Structure of X(3872) with hadronic potentials coupled to quarks

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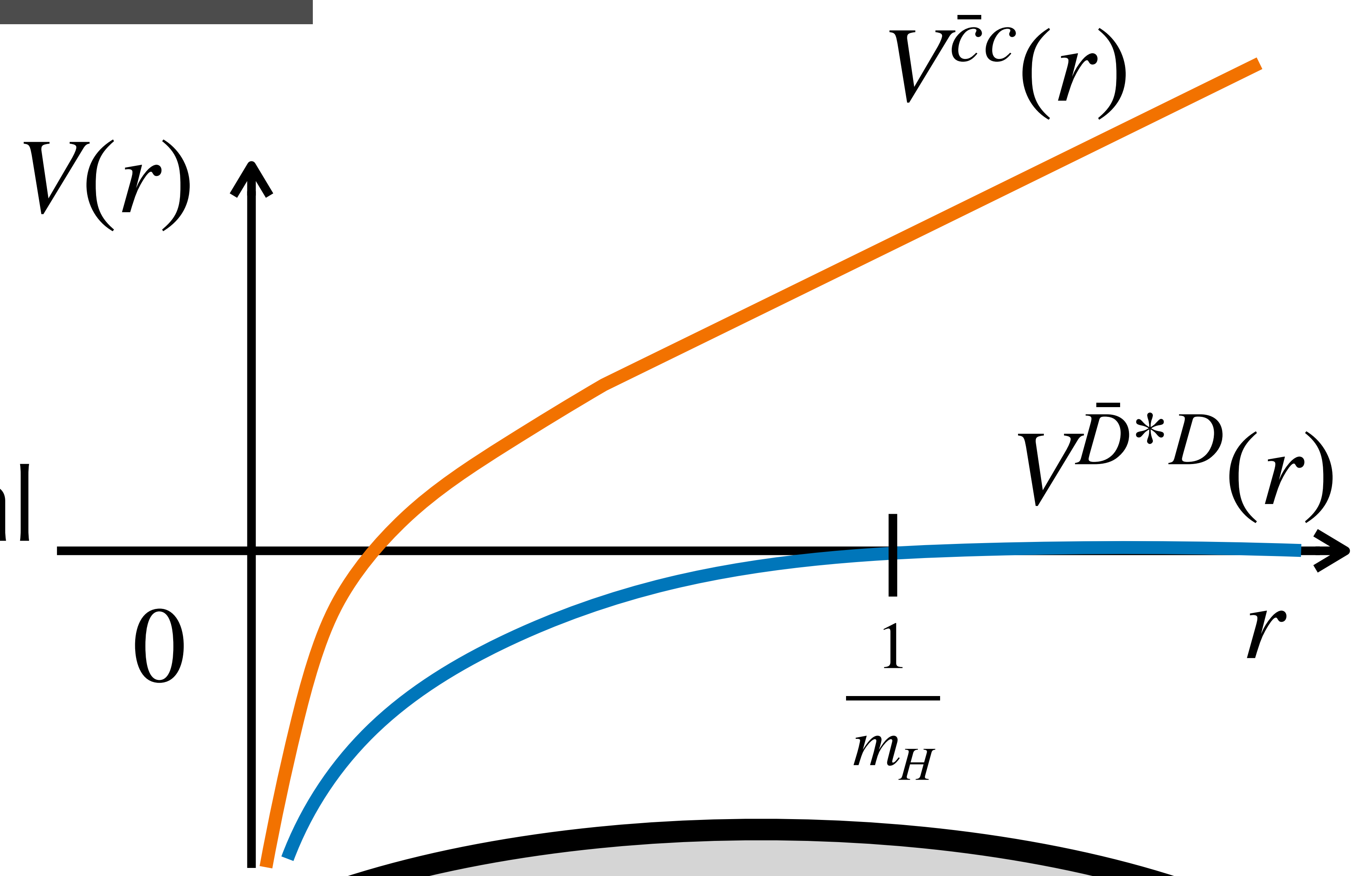
Introduction

