Sum rules and mechanical properties of quark and gluon in light-front dressed quark model

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Based on Phys.Rev.D 105 (2022),
JM, Mukherjee, Nair, Saha in preparation

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Notation [Harindranath 1996]

$$x^\mu = (x^+, x^-, \mathbf{x}^\perp)$$

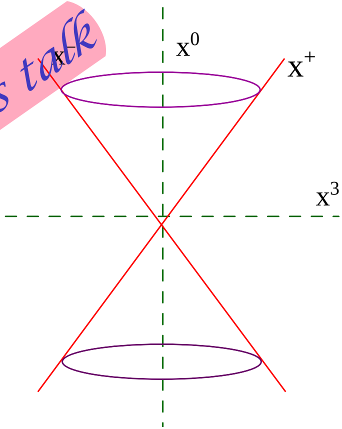
$$\text{where} \quad x^+ = x^0 + x^3, \quad x^- = x^0 - x^3, \quad \mathbf{x}^\perp = (x^1, x^2)$$

Momentum: $p^\mu = (p^+, p^-, \mathbf{p}^\perp)$

The metric tensor:

$$g^{\mu\nu} = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Xingbo Zhao's talk



Mass shell condition $p^- = \frac{\mathbf{p}_\perp^2 + m^2}{2p^+}$

Gravitational Form Factors:

Recall: The electromagnetic interaction of a nucleon with an external EM field :: $\langle p'|J^{\mu}|p\rangle A_{\mu}$

- Fundamentally one may think the gravitons interacting with the parton.
- Gravitons not feasible in collider yet. This can be thought of as a pair of vector bosons interacting with quarks and gluons.
- EMT is the source of gravitation for GTR
- If one calculates the amplitude of such a process in the quantum field theory framework it appears to be dependent on the square of the momentum transfer q^2.
- Moments of generalized parton distribution constrainted by hard scattering process.

Energy mometum tensor

The QCD Lagrangian

$$\mathcal{L}_{QCD} \quad = \quad \frac{1}{2}\overline{\psi}\,(i\gamma_\mu D^\mu - m)\,\psi - \frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu},$$

where the covariant derivative $iD^\mu = i\overleftrightarrow{\partial}^\mu + gA^\mu$.

The field strength tensor

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g\; f^{abc} A_b^\mu A_c^\nu.$$

ψ and A^μ := the fermion and boson field respectively.

Energy momentum tensor

The symmetric QCD EMT

$$\theta^{\mu\nu} = \theta_q^{\mu\nu} + \theta_g^{\mu\nu}$$

where

$$\theta_q^{\mu\nu} = \frac{1}{2}\overline{\psi}\, i\left[\gamma^\mu D^\nu + \gamma^\nu D^\mu\right]\psi - g^{\mu\nu}\overline{\psi}\left(i\gamma^\lambda D_\lambda - m\right)\psi$$

$$\theta_g^{\mu\nu} = -F^{\mu\lambda a}F^\nu_{\lambda a} + \frac{1}{4}g^{\mu\nu}\left(F_{\lambda\sigma a}\right)^2$$

Energy momentum tensor

The symmetric QCD EMT

$$\theta^{\mu\nu} = \theta_q^{\mu\nu} + \theta_g^{\mu\nu}$$

where

$$\theta_q^{\mu\nu} = \frac{1}{2}\overline{\psi}\, i\left[\gamma^\mu D^\nu + \gamma^\nu D^\mu\right]\psi - \underbrace{\cancel{g^{\mu\nu}\overline{\psi}\left(i\gamma^\lambda D_\lambda - m\right)\psi}}_{=0(EOM)}$$

$$\theta_g^{\mu\nu} = -F^{\mu\lambda a}F^\nu_{\lambda a} + \frac{1}{4}g^{\mu\nu}\left(F_{\lambda\sigma a}\right)^2$$

