Baryon-Baryon interactions in lattice QCD

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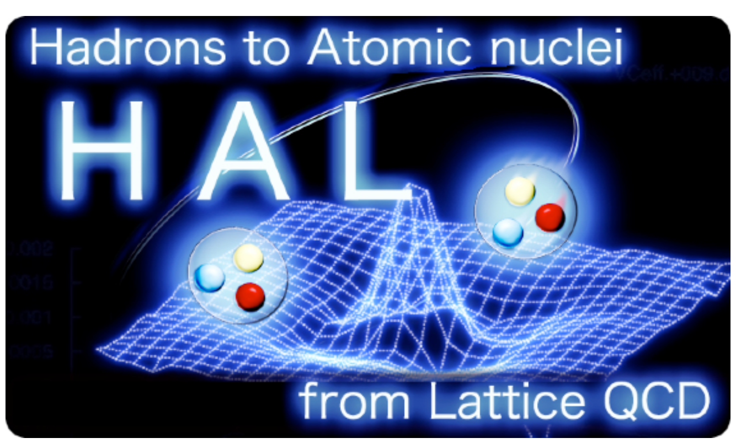

for
HAL QCD collaboration

Baryon 2022
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I. Introduction

Hadron interactions in lattice QCD

Finite volume method

spectra of two hadrons
in finite box → scattering phase shift

Luescher's finite volume formula

HAL QCD method

NBS wave functions → Potential
(Interaction kernel) → scattering
phase shift

Schrodinger equation

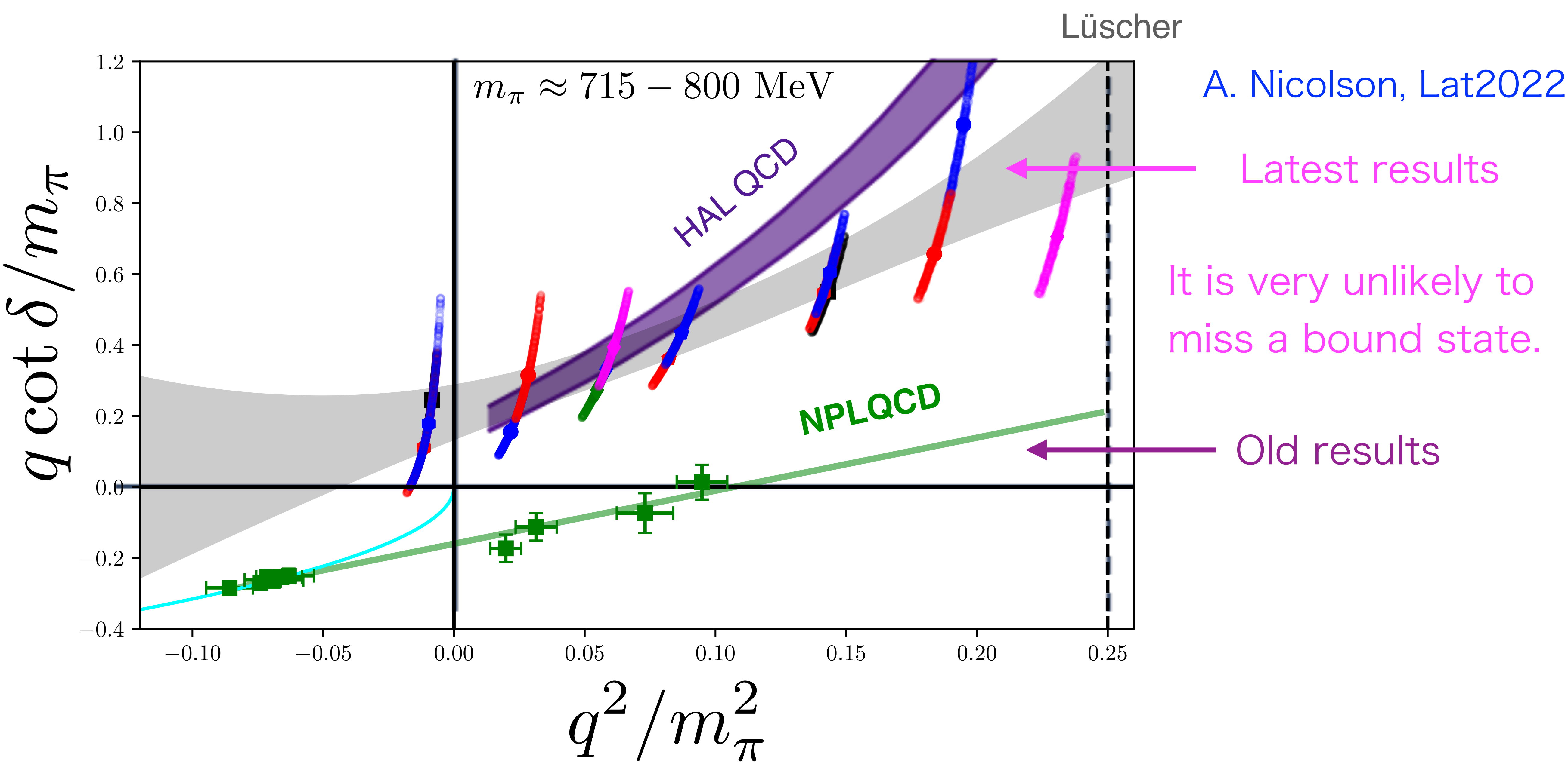
FV/HAL QCD method works better for mesons/baryons.

NN controversy

FV spectra: both deuteron and dineutron are bound at heavier pion masses.

Potential : Two nucleons are unbound at heavier pion masses.

Fortunately, lattice QCD community's efforts are resolving controversy.



Community's consensus

Latest FV spectra are consistent with potential results at low energy.

Two nucleons are unbound at heavy pion masses.

HAL QCD method

Strategy

Ishii-Aoki-Hatsuda, PRL 99 (2007) 022001.

NBS wave function

vacuum

$$\varphi^{\vec{k}}(\vec{x})e^{-W_{\vec{k}}t} = \langle 0|N(\vec{r},t)N(\vec{r}+\vec{x},t)|NN, W_{\vec{k}}\rangle$$

hadron ops. QCD eigenstate

$$W_{\vec{k}} = 2\sqrt{\vec{k}^2 + m_N^2}$$

Center of mass energy

Large r

$$\to \sum_{lm} C_{lm} \frac{\sin(kx + \delta_l(k))}{kx} Y_{lm}(\Omega_{\vec{x}})$$

energy-independent non-local potential

$$W_{\vec{k}} \leq W_{\rm th} = 2m_N + m_\pi$$

$$\left(E_{\vec{k}} - H_0\right)\varphi^{\vec{k}}(\vec{x}) = \int U(\vec{x},\vec{y})\varphi^{\vec{k}}(\vec{y})d^3y, \quad E_{\vec{k}} = \frac{\vec{k}^2}{m_N}, \ H_0 = \frac{-\nabla^2}{m_N},$$

non-local potential

derivative expansion

$$U(\vec{x},\vec{y}) = U(\vec{x},\nabla)\delta^{(3)}(\vec{x}-\vec{y}) = \sum_{n=0}^{\infty} V^{(n)}(\vec{x})\nabla^n\delta^{(3)}(\vec{x}-\vec{y})$$

$$V^{(0)}(\vec{x}) = \frac{1}{\varphi^{\vec{k}}(\vec{x})}\left(\frac{\nabla^2}{m_N} + \frac{\vec{k}^2}{m_N}\right)\varphi^{\vec{k}}(\vec{x})$$

leading order (LO) potential

4-pt correlation function → NBS wave function for a ground state

source op.

$$F_J^{NN}(\vec{x},t) := \sum_{\vec{r}} \langle 0|N(\vec{r}+\vec{x},t)N(\vec{r},t)J_{NN}^\dagger(t=0)|0\rangle \xrightarrow{t\to\infty} A_{J,0}\varphi^{\vec{k}}(\vec{x})e^{-W_0 t} + \cdots$$

It is difficult to take large t with small errors, in particular for multi-baryons.

FV method also suffers from this problem.

Time-dependent method

Ishii et al. (HAL QCD), PLB712(2012)437

normalized 4-pt function

moderately large t

$$R_J^{NN}(\vec{x},t) := \frac{F_J^{NN}(\vec{x},t)}{e^{-2m_N t}} \simeq \sum_n A_{J,n}\varphi^{\vec{k}} e^{-\Delta W_n t} \qquad \Delta W_n = W_n - 2m_n$$

$$\int d^3y\, U(\vec{x},\vec{y})R_J^{2N}(\vec{y},t) \simeq \sum_n \left(\frac{\nabla^2}{m_N} + \frac{\vec{k}^2}{m_N}\right) A_{J,n}\varphi^{\vec{k}}(\vec{x})e^{-\Delta W_n t} = \left(\frac{\nabla^2}{m_N} - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2}\right) R_J^{2N}(\vec{x},t)$$

⇒ LO potential

$$V^{(0)}(\vec{x}) = \frac{1}{R_J^{2N}(\vec{x},t)}\left(\frac{\nabla^2}{m_N} - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2}\right) R_J^{2N}(\vec{x},t)$$

A small t dependence of $V^{(0)}(\vec{x})$ ⇒

Inelastic contributions are negligible.

LO approximation works well.

