Properties of Pseudoscalar Mesons in a Contact Interaction

Bilgai Almeida Zamora

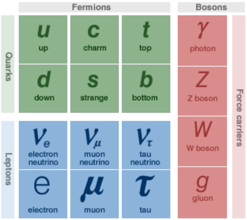
Jorge Segovia-González, Universidad Pablo de Olavide

and

Javier Cobos-Martínez, Universidad de Sonora (México)

Content

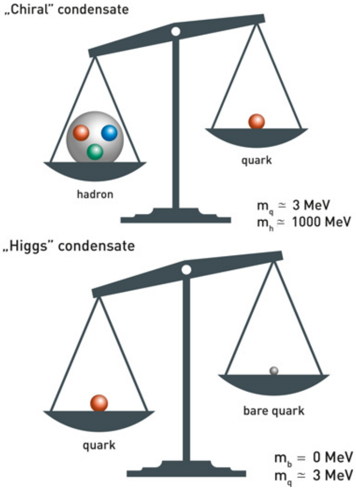
Motivation: What does mass come from?



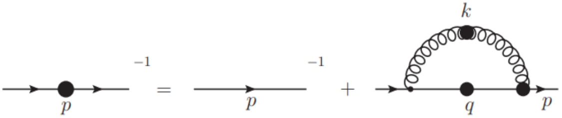
- Higgs boson gives the mass of elementary particles but only a few percent of hadron mass.
- An explanation for origin of the hadron mass: **Spontaneous Chiral Symmetry Breaking**.
- SCSB generates a **dynamical quark mass**

$$M_q \propto \langle \bar{\psi}\psi \rangle \neq 0 \quad (1)$$

- The chiral symmetry also breaks **explicitly**.



Dyson-Schwinger Equation of Dressed-quark propagator

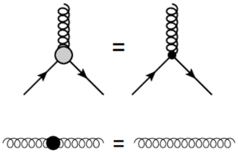


$$S_f^{-1}(p) = \underbrace{(i\gamma \cdot p + m_f)}_{free\ propagator} + \int \frac{d^4q}{(2\pi)^4} \underbrace{g^2 D_{\mu\nu}(p-q)}_{gluon} \frac{\lambda^a}{2} \gamma_\mu S_f(q) \underbrace{\Gamma^a_\nu(p,q)}_{quark-gluon}$$

- Infinitely many coupled equation **(requires truncation)**.
- Symmetry-preserving DSE treatment of a vector×vector contact interaction.
- Rainbow-Ladder truncation and contact interaction:

$$\Gamma_\nu(p,q) = \frac{\lambda_a}{2}\gamma_\nu$$

$$g^2 D_{\mu\nu}(p-q) = d_{\mu\nu}\frac{4\pi\alpha_{IR}}{m_g^2} \quad \text{momentum-independent}$$



Gap Equation

Dynamical quark mass

$$M_f = m_f + \frac{64\pi\alpha_{IR}}{3m_g^2}\int \frac{d^4q}{(2\pi)^4}\frac{M_f}{q^2+M_f^2}$$

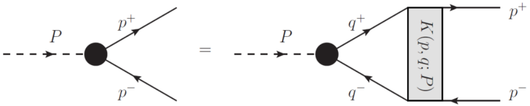
- The quark propagator is given by the gap equation.
- The integral is divergent (we need regularization techniques):

$$\int \frac{\mathrm{d}^4q}{(2\pi)^4}\frac{1}{(q^2+\Delta^2)^n} \longrightarrow \frac{1}{16\pi^2}\int_{\frac{1}{\Lambda_{UV}}}^{\frac{1}{\Lambda_{IR}}} d\tau\, \frac{e^{-\tau\Delta^2}}{\tau^{n-3}}$$

Λ_{UV}: is the regulator since the model is not renormalizable and cannot been removed.
$\Lambda_{IR} \sim \Lambda_{QCD}$: $E << \Lambda_{QCD}$ we have non-perturbative regime.

- $M \sim 0.4$ GeV implies momentum independent constituent quark mass.