Parametrization of matrix element in terms of GFFs for a spin -$1/2$ system

$$
\begin{aligned}
\langle p', s'|\theta_i^{\mu\nu}(0)|p,s\rangle &= \overline{U}(p',s')\Big[-B_i(q^2)\frac{P^\mu\ P^\nu}{M} \\
&+ \big(A_i(q^2)+B_i(q^2)\big)\frac{1}{2}(\gamma^\mu P^\nu+\gamma^\nu P^\mu) \\
&+ C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} \\
&+ \overline{C}_i(q^2)M\ g^{\mu\nu}\Big]U(p,s),
\end{aligned}
$$

where $\overline{U}(p',s')$, $U(p,s) :=$ Dirac spinors $\quad P^\mu := \frac{1}{2}(p'+p)^\mu$
$M :=$ mass of the target state, $\quad q^\mu := (p'-p)^\mu$
A_i, B_i, C_i and $\overline{C}_i :=$ quark or gluon GFFs and $i \equiv (Q,G)$
[Harindranath, Kundu, Mukherjee PLB, 728 2014]

Cédric Mezrag's Talk

Equivalent decomposition: [Harindranath, Kundu, Mukherjee, PLB 728 (2014)]

$$
\begin{aligned}
\langle p', s' | \theta_i^{\mu\nu}(0) | p, s \rangle &= \overline{U}(p', s') \bigg[A_i(q^2) \frac{P^\mu \ P^\nu}{M} + J_i(q^2) \frac{i(P^\mu \sigma^{\nu\rho} + P^\nu \sigma^{\mu\rho}) q_\rho}{2M} \\
&+ D_i(q^2) \frac{q^\mu q^\nu - g^{\mu\nu} q^2}{4M} + M \overline{C}_i(q^2) \ g^{\mu\nu} \bigg] U(p, s),
\end{aligned}
$$

✓ Sum Rules

Momentum Conservation

$$\boxed{\sum_i A_i(0) = 1}$$

Mass

δg^{++}

$$\langle p', s'|\theta_i^{\mu\nu}(0)|p, s\rangle = \overline{U}(p', s')\Big[A_i(q^2)\frac{P^\mu P^\nu}{M} + J_i(q^2)\frac{i(P^\mu\sigma^{\nu\rho} + P^\nu\sigma^{\mu\rho})q_\rho}{2M} + D_i(q^2)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{4M} + M\overline{C}_i(q^2)\, g^{\mu\nu}\Big]U(p, s),$$

[Polyakov and Schweitzer, (2018), Pagels Phys. Rev., (1966).]

Total angular momentum conservation

$$\boxed{\sum_i B_i(0) = 0}$$

Spin

δg^{+i}

$$\langle p', s'|\theta_i^{\mu\nu}(0)|p, s\rangle = \overline{U}(p', s')\Big[A_i(q^2)\frac{P^\mu P^\nu}{M} + J_i(q^2)\frac{i(P^\mu\sigma^{\nu\rho} + P^\nu\sigma^{\mu\rho})q_\rho}{2M} + D_i(q^2)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{4M} + M\overline{C}_i(q^2)\, g^{\mu\nu}\Big]U(p, s),$$

[Ji, PRL, 78:610–613,(1997)].

Unconstrained D term*

$$\langle p', s'|\theta_i^{\mu\nu}(0)|p, s\rangle = \overline{U}(p', s')\Big[A_i(q^2)\frac{P^\mu P^\nu}{M} + J_i(q^2)\frac{i(P^\mu\sigma^{\nu\rho} + P^\nu\sigma^{\mu\rho})q_\rho}{2M} + D_i(q^2)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{4M} + M\overline{C}_i(q^2)\, g^{\mu\nu}\Big]U(p, s),$$

δg^{ij}

Mechanical properities
Pressure & shear force

Related to stress tensor and internal forces

* determined from experiment

Conservation of EMT

$$\langle p', s'|\theta_i^{\mu\nu}(0)|p, s\rangle = \overline{U}(p', s')\Big[A_i(q^2)\frac{P^\mu P^\nu}{M} + J_i(q^2)\frac{i(P^\mu\sigma^{\nu\rho} + P^\nu\sigma^{\mu\rho})}{2M} + D_i(q^2)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{4M} + M\overline{C}_i(q^2)\, g^{\mu\nu}\Big]U(p, s).$$