Today's topics

I. Introduction

II. NN interactions

III. Hyperon interactions

IV. Dibaryons at almost physical point

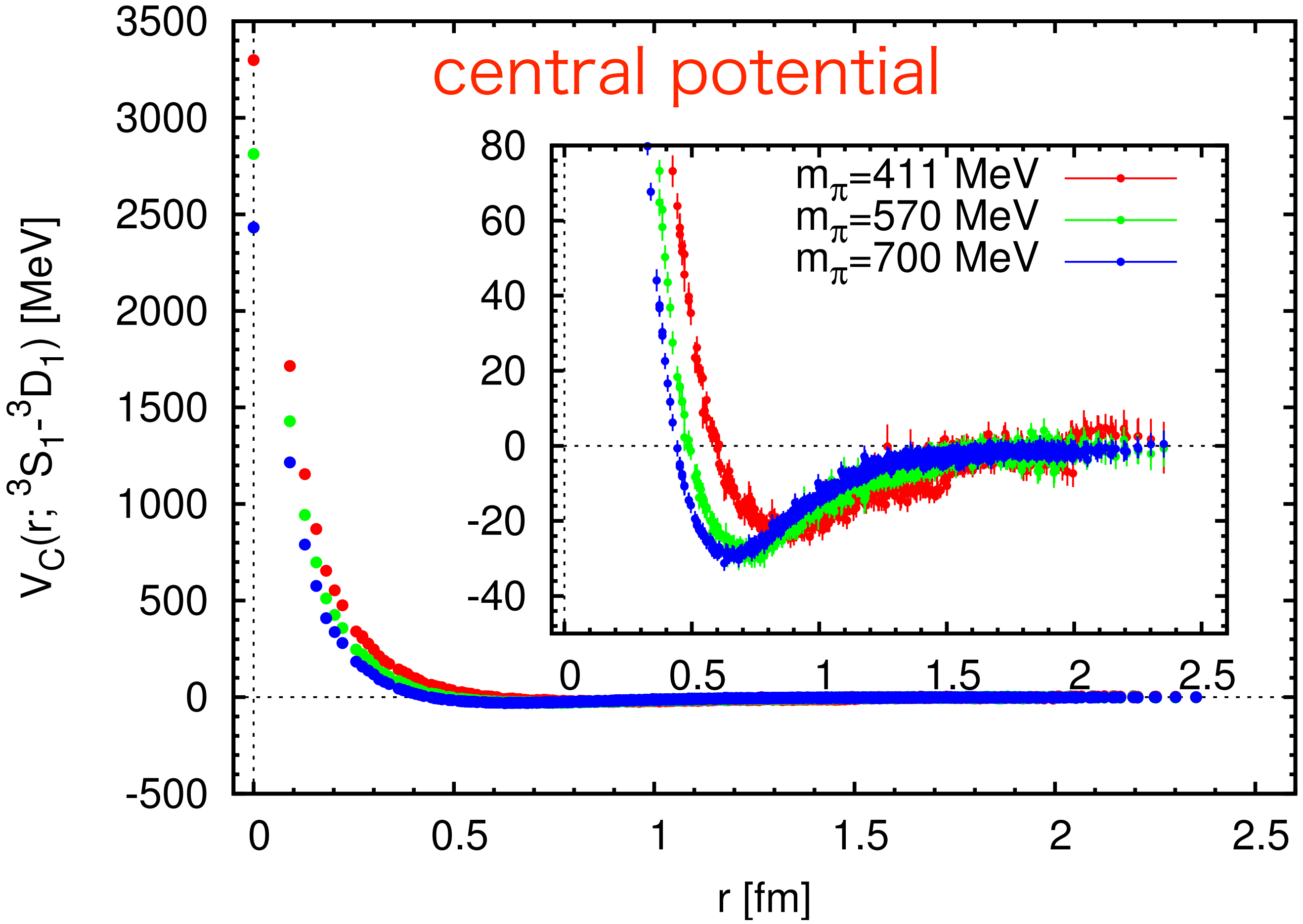
V. Summary

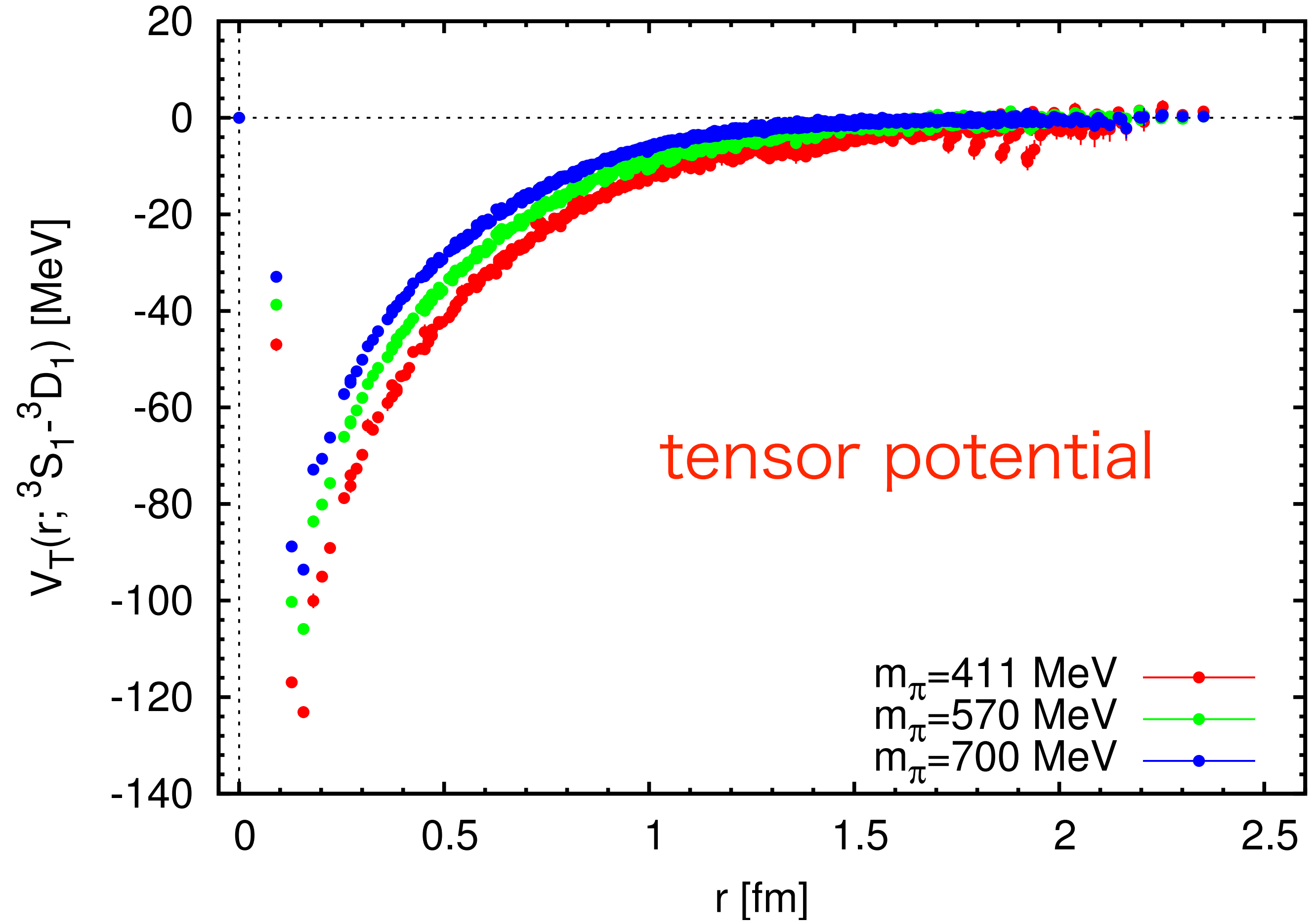
II. NN interactions

1. Spin-triplet NN potentials

heavy pions

N. Ishii[HAL QCD], PoS CD12(2013) 025.





As a pion mass decreases

(a) Range of attraction becomes wider.

one pion exchange (?)

(b) Height of repulsive core increases.

one gluon exchange in quark model (?)

(c) Strong and negative at all distances.

(d) becomes stronger as pion mass decreases.

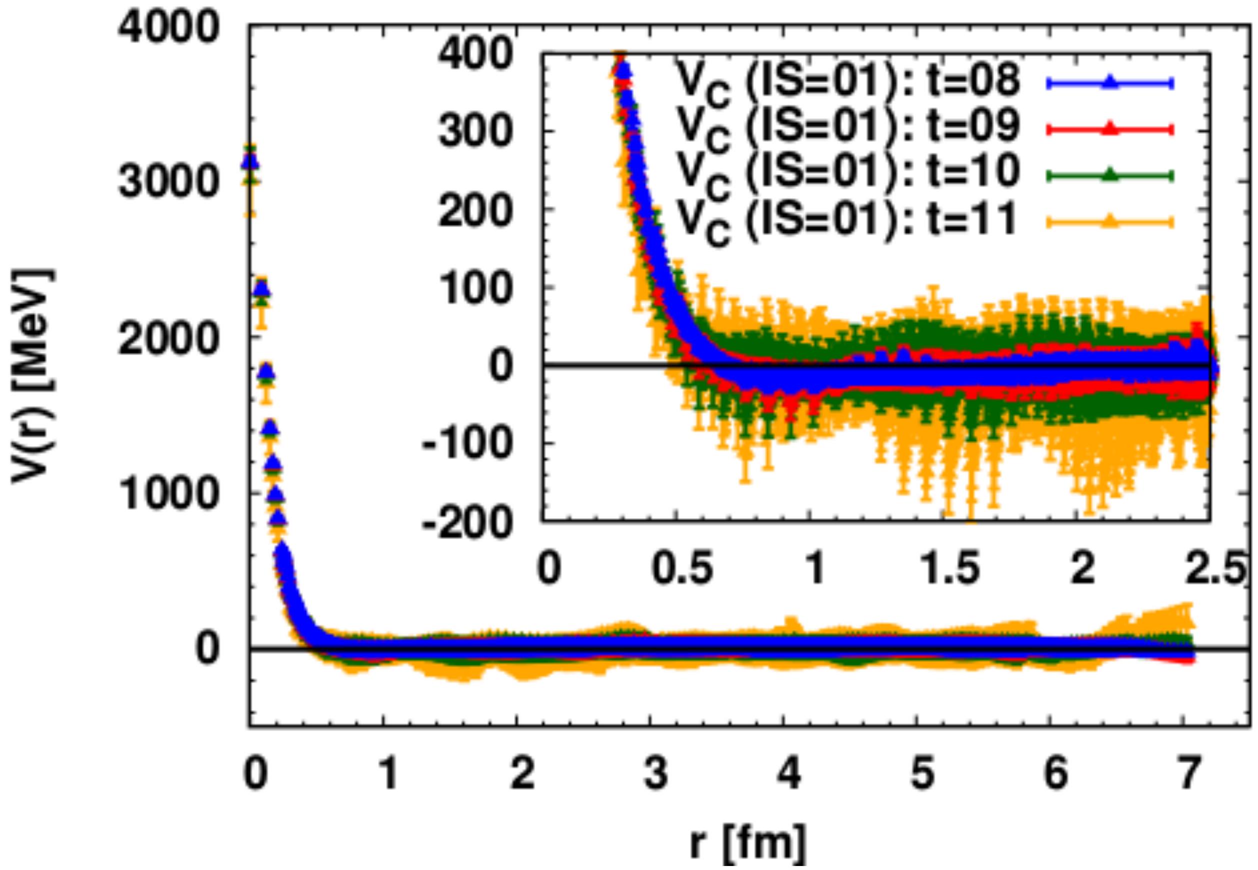
one pion exchange (?)

Almost physical point

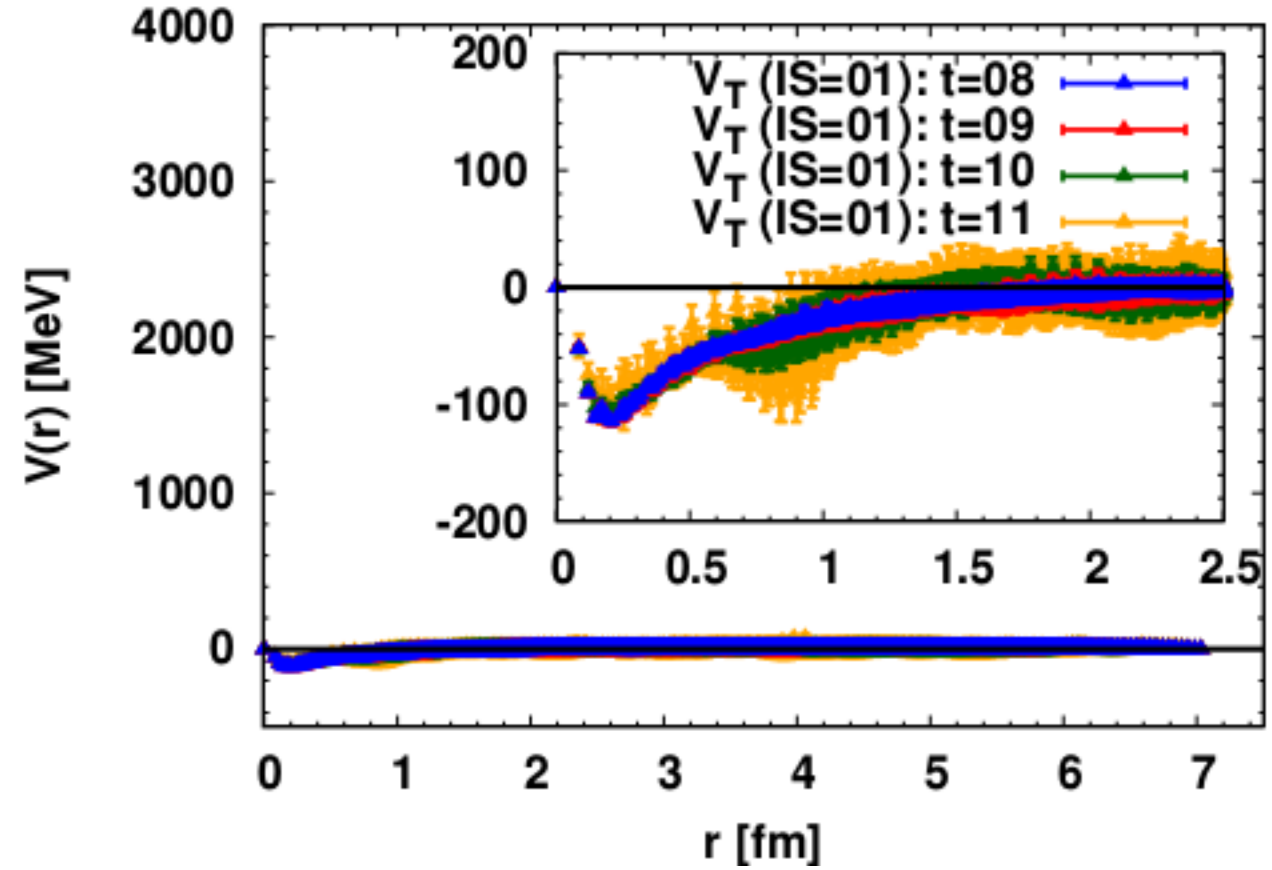
T. Doi et al. [HAL QCD], EPJ Web Conf. 175(2018)05009.

2+1 flavor QCD on 96^4 lattice $a \simeq 0.0846$ fm, $m_\pi \simeq 146$ MeV, $m_K \simeq 525$ MeV

central potential



tensor potential



(a) Much noisier than heavier pion masses.

mainly caused by higher partial waves (?)

(b) repulsive core becomes wider and stronger.

one gluon exchange in quark model (?)

(c) tensor force becomes stronger and more long-ranged .

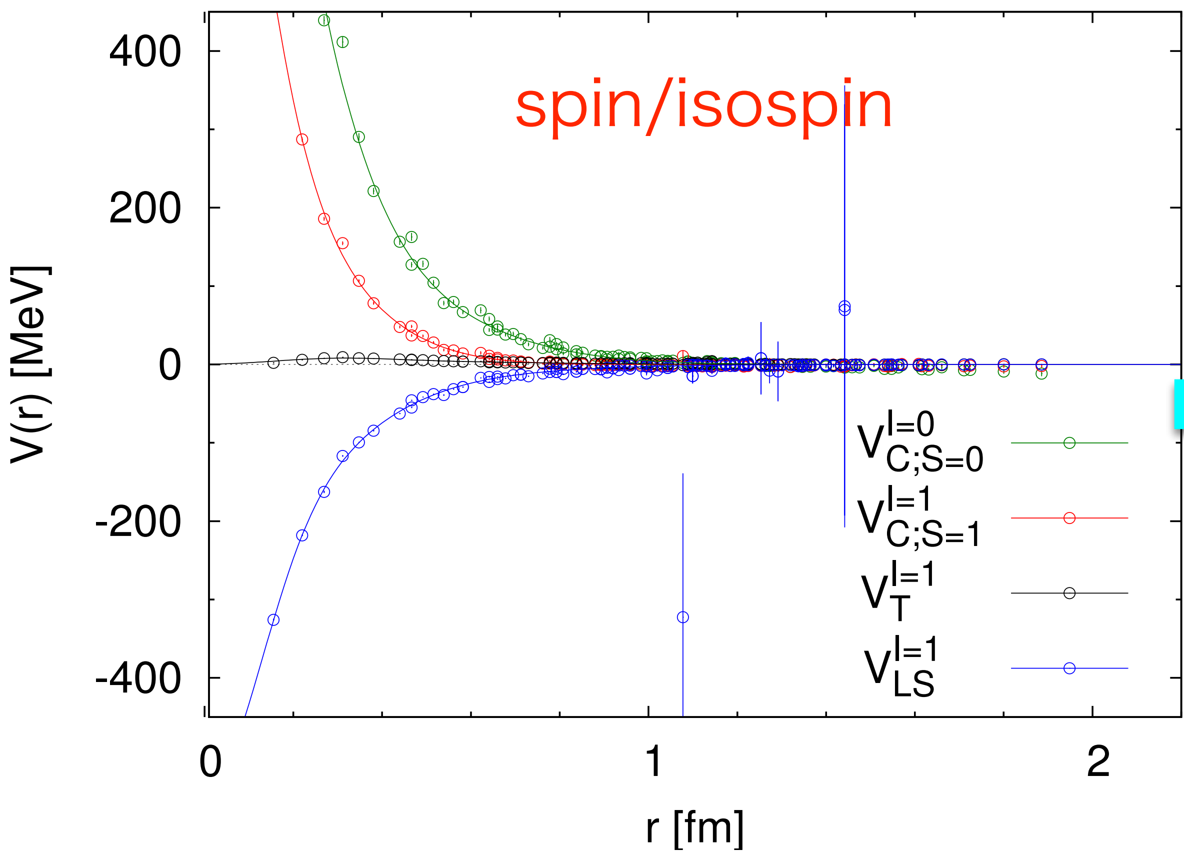
one pion exchange (?)

