB)
- Including $SU(3)$ breaking effects

Z_c and Z_{cs} form a $J^{PC} = 1^{+-}$ octet

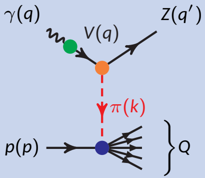
[Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)]



- $3 \otimes \bar{3} = 8 \oplus 1$
- A $\bar{Z}^0_{cs}$ has also been found by BESIII
  [PRL,129,112003('22)]
- In the case of $I = 0$ one cannot make direct identification (mixing *vs.* coupled channels)
- [Talk by E. Santopinto, Thursday 4:30pm]

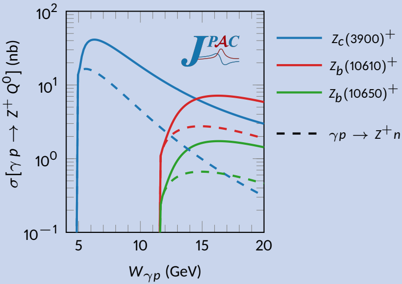
Photoproduction of Z states would be beneficial. . .

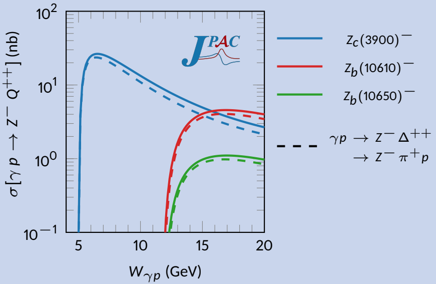
[JPAC, PR,D106,094009('22)]



- A new method to **confirm or discard** these new *XYZ* states
- In principle, photoproduction is **free of triangle-singularities** that can give rise to resonance-like effects
- Different background, easier to pin down the scattering amplitude part

[Talk by A. Hiller-Blin, Tuesday 4:30pm]





Summary and conclusions

- We have considered $D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}$ interactions with **HQSS** and **$SU(3)$ light-flavour symmetry**.
- The $X(3960)$ structure in LHCb data on $B^+ \to D_s^+ D_s^- K^+$ can be explained with a **bound or virtual** state.
- The experimental information coming from $X(3960)$, $X(3872)$, and (a virtual) $Z_c(3900)$ allows to fix the **four constants** appearing in the LO lagrangian.
- Predictions are made based on these constants for **multiplet partners** of these states in other sectors
- Considering a generalization of the interactions, the BESIII data for the Z_c and Z_{cs} states can be well reproduced, being **Z_c and Z_{cs} flavour partners** within the same **octet**

On the $Z_{cs}(3985)$ and $X(3960)$ states

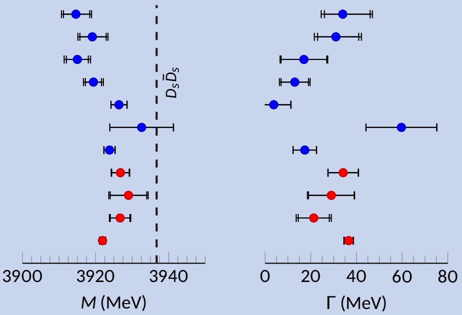
Towards HQSS and $SU(3)$ multiplet descriptions

Baryons 2022

7-11 November, Sevilla

Miguel Albaladejo (IFIC)

$\chi_{c0}(3915)$ and $\chi_{c2}(3930)$?

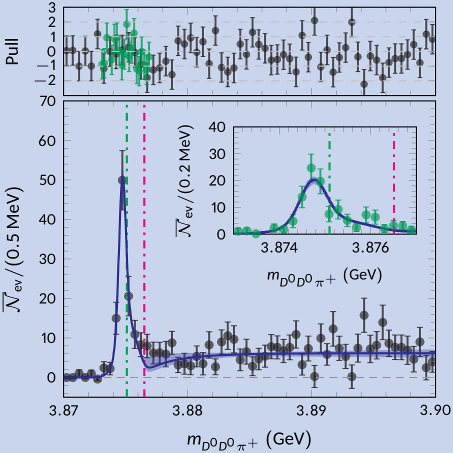


Reaction	Collab.	Ref.
$B^{(+,0)} \to J/\psi\omega K^{(+,0)}$	BaBar	BaBar:2007vxr
$B^{(+,0)} \to J/\psi\omega K^{(+,0)}$	BaBar	BaBar:2010wfc
$\gamma\gamma \to J/\psi\omega$	Belle	Belle:2009and
$\gamma\gamma \to J/\psi\omega$	BaBar	BaBar:2012nxg
$e^+e^- \to \gamma J/\psi\omega$	BESIII	BESIII:2019qvy
$e^+e^- \to \gamma J/\psi\omega$	BESIII	BESIII:2019qvy
$B^+ \to D^+D^-K^+$	LHCb	LHCb:2020pxc
$B^+ \to D^+D^-K^+$	LHCb	LHCb:2020pxc
$\gamma\gamma \to D\bar{D}$	Belle	Belle:2005rte
$\gamma\gamma \to D\bar{D}$	BaBar	BaBar:2010jfn
$pp \to D\bar{D} +$ anything	LHCb	LHCb:2019lnr