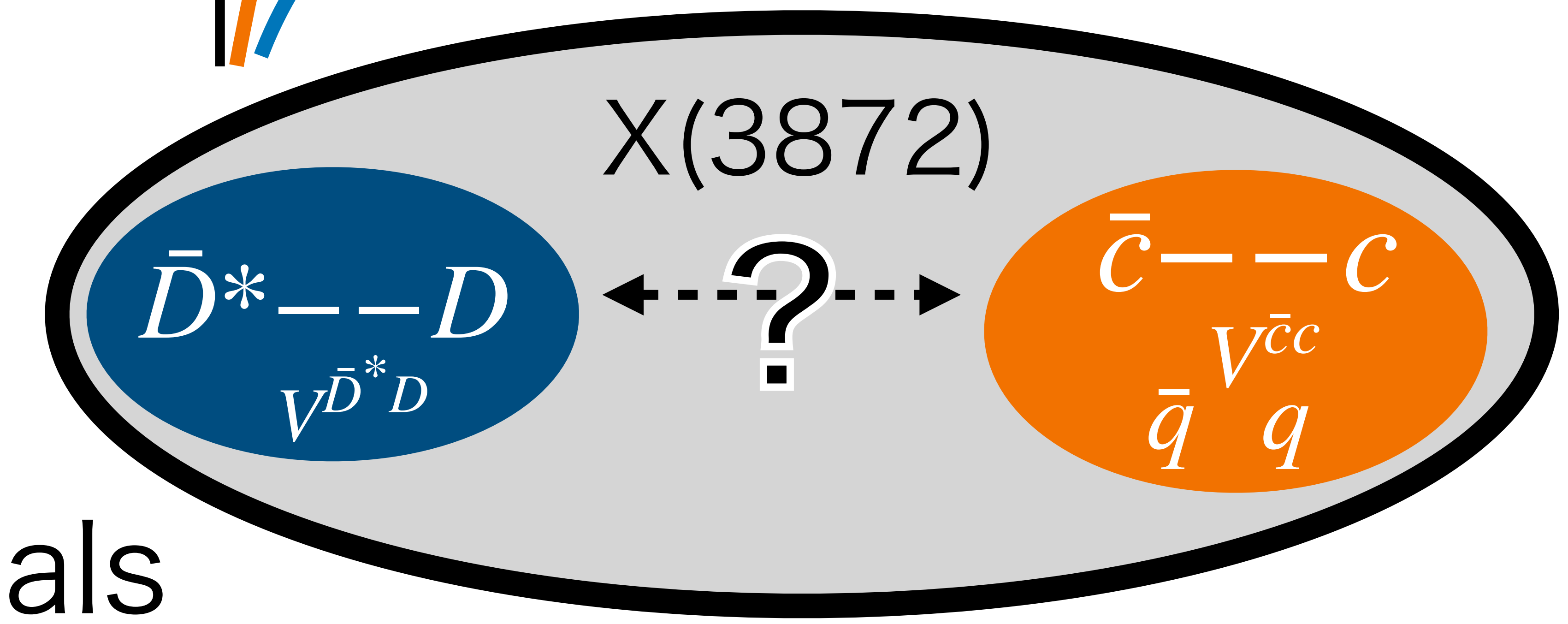
$V^{\bar{c}c}(r) = -\frac{A}{r} + \sigma r + V_0 \xrightarrow[r\to\infty]{} \infty$: Confinement potential

$V^{\bar{D}^*D}(r) = K_{\bar{D}^*D}\frac{\exp[-m_H r]}{r} + \cdots \xrightarrow[r\to\infty]{} 0$: Scattering potential



Problem

- $V^{\bar{c}c}(r)$ and $V^{\bar{D}^*D}(r)$ are calculated independently



This Study

- **Channel couplings** between $\bar{c}c$ and $\bar{D}^*D$ potentials
- Obtain the **effective potential** $V_{\text{eff}}^{\bar{D}^*D}$ as a local potential
- **Convert non-local** effective potential $V_{\text{eff}}^{\bar{D}^*D}$ to **local** ones
- Apply $V_{\text{eff}}^{\bar{D}^*D}$ to the model of **X(3872)**