Bethe-Salpeter Equations



- The BSE describes mesons as quark-antiquark bound-states.
- Properties emerge from solutions of BSE (eigenvalue problem):

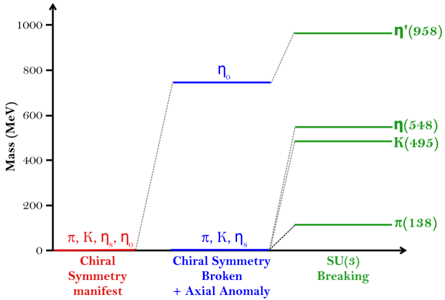
$$\Gamma_H(P;q) = \int \frac{d^4q}{(2\pi)^4}\left[S_f(q_+)\Gamma_H(q;P)S_g(q_-)K^{rs}_{tu}(q,k;P)\right] \quad (2)$$

- Scattering Kernel in contact interaction model:

$$K_L(p,q;P)_{tu;rs} = -\left[\frac{\lambda^a}{2}\gamma_\mu\right]_{ts} d_{\mu\nu}\frac{4\pi\alpha_{IR}}{m_g^2}\left[\frac{\lambda^a}{2}\gamma_\nu\right]_{ru} \quad (3)$$

- Solving BSE we can determine masses, leptonic decay constants, etc.

$\eta - \eta'$: Non-Abelian Anomaly



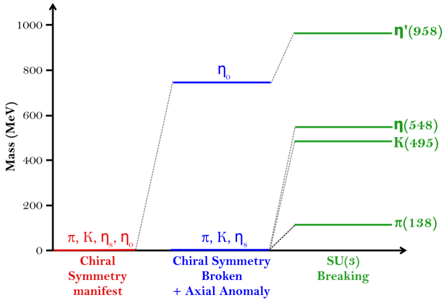
- $U_A(1)$ is not symmetry of quantized theory.
- η' is split from the Goldstone modes.
- The η and η' mesons are isosinglet made of a mixture of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ states.
- K_L **does not produce mixing of states!**

- We include a non-Abelian anomaly contribution[1] (momentum-independent)

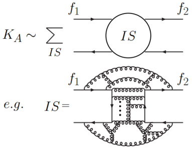
$$K_{\eta,\eta'} = K_L + K_A\left(\xi_0, \cos^2\theta\right) \tag{4}$$

[1] M. S. Bhagwat, L. Chang, Y.-X. Liu, C. D. Roberts, and P. C. Tandy, Phys. Rev. C 76, 045203 (2007).

$\eta - \eta'$: Non-Abelian Anomaly



- $U_A(1)$ is not symmetry of quantized theory.
- η' is split from the Goldstone modes.
- The η and η' mesons are isosinglet made of a mixture of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ states.
- K_L **does not produce mixing of states!**



- We include a non-Abelian anomaly contribution[1] (momentum-independent)

$$K_{\eta,\eta'} = K_L + K_A\left(\xi_0, \cos^2\theta\right) \tag{4}$$

[1] M. S. Bhagwat, L. Chang, Y.-X. Liu, C. D. Roberts, and P. C. Tandy, Phys. Rev. C 76, 045203 (2007).

Quark and Meson Properties

Parameters: $\{m_u,\, m_s,\, \Lambda_{UV},\, \Lambda_{IR},\, \xi_0,\, \cos^2\theta\}$[2]

- We fixed $\xi, \cos^2\theta$ through a least-squares fit to the experimental values of $m_{\eta,\eta'}$
- Dynamical-quark and meson masses

	M_u	M_s	m_π	m_K	m_η	$m_{\eta'}$
Herein	0.410	0.557	0.141	0.500	0.558	0.920
Exp.	-	-	0.140	0.497	0.548	0.958
Error (%)	-	-	0.7	0.6	1.8	3.9

- Width decays:

	m_η	$m_{\eta'}$	Γ_η	$\Gamma_{\eta'}$
Herein	0.558	0.920	0.418	4.16
Exp	0.548	0.948	0.516(22)	4.35(36)

- Leptonic decay constants:

	f_π	f_K	f_η^l	f_η^s	$f_{\eta'}^l$	$f_{\eta'}^s$
Herein	0.094	0.096	0.081	-0.049	0.073	0.097
Phen. & Exp.	0.093	0.110	0.090(13)	-0.093(28)	0.073(14)	0.094(8)

[2] K. Raya, L. Chang, A. Bashir, J. J. Cobos-Martinez, L. X. Gutiérrez-Guerrero, C. D. Roberts and P. C. Tandy, "Structure of the neutral pion and its electromagnetic transition form factor," Phys. Rev. D 93, no.7, 074017 (2016)

Quark and Meson Properties

Parameters: $\{m_u,\, m_s,\, \Lambda_{UV},\, \Lambda_{IR},\, \xi_0,\, \cos^2\theta\}^2$

- We fixed $\xi, \cos^2\theta$ through a least-squares fit to the experimental values of $m_{\eta,\eta'}$
- Dynamical-quark and meson masses

	M_u	M_s	m_π	m_K	m_η	$m_{\eta'}$
Herein	0.410	0.557	0.141	0.500	0.558	0.920
Exp.	-	-	0.140	0.497	0.548	0.958
Error (%)	-	-	0.7	0.6	1.8	3.9