Results: Fit

- Exp. resolution taken from LHCb ($\delta \simeq 400\,\text{keV}$):

$$\overline{\mathcal{N}}_{\rm ev}(E) = \int \mathrm{d}E'\, R_{\rm LHCb}\,\left(E, E'\right)\, \mathcal{N}_{\rm ev}(E')$$



Parameter	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV
$C_0(\Lambda)$ [fm^2]	$-0.7008(22)$	$-1.5417(121)$
$C_1(\Lambda)$ [fm^2]	$-0.440(79)$	$-0.71(27)$
β/α	0.228(108)	0.093(79)
χ^2/dof	0.95	0.92

- Good agreement ($\chi^2/\text{dof} = \{0.92, 0.95\}$)
- Check: pull of the data seems randomly distributed.
- Statistical uncertainties obtained by MC bootstrap of the data

Spectroscopy

- Bound state pole in T-matrix, $\det(\mathbb{1} - VG) = 0$:

$$T_{ij}(E) = \frac{\widetilde{g}_i \widetilde{g}_j}{E^2 - \left(M_{T_{cc}^+} - i\,\Gamma_{T_{cc}^+}/2\right)^2} + \cdots$$

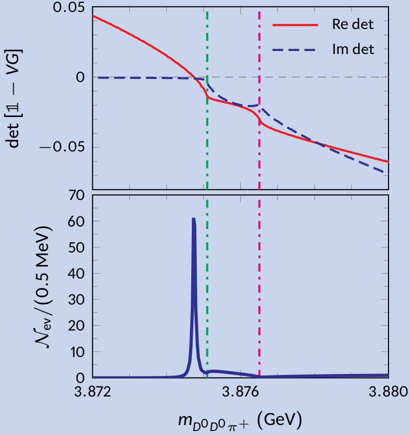
- Width: $m_{D^*} - i\,\Gamma_{D^*}/2 \Rightarrow M_{T_{cc}^+} - i\,\Gamma_{T_{cc}^+}/2$
- Pole position (wrt $D^{*+}D^0$ threshold):

Λ (GeV)	$\delta M_{T_{cc}^+}$ (keV)	$\Gamma_{T_{cc}^+}$ (keV)
1.0	$-357(29)$	77(1)
0.5	$-356(29)$	78(1)

- Good agreement with LHCb determination:

	$\delta M_{T_{cc}^+}$ (keV)	$\Gamma_{T_{cc}^+}$ (keV)
[2109.01038]	$-273(61)$	410(165)
[2109.01056]	$-360(40)$	48(2)

- Our width is somewhat larger than the ~ 50 keV obtained by LHCb and [Feijoo *et al.*, 2108.02730], [Ling *et al.*, 2108.00947].
- [Du *et al.*, 2110.13765]: $\Gamma_{T_{cc}^+}$ depending on the model used.



- Results similar to [LHCb, 2109.0156] (top) and [Feijoo *et al.*, 2108.02730; Du *et al.*, 2110.13765] (bottom).

Molecular state?

- Weinberg compositeness [Weinberg, PR,137,B672('65)]: $P = 1 - Z \simeq \dfrac{\mu^2 g^2}{2\pi\gamma_B} = -g^2 G'(E_B)$
- We get $P_{D^{*+}D^0} = 0.78(5)(2)$, $P_{D^{*0}D^+} = 0.22(5)(2) \to P_{I=0} = 1$ **purely molecular state (model built-in!)**
- Relation to ERE parameters a, r [Weinberg,PR,137,B672('65)]

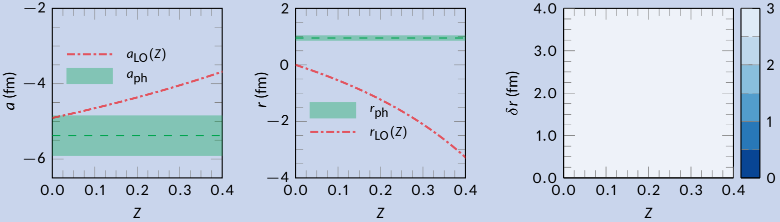
$$a = -\frac{2}{\gamma_B}\frac{1-Z}{2-Z} + \cdots\,,$$

$$r = -\frac{1}{\gamma_B}\frac{Z}{1-Z} + \cdots\,.$$

- **Single channel & isospin limit**:

Λ (GeV)	0.5	1.0
E_B (keV)	833(67)	856(53)
$a_{I=0}$ (fm)	$-5.57(25)$	$-5.18(16)$
$r_{I=0}$ (fm)	0.63	1.26

- Average values: $a_{\rm ph} = -5.38(30)$ fm, $r_{\rm ph} = 0.95(32)$ fm, $\gamma_{B\,\rm ph} = 40.4(1.7)$ MeV.



The values obtained clearly support a molecular picture for T_{cc}^+

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- We get $P_{D^{*+}D^0} = 0.78(5)(2)$, $P_{D^{*0}D^+} = 0.22(5)(2) \rightarrow P_{I=0} = 1$ **purely molecular state (model built-in!)**
- Relation to ERE parameters a, r
  [Weinberg,PR,137,B672('65)] + [MA, Nieves,EPJ,C82,724('22)]

$$a = -\frac{2}{\gamma_B}\frac{1-Z}{2-Z} - 2\delta r\left(\frac{1-Z}{2-Z}\right)^2 + \cdots,$$

$$r = -\frac{1}{\gamma_B}\frac{Z}{1-Z} + \delta r + \cdots.$$

- **Single channel & isospin limit**:

Λ (GeV)	0.5	1.0
E_B (keV)	833(67)	856(53)
$a_{I=0}$ (fm)	$-5.57(25)$	$-5.18(16)$
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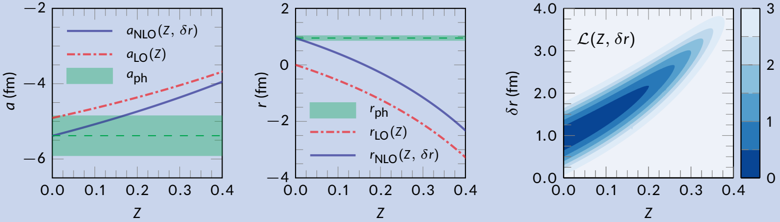
- Average values: $a_{ph} = -5.38(30)$ fm, $r_{ph} = 0.95(32)$ fm, $\gamma_{B\,ph} = 40.4(1.7)$ MeV. Minimum at $\delta r \simeq r_{ph} \simeq 1$ fm



The values obtained clearly support a molecular picture for T_{cc}^+

HQSS partner

- Heavy-Quark Spin Symmetry (HQSS) predicts that heavy-meson interactions are independent of the heavy-quark spin in the limit $m_Q \to \infty$.
- Relation between $D^*D^* \to D^*D^*$ and $D^*D \to D^*D$ amplitudes.
- The interaction kernels of the $I(J^P)$ D^*D^* systems are related to those of the D^*D ones as:

$$\left\langle D^*D^*,\, 0(1^+) \left|\hat{V}\right| D^*D^*,\, 0(1^+)\right\rangle = \left\langle D^*D,\, 0(1^+) \left|\hat{V}\right| D^*D,\, 0(1^+)\right\rangle = V_0 \,,$$
$$\left\langle D^*D^*,\, 1(2^+) \left|\hat{V}\right| D^*D^*,\, 1(2^+)\right\rangle = \left\langle D^*D,\, 1(1^+) \left|\hat{V}\right| D^*D,\, 1(1^+)\right\rangle = V_1 \,.$$

- We predict the existence of T_{cc}^{*+}, a D^*D^* molecular state, HQSS partner of T_{cc}^{+}, with a binding energy (wrt the different D^*D^* thresholds) of **1.1–1.5 MeV**.

	$\delta M_{T^*_{cc}}$ (keV)			
	Isoscalar solution		Isovector solution	
	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV
$D^{*+}D^{*+}$			$-1580(71)$	$-1156(79)$
$D^{*+}D^{*0}$	$-1561(71)$	$-1148(79)$	$-1561(71)$	$-1148(79)$
$D^{*0}D^{*0}$			$-1543(71)$	$-1140(79)$

- Similar predictions are obtained in a later work [Dai *et al.*, PR,D105,016029('22)]
- Previous works predicting D^*D^* states: [Molina *et al.*, PR,D82,014010('10); Liu *et al.*, PR,D99,094018('19)].