Formulation : $V_{\mathrm{eff}}^{\bar{D}^*D}$

- Hamiltonian H with coupled-channel between $\bar{c}c$ and $\bar{D}^*D$

$$H = \begin{pmatrix} T^{\bar{c}c} & 0 \\ 0 & T^{\bar{D}D} + \Delta \end{pmatrix} + \begin{pmatrix} V^{\bar{c}c} & V^t \\ V^t & V^{\bar{D}D} \end{pmatrix}$$

$T^{\bar{c}c}, T^{\bar{D}D}$: kinetic energy
Δ : threshold energy
V^t : transition potential

- **non-local**
- Energy dependent

- Eliminate $\bar{c}c$ to obtain effective potential $V_{\mathrm{eff}}^{\bar{D}^*D}(\boldsymbol{r}, \boldsymbol{r}', E)$[1,2]

$$V_{\mathrm{eff}}^{\bar{D}^*D}(\boldsymbol{r}, \boldsymbol{r}', E) = V^{\bar{D}^*D}(\boldsymbol{r})\delta(\boldsymbol{r}' - \boldsymbol{r}) + \sum_n \frac{\langle \boldsymbol{r}'_{\bar{D}^*D} | V^t | \phi_n \rangle \langle \phi_n | V^t | \boldsymbol{r}_{\bar{D}^*D} \rangle}{E - E_n}$$

When

- $V^{\bar{D}^*D}(\boldsymbol{r}) = 0$
- $\langle \phi_n | V^t | \boldsymbol{r}_{\bar{D}^*D} \rangle = g_0 e^{-\mu r}/r$
- only take $n = 0$

$$= \frac{g_0^2}{E - E_0} \frac{e^{-\mu r'}}{r'} \frac{e^{-\mu r}}{r}$$

g_0 : coupling constant,
E_0 : binding energy of $\bar{c}c$
μ : cut-off constant

[1] H. Feshbach, Ann. Phys. **5**, 357 (1958); ibid., **19**, 287 (1962)
[2] I. Terashima, T. Hyodo arXiv:2208.14075 [hep-ph]

Formulation : Conversion to local

① Formal derivative expansion

Express non-local potential in terms of derivative of local potential by Taylor expansion directly

$$V_{\text{eff}}^{\bar{D}^*D}(\boldsymbol{r},\boldsymbol{r}',E) = \frac{g_0^2}{E-E_0}\frac{e^{-\mu r}}{r}\frac{e^{-\mu r'}}{r'} \quad \Rightarrow \quad V^{\text{formal}}(r,E) = \frac{4\pi g_0^2}{\mu^2(E-E_0)}\frac{e^{-\mu r}}{r} + O(\nabla)$$

derivative expansion

② HAL QCD method [S.Aoki and K.Yazaki, PTEP **2022**, no.3, 033B04 (2022)]

Construct from wave function $\psi_k(r)$ obtained from non-local potentials with momentum $k=\sqrt{2mE}$