δg^{ij}

$$\boxed{\sum_i \overline{C}_i(0) = 0}$$

[Lorcé, Moutarde and Trawiński, EPJC 79(1), 89,(2019)]

✓ Dressed quark model (DQM) [Harindranath and Kundu (1999)]

Instead of a proton state, we take a quark dressed with a gluon. This is a composite spin $1/2$ state. (relativistic)

- Due to the presence of gluon dressing, the model employs a gluonic dof
- The dressed quark state can be expanded in terms of light-front wave functions (LFWFs). Although the LFWF of a bound state, like a proton, cannot be calculated analytically, the LFWF for a dressed quark can be calculated analytically in perturbation theory
- LFWFs are boost invariant and can be written in terms of relative momenta that are frame independent.

Fock state expansion of quark state dressed with a gluon

$$\left|p^+, p_\perp, s\right\rangle = \Phi^s(p) b_s^\dagger(p)|0\rangle + \sum_{s_1 s_2} \int \frac{dp_1^+ d^2 p_1^\perp}{\sqrt{16\pi^3 p_1^+}} \int \frac{dp_2^+ d^2 p_2^\perp}{\sqrt{16\pi^3 p_2^+}} \sqrt{16\pi^3 p^+}$$
$$\times \delta^3(p - p_1 - p_2) \Phi^s_{s_1 s_2}(p; p_1, p_2) b^\dagger_{s_1}(p_1) a^\dagger_{s_2}(p_2)|0\rangle$$

The Jacobi momenta:

$$p_i^+ = x_i P^+ \text{ and } q_i^\perp = p_i^\perp + x_i P^\perp$$

such that

$$\sum_i x_i = 1, \quad \sum_i q_i^\perp = 0$$

The two particle LFWF [1]

$$\Psi^{as}_{s_1 s_2}(x, q^\perp) = \frac{1}{\left[m^2 - \frac{m^2+(q^\perp)^2}{x} - \frac{(q^\perp)^2}{1-x}\right]} \frac{g}{\sqrt{2(2\pi)^3}} T^a \chi^\dagger_{s_1} \frac{1}{\sqrt{1-x}}$$

$$\times \left[-2\frac{q^\perp}{1-x} - \frac{(\sigma^\perp . q^\perp)\sigma^\perp}{x} + \frac{im\sigma^\perp(1-x)}{x}\right] \chi_s (\epsilon^\perp_{s_2})^*$$

χ: two component spinor; m: dressed quark mass= bare quark mass

[Harindranath and Kundu PRD 59 116013 (1999);
Zhang and Harindranath, PRD 48, 4881 (1993)]

[1] Independent of the momentum of the bound state.

Drell-Yan Frame $q^+ = 0$

$$
\begin{aligned}
\text{Initial momentum:} \quad p^\mu &= \left(p^+, \mathbf{0}^\perp, \ \frac{M^2}{p^+}\right), \\
\text{Final momentum:} \quad p'^\mu &= \left(p^+, \ \boldsymbol{q}^\perp, \ \frac{\boldsymbol{q}^{\perp 2} + M^2}{p^+}\right), \\
\text{Invariant momentum transfer:} \quad q^\mu &= (p' - p)^\mu = \left(0, \ \boldsymbol{q}^\perp, \frac{\boldsymbol{q}^{\perp 2}}{p^+}\right).
\end{aligned}
$$

Flag

$$\boldsymbol{p}^\perp = 0 \quad \Longrightarrow \quad q^2 = -\boldsymbol{q}^{\perp 2}.$$

Recipe: To extract the four GFFs

$$\mathcal{M}^{\mu\nu}_{ss'} = \frac{1}{2}\left[\langle p', s'|\theta^{\mu\nu}_i(0)|p, s\rangle\right]$$

where the Lorentz indices $(\mu, \nu) \equiv \{+, -, 1, 2\}$, $(s, s') \equiv \{\uparrow, \downarrow\}$ is the helicity of the initial and final state. $\uparrow$ $(\downarrow)$ positive (negative) spin projection along $z-$ axis.