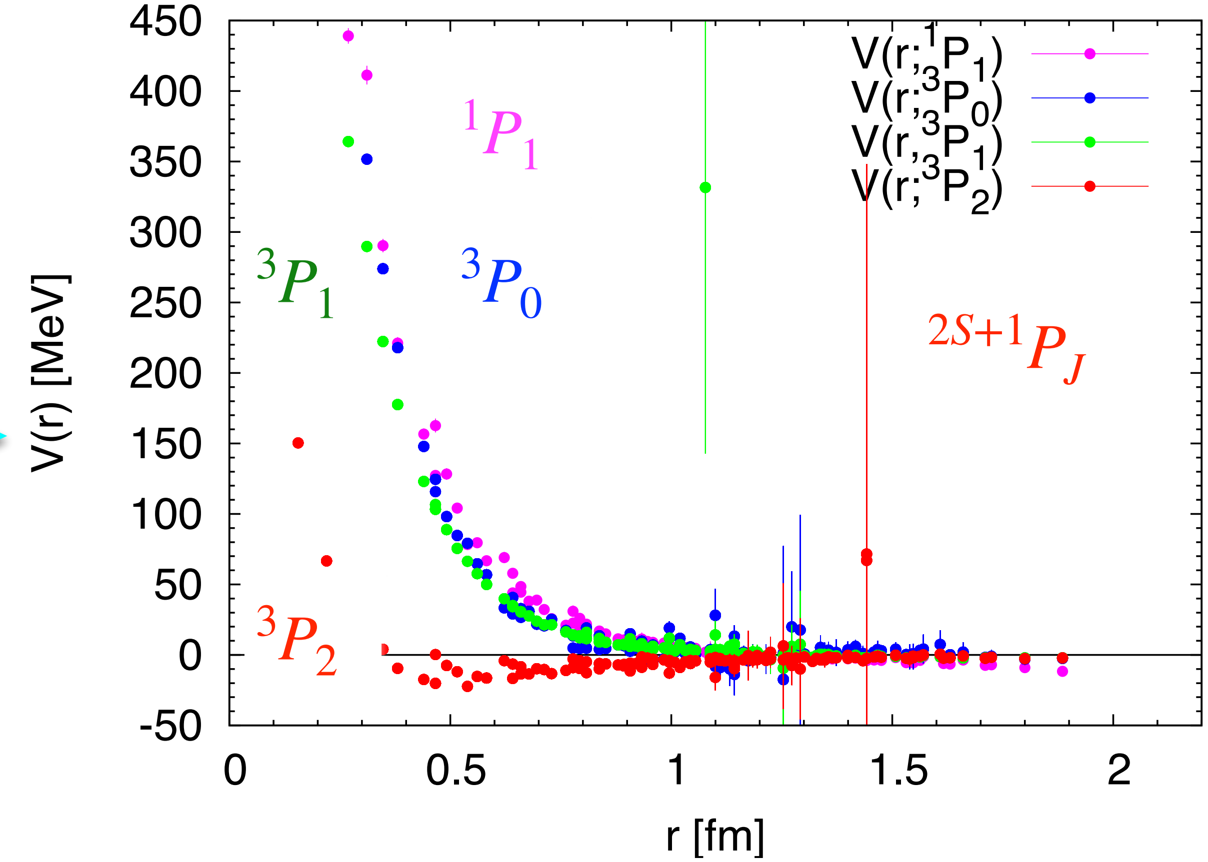
2. NN potentials in parity-odd channels

very heavy pion

K. Murano et al. [HAL QCD], PLB 735(2014)19.

$(m_\pi, m_N) \simeq (1133, 2158)$ MeV in 2flavor QCD





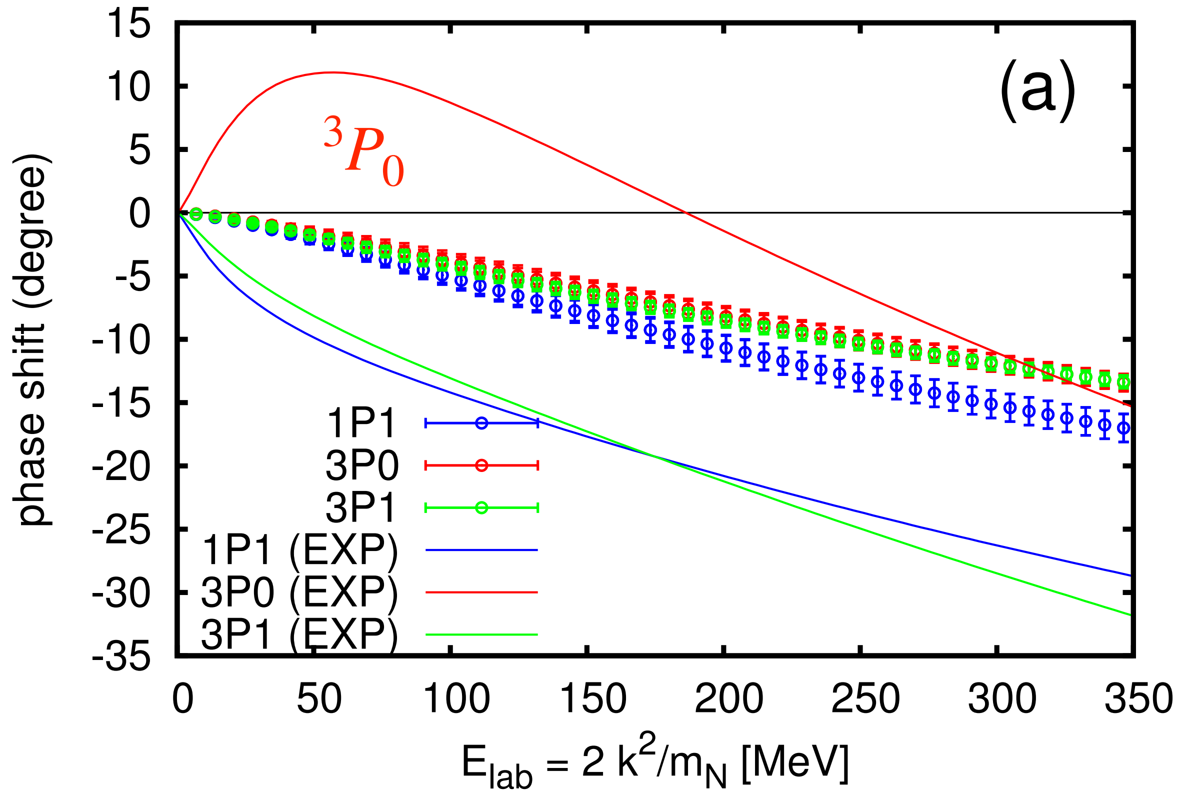
(a) $V_C^{I=0,S=0}$ (green), $V_C^{I=1,S=1}$ (red) are repulsive.

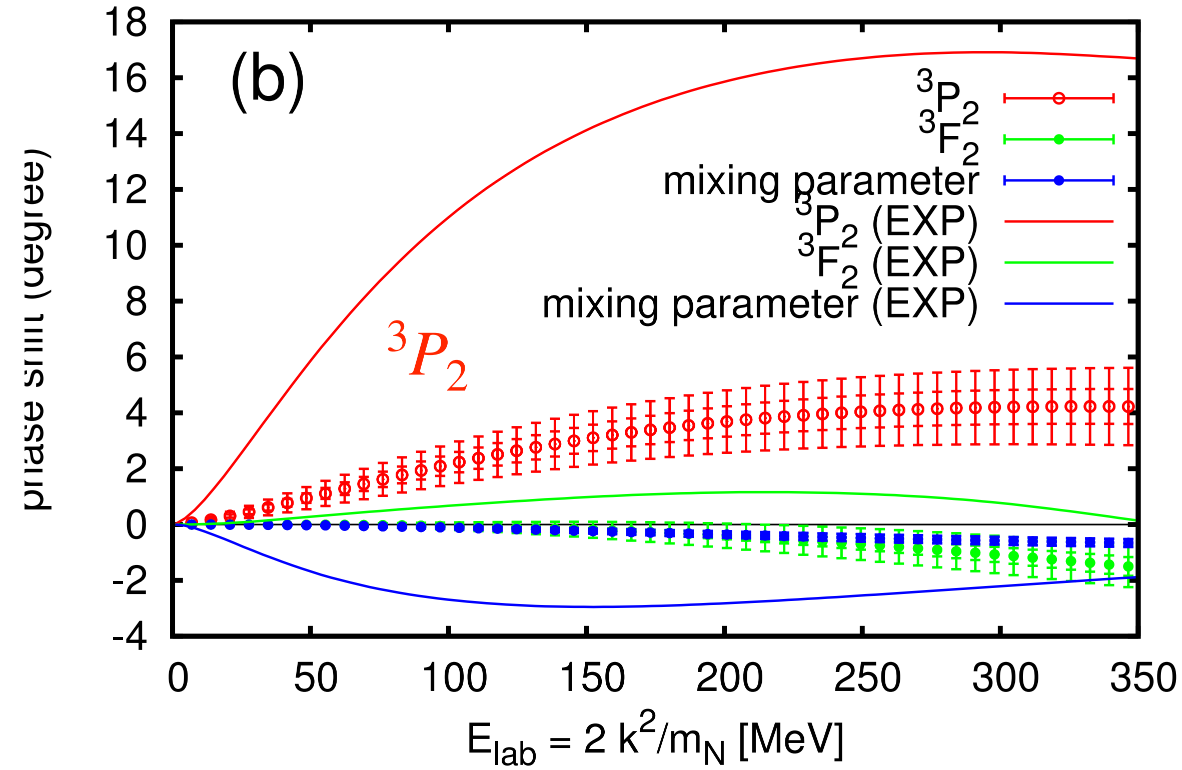
(b) tensor potential $V_T^{I=1}$ (black) is very weak. heavy pion (?)

(c) LS potential $V_{LS}^{I=1}$ (blue) at NLO is strong and negative.

neutron pair correlation/superfluidity in neutron stars scattering phase shifts

K. Murano et al. [HAL QCD], PLB 735(2014)19.





(a) Qualitative features are roughly reproduce.

(b) Attractive behavior at low energy is missing in $^3P_0$ (red, left).

weak tensor force due to the heavy pion

(c) Attractive behavior in $^3P_2$ (red, right).

strong attraction of LS potential

physical point ?

(stay tuned)

III. Hyperon interactions

1. Baryon interactions in the flavor SU(3) limit

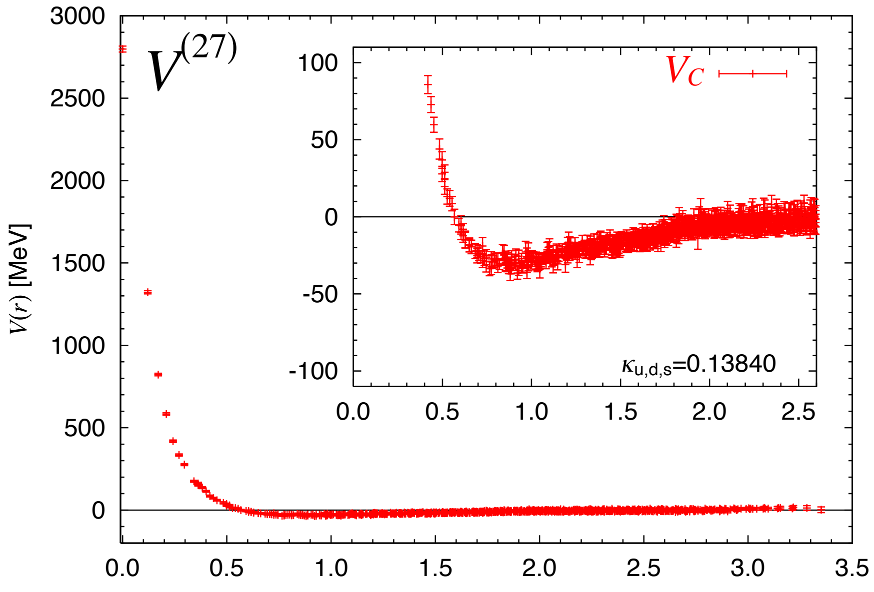
T. Inoue et al. [HAL QCD] NPA 881 (2012) 28.

Two octet baryons

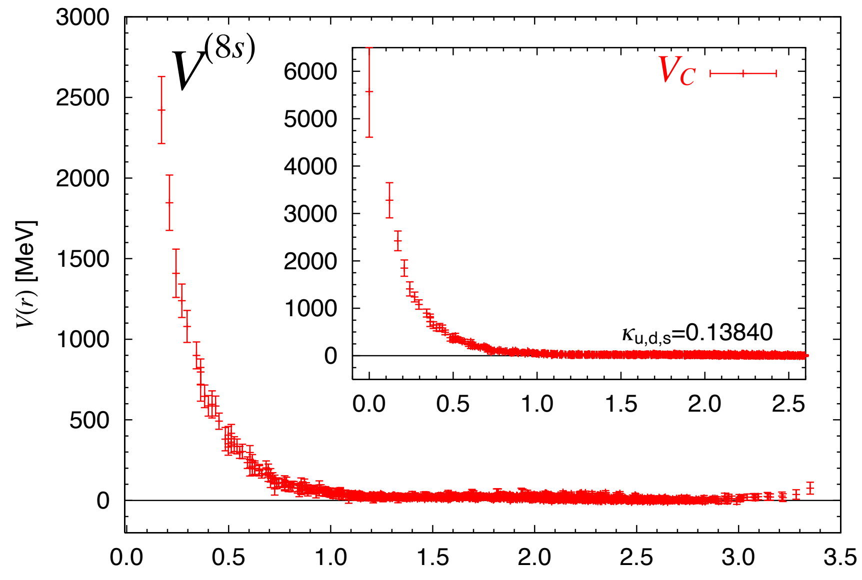
$$\mathbf{8} \otimes \mathbf{8} = \underbrace{\mathbf{27} \oplus \mathbf{8_s} \oplus \mathbf{1}}_{\text{symmetric}} \oplus \underbrace{\overline{\mathbf{10}} \oplus \mathbf{10} \oplus \mathbf{8_a}}_{\text{anti-symmetric}}$$

$$m_u = m_d = m_s$$

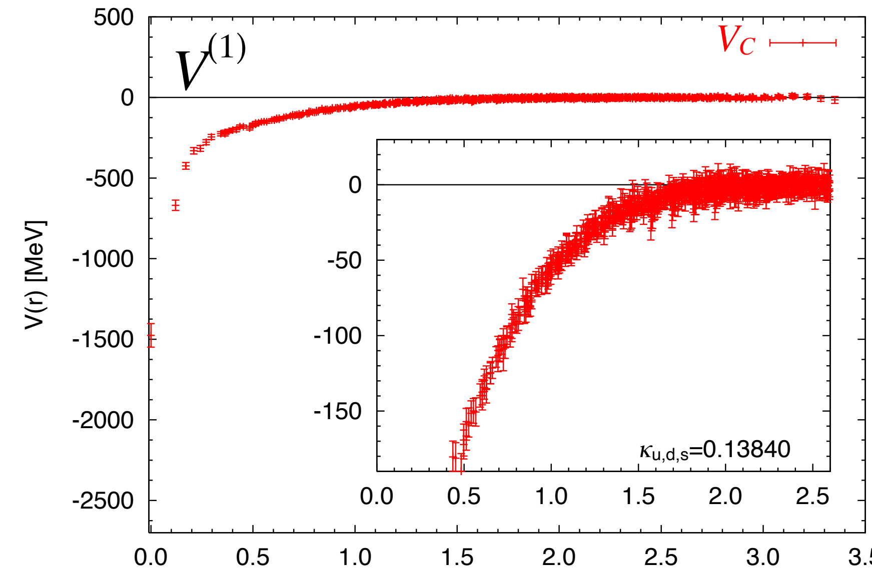
$$(M_{\rm PS}, M_B) \simeq (470, 1160)\text{MeV}$$