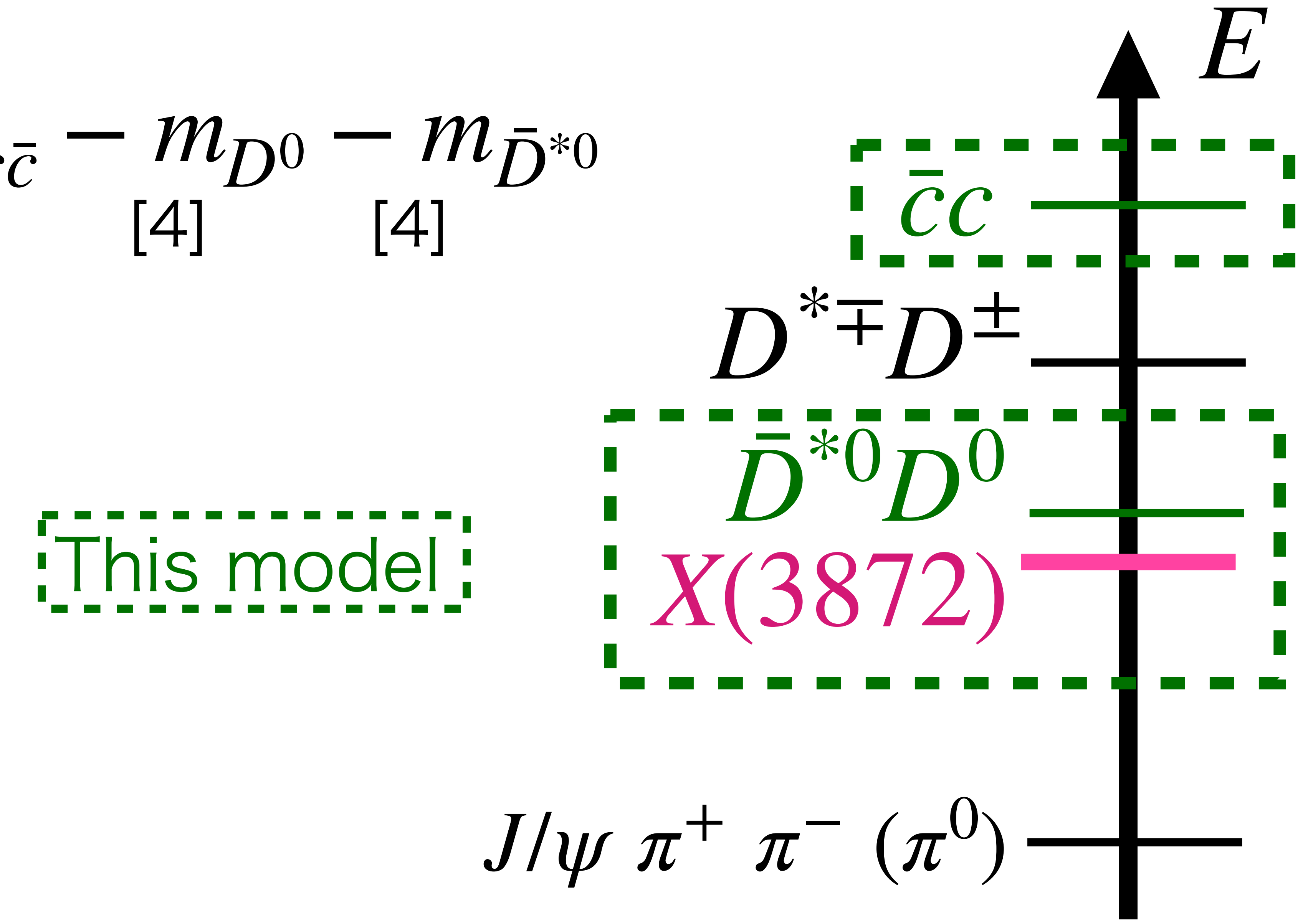
$$V^{\text{HAL}}(r,E) = \frac{k^2}{2m} + \frac{1}{2mr\psi_k(r)}\frac{d^2}{dr^2}\left[r\psi_k(r)\right] + O(\nabla^2)$$