✓ *Ex : Diagonal component of EMT*

$$\left[\mathcal{M}^{++}_{\sigma'\sigma}\right]_{2,\mathrm{D}} = 2P^{+2}\sum_{\lambda_2,\lambda_2',\sigma_1}\int [x\kappa^{\perp}]\phi^{*\sigma_1,\lambda_2'}_{2\sigma'}\left((1-x),-\boldsymbol{\kappa}'^{\perp}\right)\left[x\epsilon^{i*}_{\lambda_2'}\epsilon^{i}_{\lambda_2}\right]\phi^{\sigma_1,\lambda_2}_{2\sigma}\left((1-x),-\boldsymbol{\kappa}^{\perp}\right.$$

The quark GFFs: [JM, Mukherjee, Nair, Saha, PRD 105, (2022)]

$$
\begin{aligned}
A_Q(q^2) &= 1 + \frac{g^2\, C_F}{2\pi^2}\left[\frac{11}{10} - \frac{4}{5}\left(1 + \frac{2m^2}{q^2}\right)\frac{f_2}{f_1} - \frac{1}{3}\log\left(\frac{\Lambda^2}{m^2}\right)\right] \\
B_Q(q^2) &= \frac{g^2 C_F}{12\pi^2}\,\frac{m^2}{q^2}\,\frac{f_2}{f_1}, \\
D_Q(q^2) &= \frac{5g^2 C_F}{6\pi^2}\,\frac{m^2}{q^2}\left(1 - f_1 f_2\right) = 4\; C_Q(q^2), \\
\overline{C}_Q(q^2) &= \frac{g^2 C_F}{72\pi^2}\left(29 - 30\; f_1\; f_2 + 3\,\log\left(\frac{\Lambda^2}{m^2}\right)\right),
\end{aligned}
$$

where

$$
\begin{aligned}
f_1 &:= \frac{1}{2}\sqrt{1 + \frac{4m^2}{q^2}}, \\
f_2 &:= \log\left(1 + \frac{q^2\,(1 + 2f_1)}{2m^2}\right).
\end{aligned}
$$

The gluon GFFs: [JM, Mukherjee, Nair, Saha, in preparation]

$$A_G(q^2) = \frac{g^2 C_F}{8\pi^2}\left[\frac{29}{9} + \frac{4}{3}\ln\left(\frac{\Lambda^2}{m^2}\right) - \int dx \left(\left(1+(1-x)^2\right) + \frac{4m^2x^2}{q^2(1-x)}\right)\frac{\tilde{f}_2}{\tilde{f}_1}\right]$$

$$B_G(q^2) = -\frac{g^2 C_F}{2\pi^2}\int dx\, \frac{m^2x^2}{q^2}\frac{\tilde{f}_2}{\tilde{f}_1}$$

$$D_G(q^2) = \frac{g^2 C_F}{6\pi^2}\left[\frac{2m^2}{3q^2} + \int dx \frac{m^2}{q^4}\left(x\left((2-x)\,q^2 - 4m^2x\right)\right)\right]\frac{\tilde{f}_2}{\tilde{f}_1}$$

$$\overline{C}_G(q^2) = \frac{g^2 C_F}{72\pi^2}\left[10 + 9\int dx\, \left(x - \frac{4m^2x^2}{q^2(1-x)}\right)\frac{\tilde{f}_2}{\tilde{f}_1} - 3\ln\left(\frac{\Lambda^2}{m^2}\right)\right]$$

where,

$$\tilde{f}_1 := \sqrt{1 + \frac{4m^2x^2}{q^2(1-x)^2}}$$

$$\tilde{f}_2 := ln\left(\frac{1+\tilde{f}_1}{-1+\tilde{f}_1}\right)$$

Total A and $\bar{C}$ are cutoff independent

Plots of GFFs

Plot of $A(q^2)$:

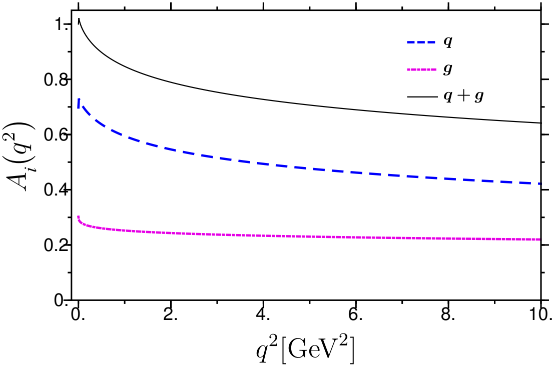
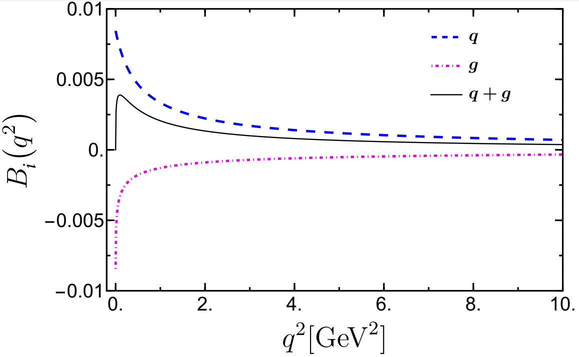


Figure: Plot of the GFFs $A_i(q^2)$ a as function of q^2, with $m = 0.3$ GeV and $g = 1$.

Infer: Conservation of mometum

$$\boxed{\sum_i A_i(0) = 1}$$

Plot of $B(q^2)$:



Infer: anomalous gravitomagnetic moment

$$\boxed{\sum_i B_i(0) = 0}$$

Conservation of total angular momentum:

$$\boxed{J(0) = \frac{1}{2}[(A(0) + B(0)]}$$

Plot of $D(q^2)$:

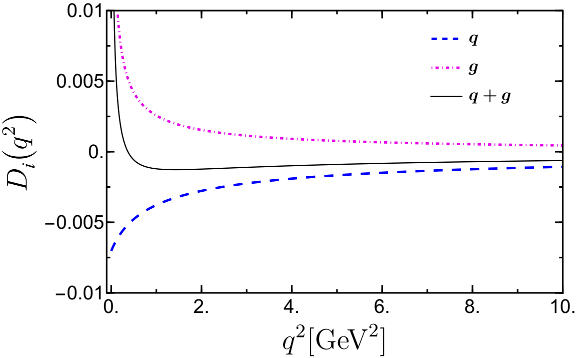


Figure: Plot of the GFFs $D_i(q^2)$ a as function of q^2, with $m = 0.3$ GeV and $g = 1$.

A comparision: [Metz etal Phys. Lett. B, 820:136501(2021)]

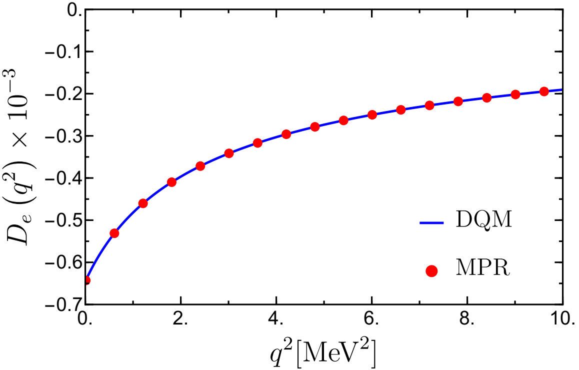


Figure: Plot of electron GFF $D_e(q^2)$ as function of q^2, here we set $m = 0.511$ MeV, $\alpha = \frac{1}{137}$.