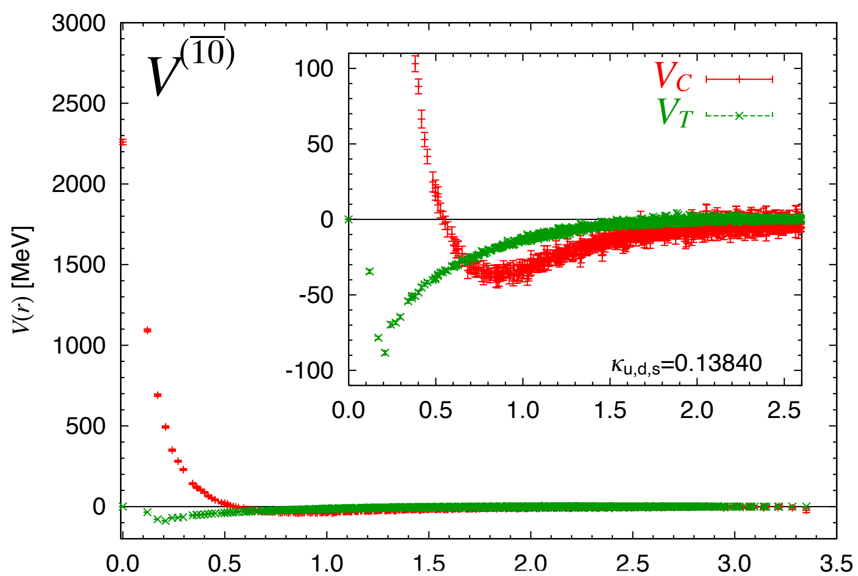
NN $^1S_0$



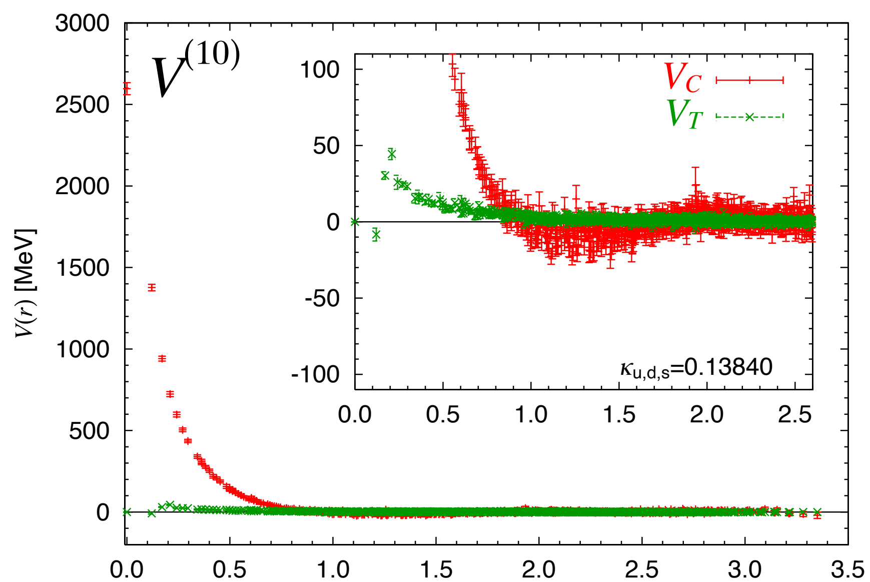
strongest repulsion



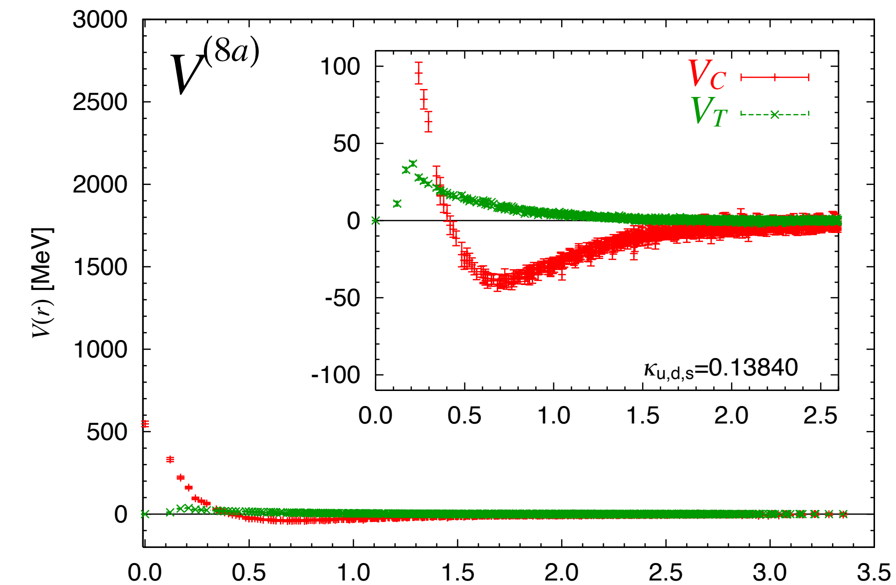
attractive core



NN $^3S_1$



stronger repulsion

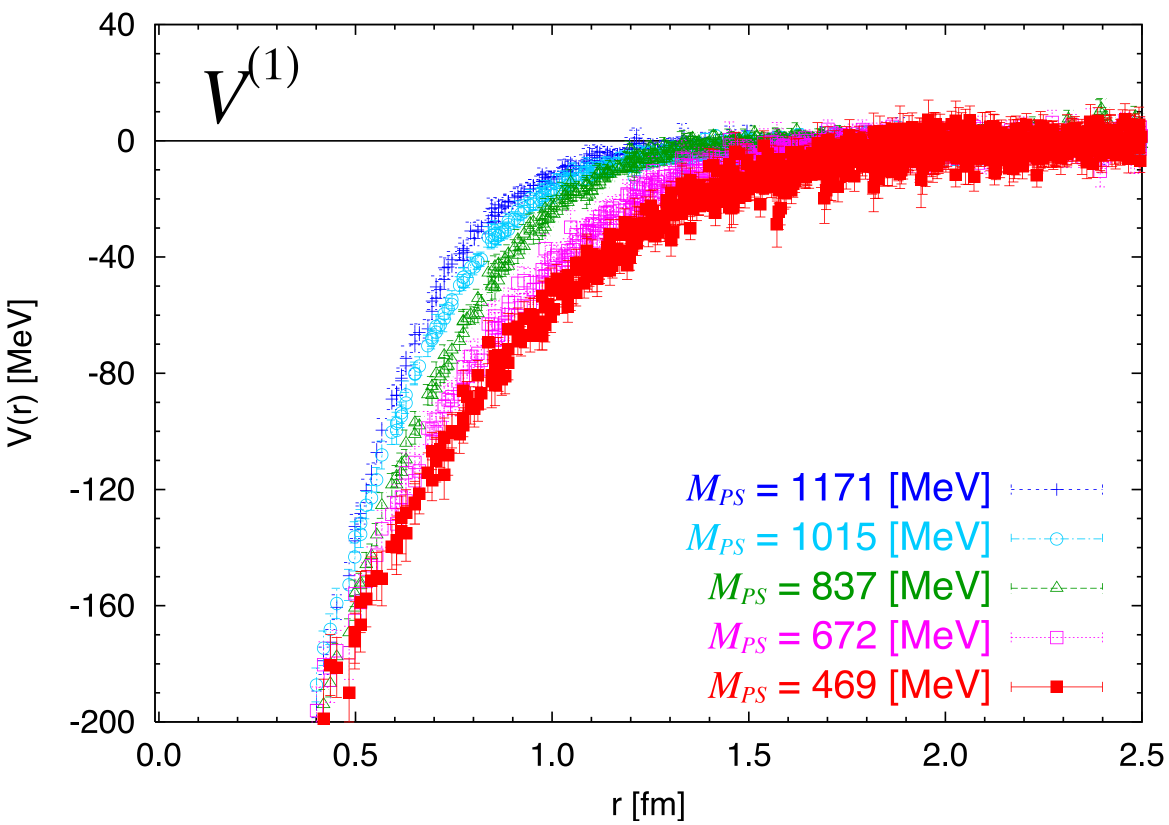


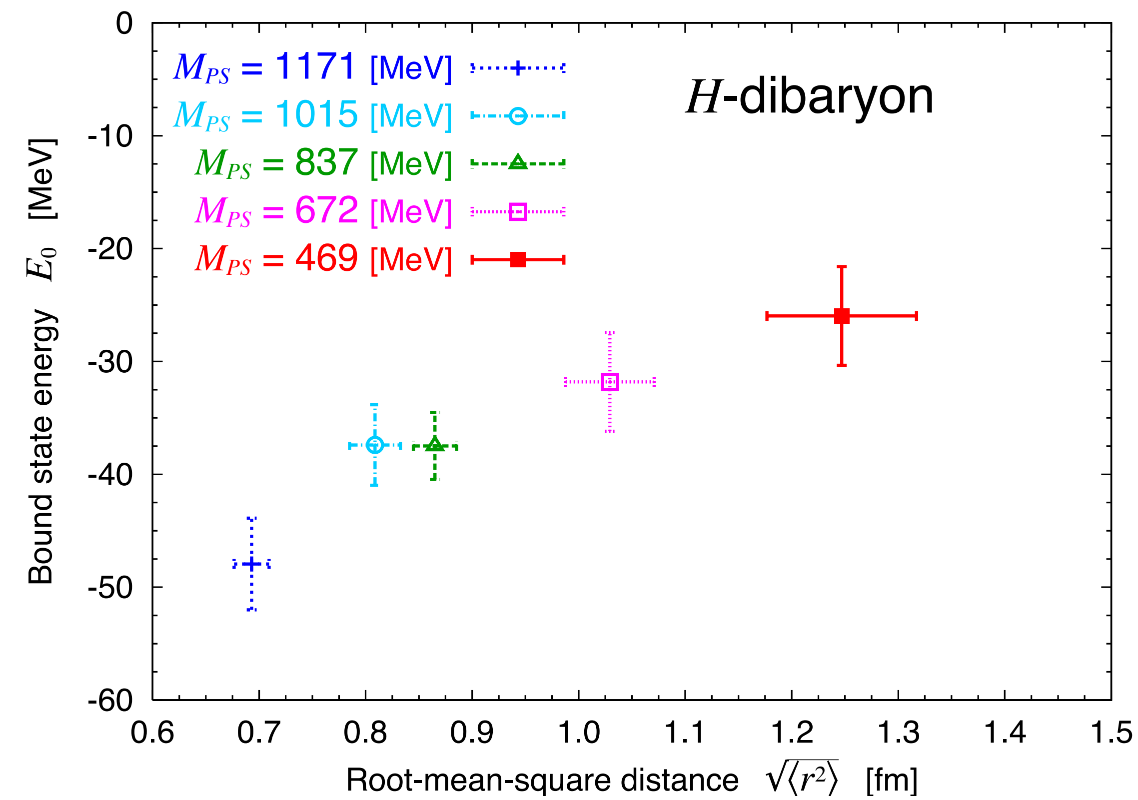
deep attraction

Potentials depend on irreducible representation in the flavor SU(3).

2. H dibaryon in the flavor SU(3) limit

T. Inoue et al. [HAL QCD] NPA 881 (2012) 28.





(a) Attraction becomes stronger at lighter $M_{\rm ps}$.

(b) Binding energy decreases at lighter $M_{\rm ps}$.

(c) Size of bound state increases at lighter $M_{\rm ps}$.

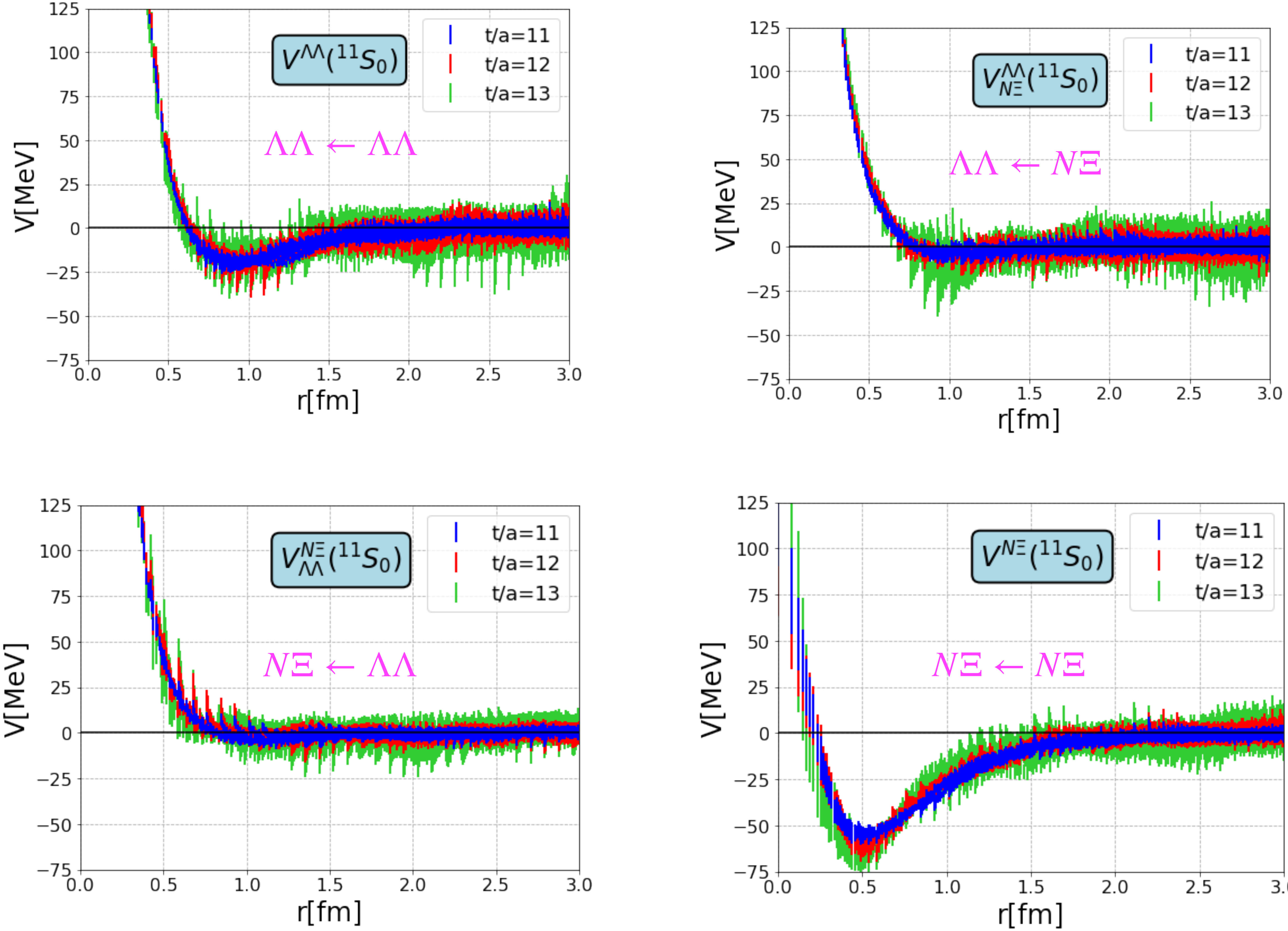
➡ physical point ?