- $\psi_k(r)$ can be solved analytically by virtue of Yukawa potential

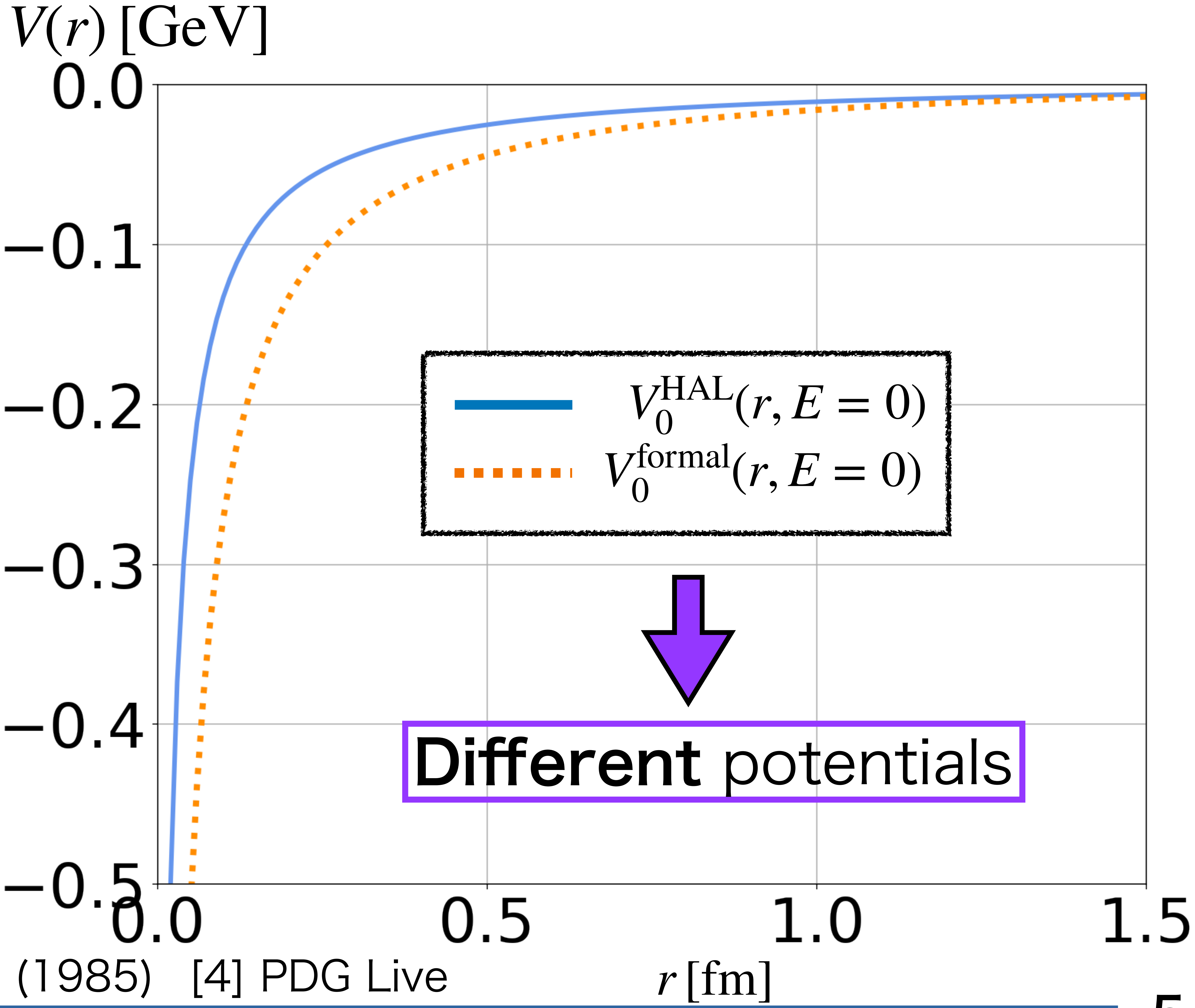
Conclusion : compare in $X(3872)$

Model of $X(3872)$

- g_0 is determined to reproduce mass of X(3872) [4]
- cut-off μ takes as energy of π
- $E_0 = m_{c\bar{c}} - m_{D^0} - m_{\bar{D}^{*0}}$ ([3], [4], [4])



‣Comparing approximated potentials



[3] S. Godfrey and N. Isgur, Phys. Rev. D, **32**, 189 (1985) [4] PDG Live

Summary

$$V_{\rm eff}^{\bar{D}^*D}(\boldsymbol{r},\boldsymbol{r}',E) = \frac{g_0^2}{E-E_0} v(\boldsymbol{r})v(\boldsymbol{r}')$$

- Consider **channel coupling** of $X(3872)$ between $V^{\bar{c}c}(r)$ and $V^{\bar{D}^*D}(r)$
- Convert $V_{\rm eff}^{\bar{D}D}(E)$ obtained by eliminating $c\bar{c}$ channel, non-local to local
- compare of two conversion methods ①formal derivative expansion $V_0^{\rm formal}$ and ②HAL QCD method $V_0^{\rm HAL}$

➡ • $V_0^{\rm formal}$ and $V_0^{\rm HAL}$ are different for small r

⬌ • Investigating in physical quantities

Future outlook

- Comparison of what physical quantities (e.g., scattering amplitude) are obtained for each conversion
- Add $D^{*\mp}D^{\pm}$ channel [M. Takizawa, PTEP **2013**, 093 D 01 (2013)]