A comparision: [Metz etal Phys. Lett. B, 820:136501(2021)]

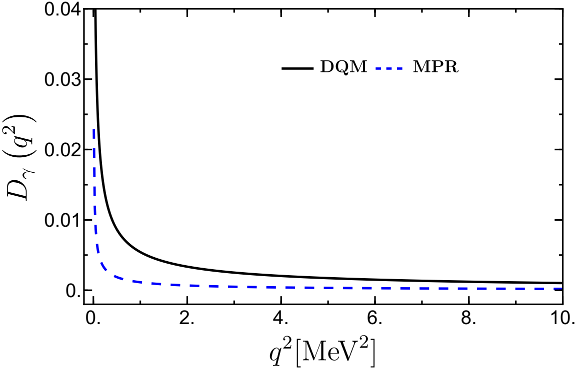


Figure: Plot of electron GFF $D_e(q^2)$ as function of q^2, here we set $m = 0.511$ MeV, $\alpha = \frac{1}{137}$.

Plot of $\overline{C}(q^2)$:

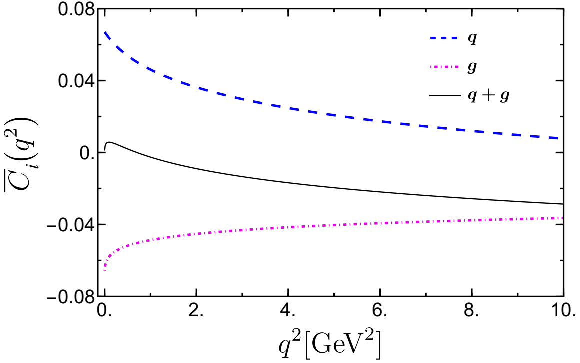


Figure: Plot of the GFFs $\overline{C}_i(q^2)$ a as function of q^2, with $m = 0.3$ GeV and $g = 1$.

Infer: $\boxed{\sum_i \overline{C}_i(0) = 0}$

Wave packets [Chakrabarti and Mukherjee (2005), Diehl (2002)]

Densities corresponds to probablity hence two momentum integral

Dependence of average momentum and momentum transfer

These probablities are preferably studied in impact parameter space and so is pressure distributions

Not only yields the Fourier transformed pressure in the impact parameter space but also gives smooth plots for the distribution

Use Gaussian wave function with a rational choice of width

Pressure and Shear force distributions

$$\theta_a^{ij}(r) = p_a(r)\,\delta^{ij} + s_a(r)\left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)$$

[Polyakov & Schweitzer 2018]

$$\frac{1}{2M\,\boldsymbol{b}^{\perp}}\frac{d}{d\boldsymbol{b}^{\perp}}\left[\boldsymbol{b}^{\perp}\frac{d}{d\boldsymbol{b}^{\perp}}\widetilde{D}_a(\boldsymbol{b}^{\perp})\right] - M\,\widetilde{\overline{C}}_a(\boldsymbol{b}^{\perp})$$

$$-\frac{\boldsymbol{b}^{\perp}}{M}\frac{d}{d\boldsymbol{b}^{\perp}}\left[\frac{1}{\boldsymbol{b}^{\perp}}\frac{d}{d\boldsymbol{b}^{\perp}}\widetilde{D}_a(\boldsymbol{b}^{\perp})\right]$$

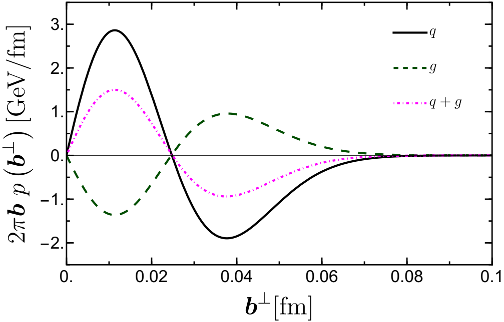
[Freese, Miller (2021)]