lighter pion mass

flavor SU(3) breaking $\Lambda\Lambda - N\Xi$s coupled channel

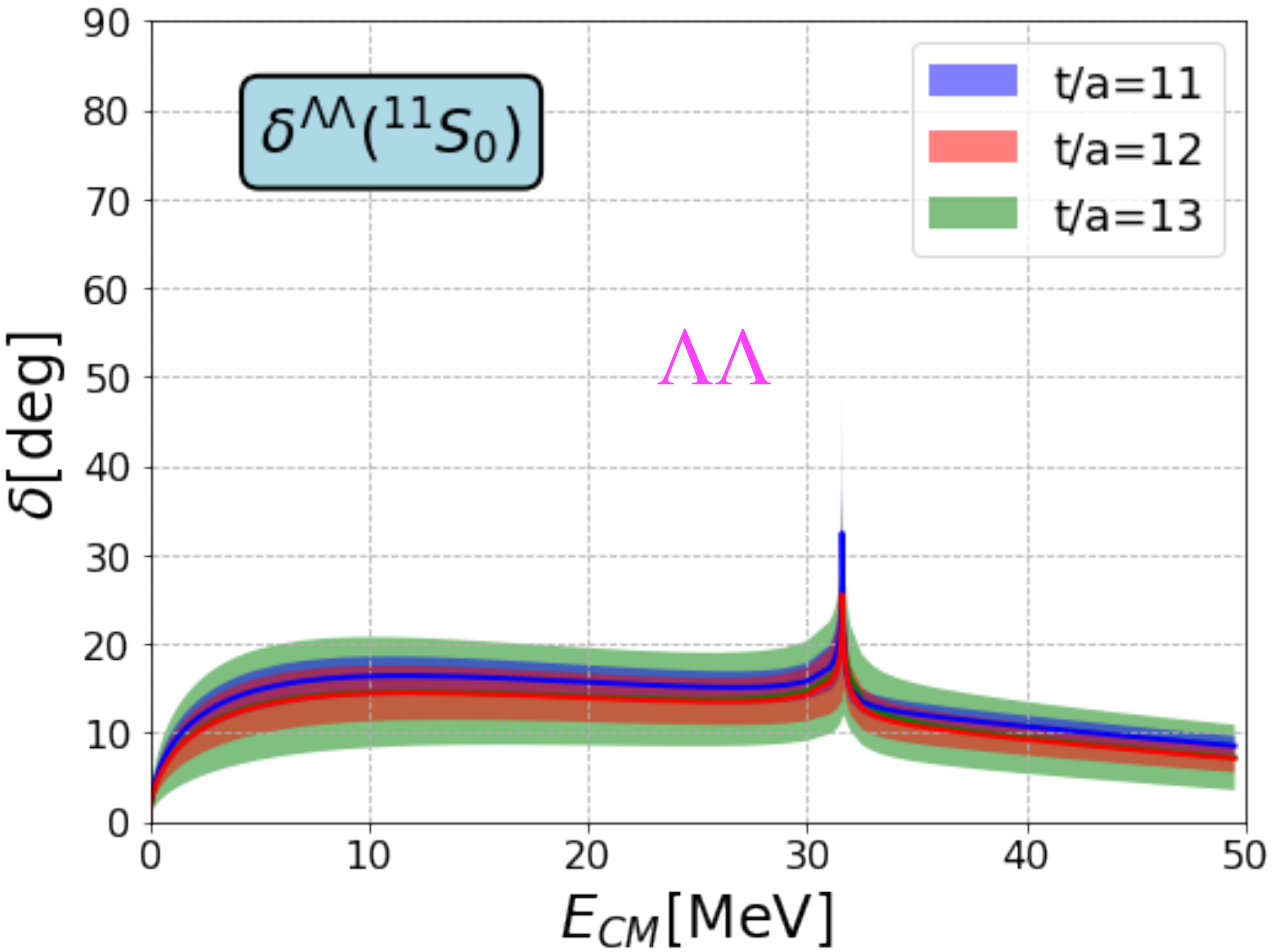
3. H dibaryon at almost physical point

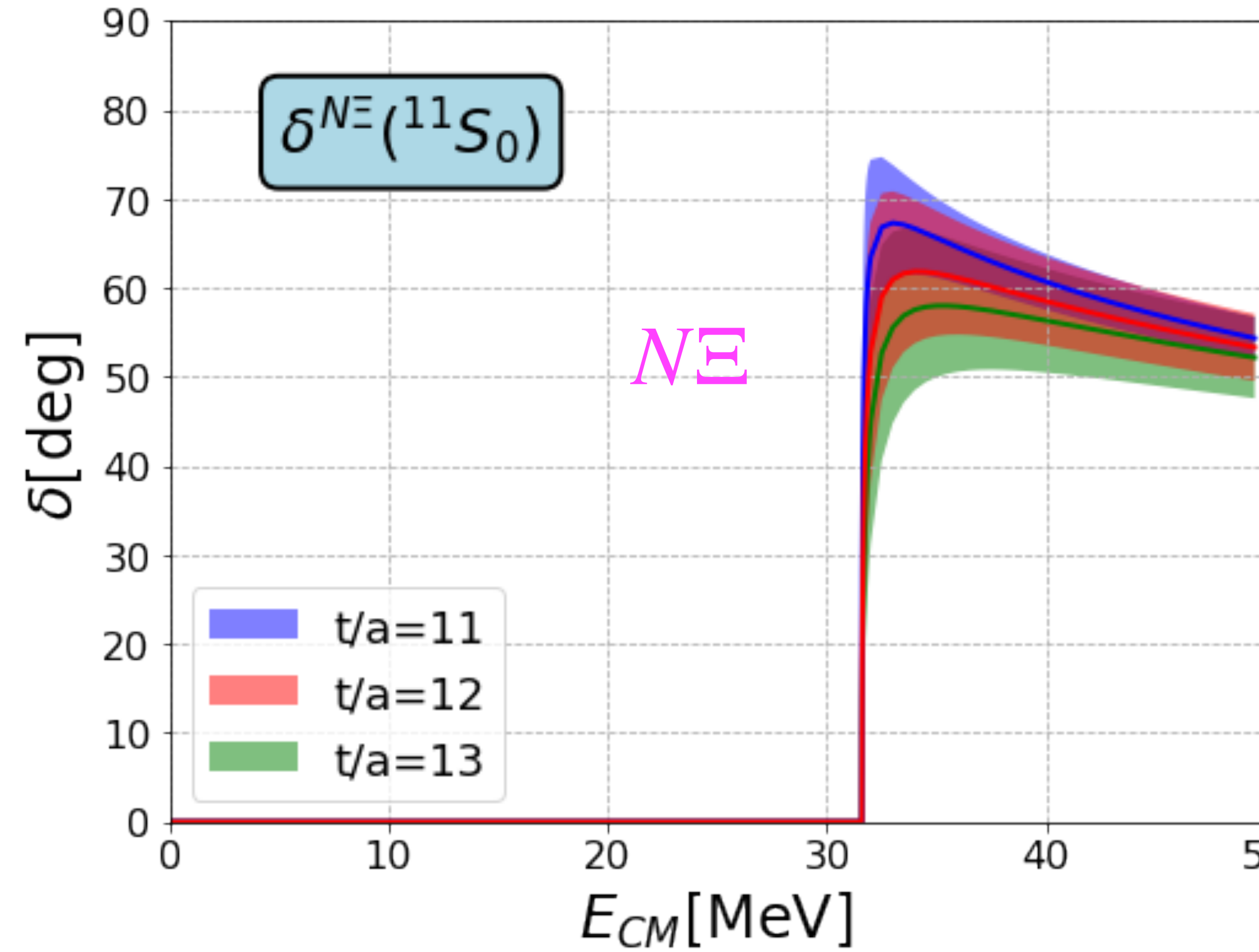
Coupled channel potential $^{2I+1,2s+1}S_J$ K. Sasaki et al. [HAL QCD] NPA 998 (2020) 121737.



scattering phase shifts

K. Sasaki et al. [HAL QCD] NPA 998 (2020) 121737.





(a) No bound state/resonance in $\Lambda\Lambda$.

weak attraction in $\Lambda\Lambda$ potential

(b) No H dibaryon near $\Lambda\Lambda$ threshold.

$$a_0^{\Lambda\Lambda} = -0.8(3)\,\mathrm{fm}$$

(c) Sharpe enhancement and rapid drop near $N\Xi$ threshold.

off-diagonal $\Lambda\Lambda \to N\Xi$ potential

(d) Sharpe increase near $N\Xi$ threshold.

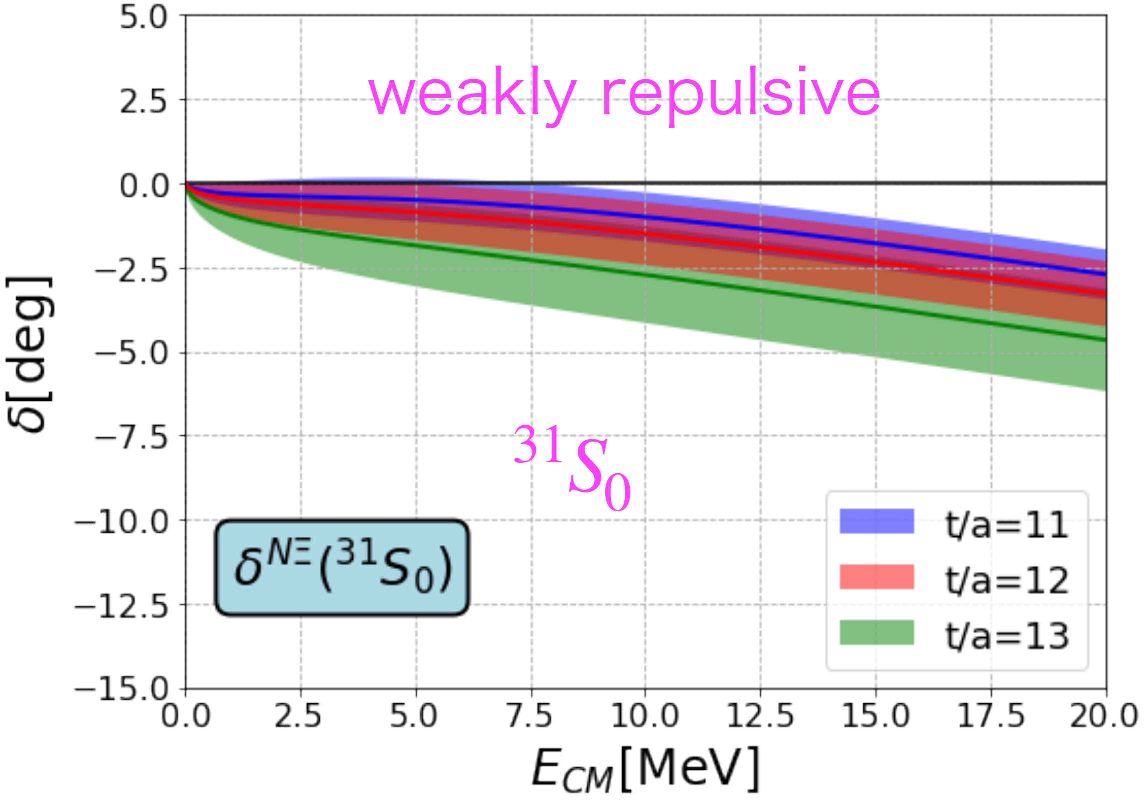
significant $N\Xi$ attraction

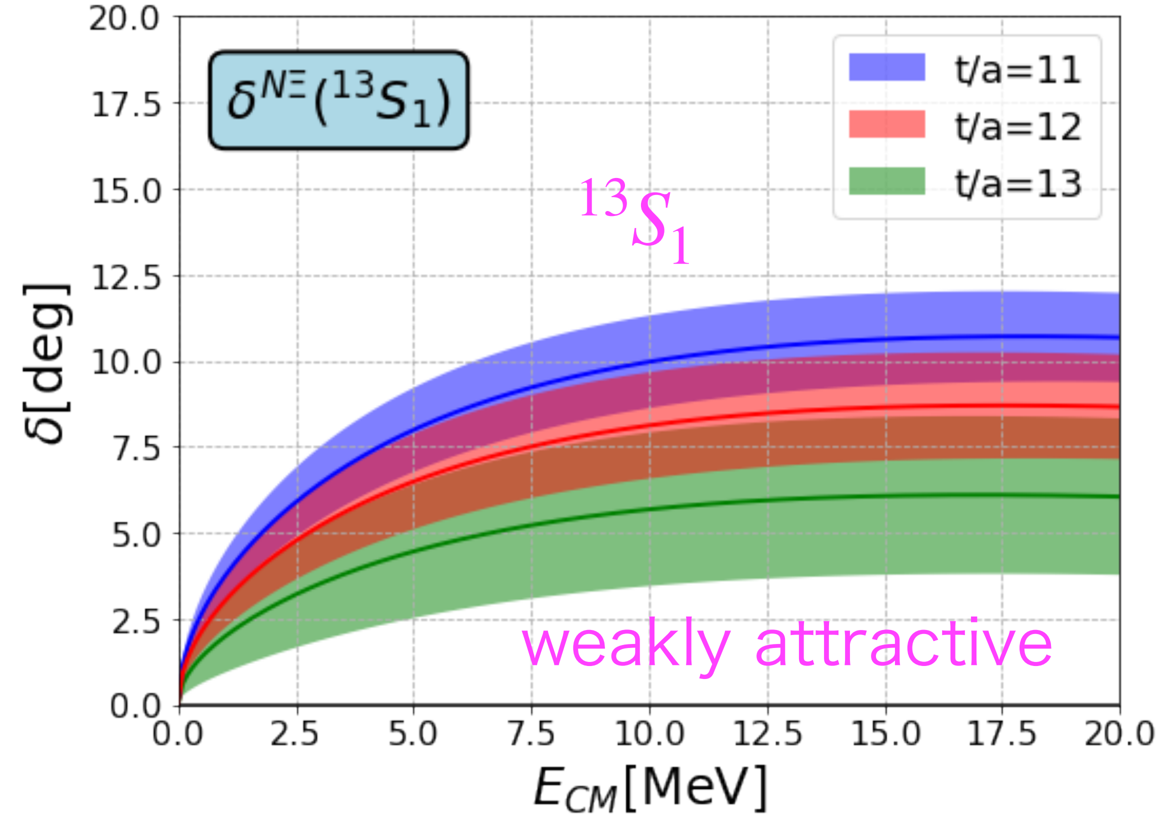
(e) H appears as a virtual state of $N\Xi$ at almost physical point.