- Width decays:

	m_η	$m_{\eta'}$	Γ_η	$\Gamma_{\eta'}$
Herein	0.558	0.920	0.418	4.16
Exp	0.548	0.948	0.516(22)	4.35(36)

- Leptonic decay constants:

	f_π	f_K	f_η^l	f_η^s	$f_{\eta'}^l$	$f_{\eta'}^s$
Herein	0.094	0.096	0.081	-0.049	0.073	0.097
Phen. & Exp.	0.093	0.110	0.090(13)	-0.093(28)	0.073(14)	0.094(8)

[2] K. Raya, L. Chang, A. Bashir, J. J. Cobos-Martinez, L. X. Gutiérrez-Guerrero, C. D. Roberts and P. C. Tandy, "Structure of the neutral pion and its electromagnetic transition form factor," Phys. Rev. D 93, no.7, 074017 (2016)

Quark and Meson Properties

Parameters: $\{m_u,\ m_s,\ \Lambda_{UV},\ \Lambda_{IR},\ \xi_0,\ \cos^2\theta\}^2$

- We fixed $\xi, \cos^2\theta$ through a least-squares fit to the experimental values of $m_{\eta,\eta'}$
- Dynamical-quark and meson masses

	M_u	M_s	m_π	m_K	m_η	$m_{\eta'}$
Herein	0.410	0.557	0.141	0.500	0.558	0.920
Exp.	-	-	0.140	0.497	0.548	0.958
Error (%)	-	-	0.7	0.6	1.8	3.9

- Width decays:

	m_η	$m_{\eta'}$	Γ_η	$\Gamma_{\eta'}$
Herein	0.558	0.920	0.418	4.16
Exp	0.548	0.948	0.516(22)	4.35(36)

- Leptonic decay constants:

	f_π	f_K	f_η^l	f_η^s	$f_{\eta'}^l$	$f_{\eta'}^s$
Herein	0.094	0.096	0.081	-0.049	0.073	0.097
Phen. & Exp.	0.093	0.110	0.090(13)	-0.093(28)	0.073(14)	0.094(8)

[2] K. Raya, L. Chang, A. Bashir, J. J. Cobos-Martinez, L. X. Gutiérrez-Guerrero, C. D. Roberts and P. C. Tandy, "Structure of the neutral pion and its electromagnetic transition form factor," Phys. Rev. D 93, no.7, 074017 (2016)

Transition Form Factors

- Quark-Photon vertex in CI:

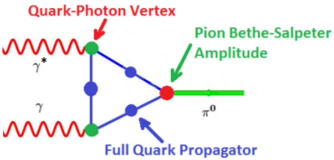
$$\Gamma_\mu(Q) = \gamma_\mu^T P_T(Q^2) + \gamma_\mu^L$$

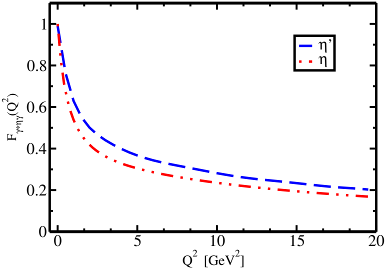
- Transition form factor can be compute from:

$$T_{\mu\nu}(Q_1,Q_2) = \frac{\alpha_{em}}{\pi f_N} V_{\mu\nu} G^{\gamma^* N\gamma}(Q_1^2,Q_2^2),$$

$$= \mathrm{Tr}\int \frac{d^4k}{(2\pi)^4} S(k_1)\Gamma_N(k_1,k_2;P) i\mathcal{Q}\Gamma_\mu(Q_2;k_2,k_3)S(k_2)\times$$

$$\times\, S(k_3) i\mathcal{Q}\Gamma_\nu(Q_1;k_3,k_1)$$





Summary and Perspective

- We were able to calculate static properties of pseudoscalar mesons:
  - Pion and Kaon
  - axial anomaly contribution: η and η'
- We can study:
  - $\gamma\gamma^* \rightarrow \pi^0, \eta, \eta'$ transition form factors
  - Parton distribution functions, etc.
  - the same for vector mesons...
  - and extend the study to finite temperature and density.

Summary and Perspective

- We were able to calculate static properties of pseudoscalar mesons:
  - Pion and Kaon
  - axial anomaly contribution: η and η'
- We can study:
  - $\gamma\gamma^* \to \pi^0, \eta, \eta'$ transition form factors
  - Parton distribution functions, etc.
  - the same for vector mesons...
  - and extend the study to finite temperature and density.