FT: $$\boxed{\widetilde{\mathcal{F}}_a(\boldsymbol{b}^{\perp}) = \frac{1}{2\pi}\int_o^{\infty} d\,\boldsymbol{q}^{\perp 2}\, J_0(\boldsymbol{q}^{\perp}\,\boldsymbol{b}^{\perp})\,\mathcal{F}_a(q^2)}$$

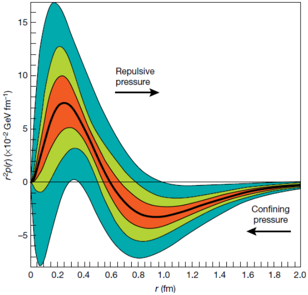
$\mathcal{F} := (A, B, D, \overline{C})$ $\quad$ J_0 := Bessel function of the zeroth order

$\boldsymbol{b}^{\perp}$:= Impact parameter $\quad$ M := mass of the dressed quark state.

✓ Analysis of Pressure distributions



[Burkert, Nature (2018)]

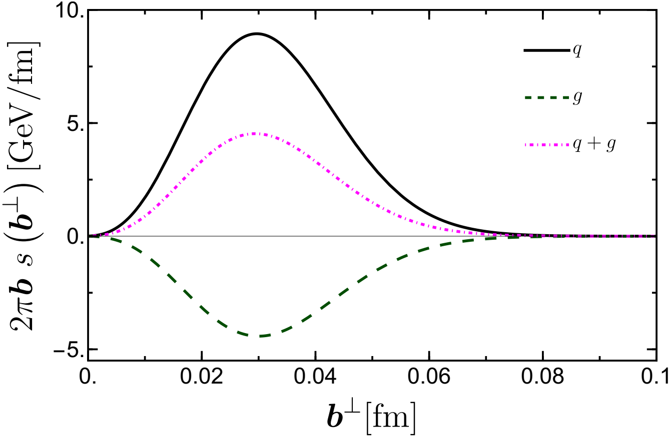


Infer: 1) The net repulsive force (inner region) and the attractive force (outer region) are balanced

2) Satisfies Von-Laue condition:

$$\int_0^{\infty} d^2 b^{\perp} p(b^{\perp}) = 0$$

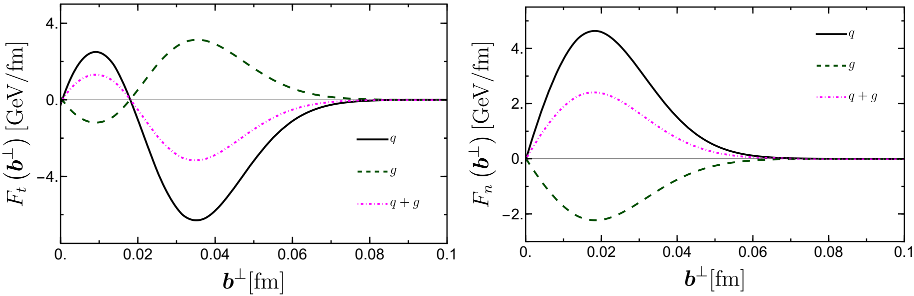
Analysis of Shear Force



Analysis of Force

$$F_n(\boldsymbol{b}^\perp) = 2\pi \boldsymbol{b}^\perp \left(p(\boldsymbol{b}^\perp) + \frac{1}{2} s(\boldsymbol{b}^\perp) \right),$$

$$F_t(\boldsymbol{b}^\perp) = 2\pi \boldsymbol{b}^\perp \left(p(\boldsymbol{b}^\perp) - \frac{1}{2} s(\boldsymbol{b}^\perp) \right).$$



Concluding remarks

EMT encapsules the momentum, energy and pressure distributions.

Though nucleon scattering by gravitational field is not feasible, its noteworthy that the GFFs can be extracted from experimental data.

Analysis of quark and gluon GFFs in dressed quark model satifies the sum rule.

Some of the interesting pressure distributions of quark and gluon are studied.

The GFFs, pressure and shear force exhibit similar qualitative nature when compared to existing phenomenological models.