4. $N\Xi$ interactions at almost physical point

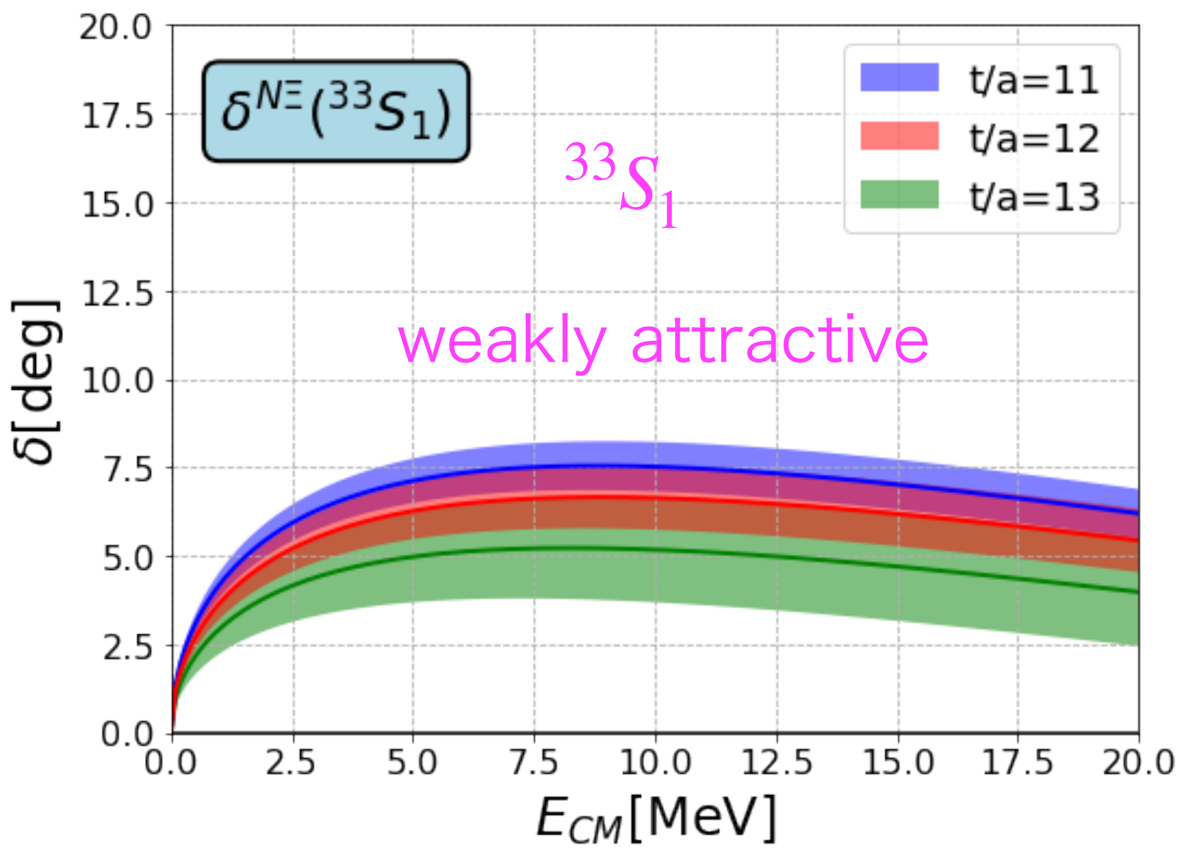
scattering phase shifts

K. Sasaki et al. [HAL QCD] NPA 998 (2020) 121737.



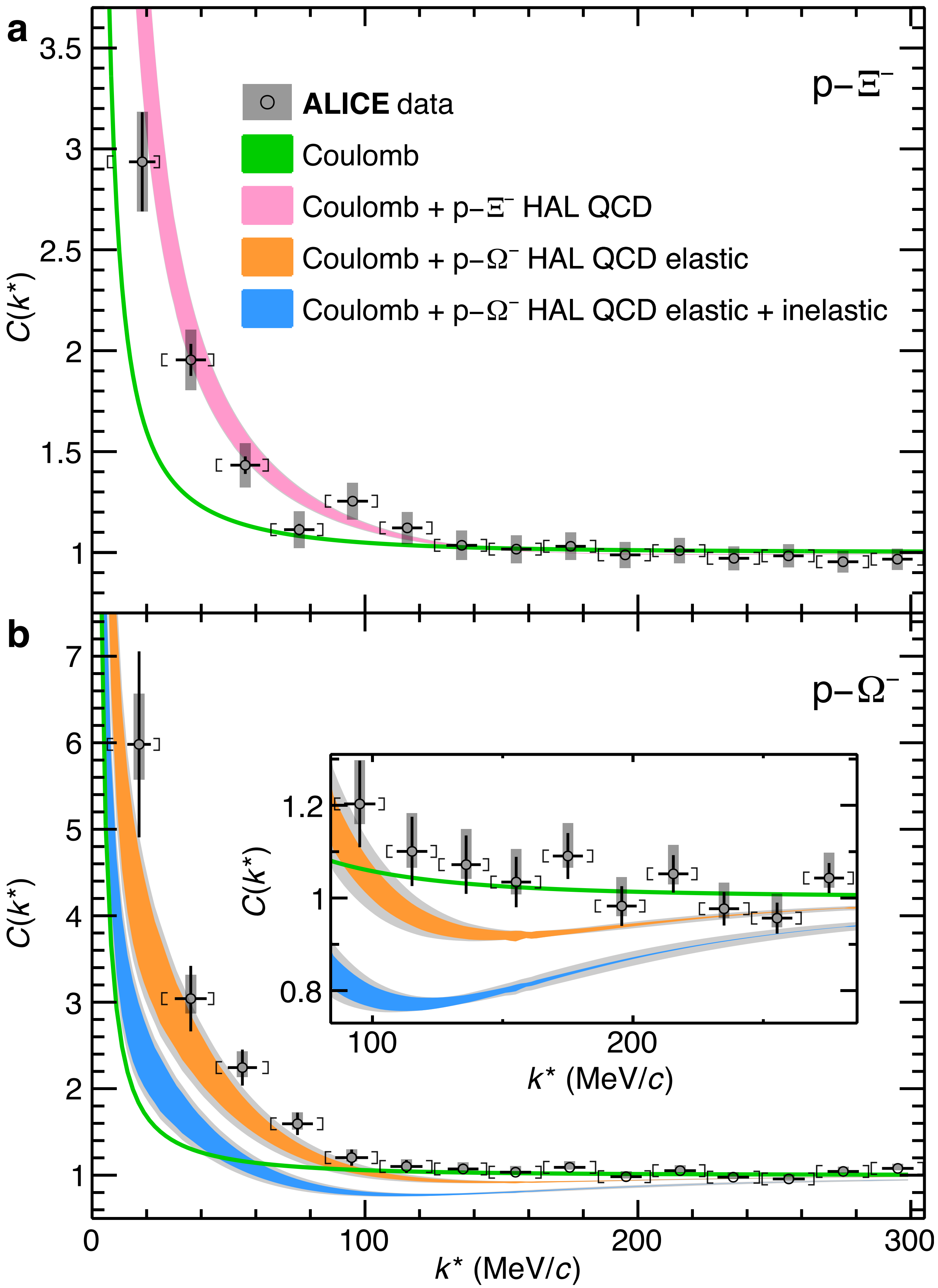


$^{2I+1,2s+1}S_J$



A comparison with experiments

Allice collaboration, Nature 558 (2020), 232-238.



(a) LHC $p-\Xi^-$ data can not be explained by Coulomb interaction alone.

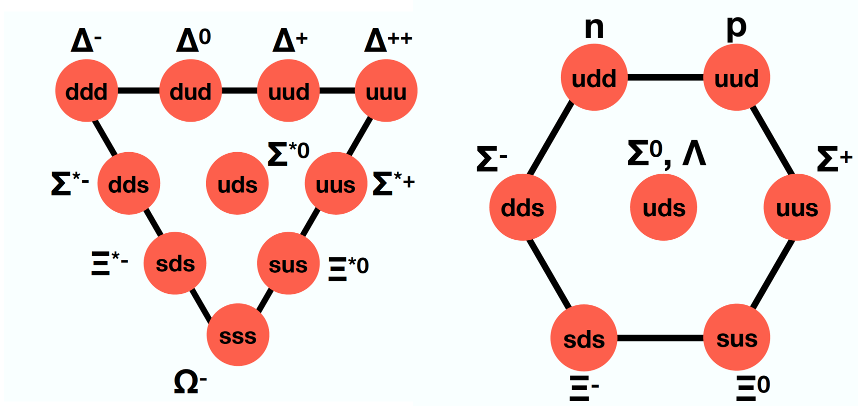
(b) Data are consistent with Coulomb + HAL QCD potential.

(c) A similar comparison has been made for $p-\Omega^-$ data.

IV. Dibaryons at almost physical point

Y. Lyu, H. Tong, T. Sugiura, S. Aoki, T. Doi, T. Hatsuda, J. Meng, T. Miyamoto,
"Dibaryon with highest charm number near unitarity from lattice QCD",
Phys. Rev. Lett. 127 (2021) 072003 (arXiv:2102.0081).

SU(3) classification for Dibaryon candidates (B=2)



Jaffe (1977)

H-dibaryon(J=0)

1) octet-octet system

$$8 \otimes 8 = 27 \oplus 8_s \oplus \boxed{1} \oplus \boxed{\overline{10}} \oplus 10 \oplus 8_a$$

Deuteron(J=1)

2) decuplet-octet system

NΩ system and NΔ system (J=2)

$$10 \otimes 8 = 35 \oplus \boxed{8} \oplus 10 \oplus 27$$

Goldman et al (1987)
Dyson, Xuong (1964)

3) decuplet-decuplet system

$$10 \otimes 10 = \boxed{28} \oplus 27 \oplus 35 \oplus \boxed{\overline{10}}$$

$d^*(2380)$ resonance

ΩΩ system (J=0)

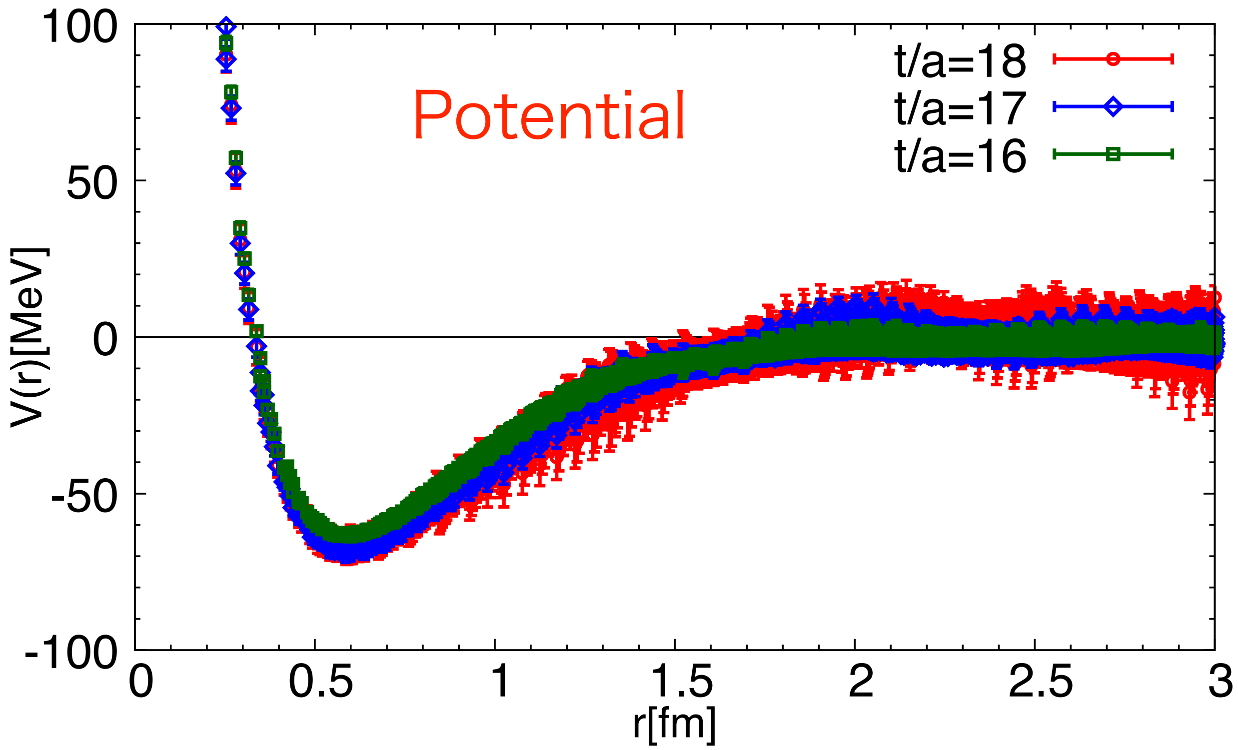
Zhang et al(1997)

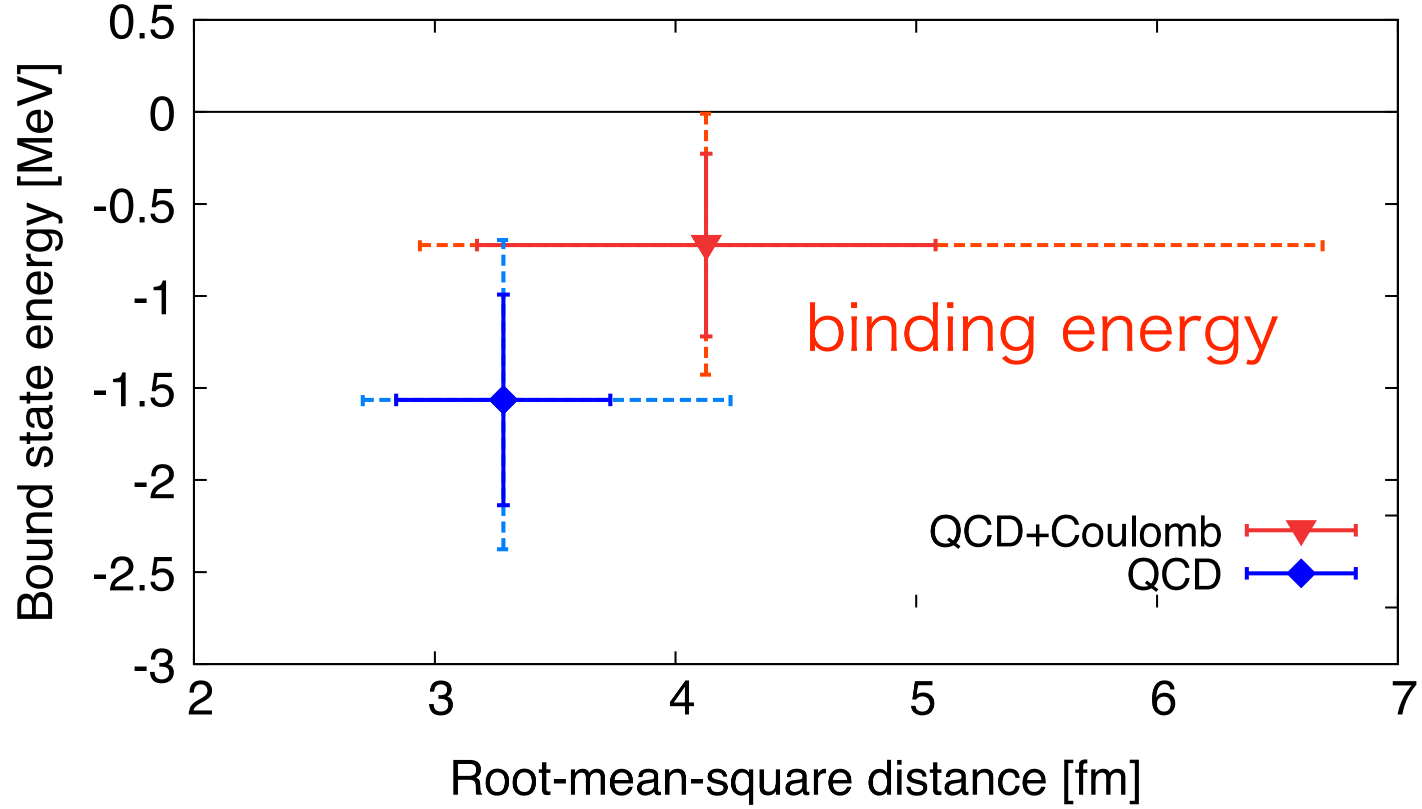
ΔΔ system (J=3)

Dyson, Xuong (1964)
Kamae, Fujita(1977)
Oka, Yazaki(1980)

1. The most strange dibaryon $\Omega^-_{sss}\Omega^-_{sss}$

S. Gongyo et al. [HAL QCD] PRL 120(2018) 212001.





(a) Potential is very similar to NN central potentials.

repulsive core + attractive pocket

binding energy

$$B_{\Omega^-_{sss}\Omega^-_{sss}} = 1.6(9)\,\mathrm{MeV}$$ w/o Coulomb

$$B_{\Omega^-_{sss}\Omega^-_{sss}} = 0.7(7)\,\mathrm{MeV}$$ with Coulomb repulsion

Effective Range Expansion (ERE) w/o Coulomb

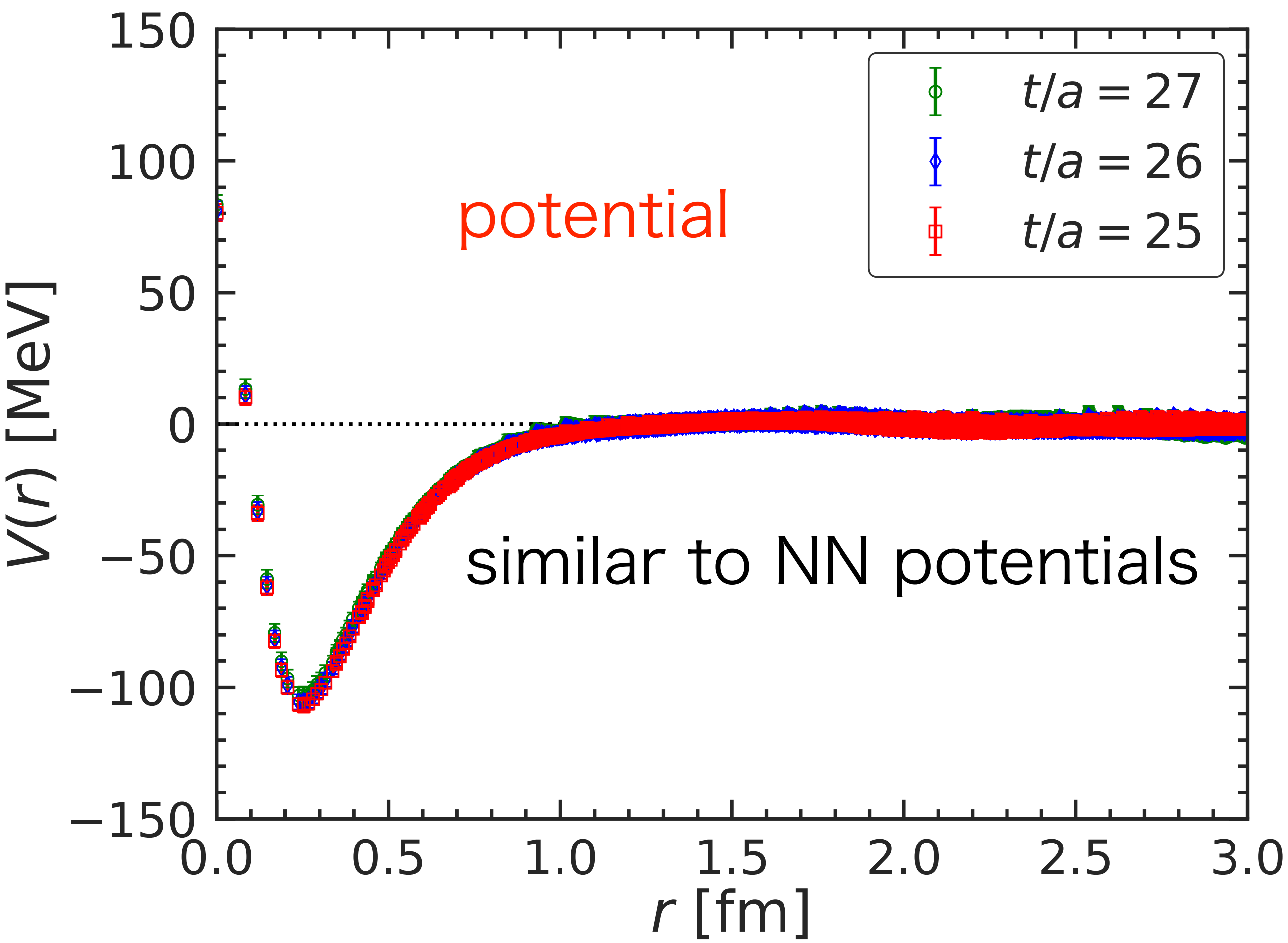
$$k\cot\delta_0(k) = -\frac{1}{a_0} + \frac{1}{2} r_{\mathrm{eff}} k^2 + O(k^4)$$

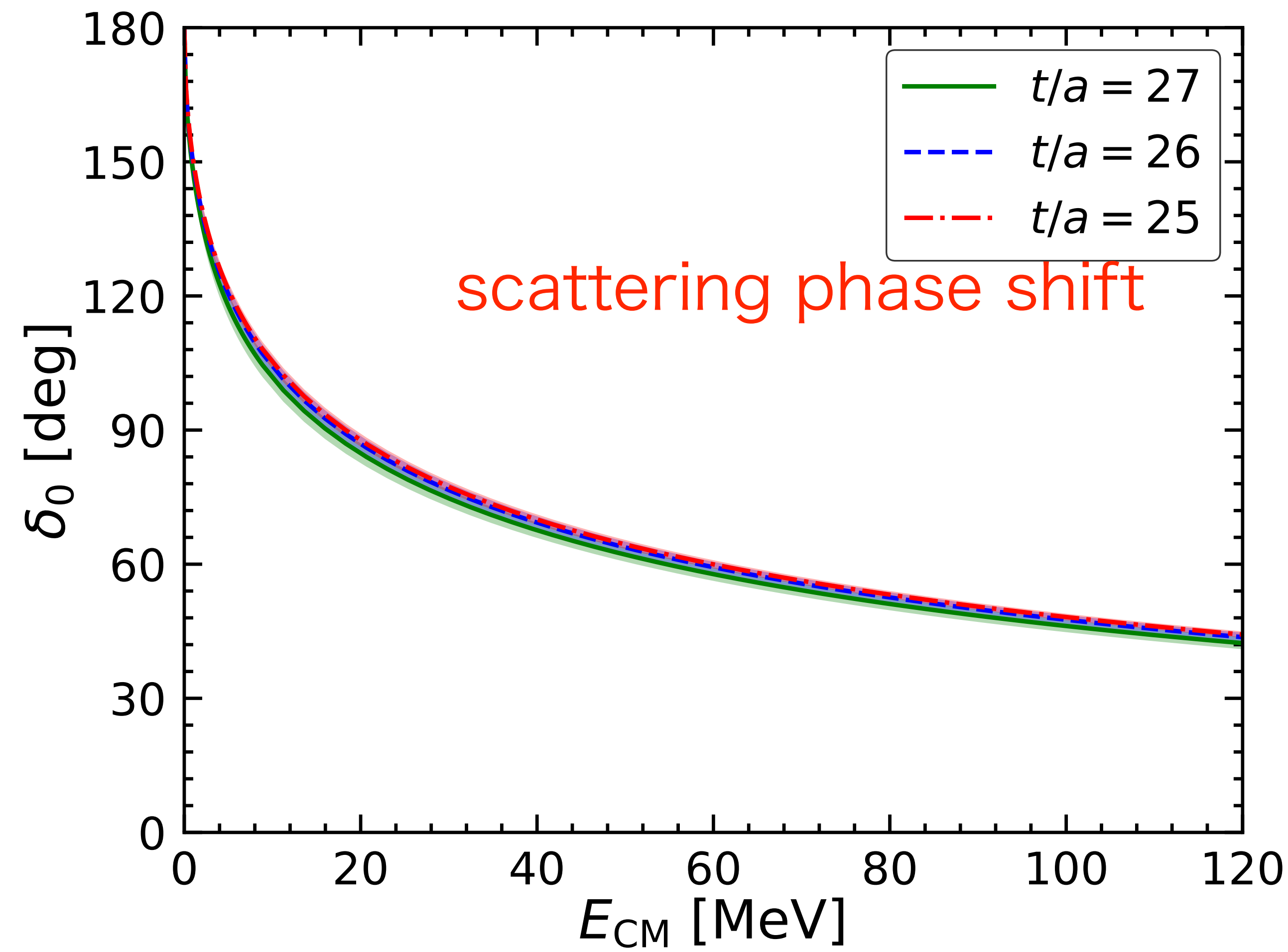
$$a_0^{\Omega^-_{sss}\Omega^-_{sss}} = 4.6(1.3)\,\mathrm{fm} \qquad r_{\mathrm{eff}}^{\Omega^-_{sss}\Omega^-_{sss}} = 1.27(7)\,\mathrm{fm}$$

The most strange dibaryon

2. The most charming dibaryon $\Omega_{ccc}^{++}\Omega_{ccc}^{++}$

Y. Lyu et al.[HAL QCD], PRL 127 (2021) 072003.





one bound state

$$B = 5.68(0.77)(^{+0.46}_{-1.02})\ \text{MeV},$$ BE

$$\sqrt{\langle r^2 \rangle} = 1.13(0.06)(^{+0.08}_{-0.03})\ \text{fm}.$$ size

$$a_0 = 1.57(0.08)(^{+0.12}_{-0.04})\ \text{fm},$$

$$r_{\text{eff}} = 0.57(0.02)(^{+0.01}_{-0.00})\ \text{fm}.$$

+ Coulomb repulsion

The most charming dibaryon

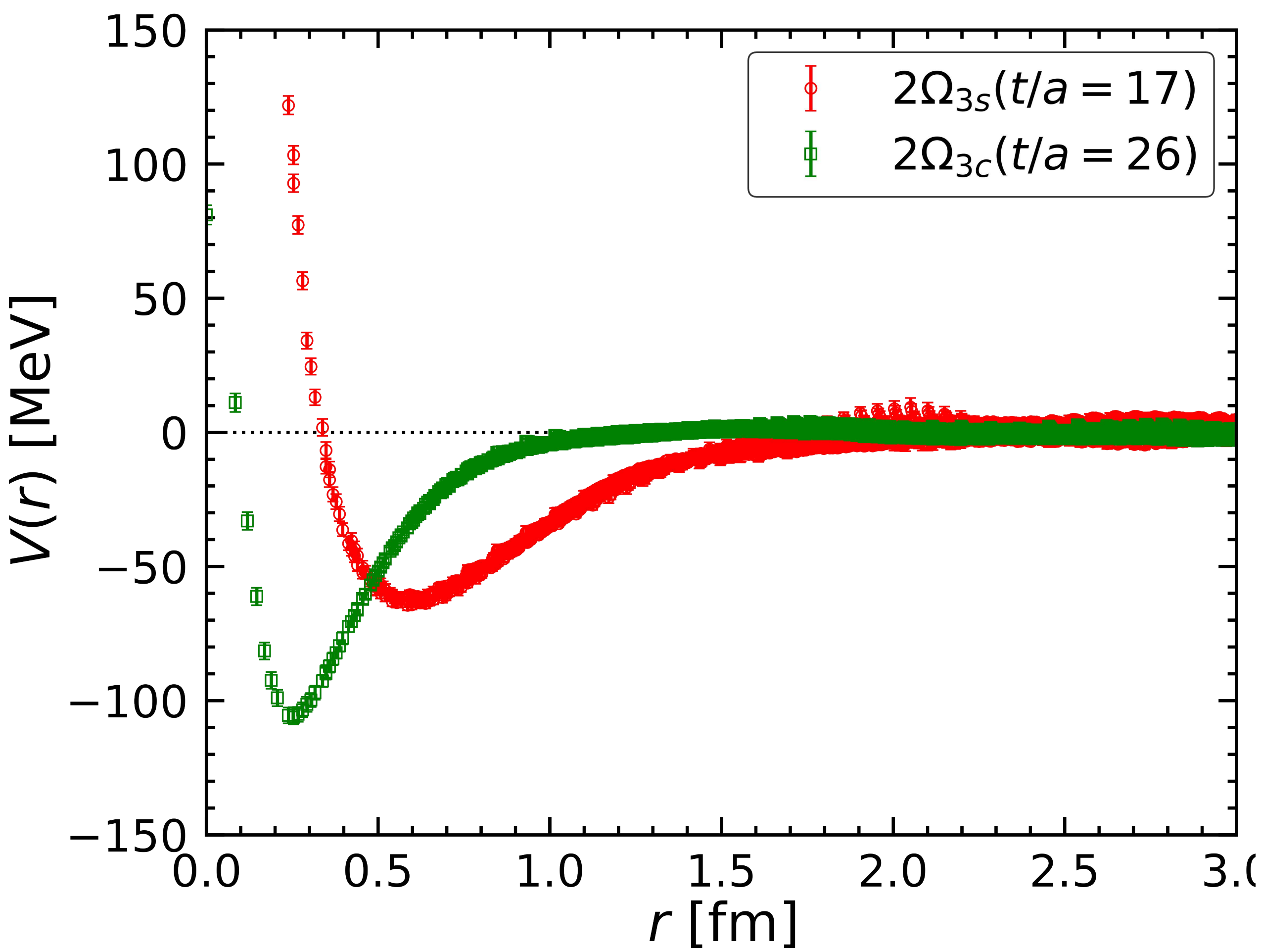
$$a_0^{\text{C}} = -19(7)(^{+7}_{-6})\ \text{fm},$$ unitary region

$$r_{\text{eff}}^{\text{C}} = 0.45(0.01)(^{+0.01}_{-0.00})\ \text{fm}.$$

bound state disappears.

3. Comparisons of two systems

Y. Lyu et al.[HAL QCD], PRD 105 (2022) 074512.



(a) height of the repulsive core is an order of magnitude smaller for Ω_{ccc}

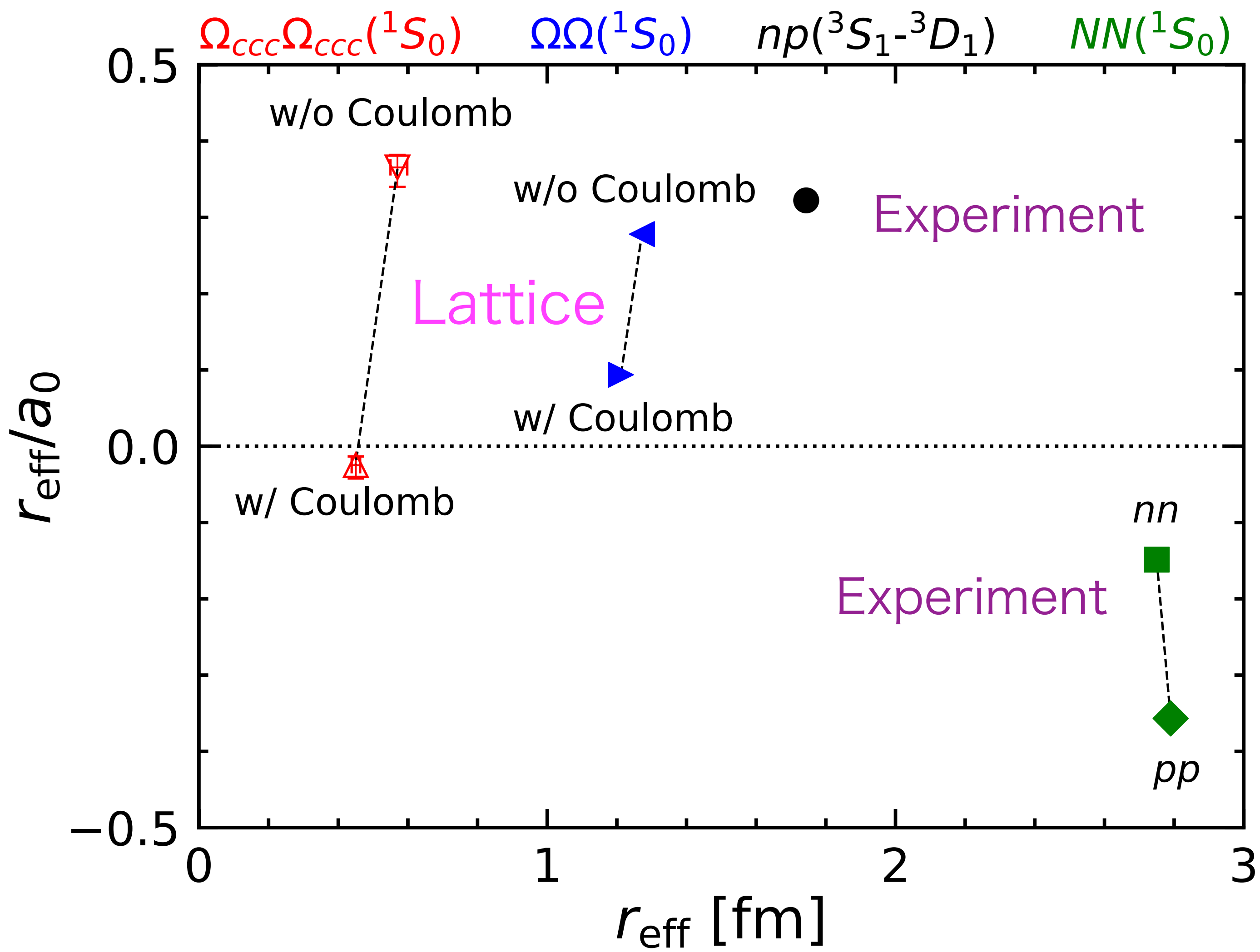
color magnetic interaction of quarks

(b) range of attraction

$\Omega_{ccc}\Omega_{ccc}$: 0.25 fm ~ 1.0 fm $\Omega_{sss}\Omega_{sss}$: 0.5 fm ~ 2.0 fm

meson exchange

Y. Lyu et al.[HAL QCD], PRL 127 (2021) 072003.

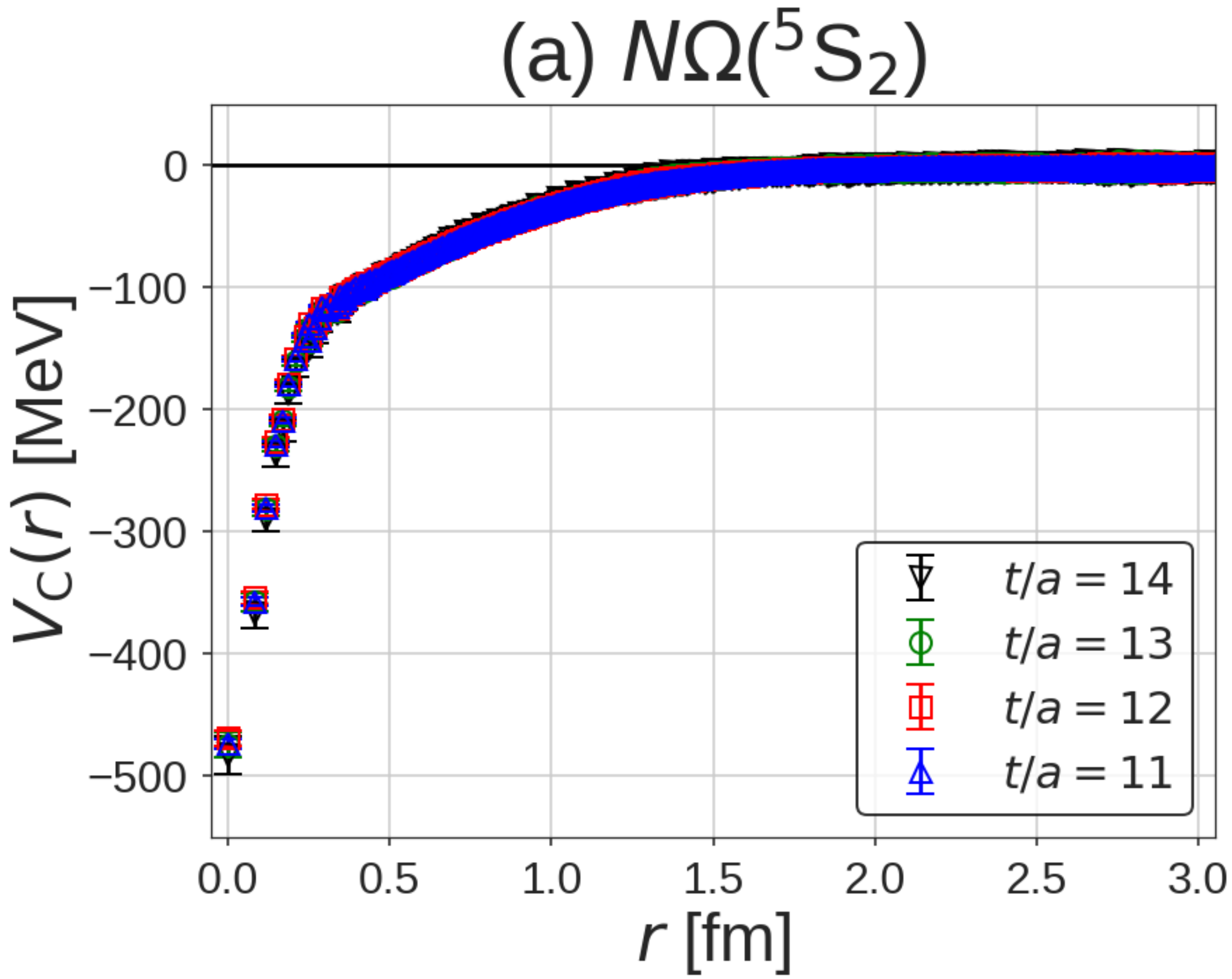


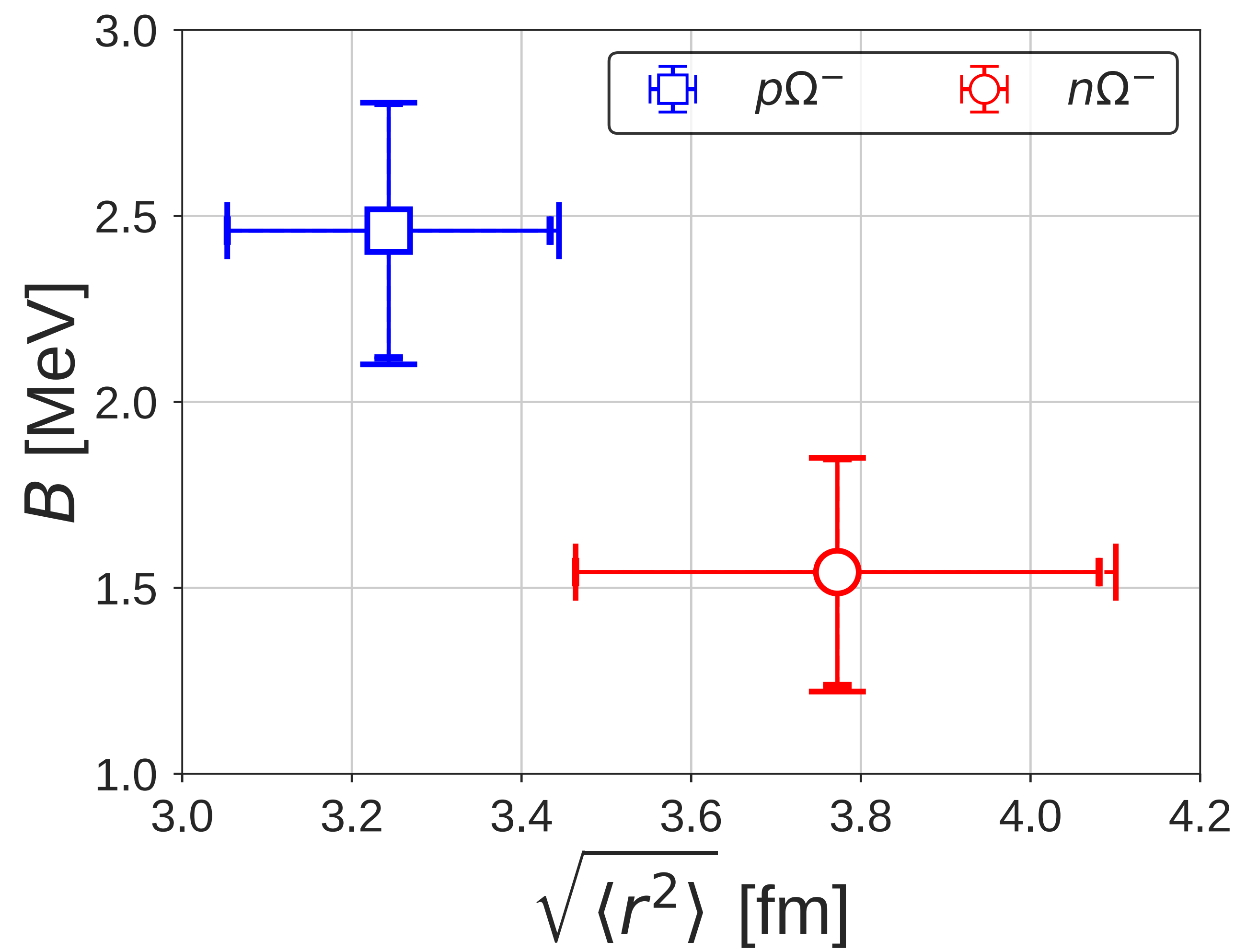
All “dibaryons” appear near unitarity. Why ?

$\Omega_{ccc}\Omega_{ccc}(^1S_0)$ dibaryon is closest to unitarity among these.

4. $N\Omega_{sss}(^5S_2)$ dibaryon

T. Iritani et al.[HAL QCD], PLB 792 (2019) 284.





(a) attractive at all distances without repulsive core like H-dibaryon.

(b) one bound state in each channel.

(c) a comparison of interaction with LHC data, as mentioned before.

$B = 1.5(3)$ MeV $\quad n\Omega^-_{sss}$

no Coulomb attraction

$B = 2.5(4)$ MeV $\quad p^+\Omega^-_{sss}$

with Coulomb attraction

V. Summary

- HAL QCD potential method provides useful tools to investigate baryon-baryon interactions.
- Qualitative features of NN interactions are roughly reproduced. NN potentials at physical point are needed at quantitative level, though current ones are still noisy.
- Hyperon interactions in the flavor SU(3) limit reveals their flavor dependences.
- Lattice $N\Xi$ interactions are compared with LHC data.
- Several candidates for dibaryons are investigated at almost physical point, and $p^+\Omega^-_{sss}$ interactions are compared with LHC data.
- All dibaryons appear in unitary regions. A reason for this should be investigated.

More results at physical point will be expected. Stay tuned.

Thank you !